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SOME MODIFIED TEST STATISTICS FOR TESTING THE POPULATION SIGNAL TO NOISE RATIO (SNR)

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To: Dean Michael Heithaus
College of Arts, Sciences and Education

This thesis, written by Samantha Menendez, and entitled Some Modified Test Statistics for Testing the Population Signal to Noise Ratio (SNR), having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this thesis and recommend that it be approved.

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ABSTRACT OF THE THESIS

ON SOME MODIFIED TEST STATISTICS FOR TESTING THE POPULATION SIGNAL TO NOISE RATIO (SNR)

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SNR is a measure of the strength of desired data relative to undesigned data. Population SNR is equal to the population mean divided by the population standard deviation. In practice, commonly in image processing, a high SNR means that the signal strength is stronger in relation to the noise. Having higher SNR provides more useful information.

This thesis considers fifteen existing and proposed test statistics for testing the population SNR. A theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study has been conducted. The performance of the test statistics is based on the empirical size and power of the tests considering a significance level 0.05. The simulation study resulted that some existing and proposed methods are performing well in some conditions. However, Method 10 performed the best in all simulation conditions. Three real life data are analyzed to illustrate the performance of the test statistics.

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CHAPTER 1

INTRODUCTION

1.1 Literature Review

Mean is known as an expected value that represents the central tendency of a probability distribution which is known as the average of a data set. When we are interested in the measure of how dispersed the distribution of data is in relation to the mean, we refer to the standard deviation. Standard deviation can vary between high and low values, when the standard deviation is high the values fall far from the mean and when standard deviation is low the values are closer to the mean.

Standard deviation and noise relate vice versa as variability within a gathered data sample. Improper filtering where data was measured, and random errors can be introduced can result in unexpected noise. Noise has two main sources; errors introduced by measurement tools and random errors introduced by processing. Noise can be any data that has been received and changed in a manner that it cannot be read which adversely affects results of any data analysis.

While the mean describes what is being measured, standard deviation represents noise. Signal to noise ratio (SNR) is a measure of the strength of the desired data relative to the undesigned data or signal known as noise. Population SNR is equal to the population mean divided by the population standard deviation ($\frac{\mu}{\sigma}$). In practice, commonly in digital communications and image processing, a high SNR means that the signal strength is stronger in relation to the noise level. Having higher SNR means there is more useful information than unwanted noise data in the output. The application on SNR can be found in John (2007) and Tania (2008) among others.

SNR is the reciprocal of commonly used coefficient of variation (CV) which is unitless and is defined as the standard deviation divided by the mean. Since SNR is the reciprocal, when there is better data there is a higher value for the SNR and a lower value for the CV. This can also be seen when, putting SNR into practice, the larger the standard deviation (noise) the smaller the SNR. Therefore, it is possible to construct both confidence interval and hypothesis testing from CV for SNR. The information on testing the population SNR is limited. Nevertheless, there are various methods available for estimating the confidence interval (CI) for a population CV, such as parametric, nonparametric, modified, and bootstrapping (Banik and Kibria, 2010). For more information on the CI for the CV, we refer Koopmans et al. (1964), Miller (1991), Sharma and Krishna (1994), McKay (1932), Vangel (1996), and Curto and Pinto (2009), George and Kibria (2012) among others. First Kibria and George (2014) consider various test statistics based on the confidence interval for testing the population SNR.

1.2 Objective of the Thesis

The objective of this thesis is to review and propose some test statistics for testing the population SNR based on parametric, and modified method for confidence intervals for SNR. The organization of the thesis is as follows: We will review and propose some test statistics for testing the SNR in Chapter 2. A simulation will be conducted in Chapter 3. Three real life data will be analyzed in chapter 4. Finally, some concluding remarks will be given in Chapter 5.

CHAPTER 2

STATISTICAL METHODOLOGY

Let $X_{1,}\ X_2, \ldots, X_n$ be an independently and identically distributed (iid) random sample of size n from a population from a population with finite mean, μ, and finite variance, σ^2. Let $\bar{x}$ be the sample mean and s be the sample standard deviation. Then $S\widehat{N}R = \frac{\bar{x}}{s}$ would be the estimated values of the population SNR ($\frac{\mu}{\sigma}$) and $\widehat{CV} = \frac{s}{\bar{x}}$ would be the estimated values of the population CV ($\frac{\sigma}{\mu}$). The objective of this thesis is to test the population SNR. The null and alternative hypothesis are defined as follows:

$$H_0: SNR = SNR_0$$

$$H_a: SNR \neq SNR_0$$

Following Kibria and George (2014), we have considered following eleven (11) promising test statistics, which are summarized along with critical values in the following Table 2.1. For details about these tests, we refer to Kibria and George (2014).

2.1 Miller (1991) Method

Method 1. Miller demonstrated that the estimator, $\frac{s}{\bar{x}}$ has an approximate normal distribution with mean $\frac{\sigma}{\mu}$ and variance of (1/(n-1))($\frac{\sigma}{\mu})^2[0.5+(\frac{\sigma}{\mu})^2]$. Then the approximate upper and lower confidence limits the population SNR can be constructed. From the confidence intervals, we will reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit. The upper and lower limits for SNR_0 are given as follows:

$$SNR_0 < (\frac{s}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2[0.5 + (\frac{s}{\bar{x}})^2]}\ )^{-1}$$

$$or$$

$$SNR_0 > (\frac{s}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2[0.5 + (\frac{s}{\bar{x}})^2]}\ )^{-1}$$

We will reject the null hypothesis when SNR is more than the above upper limit or less than the lower limit.

2.2 Sharma and Krishna's (1994) Method

Method 2. Sharma and Krishna (1994) developed the asymptotic sampling distribution of the inverse of the coefficient of variation for making statistical inferences about population coefficient of variation (CV). Following Sharma and Krishna (1994), based on the CI for inverted SNR, we reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit, which are given below:

$$SNR_0 < \frac{\bar{x}}{s} - \frac{Z_{\alpha/2}}{\sqrt{n}}$$

$$or$$

$$SNR_0 > \frac{\bar{x}}{s} + \frac{Z_{\alpha/2}}{\sqrt{n}}$$

2.3 Curto and Pinto's (2009) Method

Method 3. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < \left(\frac{s}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{n}\left(\left(\frac{s}{\bar{x}}\right)^4 + 0.5\left(\frac{s}{\bar{x}}\right)^2\right.}\right)^{-1}$$

$$or$$

$$SNR_0 > \left(\frac{s}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{n}\left(\left(\frac{s}{\bar{x}}\right)^4 + 0.5\left(\frac{s}{\bar{x}}\right)^2\right.}\right)^{-1}$$

2.4 McKay's (1932) Method

Method 4. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{s}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

$$or$$

$$SNR_0 > [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{s}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

where $\chi^2_{n-1,\frac{\alpha}{2}}$ and $\chi^2_{n-1,1-\frac{\alpha}{2}}$ are (α/2)*th* and (1-α/2)*th* percentile points for a chi-square distribution with (n-1) degrees of freedom

2.5 Modified McKay Confidence Interval (MMcK)

Method 5. Modified McKay confidence interval is Vangel (1996) modifying McKay's original 1932 interval. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

$$or$$

$$SNR_0 > [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

2.6 Panichkitkosolkul's (2009) Method

Method 6. Panichkitkosolkul's (2009) modified the Modified Mckay (in section 2.5) method by replacing the sample inverted SNR with k the maximum likelihood estimator for a normal distribution, where $k = \frac{\sqrt{\sum(x-\bar{x})^2}}{\sqrt{n}\bar{x}}$. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < [k\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

$$or$$

$$SNR_0 > [k\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

2.7 Median Modified Miller

For skewed data the median describes the center of the distribution better than the mean (Kibria (2006), and Shi and Kibria (2007)) expressing that it makes more sense to measure sample variability in terms of median. The following median modifications are proposed by Kibria and George (2014) to improve performance for skewed distributions and represent both parametric and nonparametric methods.

Method 7. Null hypothesis will be rejected, when,

$$SNR_0 < \left(\frac{\tilde{s}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5 + (\frac{\tilde{s}}{\bar{x}})^2]}\right)^{-1}$$

$$or$$

$$SNR_0 > \left(\frac{\tilde{s}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5 + (\frac{\tilde{s}}{\bar{x}})^2]}\right)^{-1}$$

where $\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x - med(x))^2}$ is the modified sample variance.

2.8 Median Modification of McKay

Method 8. Reject the null hypothesis when,

$$SNR_0 < [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

$$or$$

$$SNR_0 > [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

2.9 Median Modification of Modified McKay

Method 9. Reject the null hypothesis when,

$$SNR_0 < [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n} - 1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

$$or$$

$$SNR_0 \ > [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1}$$

2.10 Median Modification of Curto and Pinto (2009)

Method 10. Reject the null hypothesis when,

$$SNR_0 \ < \ (\frac{\tilde{s}}{\bar{x}}+Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2}\ )^{-1}$$

$$or$$

$$SNR_0 > \ (\frac{\tilde{s}}{\bar{x}}-Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2}\ )^{-1}$$

2.11 Kibria and George (2014)

Kibria and George (2014) developed Method 11 based on the normality assumption. First they developed the confidence interval for inverse of the population variance and then after some algebraic simplification they obtained the confidence interval for population SNR. From the confidence interval they found the upper and lower limits for the SNR_0, which are given below.

Method 11. Reject the null hypothesis when,

$$SNR_0 \ < \ \sqrt{\frac{\chi^2_{\upsilon,\frac{\alpha}{2}}}{(n-1)}}S\widehat{N}R$$

$$or$$

$$SNR_0 > \ \sqrt{\frac{\chi^2_{\upsilon,1-\alpha/2}}{(n-1)}}S\widehat{N}R$$

Some Proposed Methods

Kibria and George (2014) modified Sharma and Krishna's method using bootstrap technique, which is computationally expensive and time consuming and did not improve that much. In this thesis, we will modify Sharma and Krishna's method using the modified standard deviation and MAD. We also modified Miller's methods and provided them in the following subsections.

2.12 Modified Kibria and George (2014)

Method 12. Reject the null hypothesis when,

$$SNR_0 < \sqrt{\frac{\chi^2_{\upsilon,\frac{\alpha}{2}}}{(n-1)}}SNR^*$$

$$or$$

$$SNR_0 > \sqrt{\frac{\chi^2_{\upsilon,1-\alpha/2}}{(n-1)}}SNR^*$$

where $SNR^* = \frac{\bar{x}}{\tilde{s}}$ and $\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x - med(x))^2}$.

2.13 Modification of Miller (1991)

Method 13. Reject the null hypothesis when

$$SNR_0 < (\frac{s_{MAD}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s_{MAD}}{\bar{x}})^2[0.5 + (\frac{s_{MAD}}{\bar{x}})^2]})^{-1}$$

$$or$$

$$SNR_0 > (\frac{s_{MAD}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s_{MAD}}{\bar{x}})^2[0.5 + (\frac{s_{MAD}}{\bar{x}})^2]})^{-1}$$

where,

$$s_{MAD} = 1.4826 \times Median(|x_i - Mean(x_j)|)$$

This formula is little modification of Rousseeuw and Croux (1993).

2.14 Modification of Sharma and Krishna's (1994) Method

Method 14. We will reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit.

$$SNR_0 < \frac{\bar{x}}{s_{MAD}} - \frac{Z_{\alpha/2}}{\sqrt{n}}$$

$$or$$

$$SNR_0 > \frac{\bar{x}}{s_{MAD}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$$

2.15 Median Modification of Sharma and Krishna's (1994) Method

Method 15. We reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit.

$$SNR_0 < \frac{\bar{x}}{\tilde{s}} - \frac{Z_{\alpha/2}}{\sqrt{n}}$$

$$or$$

$$SNR_0 > \frac{\bar{x}}{\tilde{s}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$$

$\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x - med(x))^2}$. We summarized all lower and upper critical values in Table 2.1.

Table 2.1: Lower and upper critical values for testing the SNR for all methods

Method #	Lower	Upper
1	$(\frac{s}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2[0.5+(\frac{s}{\bar{x}})^2]}\ )^{-1}$	$(\frac{s}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s}{\bar{x}})^2[0.5+(\frac{s}{\bar{x}})^2]}\ )^{-1}$
2	$\frac{\bar{x}}{s} - \frac{Z_{\alpha/2}}{\sqrt{n}}$	$\frac{\bar{x}}{s} + \frac{Z_{\alpha/2}}{\sqrt{n}}$
3	$(\frac{s}{\bar{x}}$ $+ Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{s}{\bar{x}})^4 + 0.5(\frac{s}{\bar{x}})^2}\ )^{-1}$	$(\frac{s}{\bar{x}}$ $- Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{s}{\bar{x}})^4 + 0.5(\frac{s}{\bar{x}})^2}\ )^{-1}$
4	$[\ \frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}\ ]]^{-1}$	$[\ \frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}\ ]]^{-}$
5	$[\ \frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}\ ]]$	$[\ \frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}$
6	$[\ k\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}\ ]]^{-1}$	$[\ k\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}\ ]]^{-}$
7	$(\frac{\tilde{s}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5+(\frac{\tilde{s}}{\bar{x}})^2]}\ )^{-1}$	$(\frac{\tilde{s}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5+(\frac{\tilde{s}}{\bar{x}})^2]}\ )^{-1}$
8	$[\ \frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}\ ]]^{-1}$	$[\ \frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}\ ]]^{-}$

9	$[\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}]]}$	$[\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}]}$
10	$(\frac{\tilde{s}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2}\ )^{-1}$	$(\frac{\tilde{s}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2}\ )^{-1}$
11	$\sqrt{\frac{\chi^2_{\upsilon,\alpha/2}}{(n-1)}}S\widehat{N}R$	$\sqrt{\frac{\chi^2_{\upsilon,1-\alpha/2}}{(n-1)}}S\widehat{N}R$
12	$\sqrt{\frac{\chi^2_{\upsilon,\frac{\alpha}{2}}}{(n-1)}}SNR^*$	$\sqrt{\frac{\chi^2_{\upsilon,1-\frac{\alpha}{2}}}{(n-1)}}SNR^*$
13	$(\frac{s_{MAD}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s_{MAD}}{\bar{x}})^2[0.5+(\frac{s_{MAD}}{\bar{x}})^2]}$	$(\frac{s_{MAD}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{s_{MAD}}{\bar{x}})^2[0.5+(\frac{s_{MAD}}{\bar{x}})^2]}\ )^{-}$
14	$\frac{\bar{x}}{s_{MAD}} - \frac{Z_{\alpha/2}}{\sqrt{n}}$	$\frac{\bar{x}}{s_{MAD}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$
15	$\frac{\bar{x}}{\tilde{s}} - \frac{Z_{\alpha/2}}{\sqrt{n}}$	$\frac{\bar{x}}{\tilde{s}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$

Since, a theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study using R 4.2.1 to find the empirical size and power of the tests is conducted in the following Chapter.

CHAPTER 3

SIMULATION STUDY

In this chapter, we will compare the performance of the test statistics that are given in section 2. Simulation study is done under the symmetric (normal), left skewed (Beta) and right skewed (Gamma) distributional conditions.

3.1 Simulation Techniques

The performance of the test statistics is examined for both small and large sample sizes (n = 15, 30, 50, 100). Simulation will be run for 5,000. The simulation results are presented only for α=0.05, a widely used significance level. The simulation results of empirical type I error rate and the power of the tests are presented in Tables 3.1 to 3.28 for various parametric conditions. For a visual representation some power functions are plotted in Figures 3.1-3.6 for some selected parameters.

In all Tables (3.1 to 3.28), column number 6 represents the empirical size and rest of the columns are empirical power of the tests. For more on simulation studies, we refer the readers to Shi and Kirbia (2007), Banik and Kibria (2010), Kibria and George (2014) and very recently Panichkitosolkul (2022) among others.

Table 3.1: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 15

SNR0	**0.400**	**0.600**	**0.700**	**0.800**	**1.000**	**1.200**	**1.300**	**1.400**	**1.500**
Method1	0.896	0.592	0.437	0.307	0.149	0.087	0.073	0.065	0.061
Method2	0.643	0.396	0.295	0.213	0.128	0.177	0.253	0.362	0.482
Method3	0.894	0.597	0.442	0.312	0.147	0.081	0.067	0.059	0.053
Method4	1.000	1.000	1.000	1.000	0.404	0.141	0.083	0.049	0.027
Method5	1.000	1.000	0.963	0.644	0.245	0.086	0.050	0.029	0.017
Method6	1.000	1.000	0.951	0.667	0.266	0.105	0.056	0.037	0.022
Method7	0.888	0.575	0.419	0.293	0.145	0.088	0.075	0.069	0.063
Method8	1.000	1.000	1.000	1.000	0.401	0.140	0.078	0.048	0.026
Method9	1.000	1.000	0.949	0.627	0.236	0.081	0.046	0.028	0.015
Method10	0.887	0.580	0.426	0.295	0.141	0.082	0.068	0.061	0.056
Method11	0.919	0.610	0.435	0.305	0.236	0.344	0.428	0.515	0.596
Method12	0.911	0.593	0.418	0.293	0.242	0.364	0.446	0.531	0.618
Method13	0.879	0.616	0.488	0.381	0.236	0.157	0.133	0.115	0.104
Method14	0.633	0.427	0.344	0.278	0.209	0.259	0.331	0.415	0.507
Method15	0.623	0.376	0.270	0.194	0.121	0.182	0.269	0.379	0.503

Table 3.2: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 30

SNR0	**0.400**	**0.600**	**0.700**	**0.800**	**1.000**	**1.200**	**1.300**	**1.400**	**1.500**
Method1	0.962	0.705	0.491	0.298	0.082	0.016	0.007	0.003	0.001
Method2	0.882	0.584	0.410	0.258	0.122	0.246	0.398	0.566	0.723
Method3	0.964	0.710	0.498	0.305	0.084	0.017	0.007	0.003	0.001
Method4	1.000	1.000	0.741	0.456	0.128	0.025	0.011	0.004	0.001
Method5	1.000	0.877	0.610	0.366	0.102	0.020	0.008	0.003	0.001
Method6	1.000	0.888	0.629	0.396	0.114	0.023	0.010	0.004	0.001
Method7	0.960	0.694	0.477	0.285	0.076	0.015	0.006	0.002	0.001
Method8	1.000	1.000	0.736	0.445	0.123	0.023	0.010	0.003	0.001
Method9	1.000	0.868	0.596	0.354	0.097	0.019	0.008	0.002	0.001
Method10	0.961	0.700	0.484	0.290	0.079	0.015	0.007	0.002	0.001
Method11	0.992	0.831	0.627	0.415	0.252	0.425	0.557	0.680	0.787
Method12	0.991	0.820	0.613	0.403	0.255	0.441	0.571	0.696	0.799
Method13	0.942	0.678	0.500	0.340	0.134	0.051	0.031	0.019	0.012
Method14	0.850	0.581	0.432	0.311	0.195	0.320	0.439	0.573	0.693
Method15	0.877	0.570	0.396	0.247	0.120	0.251	0.411	0.580	0.736

Table 3.3: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 50

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.995	0.850	0.632	0.373	0.064	0.005	0.061	0.217	0.434
Method2	0.981	0.774	0.568	0.341	0.115	0.323	0.543	0.745	0.882
Method3	0.995	0.855	0.635	0.378	0.065	0.008	0.071	0.233	0.453
Method4	1.000	0.882	0.654	0.380	0.068	0.006	0.002	0.001	0.000
Method5	1.000	0.844	0.614	0.352	0.059	0.005	0.002	0.001	0.000
Method6	1.000	0.854	0.636	0.376	0.069	0.006	0.002	0.001	0.000
Method7	0.994	0.842	0.622	0.358	0.059	0.005	0.064	0.226	0.445
Method8	1.000	0.877	0.644	0.366	0.063	0.006	0.002	0.001	0.000
Method9	1.000	0.836	0.601	0.340	0.055	0.005	0.002	0.001	0.000
Method10	0.995	0.846	0.627	0.364	0.060	0.008	0.074	0.241	0.465
Method11	0.999	0.945	0.785	0.535	0.248	0.501	0.678	0.824	0.909
Method12	0.999	0.943	0.777	0.523	0.251	0.514	0.689	0.831	0.913
Method13	0.988	0.808	0.612	0.397	0.112	0.023	0.098	0.256	0.445
Method14	0.964	0.741	0.558	0.378	0.188	0.375	0.541	0.705	0.830
Method15	0.980	0.765	0.557	0.330	0.111	0.331	0.554	0.757	0.889

Table 3.4: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 100

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	1.000	0.979	0.842	0.533	0.055	0.160	0.428	0.728	0.908
Method2	1.000	0.963	0.817	0.528	0.112	0.499	0.791	0.936	0.985
Method3	1.000	0.979	0.845	0.537	0.056	0.164	0.433	0.735	0.908
Method4	1.000	0.964	0.812	0.499	0.050	0.000	0.000	0.132	0.695
Method5	1.000	0.960	0.799	0.481	0.047	0.000	0.000	0.371	0.778
Method6	1.000	0.963	0.810	0.498	0.051	0.000	0.000	0.357	0.767
Method7	1.000	0.978	0.838	0.526	0.053	0.164	0.436	0.738	0.910
Method8	1.000	0.963	0.807	0.489	0.048	0.000	0.000	0.134	0.704
Method9	1.000	0.959	0.795	0.472	0.045	0.000	0.000	0.383	0.785
Method10	1.000	0.978	0.841	0.529	0.054	0.169	0.441	0.743	0.914
Method11	1.000	0.998	0.951	0.743	0.250	0.660	0.864	0.957	0.987
Method12	1.000	0.997	0.949	0.736	0.251	0.666	0.869	0.959	0.988
Method13	1.000	0.957	0.799	0.523	0.116	0.216	0.453	0.677	0.844
Method14	1.000	0.935	0.775	0.519	0.200	0.520	0.740	0.883	0.954
Method15	1.000	0.961	0.812	0.519	0.111	0.508	0.797	0.940	0.985

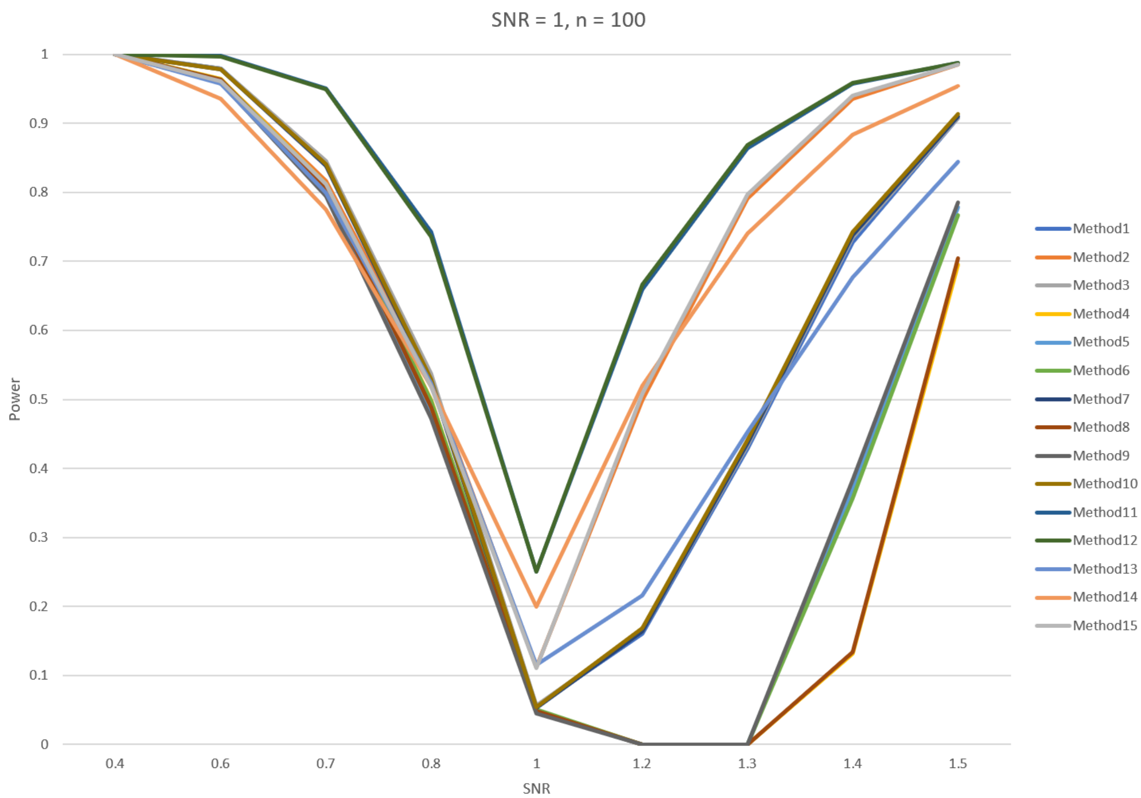

Figure 3.1: Plot of Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 100

Table 3.5: Empirical type I error rate and power of tests for Normal (10, 25), SNR = 2, n = 15

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.736	0.504	0.300	0.167	0.095	0.049	0.024	0.011	0.015
Method2	0.781	0.610	0.441	0.319	0.280	0.330	0.458	0.599	0.728
Method3	0.751	0.521	0.314	0.178	0.100	0.053	0.027	0.013	0.041
Method4	0.768	0.535	0.325	0.183	0.102	0.054	0.027	0.013	0.006
Method5	0.748	0.515	0.311	0.175	0.098	0.052	0.026	0.012	0.006
Method6	0.789	0.571	0.364	0.213	0.119	0.067	0.036	0.017	0.008
Method7	0.708	0.476	0.276	0.151	0.086	0.043	0.022	0.009	0.016
Method8	0.745	0.509	0.302	0.165	0.094	0.047	0.024	0.010	0.006
Method9	0.721	0.489	0.288	0.158	0.090	0.046	0.024	0.009	0.005
Method10	0.724	0.492	0.291	0.161	0.091	0.046	0.024	0.010	0.048
Method11	0.622	0.362	0.194	0.123	0.116	0.166	0.261	0.391	0.514
Method12	0.594	0.335	0.175	0.122	0.123	0.183	0.287	0.415	0.544
Method13	0.688	0.520	0.381	0.267	0.186	0.124	0.082	0.060	0.073
Method14	0.725	0.602	0.491	0.447	0.445	0.486	0.557	0.642	0.706
Method15	0.762	0.585	0.418	0.302	0.275	0.341	0.480	0.620	0.748

Table 3.6: Empirical type I error rate and power of tests for Normal (10, 25), SNR = 2, n = 30

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.926	0.712	0.413	0.186	0.069	0.028	0.061	0.169	0.349
Method2	0.951	0.813	0.584	0.365	0.269	0.355	0.555	0.745	0.877
Method3	0.929	0.719	0.421	0.191	0.073	0.032	0.069	0.187	0.369
Method4	0.931	0.724	0.426	0.195	0.073	0.022	0.008	0.056	0.218
Method5	0.925	0.715	0.415	0.190	0.072	0.021	0.022	0.109	0.285
Method6	0.939	0.748	0.457	0.220	0.084	0.027	0.020	0.094	0.252
Method7	0.916	0.692	0.394	0.174	0.063	0.028	0.066	0.183	0.367
Method8	0.922	0.708	0.409	0.181	0.068	0.020	0.007	0.062	0.232
Method9	0.915	0.694	0.398	0.176	0.065	0.019	0.025	0.119	0.306
Method10	0.920	0.703	0.403	0.178	0.067	0.033	0.076	0.201	0.386
Method11	0.923	0.678	0.358	0.152	0.107	0.193	0.360	0.557	0.724
Method12	0.914	0.658	0.337	0.145	0.111	0.208	0.379	0.576	0.744
Method13	0.854	0.655	0.442	0.276	0.161	0.117	0.161	0.267	0.402
Method14	0.890	0.740	0.568	0.451	0.431	0.503	0.619	0.721	0.816
Method15	0.945	0.799	0.561	0.346	0.265	0.365	0.570	0.762	0.885

Table 3.7: Empirical type I error rate and power of tests for Normal (10, 25), SNR = 2, n = 50

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.990	0.866	0.532	0.206	0.058	0.050	0.178	0.429	0.688
Method2	0.994	0.934	0.718	0.409	0.256	0.404	0.682	0.882	0.964
Method3	0.990	0.871	0.539	0.210	0.061	0.053	0.184	0.441	0.700
Method4	0.990	0.873	0.542	0.212	0.058	0.025	0.112	0.344	0.626
Method5	0.989	0.862	0.531	0.206	0.057	0.034	0.142	0.382	0.657
Method6	0.991	0.883	0.566	0.231	0.065	0.031	0.121	0.350	0.623
Method7	0.988	0.856	0.513	0.192	0.056	0.054	0.188	0.447	0.703
Method8	0.988	0.863	0.523	0.201	0.056	0.026	0.119	0.362	0.646
Method9	0.988	0.853	0.512	0.194	0.054	0.036	0.152	0.405	0.677
Method10	0.988	0.861	0.520	0.198	0.058	0.056	0.198	0.459	0.713
Method11	0.993	0.879	0.523	0.193	0.108	0.236	0.495	0.737	0.888
Method12	0.992	0.870	0.504	0.182	0.111	0.251	0.514	0.749	0.896
Method13	0.954	0.780	0.530	0.293	0.163	0.164	0.279	0.452	0.634
Method14	0.969	0.858	0.657	0.477	0.430	0.530	0.689	0.817	0.900
Method15	0.994	0.928	0.703	0.395	0.255	0.415	0.699	0.889	0.967

Table 3.8: Empirical type I error rate and power of tests for Normal (10,25), SNR = 2, n = 100

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	1.000	0.989	0.777	0.301	0.052	0.122	0.466	0.808	0.961
Method2	1.000	0.997	0.903	0.526	0.262	0.523	0.856	0.980	0.998
Method3	1.000	0.989	0.779	0.304	0.053	0.124	0.471	0.810	0.962
Method4	1.000	0.989	0.778	0.304	0.049	0.087	0.419	0.779	0.956
Method5	1.000	0.988	0.771	0.298	0.049	0.097	0.436	0.793	0.960
Method6	1.000	0.990	0.790	0.314	0.053	0.089	0.415	0.773	0.954
Method7	1.000	0.987	0.767	0.292	0.050	0.129	0.479	0.814	0.963
Method8	1.000	0.987	0.768	0.295	0.047	0.092	0.431	0.790	0.960
Method9	1.000	0.986	0.763	0.289	0.046	0.104	0.447	0.802	0.961
Method10	1.000	0.988	0.769	0.295	0.051	0.132	0.483	0.818	0.964
Method11	1.000	0.994	0.811	0.319	0.105	0.336	0.708	0.929	0.989
Method12	1.000	0.993	0.802	0.312	0.105	0.348	0.718	0.933	0.990
Method13	0.999	0.939	0.708	0.365	0.169	0.237	0.478	0.725	0.889
Method14	0.999	0.967	0.817	0.556	0.439	0.576	0.789	0.926	0.978
Method15	1.000	0.996	0.897	0.512	0.262	0.534	0.863	0.981	0.998

Table 3.9: Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 15

SNR0	**4.000**	**4.400**	**4.600**	**4.800**	**5.000**	**5.200**	**5.400**	**5.600**	**6.400**
Method1	0.347	0.199	0.150	0.112	0.084	0.064	0.049	0.041	0.076
Method2	0.772	0.661	0.630	0.607	0.596	0.612	0.646	0.675	0.839
Method3	0.367	0.211	0.161	0.120	0.091	0.068	0.056	0.048	0.096
Method4	0.358	0.205	0.155	0.115	0.086	0.065	0.050	0.039	0.068
Method5	0.356	0.204	0.154	0.115	0.085	0.065	0.051	0.042	0.080
Method6	0.429	0.256	0.193	0.145	0.111	0.083	0.064	0.050	0.058
Method7	0.315	0.179	0.135	0.103	0.076	0.058	0.044	0.041	0.091
Method8	0.324	0.186	0.140	0.105	0.078	0.058	0.044	0.038	0.082
Method9	0.322	0.184	0.139	0.105	0.078	0.059	0.045	0.041	0.094
Method10	0.333	0.191	0.145	0.108	0.081	0.062	0.052	0.046	0.114
Method11	0.157	0.084	0.066	0.055	0.056	0.072	0.095	0.128	0.322
Method12	0.141	0.078	0.062	0.057	0.063	0.084	0.110	0.150	0.357
Method13	0.413	0.299	0.261	0.228	0.202	0.182	0.176	0.170	0.212
Method14	0.757	0.732	0.723	0.728	0.737	0.743	0.753	0.772	0.855
Method15	0.742	0.643	0.616	0.600	0.596	0.624	0.654	0.692	0.857

Table 3.10: Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 30

SNR0	**4.000**	**4.400**	**4.600**	**4.800**	**5.000**	**5.200**	**5.400**	**5.600**	**6.400**
Method1	0.489	0.246	0.165	0.105	0.069	0.048	0.047	0.066	0.297
Method2	0.869	0.707	0.643	0.610	0.611	0.631	0.680	0.729	0.916
Method3	0.498	0.254	0.172	0.109	0.072	0.050	0.052	0.072	0.313
Method4	0.495	0.252	0.170	0.109	0.070	0.048	0.044	0.063	0.290
Method5	0.493	0.250	0.169	0.108	0.070	0.049	0.048	0.067	0.299
Method6	0.544	0.292	0.197	0.127	0.084	0.056	0.045	0.058	0.257
Method7	0.467	0.227	0.153	0.098	0.064	0.046	0.052	0.072	0.319
Method8	0.472	0.231	0.157	0.100	0.065	0.047	0.049	0.067	0.313
Method9	0.470	0.230	0.156	0.099	0.066	0.047	0.051	0.072	0.321
Method10	0.473	0.235	0.158	0.102	0.067	0.051	0.056	0.079	0.336
Method11	0.329	0.139	0.090	0.062	0.062	0.083	0.122	0.184	0.528
Method12	0.312	0.131	0.083	0.060	0.065	0.087	0.136	0.205	0.548
Method13	0.503	0.337	0.281	0.239	0.214	0.198	0.200	0.212	0.377
Method14	0.818	0.750	0.730	0.726	0.737	0.745	0.769	0.798	0.888
Method15	0.854	0.691	0.631	0.612	0.617	0.637	0.691	0.741	0.922

Table 3.11: Empirical type I error rate and power of tests for Normal (25,25), SNR = 5, n = 50

SNR0	**4.000**	**4.400**	**4.600**	**4.800**	**5.000**	**5.200**	**5.400**	**5.600**	**6.400**
Method1	0.659	0.312	0.181	0.100	0.057	0.044	0.067	0.109	0.541
Method2	0.944	0.785	0.689	0.614	0.598	0.623	0.696	0.774	0.970
Method3	0.667	0.320	0.186	0.104	0.059	0.047	0.070	0.114	0.548
Method4	0.667	0.318	0.186	0.103	0.058	0.044	0.064	0.107	0.536
Method5	0.665	0.315	0.185	0.102	0.058	0.045	0.066	0.109	0.541
Method6	0.701	0.351	0.207	0.118	0.064	0.045	0.058	0.095	0.501
Method7	0.640	0.297	0.170	0.094	0.053	0.046	0.073	0.119	0.560
Method8	0.645	0.302	0.174	0.096	0.053	0.045	0.071	0.117	0.555
Method9	0.644	0.300	0.173	0.095	0.054	0.046	0.072	0.119	0.560
Method10	0.646	0.302	0.175	0.097	0.056	0.047	0.076	0.124	0.568
Method11	0.536	0.206	0.115	0.064	0.057	0.085	0.146	0.240	0.714
Method12	0.516	0.193	0.108	0.062	0.061	0.092	0.158	0.257	0.729
Method13	0.605	0.389	0.300	0.236	0.204	0.197	0.216	0.256	0.530
Method14	0.880	0.774	0.737	0.731	0.735	0.756	0.775	0.804	0.928
Method15	0.937	0.771	0.672	0.605	0.596	0.634	0.707	0.782	0.975

Table 3.12: Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 100

SNR0	**4.000**	**4.400**	**4.600**	**4.800**	**5.000**	**5.200**	**5.400**	**5.600**	**6.400**
Method1	0.892	0.480	0.263	0.130	0.064	0.059	0.126	0.263	0.867
Method2	0.993	0.892	0.761	0.634	0.598	0.667	0.777	0.862	0.997
Method3	0.894	0.483	0.265	0.132	0.065	0.061	0.129	0.268	0.868
Method4	0.895	0.483	0.265	0.131	0.064	0.058	0.124	0.259	0.866
Method5	0.894	0.482	0.263	0.130	0.064	0.059	0.126	0.262	0.867
Method6	0.906	0.506	0.284	0.145	0.071	0.053	0.115	0.241	0.852
Method7	0.885	0.467	0.255	0.126	0.061	0.061	0.134	0.276	0.872
Method8	0.887	0.470	0.256	0.127	0.061	0.059	0.131	0.273	0.871
Method9	0.886	0.469	0.255	0.127	0.061	0.061	0.133	0.275	0.871
Method10	0.886	0.470	0.256	0.128	0.062	0.064	0.136	0.281	0.873
Method11	0.852	0.403	0.212	0.104	0.061	0.106	0.226	0.397	0.917
Method12	0.843	0.390	0.200	0.100	0.060	0.113	0.238	0.411	0.923
Method13	0.786	0.496	0.365	0.276	0.225	0.225	0.289	0.366	0.757
Method14	0.951	0.833	0.775	0.751	0.737	0.765	0.810	0.849	0.966
Method15	0.992	0.885	0.751	0.628	0.598	0.672	0.785	0.866	0.998

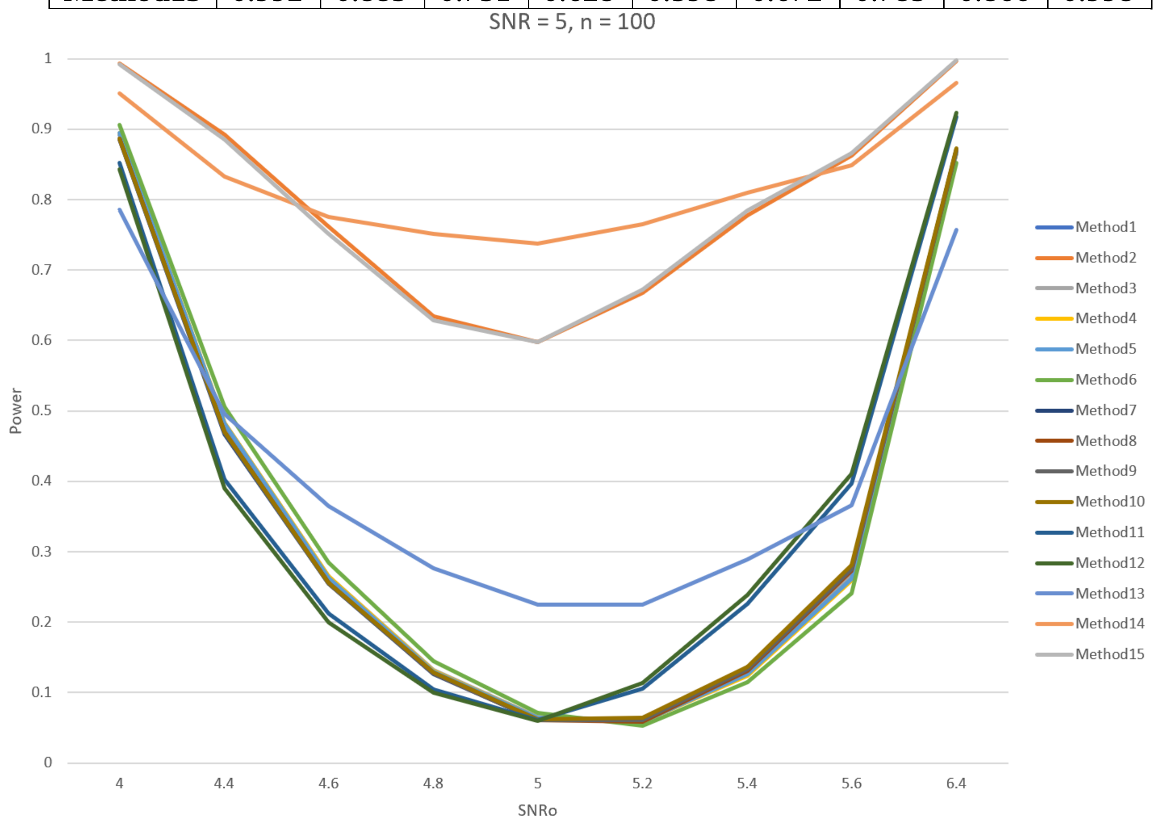

Figure 3.2: Plot of Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 100

Table 3.13: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 15

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.966	0.671	0.437	0.247	0.063	0.015	0.009	0.005	0.003
Method2	0.795	0.460	0.299	0.184	0.061	0.044	0.092	0.194	0.345
Method3	0.968	0.686	0.461	0.261	0.068	0.015	0.008	0.004	0.003
Method4	1.000	1.000	1.000	1.000	0.282	0.061	0.029	0.013	0.005
Method5	1.000	1.000	0.931	0.532	0.136	0.028	0.013	0.006	0.002
Method6	1.000	1.000	0.956	0.577	0.155	0.035	0.016	0.008	0.004
Method7	0.934	0.570	0.363	0.209	0.059	0.017	0.012	0.008	0.007
Method8	1.000	1.000	1.000	1.000	0.287	0.062	0.028	0.011	0.005
Method9	1.000	1.000	0.946	0.545	0.145	0.031	0.014	0.005	0.002
Method10	0.938	0.581	0.375	0.219	0.059	0.015	0.009	0.005	0.004
Method11	0.991	0.734	0.465	0.233	0.082	0.176	0.276	0.398	0.526
Method12	0.981	0.635	0.378	0.199	0.113	0.258	0.376	0.501	0.614
Method13	0.998	0.666	0.438	0.281	0.112	0.050	0.030	0.018	0.012
Method14	0.807	0.454	0.325	0.230	0.114	0.060	0.083	0.196	0.364
Method15	0.704	0.373	0.252	0.152	0.054	0.065	0.150	0.281	0.448

Table 3.14: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 30

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.998	0.847	0.589	0.314	0.041	0.002	0.001	0.000	0.000
Method2	0.972	0.718	0.476	0.260	0.048	0.096	0.248	0.491	0.711
Method3	0.998	0.852	0.598	0.323	0.044	0.003	0.001	0.000	0.000
Method4	1.000	1.000	0.725	0.393	0.055	0.004	0.001	0.000	0.000
Method5	1.000	0.910	0.613	0.325	0.044	0.002	0.001	0.000	0.000
Method6	1.000	0.916	0.647	0.358	0.053	0.004	0.001	0.000	0.000
Method7	0.995	0.751	0.471	0.235	0.032	0.002	0.001	0.000	0.000
Method8	1.000	1.000	0.676	0.344	0.051	0.003	0.001	0.000	0.000
Method9	1.000	0.857	0.527	0.265	0.037	0.002	0.001	0.000	0.000
Method10	0.995	0.761	0.480	0.242	0.035	0.002	0.001	0.000	0.000
Method11	1.000	0.944	0.756	0.435	0.102	0.280	0.474	0.657	0.803
Method12	1.000	0.895	0.631	0.333	0.133	0.411	0.590	0.744	0.858
Method13	1.000	0.914	0.656	0.380	0.085	0.018	0.008	0.004	0.002
Method14	1.000	0.793	0.537	0.332	0.092	0.057	0.190	0.433	0.645
Method15	0.943	0.586	0.365	0.193	0.041	0.164	0.373	0.608	0.783

Table 3.15: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 50

SNR0	**0.400**	**0.600**	**0.700**	**0.800**	**1.000**	**1.200**	**1.300**	**1.400**	**1.500**
Method1	0.999	0.941	0.747	0.397	0.032	0.000	0.012	0.098	0.337
Method2	0.998	0.894	0.664	0.360	0.047	0.192	0.478	0.749	0.916
Method3	0.999	0.943	0.753	0.404	0.033	0.001	0.017	0.112	0.363
Method4	1.000	0.941	0.733	0.388	0.032	0.001	0.000	0.000	0.000
Method5	1.000	0.926	0.700	0.361	0.029	0.000	0.000	0.000	0.000
Method6	1.000	0.931	0.728	0.388	0.034	0.001	0.000	0.000	0.000
Method7	0.999	0.887	0.593	0.285	0.022	0.000	0.027	0.193	0.491
Method8	1.000	0.887	0.584	0.280	0.022	0.000	0.000	0.000	0.000
Method9	1.000	0.853	0.549	0.256	0.020	0.000	0.000	0.000	0.000
Method10	0.999	0.890	0.599	0.289	0.023	0.001	0.037	0.213	0.516
Method11	1.000	0.989	0.901	0.605	0.122	0.415	0.662	0.844	0.946
Method12	1.000	0.975	0.807	0.460	0.154	0.562	0.763	0.897	0.964
Method13	1.000	0.993	0.852	0.510	0.084	0.009	0.003	0.027	0.222
Method14	1.000	0.970	0.779	0.469	0.100	0.108	0.367	0.658	0.842
Method15	0.994	0.795	0.511	0.255	0.042	0.323	0.618	0.827	0.945

Table 3.16: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 100

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	1.000	0.995	0.921	0.605	0.022	0.080	0.352	0.736	0.946
Method2	1.000	0.989	0.900	0.595	0.049	0.437	0.810	0.971	0.998
Method3	1.000	0.995	0.922	0.608	0.022	0.084	0.360	0.741	0.948
Method4	1.000	0.989	0.894	0.559	0.019	0.000	0.000	0.082	0.701
Method5	1.000	0.988	0.885	0.541	0.017	0.000	0.000	0.289	0.798
Method6	1.000	0.989	0.893	0.558	0.020	0.000	0.000	0.275	0.780
Method7	1.000	0.983	0.822	0.413	0.009	0.180	0.540	0.850	0.973
Method8	1.000	0.976	0.778	0.363	0.007	0.000	0.000	0.168	0.825
Method9	1.000	0.972	0.761	0.343	0.006	0.000	0.000	0.481	0.896
Method10	1.000	0.983	0.824	0.416	0.010	0.187	0.548	0.856	0.975
Method11	1.000	1.000	0.984	0.827	0.138	0.646	0.899	0.985	0.999
Method12	1.000	0.999	0.958	0.677	0.193	0.786	0.951	0.995	0.999
Method13	1.000	1.000	0.992	0.803	0.076	0.010	0.161	0.551	0.852
Method14	1.000	1.000	0.987	0.795	0.101	0.234	0.652	0.911	0.985
Method15	1.000	0.974	0.789	0.405	0.047	0.631	0.905	0.989	0.999

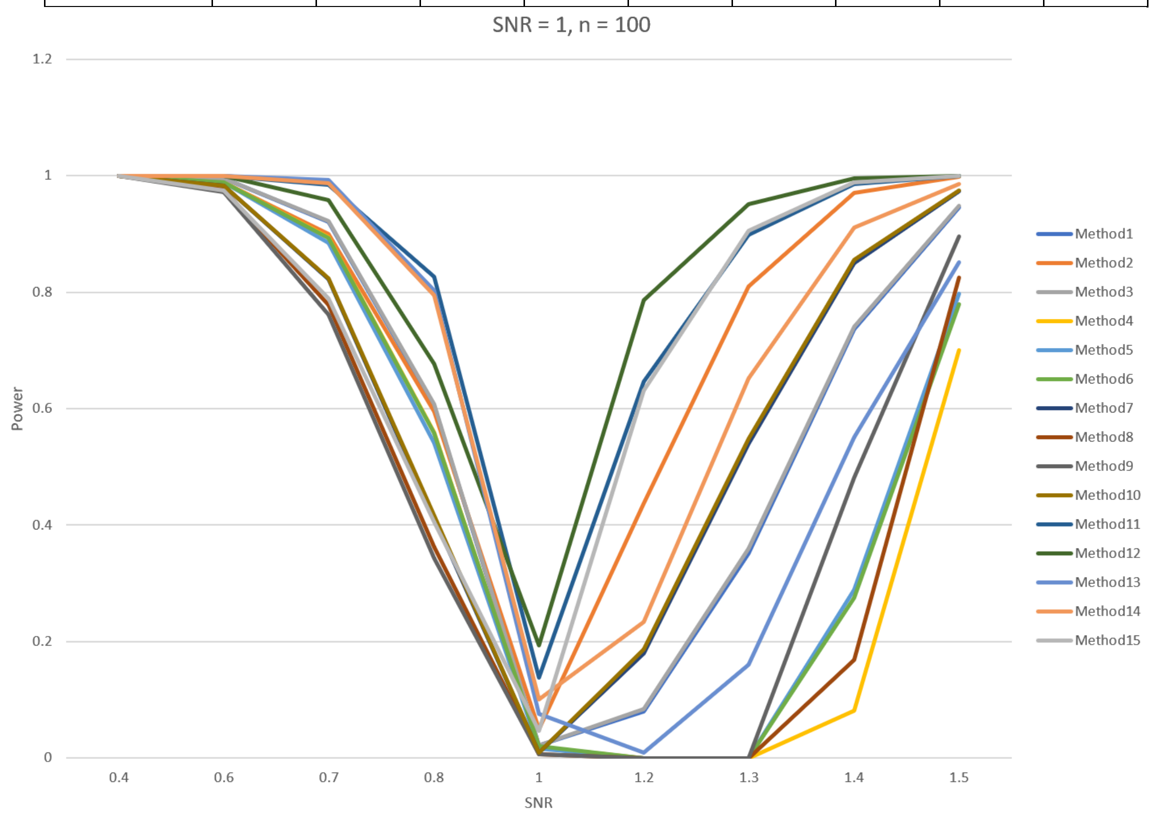

Figure 3.3: Plot of Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 100

Table 3.17: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 15

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.812	0.564	0.317	0.158	0.077	0.036	0.015	0.006	0.005
Method2	0.862	0.695	0.487	0.316	0.227	0.241	0.353	0.537	0.699
Method3	0.828	0.585	0.337	0.171	0.084	0.038	0.016	0.007	0.013
Method4	0.838	0.596	0.343	0.173	0.085	0.038	0.016	0.007	0.003
Method5	0.824	0.578	0.330	0.167	0.081	0.037	0.015	0.007	0.003
Method6	0.865	0.653	0.390	0.213	0.105	0.046	0.023	0.010	0.005
Method7	0.769	0.512	0.290	0.143	0.065	0.031	0.013	0.005	0.006
Method8	0.797	0.546	0.307	0.155	0.071	0.033	0.014	0.005	0.003
Method9	0.780	0.526	0.298	0.150	0.068	0.031	0.014	0.005	0.003
Method10	0.784	0.533	0.302	0.152	0.070	0.033	0.014	0.005	0.024
Method11	0.706	0.387	0.178	0.081	0.057	0.095	0.180	0.305	0.457
Method12	0.651	0.350	0.161	0.078	0.074	0.127	0.223	0.360	0.506
Method13	0.735	0.535	0.373	0.249	0.163	0.109	0.073	0.049	0.035
Method14	0.782	0.629	0.484	0.385	0.355	0.408	0.507	0.614	0.706
Method15	0.818	0.640	0.444	0.297	0.225	0.268	0.402	0.577	0.724

Table 3.18: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 30

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.955	0.749	0.421	0.167	0.051	0.018	0.032	0.128	0.314
Method2	0.974	0.856	0.614	0.350	0.213	0.296	0.525	0.755	0.898
Method3	0.957	0.759	0.433	0.174	0.054	0.020	0.040	0.142	0.337
Method4	0.959	0.763	0.438	0.178	0.054	0.015	0.004	0.029	0.173
Method5	0.955	0.750	0.426	0.172	0.052	0.015	0.011	0.071	0.249
Method6	0.964	0.792	0.472	0.206	0.066	0.019	0.009	0.058	0.208
Method7	0.930	0.697	0.380	0.144	0.043	0.020	0.053	0.169	0.365
Method8	0.936	0.712	0.397	0.154	0.047	0.014	0.003	0.050	0.223
Method9	0.929	0.699	0.385	0.148	0.045	0.014	0.017	0.105	0.300
Method10	0.934	0.707	0.391	0.151	0.047	0.024	0.061	0.187	0.388
Method11	0.954	0.712	0.352	0.123	0.069	0.146	0.328	0.543	0.737
Method12	0.929	0.662	0.316	0.116	0.087	0.190	0.376	0.586	0.768
Method13	0.917	0.715	0.475	0.267	0.142	0.083	0.097	0.193	0.340
Method14	0.942	0.805	0.610	0.436	0.360	0.426	0.558	0.707	0.824
Method15	0.954	0.811	0.562	0.320	0.225	0.340	0.570	0.782	0.910

Table 3.19: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 50

SNR0	**1.200**	**1.400**	**1.600**	**1.800**	**2.000**	**2.200**	**2.400**	**2.600**	**2.800**
Method1	0.995	0.913	0.567	0.198	0.043	0.026	0.128	0.389	0.678
Method2	0.997	0.959	0.761	0.423	0.208	0.348	0.665	0.892	0.978
Method3	0.995	0.916	0.574	0.201	0.044	0.029	0.138	0.404	0.690
Method4	0.995	0.917	0.578	0.204	0.044	0.013	0.074	0.307	0.611
Method5	0.994	0.911	0.567	0.199	0.042	0.017	0.091	0.344	0.645
Method6	0.995	0.923	0.605	0.225	0.053	0.015	0.079	0.312	0.608
Method7	0.990	0.873	0.512	0.172	0.038	0.041	0.175	0.450	0.715
Method8	0.990	0.878	0.522	0.178	0.037	0.018	0.106	0.364	0.659
Method9	0.989	0.869	0.511	0.173	0.036	0.027	0.134	0.406	0.687
Method10	0.990	0.877	0.518	0.175	0.040	0.045	0.186	0.461	0.728
Method11	0.996	0.922	0.559	0.181	0.066	0.183	0.461	0.738	0.900
Method12	0.992	0.883	0.503	0.162	0.086	0.238	0.517	0.771	0.913
Method13	0.986	0.852	0.571	0.300	0.134	0.103	0.207	0.405	0.606
Method14	0.994	0.917	0.717	0.475	0.373	0.454	0.645	0.808	0.912
Method15	0.993	0.934	0.705	0.378	0.220	0.401	0.705	0.906	0.980

Table 3.20: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 100

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	1.000	0.993	0.804	0.289	0.035	0.095	0.438	0.826	0.976
Method2	1.000	0.998	0.925	0.550	0.221	0.488	0.873	0.988	0.999
Method3	1.000	0.993	0.807	0.293	0.035	0.098	0.443	0.829	0.976
Method4	1.000	0.993	0.806	0.294	0.033	0.066	0.381	0.797	0.971
Method5	1.000	0.992	0.799	0.286	0.032	0.075	0.400	0.808	0.972
Method6	1.000	0.994	0.816	0.309	0.038	0.067	0.378	0.792	0.969
Method7	1.000	0.984	0.741	0.242	0.035	0.143	0.513	0.856	0.981
Method8	1.000	0.983	0.741	0.246	0.029	0.108	0.457	0.832	0.977
Method9	1.000	0.982	0.737	0.239	0.030	0.119	0.477	0.842	0.979
Method10	1.000	0.984	0.743	0.245	0.036	0.145	0.518	0.859	0.982
Method11	1.000	0.996	0.840	0.314	0.076	0.304	0.725	0.946	0.993
Method12	1.000	0.990	0.781	0.266	0.100	0.374	0.768	0.957	0.994
Method13	1.000	0.980	0.769	0.387	0.129	0.162	0.426	0.716	0.896
Method14	1.000	0.995	0.882	0.577	0.373	0.525	0.785	0.931	0.979
Method15	1.000	0.996	0.879	0.477	0.244	0.560	0.899	0.989	0.999

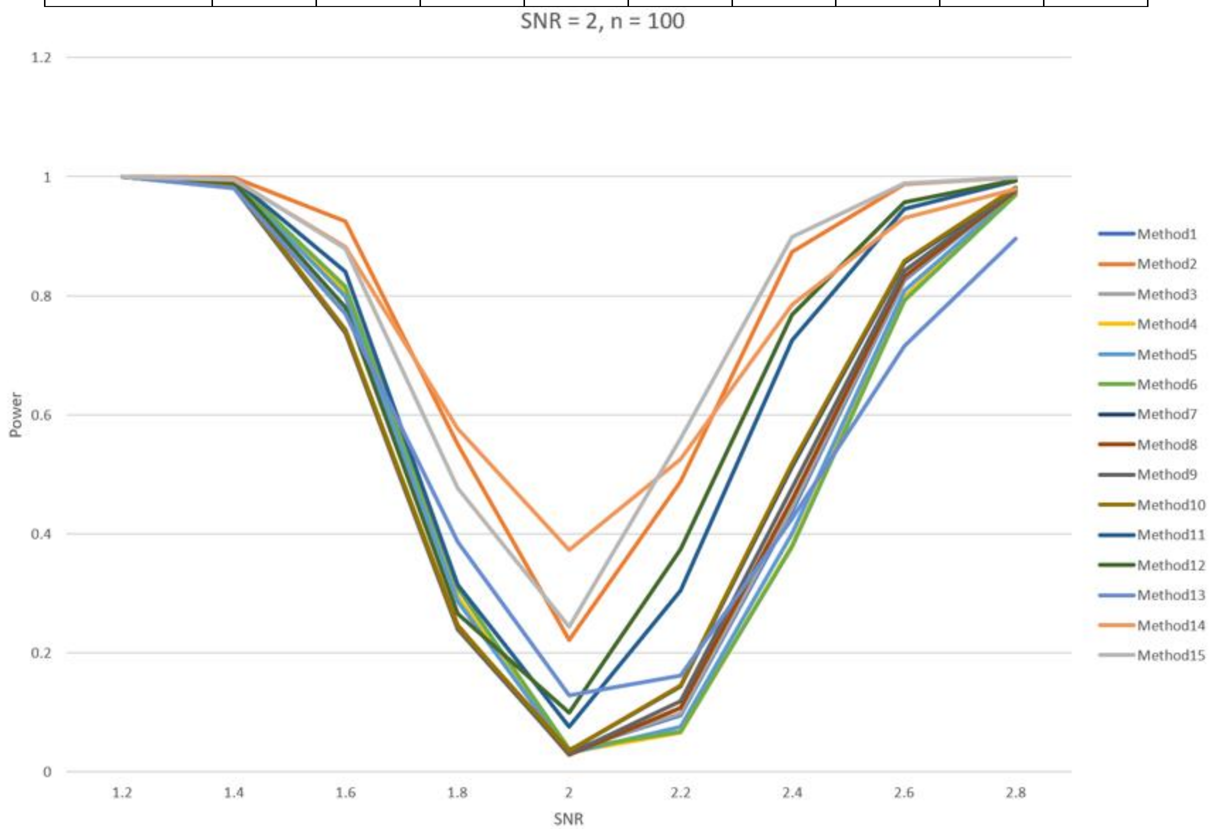

Figure 3.4: Plot of Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 100

Table 3.21: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 15

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.349	0.197	0.147	0.111	0.082	0.061	0.047	0.040	0.065
Method2	0.770	0.651	0.605	0.590	0.594	0.614	0.643	0.680	0.837
Method3	0.367	0.212	0.154	0.118	0.087	0.067	0.053	0.046	0.084
Method4	0.359	0.204	0.151	0.115	0.083	0.063	0.048	0.039	0.058
Method5	0.357	0.203	0.151	0.114	0.084	0.063	0.048	0.041	0.069
Method6	0.421	0.255	0.189	0.143	0.110	0.082	0.061	0.048	0.050
Method7	0.319	0.175	0.130	0.098	0.072	0.055	0.044	0.039	0.082
Method8	0.329	0.182	0.135	0.102	0.073	0.056	0.043	0.037	0.074
Method9	0.327	0.181	0.134	0.102	0.074	0.057	0.045	0.040	0.086
Method10	0.335	0.188	0.136	0.106	0.077	0.060	0.051	0.045	0.104
Method11	0.153	0.080	0.062	0.052	0.054	0.065	0.083	0.113	0.317
Method12	0.137	0.073	0.059	0.057	0.061	0.077	0.100	0.142	0.360
Method13	0.406	0.296	0.254	0.221	0.192	0.170	0.159	0.157	0.205
Method14	0.750	0.719	0.712	0.717	0.724	0.738	0.750	0.778	0.852
Method15	0.738	0.631	0.597	0.587	0.602	0.630	0.657	0.697	0.850

Table 3.22: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 30

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.492	0.249	0.158	0.095	0.062	0.043	0.042	0.058	0.275
Method2	0.882	0.729	0.647	0.609	0.600	0.617	0.671	0.712	0.921
Method3	0.502	0.258	0.164	0.101	0.065	0.046	0.046	0.063	0.289
Method4	0.500	0.256	0.162	0.098	0.064	0.042	0.039	0.056	0.269
Method5	0.497	0.254	0.161	0.098	0.064	0.043	0.042	0.059	0.276
Method6	0.552	0.297	0.194	0.120	0.075	0.049	0.040	0.049	0.236
Method7	0.467	0.231	0.147	0.088	0.057	0.045	0.048	0.066	0.302
Method8	0.474	0.236	0.149	0.090	0.058	0.043	0.046	0.063	0.296
Method9	0.472	0.235	0.149	0.090	0.058	0.045	0.048	0.067	0.304
Method10	0.475	0.239	0.150	0.092	0.061	0.047	0.054	0.073	0.317
Method11	0.336	0.133	0.081	0.055	0.054	0.073	0.111	0.170	0.521
Method12	0.315	0.122	0.076	0.056	0.062	0.084	0.129	0.193	0.546
Method13	0.501	0.336	0.279	0.235	0.199	0.186	0.182	0.195	0.364
Method14	0.823	0.748	0.719	0.713	0.726	0.738	0.771	0.800	0.882
Method15	0.861	0.708	0.636	0.610	0.604	0.630	0.680	0.727	0.927

Table 3.23: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 50

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.668	0.307	0.183	0.097	0.053	0.042	0.057	0.101	0.540
Method2	0.954	0.792	0.684	0.607	0.577	0.621	0.700	0.783	0.972
Method3	0.672	0.312	0.186	0.099	0.055	0.044	0.060	0.106	0.552
Method4	0.672	0.311	0.186	0.098	0.054	0.042	0.055	0.097	0.534
Method5	0.670	0.309	0.184	0.097	0.054	0.042	0.056	0.100	0.540
Method6	0.707	0.340	0.211	0.116	0.064	0.042	0.048	0.088	0.498
Method7	0.638	0.286	0.167	0.088	0.051	0.044	0.066	0.119	0.568
Method8	0.645	0.290	0.170	0.090	0.051	0.045	0.064	0.115	0.564
Method9	0.642	0.288	0.170	0.089	0.051	0.045	0.066	0.118	0.568
Method10	0.646	0.291	0.171	0.091	0.054	0.048	0.069	0.126	0.580
Method11	0.544	0.210	0.111	0.062	0.052	0.075	0.138	0.231	0.721
Method12	0.514	0.193	0.104	0.060	0.057	0.088	0.157	0.255	0.740
Method13	0.614	0.387	0.299	0.233	0.200	0.189	0.208	0.247	0.523
Method14	0.881	0.777	0.738	0.725	0.728	0.748	0.773	0.808	0.924
Method15	0.944	0.773	0.668	0.596	0.583	0.635	0.715	0.793	0.975

Table 3.24: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 100

SNR0	**4.000**	**4.400**	**4.600**	**4.800**	**5.000**	**5.200**	**5.400**	**5.600**	**6.400**
Method1	0.904	0.480	0.264	0.115	0.053	0.052	0.119	0.257	0.880
Method2	0.995	0.902	0.764	0.636	0.597	0.649	0.759	0.869	0.997
Method3	0.905	0.484	0.266	0.118	0.054	0.053	0.122	0.262	0.883
Method4	0.905	0.485	0.266	0.118	0.054	0.052	0.117	0.253	0.880
Method5	0.905	0.483	0.266	0.117	0.053	0.052	0.118	0.256	0.880
Method6	0.917	0.511	0.286	0.128	0.055	0.048	0.103	0.233	0.867
Method7	0.888	0.458	0.246	0.108	0.052	0.060	0.134	0.285	0.889
Method8	0.891	0.463	0.248	0.111	0.053	0.059	0.132	0.281	0.887
Method9	0.889	0.461	0.247	0.110	0.052	0.060	0.134	0.284	0.889
Method10	0.890	0.462	0.248	0.111	0.053	0.062	0.137	0.289	0.890
Method11	0.865	0.403	0.202	0.083	0.054	0.095	0.217	0.392	0.934
Method12	0.848	0.383	0.187	0.080	0.060	0.109	0.242	0.415	0.940
Method13	0.798	0.494	0.353	0.257	0.211	0.212	0.267	0.354	0.769
Method14	0.951	0.845	0.780	0.736	0.729	0.759	0.801	0.850	0.972
Method15	0.993	0.887	0.744	0.627	0.604	0.660	0.774	0.878	0.997

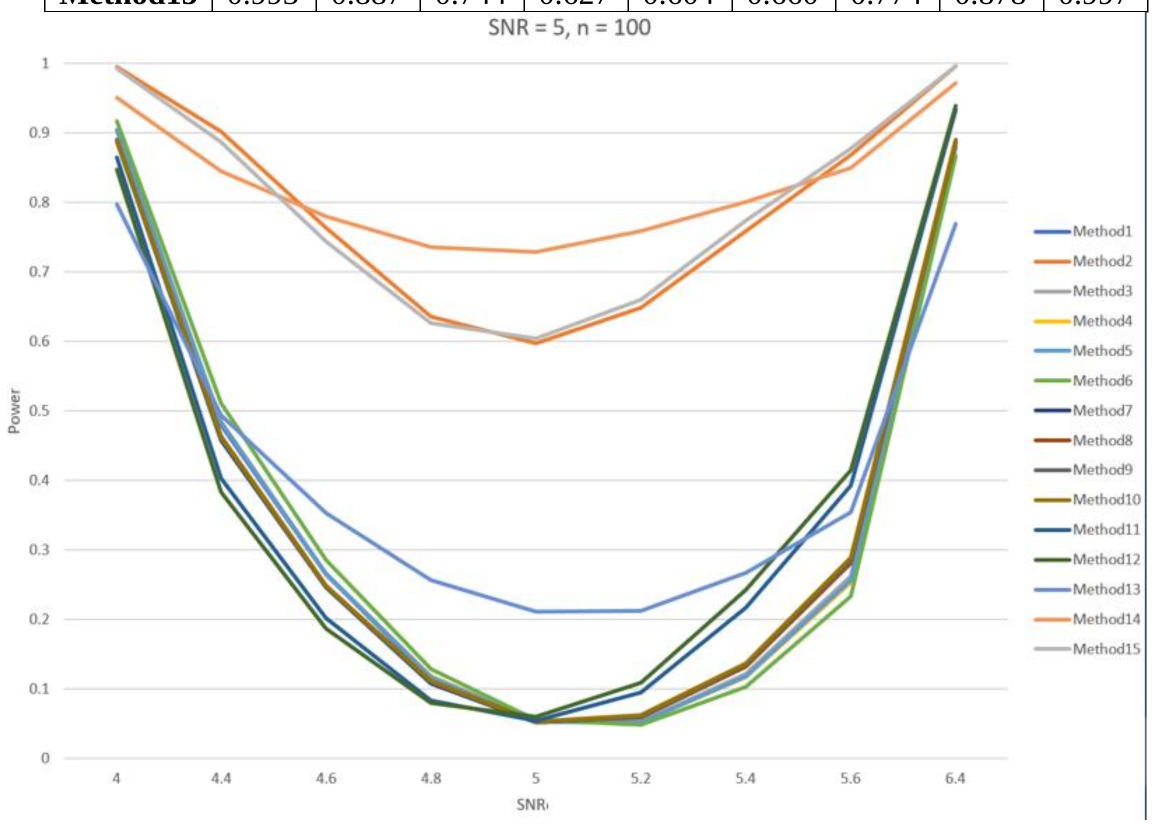

Figure 3.5: Plot of Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 100

Table 3.25: Empirical type I error rate and power of tests for Beta (1, 0.67), SNR = 2, n = 15

SNR0	**1.200**	**1.400**	**1.600**	**1.800**	**2.000**	**2.200**	**2.400**	**2.600**	**2.800**
Method1	0.709	0.450	0.251	0.131	0.066	0.034	0.021	0.013	0.009
Method2	0.768	0.572	0.387	0.256	0.231	0.312	0.469	0.637	0.771
Method3	0.725	0.469	0.267	0.139	0.071	0.037	0.023	0.014	0.028
Method4	0.738	0.477	0.270	0.141	0.072	0.037	0.023	0.013	0.007
Method5	0.720	0.462	0.261	0.137	0.070	0.036	0.022	0.013	0.007
Method6	0.774	0.526	0.316	0.168	0.089	0.045	0.027	0.016	0.009
Method7	0.651	0.389	0.208	0.105	0.054	0.029	0.017	0.010	0.011
Method8	0.685	0.418	0.223	0.114	0.058	0.031	0.018	0.011	0.006
Method9	0.663	0.400	0.215	0.109	0.056	0.029	0.018	0.010	0.006
Method10	0.668	0.406	0.218	0.111	0.057	0.030	0.018	0.011	0.040
Method11	0.587	0.312	0.146	0.079	0.096	0.174	0.287	0.431	0.569
Method12	0.523	0.259	0.119	0.076	0.116	0.215	0.343	0.492	0.628
Method13	0.398	0.259	0.163	0.101	0.063	0.041	0.028	0.019	0.121
Method14	0.436	0.325	0.312	0.391	0.514	0.636	0.735	0.810	0.866
Method15	0.711	0.510	0.335	0.224	0.231	0.344	0.525	0.686	0.810

Table 3.26: Empirical type I error rate and power of tests for Beta (1, 0.67), SNR = 2, n = 30

SNR0	**1.200**	**1.400**	**1.600**	**1.800**	**2.000**	**2.200**	**2.400**	**2.600**	**2.800**
Method1	0.931	0.673	0.364	0.152	0.056	0.023	0.051	0.181	0.392
Method2	0.959	0.799	0.540	0.312	0.232	0.376	0.598	0.791	0.903
Method3	0.935	0.682	0.376	0.157	0.058	0.026	0.061	0.199	0.415
Method4	0.938	0.688	0.380	0.160	0.059	0.020	0.007	0.046	0.237
Method5	0.931	0.675	0.368	0.154	0.057	0.019	0.017	0.109	0.325
Method6	0.948	0.721	0.412	0.180	0.067	0.024	0.016	0.086	0.279
Method7	0.910	0.617	0.306	0.122	0.041	0.022	0.064	0.224	0.450
Method8	0.916	0.633	0.324	0.130	0.043	0.015	0.006	0.062	0.286
Method9	0.909	0.619	0.311	0.124	0.042	0.015	0.020	0.140	0.381
Method10	0.914	0.626	0.316	0.126	0.043	0.024	0.077	0.245	0.473
Method11	0.929	0.636	0.308	0.117	0.092	0.206	0.407	0.604	0.767
Method12	0.908	0.578	0.258	0.097	0.103	0.249	0.462	0.662	0.815
Method13	0.458	0.250	0.125	0.068	0.129	0.309	0.495	0.658	0.785
Method14	0.523	0.345	0.320	0.454	0.630	0.770	0.866	0.929	0.962
Method15	0.947	0.754	0.478	0.264	0.231	0.413	0.648	0.833	0.926

Table 3.27: Empirical type I error rate and power of tests for Beta (1, 0.67), SNR = 2, n = 50

SNR0	**1.200**	**1.400**	**1.600**	**1.800**	**2.000**	**2.200**	**2.400**	**2.600**	**2.800**
Method1	0.994	0.879	0.501	0.183	0.049	0.036	0.167	0.459	0.719
Method2	0.998	0.943	0.705	0.365	0.224	0.418	0.711	0.899	0.971
Method3	0.994	0.883	0.508	0.188	0.050	0.039	0.178	0.473	0.729
Method4	0.994	0.883	0.510	0.189	0.050	0.016	0.100	0.368	0.665
Method5	0.994	0.876	0.499	0.184	0.049	0.021	0.127	0.405	0.692
Method6	0.995	0.893	0.534	0.206	0.055	0.022	0.112	0.374	0.663
Method7	0.992	0.843	0.433	0.141	0.035	0.045	0.216	0.527	0.775
Method8	0.993	0.848	0.444	0.147	0.036	0.016	0.128	0.432	0.727
Method9	0.991	0.842	0.432	0.142	0.035	0.024	0.163	0.477	0.752
Method10	0.993	0.847	0.439	0.144	0.036	0.048	0.226	0.538	0.784
Method11	0.997	0.887	0.492	0.166	0.092	0.233	0.528	0.766	0.905
Method12	0.996	0.858	0.423	0.128	0.097	0.287	0.596	0.815	0.929
Method13	0.543	0.258	0.108	0.142	0.357	0.598	0.781	0.891	0.950
Method14	0.613	0.361	0.326	0.522	0.734	0.884	0.951	0.980	0.991
Method15	0.997	0.924	0.634	0.306	0.215	0.472	0.767	0.925	0.980

Table 3.28: Empirical type I error rate and power of tests for Beta (1, 0.67), SNR = 2, n = 100

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	1.000	0.993	0.778	0.266	0.043	0.110	0.487	0.837	0.968
Method2	1.000	0.999	0.911	0.500	0.226	0.539	0.883	0.984	0.998
Method3	1.000	0.994	0.780	0.267	0.043	0.114	0.492	0.841	0.968
Method4	1.000	0.993	0.780	0.268	0.041	0.078	0.427	0.815	0.961
Method5	1.000	0.992	0.773	0.260	0.041	0.089	0.446	0.825	0.964
Method6	1.000	0.994	0.793	0.281	0.045	0.079	0.421	0.812	0.960
Method7	1.000	0.989	0.721	0.200	0.028	0.145	0.573	0.885	0.981
Method8	1.000	0.989	0.725	0.204	0.025	0.102	0.514	0.866	0.978
Method9	1.000	0.988	0.713	0.198	0.024	0.118	0.537	0.874	0.980
Method10	1.000	0.989	0.726	0.203	0.028	0.149	0.578	0.888	0.982
Method11	1.000	0.997	0.818	0.286	0.087	0.337	0.749	0.940	0.991
Method12	1.000	0.995	0.764	0.221	0.093	0.411	0.808	0.962	0.995
Method13	0.722	0.276	0.117	0.320	0.686	0.895	0.974	0.994	0.998
Method14	0.795	0.398	0.334	0.659	0.893	0.977	0.996	1.000	1.000
Method15	1.000	0.998	0.879	0.414	0.222	0.616	0.921	0.990	0.999

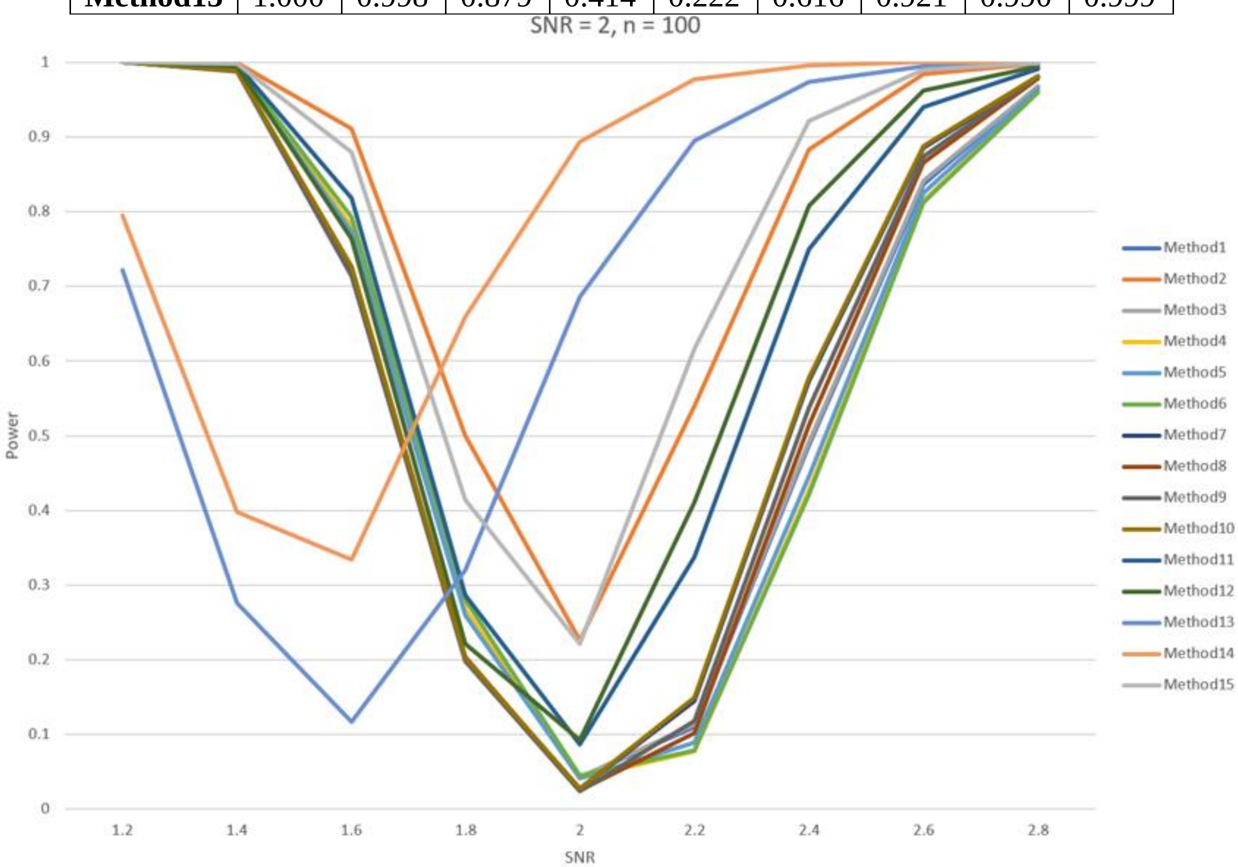

Figure 3.6: Plot of Empirical type I error rate and power of tests for Beta (1, 0.67), SNR = 2, n = 100

3.2 Simulation Results Discussion

3.2.1 When Data is Generated from a Normal Distribution.

We will consider a test is good test when the test attains the empirical nominal level at most 0.06 [(05-1.96*sqrt((.95*.05)/5000) =0.04 and 05+1.96*sqrt((.95*.05)/5000)=0.06)] for 5% level of significance test. When we review Tables 3.1 to 3.4, we can see that for testing SNR=1, and small sample sizes (15 and 30) none of the tests achieves the empirical nominal level 0.05 and cannot be used for testing for SNR. However, when the sample sizes are large (50 and 100), all methods except methods 2, 11 to 15 achieved the nominal level 0.05.

If we review Tables 3.3 and 3.4, and consider lower and upper tail powers, it appears that methods 1, 3, 7 and 10 are performing better than Methods 4, 5, 6, 8 and 9. However, method 10 proposed by Kibria and George (2014) has performed the best in the sense of attaining the nominal size and high empirical power. When we review Tables 3.5 to 3.8, we can see that for testing SNR=2, and small sample size (15) none of the tests achieves the nominal level 0.05 and cannot be used for testing for SNR.

For n=30, only method 7 attained the nominal level. When the sample sizes are large (50 and 100), all methods except methods 2, 11 to 15 achieve nominal level 0.05 and giving high empirical power. Again, method 10 has performed the best in the sense of attaining the nominal size and high empirical power. When we review Tables 3.9 to 3.12, we can see that for testing SNR=5, and small sample size (15) none of the tests except methods 11 and 12 (proposed) achieves the nominal level 0.05 and both performed equivalently well in the sense empirical power. Proposed method 12 performed better than the method 11 for testing the high values of SNR, while method 11 for low values. Table 3.10 indicates that methods 1, 7 to 12 attained the nominal level. However, methods 11 & 12 performed better in the high values of SNR, while methods 1, 7, 8, 9 and 10 for low values. From tables 3.11 and 3.12 we observed that methods 1 and 3 through 12 attained the nominal level and performed almost equivalently in the sense of power. Overall, proposed method 12 produces the highest power at the high value of SNR and method 6 for the low value of SNR.

3.2.2 When Data is Generated from a Gamma Distribution.

The simulated empirical sizes and powers are tabulated in Tables 3.13 to 3.24 when data is generated from a Gamma distribution. When we review Table 3.13, we can see that for testing SNR=1, and small sample size (15), only methods 1, 2 and proposed method 15 attained the nominal level. Proposed method 15 produces the highest power at the high value of SNR and methods 1 & 2 for the low values. Tables 3.14 and 3.15 indicate that for testing SNR=1, and for n=30, 50 all methods but methods 11-14 attained the nominal level.

Methods 1 through 10 performed poorly at the high values of SNR. However, proposed method 15 performed the best in the sense of high power at both ends. From Tables 3.16, we can see that for testing SNR=1, and large sample size (100), all methods but methods 11 through14 attained the nominal level and performed equivalently well. The proposed method 15 performed the best in the sense of high power of both ends. When we review Table 3.17, we can see that for testing SNR=2, and small sample size (15), only method 11 attained the nominal level. We also observe from Table 3.18 through3.20 for testing SNR=2, and n=30, 50 and 100, methods 1, 3, 4 through 10 attained the nominal level and produces high power for large sample sizes. Method 10 performed the best in the sense of highest power at both ends.

Table 3.21, for testing SNR=5, and small sample size (15), only method 11 and the proposed method 12 attained the nominal level. However, their powers are not that high to consider a good test. When we review Table 3.22, we can see that for testing SNR=5, and n=30, methods 1, 3, 4, 5, 7 through 12 attained the nominal level. Proposed method 15 produces the highest power at the high value of SNR and method 3 for the low values. All empirical powers are lower than 55%. When we examine Table 3.24, we can see that for testing SNR=5, and n=100, methods 1, 3, 4, 5, 7 through 12 attained the nominal level. Proposed method 15 produces the highest power at the high value of SNR and method 6 for the low values of SNR.

3.2.3 When Data is Generated from a Beta Distribution

Kibria and George (2014) did not compare the performance of the test methods 1 through 11 under left skewed conditions. In this section we want to see the performance of the test statistics when the data are generated from a Beta distribution with parameter (1, 0.67). That means data is generated from a left skewed distribution. From Tables 3.25, we can see that for testing SNR=2, and small sample size (15), methods 1, 7, 8, 9, 10 and proposed method 13 attained the nominal level and performed equivalently well. All methods produce low power at the upper end and high power at the lower end of the alternative hypothesis. From Tables 3.26 through3.28, we can find that for testing SNR=2, and sample sizes 30, 50 and 100 methods 1 through10 attained the nominal level and performed equivalently well. All methods produce high power at both ends of alternative hypotheses. Overall, method 10 performed the best.

CHAPTER 4

APPLICATIONS

In this chapter we will consider and analyze three (3) real life data sets to illustrate the performance of the test statistics.

4.1 Failure Time

In this section we analyze the failure times data sets from Aarset (1987). It contains the following 50 failure time data. 0.1, 0.2, 1.0, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 6.0, 7.0, 11.0, 12.0, 18.0, 18.0, 18.0, 18.0, 18.0, 21.0, 32.0, 36.0, 40.0, 45.0, 45.0, 47.0, 50.0, 55.0, 60.0, 63.0, 63.0, 67.0, 67.0, 67.0, 67.0, 72.0, 75.0, 79.0, 82.0, 82.0, 83.0, 84.0, 84.0, 84.0, 85.0, 85.0, 85.0, 85.0, 85.0, 86.0, 86.0.

The histogram of the failure time data is presented below, which looks like a uniform distribution.

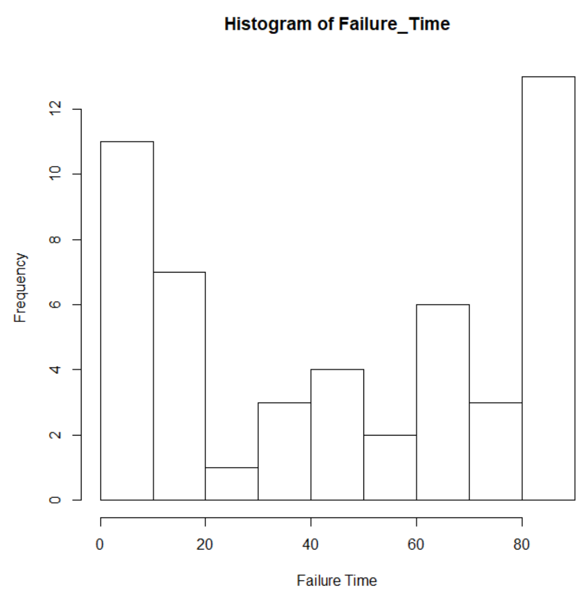

Figure 4.1: Histogram of the Failure time data

Using the library (fitdistrplus) in R we have the following analysis.

```
> library(fitdistrplus)
Loading required package: MASS
Loading required package: survival
> library(MASS)
> fit=fitdist(Failure_Time, dist="unif",method="mle")
> fit
Fitting of the distribution ' unif ' by maximum likelihood
Parameters:
    estimate Std. Error
min      0.1         NA
max     86.0         NA
```

```
> ks.test(Failure_Time, dist="unif", 0.1, 86)

        Two-sample Kolmogorov-Smirnov test

data:  Failure_Time and 0.1
D = 0.98, p-value = 0.3032
alternative hypothesis: two-sided
```

The estimated parameters of the uniform distribution are obtained as 0.1 and 86.0. Using KS test, we found the P-value as 0.3032 and concluded that the failure data follow a uniform distribution with parameters 0.1 and 86.0. The histogram, density plot, Q-Q plot, empirical and theoretical CDF plot and P-P plot are provided in Figure 4.2, which supported the uniform distribution. That means the population signal to ratio will be $\frac{42.95}{24.80} = 1.73$. Then it is logical to test the true signal to ratio as, H_0: SNR=1.73. The lower and upper critical values for all tests are presented in Table 4.1.

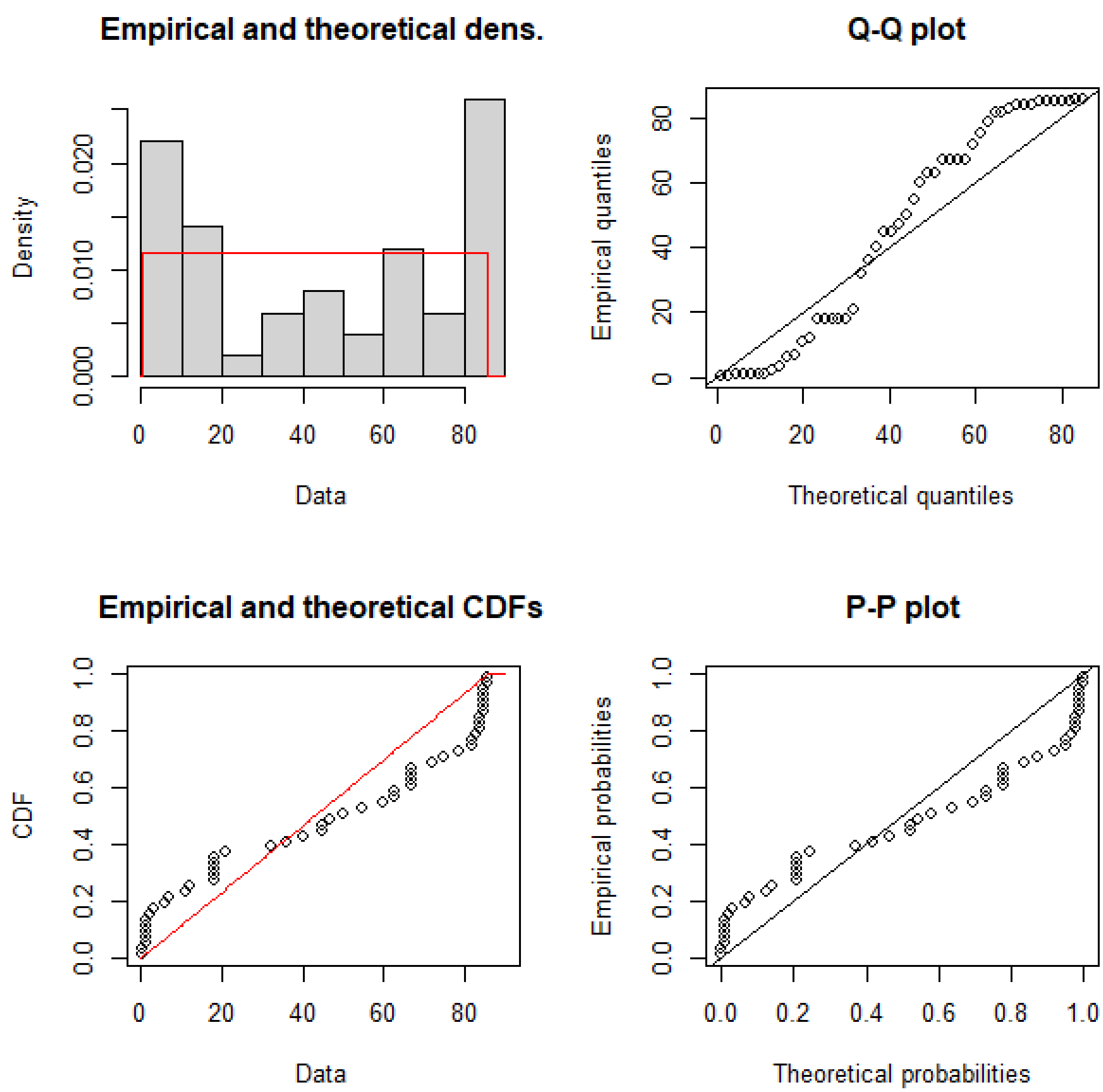

Figure 4.2: Empirical and Theoretical Density, QQ plot, CDF and P-P plots of Failure Time Data

From Table 4.1, we can see that all tests but the methods 12 through 15 do not reject the null hypothesis. It is noted that methods 12 to 15 did not achieve the nominal level 0.05 and can not be recommended for the practitioners.

Table 4.1: Lower and upper critical values for the failure time data

Method	Lower Limit	Upper Limit
1	1.139	2.009
2	1.177	1.731
3	1.142	2.001
4	1.141	2.122
5	1.134	2.081
6	1.147	2.095
7	1.117	1.994
8	1.122	2.102
9	1.115	2.059
10	0.911	3.346
11	1.167	1.741
12	1.149	1.714
13	0.696	1.467
14	0.666	1.221
15	1.155	1.709

4.2 Chemotherapy Treatment

In this section, we will consider the chemotherapy data, which was given on a group of individuals who had chemotherapy treatment exclusively for 45 years (Bekker *et al*. 2000). The observations are as follows: 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033.

The histogram of the failure time data is presented in Figure 4.3, which looks like a gamma distribution. The skewness and kurtosis are obtained as 0.97 and 2.66 respectively.

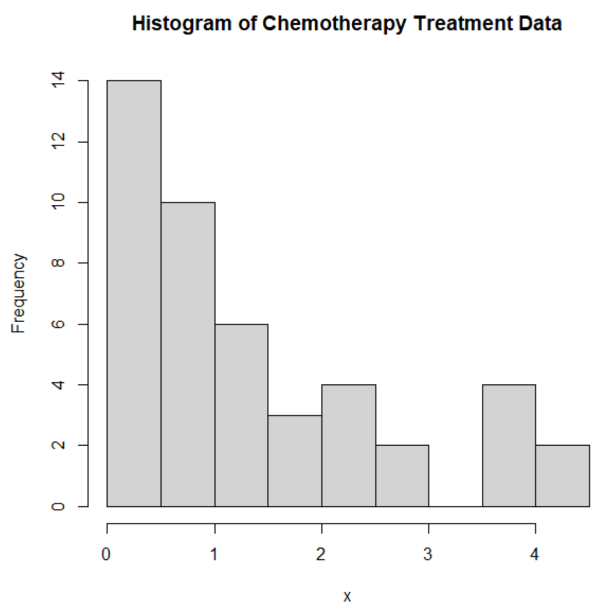

Figure 4.3: Histogram of chemotherapy data

Using R 4.2.2, we obtained the following results.

```
> fitg=fitdist(x, dist="gamma", method="mle")
> fitg
Fitting of the distribution ' gamma ' by maximum likelihood
Parameters:
       estimate Std. Error
shape 1.1006092  0.2060163
rate  0.8204196  0.1928045
> ks.test(x, "pgamma", 1.1, 0.82)

        One-sample Kolmogorov-Smirnov test

data:  x
D = 0.11048, p-value = 0.6029
alternative hypothesis: two-sided
```

The estimated parameters of the gamma distribution are obtained as 1.1 and 0.82. Now using the KS test, we found the P-value as 0.6029 and concluded that the chemotherapy data follow a gamma distribution with parameters 1.1 and 0.82. The histogram, density plot, Q-Q plot, empirical and theoretical cdf plot and P-P plot are provided in Figure 4.3, which also supported the gamma distribution. That means the population signal to ratio will be $\sqrt{1.1} = 1.05$. Then it is logical to test the true signal to ratio as, H_0: SNR=1.05. The lower and upper critical values for all tests are presented in Table 4.2.

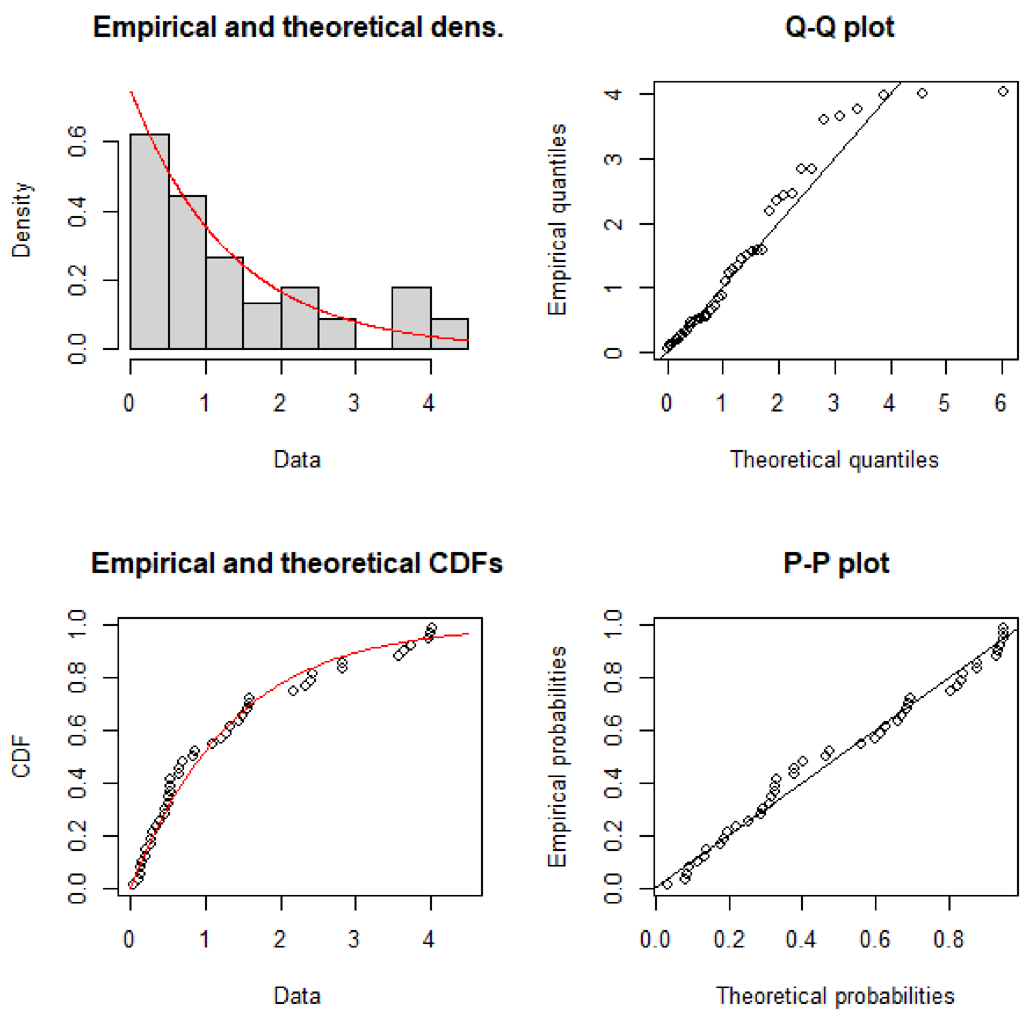

Figure 4.4: Emperical and theoretical density, QQ plot, CDF and P-P plots of chemotherap data

Table 4.2: Lower and upper critical values of the chemotherapy treatment data

Method	**Lower Limit**	**Upper Limit**
1	0.800	1.643
2	0784	1.368
3	0.802	1.633
4	0.796	1.990
5	0.788	1.871
6	0.799	1.874
7	0.804	1.313
8	0.726	2.047
9	0.717	1.878
10	0.806	1.306
11	0.852	1.300
12	0.789	1.204
13	0.696	1.525
14	0.664	1.248
15	0.705	1.289

From Table 4.2, we observed that all methods including our proposed methods 12 to 15 have accepted the null hypothesis H_0: SNR=1.05. These results are consistent with the simulation results. It is noted that the proposed method 15 performed very well in the simulation results and it is consistent with the real data.

4.3 Mathematics Grades

The below datasets contain the 2013 mathematics grades for 48 students enrolled in the slow-paced program. From Linhart and Zucchini (1986), the following observations were obtained: 29, 25, 50, 15, 13, 27, 15, 18, 7, 7, 8, 19, 12, 18, 5, 21, 15, 86, 21, 15, 14, 39, 15, 14, 70, 44, 6, 2,58, 19, 50, 23, 11, 6, 34, 18, 28, 34, 12, 37, 4, 60, 20, 23, 40, 65, 19, 31.

The histogram of the mathematics grade data is presented in Figure-4.5, which looks like a gamma distribution. The skewness and kurtosis are obtained as 1.29 and 4.22 respectively.

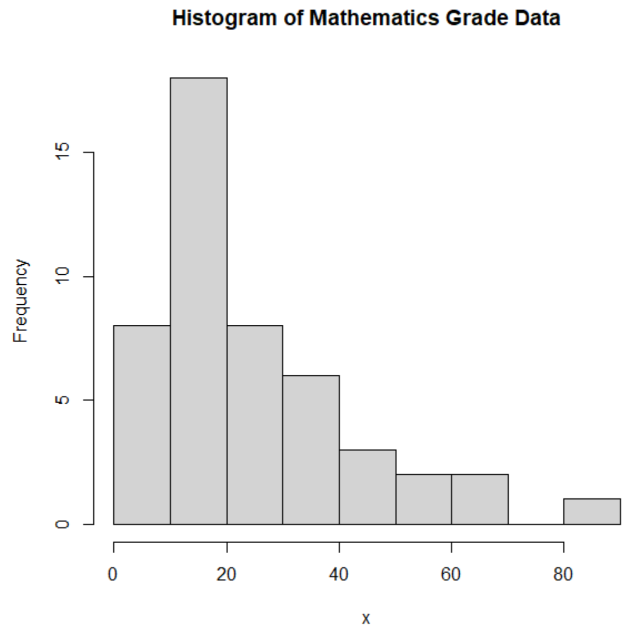

Figure 4.5: Histogram of mathematics grade data

```
> fitg2=fitdist(x, dist="gamma", method="mle")
> fitg2
Fitting of the distribution ' gamma ' by maximum likelihood
Parameters:
        estimate Std. Error
shape 1.96553342 0.37204811
rate  0.07719837 0.01662992
```

```
> ks.test(x, "pgamma", 1.97, 0.08)

        One-sample Kolmogorov-Smirnov test

data:  x
D = 0.073453, p-value = 0.958
alternative hypothesis: two-sided
```

The estimated parameters of the gamma distribution are obtained as 1.97 and 0.08. Now using the KS test, we found the P-value as 0.958 and concluded that the mathematics grade data follow a gamma distribution with parameters 1.97 and 0.82. The histogram, density plot, Q-Q plot, empirical and theoretical cdf plot and P-P plot are provided in Figure 4.6, which also supported the gamma distribution. That means the population signal to ratio will be $\sqrt{1.97} = 1.40$. Then it is logical to test the true signal to ratio as, H_0: SNR=1.40. The lower and upper critical values for all tests are presented in Table 4.3.

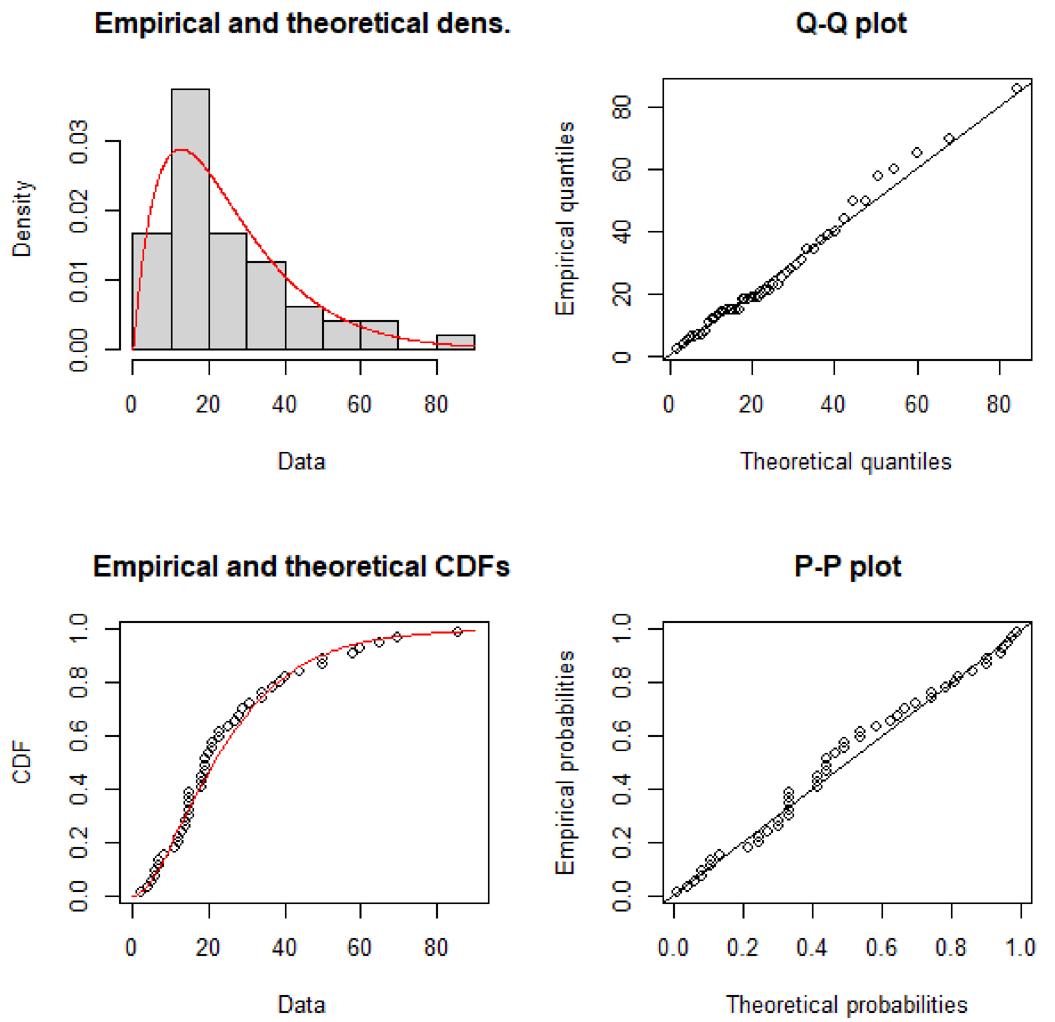

Figure 4.6: Emperical and theoretical density, QQ plot, CDF and P-P plots of mathematics grade data

Table 4.3: Lower and upper critical values of the mathematics grades data

Method	**Lower Limit**	**Upper Limit**
1	1.041	1.904
2	1.063	1.629
3	1.043	1.896
4	1.041	2.053
5	1.034	1.999
6	1.046	2.012
7	1.001	1.744
8	0.976	1.999
9	0.969	1.938
10	0.888	2.243
11	1.074	1.616
12	1.016	1.528
13	1.173	2.075
14	1.216	1.781
15	0.989	1.555

From Table 4.3, we observed that all methods including our proposed methods 12 to 15 have accepted the null hypothesis H_0: SNR=1.40. These results are consistent with the simulation results.

CHAPTER 5

SUMMARY AND CONCLUDING REMARKS

In this thesis we consider fifteen different test statistics (Methods 1 to 15) for testing the population SNR. We consider some existing test statistics, which were developed from the confidence interval of SNR and from the work of Miller (1991), Sharma and Krishna (1994), Curto and Pinto (2009), McKay's (1932), Panichkitkosolkul (2009) and Kibria and George (2014) among others. We also propose a few new test statistics based on the robust estimator of population standard deviation σ. Since a theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study has been conducted to compare the performance of the test statistics. The performance of the test statistics is determined based on the empirical size and power of the tests. We have considered the most popular and widely used significance level 0.05 for finding the size and power of the test. To see the impact of sample size on the test statistics, we considered n=15, 30, 50 & 100. From simulation study it appears that Methods 1, 3, 4, 5, 6, 7, 8, 9,10 and proposed Methods 12 through 15 are promising and performed well in some conditions. However, Method 10 proposed by Kibria and George (2014) performed the best in all simulation conditions both at lower and upper end of the alternative hypotheses. Three real life data on failure time, chemotherapy treatment and mathematics grades are analyzed to illustrate the performance of the test statistics. It appears that the simulation results and applications are consistent to some extent. The conclusions of this thesis are restricted to the given simulation conditions of this thesis. For a definite statement one might need more simulation conditions and more sample sizes and do a simulation under various distributional conditions. Hope the findings of the thesis will be a valuable asset for the practitioners.

We also note that methods 1, 2, 3, 7, 10, 13, 14, and 15 depend on the percentile points from a normal distribution. It is well known that for small sample t-test perform better than the Z-test. Moreover, one of the professors asked to see whether t-critical value gives better size and power of the tests when sample sizes are small. In future research, one can consider these methods with t-critical values. Perform the simulation under different parametric conditions and see if the performance of the test statistics, mentioned within this thesis, improved or not. It will also be interesting to see the performance of the test statistics under a one sided alternative and under the presence of outliers.

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