Lattice Boltzmann Modeling and Specialized Laboratory Techniques to Determine the Permeability of Megaporous Karst Rock

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LATTICE BOLTZMANN MODELING AND SPECIALIZED LABORATORY TECHNIQUES TO DETERMINE THE PERMEABILITY OF MEGAPOROUS KARST ROCK

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

GEOSCIENCES

by

Sadé Maria Garcia

2013
To:  Dean Kenneth G. Furton  
     College of Arts and Sciences

This thesis, written by Sadé Maria Garcia, and entitled Lattice Boltzmann Modeling and Specialized Laboratory Techniques to determine the Permeability of Megaporous Karst Rock, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this thesis and recommend that it be approved.

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Date of Defense: June 27, 2013

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Florida International University, 2013
DEDICATION

Without the understanding, love and support of my entire family at home in Trinidad this work would not have been possible. I dedicate this thesis especially to those in my life who have taught me to believe in myself and who inspire me to strive for better. I am forever motivated by my mother Lisa, my brother Ché, my sister Taylor, and my grandmother Tenty. To my family here with me in Miami, for their continued support throughout my studies. Last but not least, to my best friend and confidant Rohan, for his positivity and encouragement.
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ABSTRACT OF THE THESIS

LATTICE BOLTZMANN MODELING AND SPECIALIZED LABORATORY TECHNIQUES TO DETERMINE THE PERMEABILITY OF MEGAPOROUS KARST ROCK

by

Sadé Maria Garcia

Florida International University, 2013

Miami, Florida

Professor Michael C. Sukop, Major Professor

The Pleistocene carbonate rock Biscayne Aquifer of south Florida contains laterally-extensive bioturbated oolitic zones characterized by interconnected touching-vug megapores that channelize most flow and make the aquifer extremely permeable. Standard petrophysical laboratory techniques may not be capable of accurately measuring such high permeabilities. Instead, innovative procedures that can measure high permeabilities were applied. These fragile rocks cannot easily be cored or cut to shapes convenient for conducting permeability measurements. For the laboratory measurement, a 3D epoxy-resin printed rock core was produced from computed tomography data obtained from an outcrop sample. Permeability measurements were conducted using a viscous fluid to permit easily observable head gradients (~2 cm over 1 m) simultaneously with low Reynolds number flow. For a second permeability measurement, Lattice Boltzmann Method flow simulations were computed on the 3D core renderings. Agreement between the two estimates indicates an accurate permeability was obtained that can be applied to future studies.
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1 Introduction

1.1 Karst Aquifers

Karstification of rocks is caused by the dissolution of soluble rock. Mostly, it refers to the weathering of carbonate rock. The dissolution of carbonate generally creates conduits resulting in highly productive aquifers. Karst rock is generally characterized by a network of interconnected fissures, fractures, and conduits in relatively low-permeability rock matrix (http://water.usgs.gov/ogw/karst/pages/whatiskarst). Several sources define karst aquifers and the general consensus is that the underground water in karst aquifers does not move chiefly through intergranular pores but through narrow fissures and large caves (Jennings, 1971). The result is a set of flow zones constrained to enlarged channels throughout the aquifer with the rock matrix responsible for most of the groundwater storage (Freeze and Cherry 1979).

1.2 Biscayne Aquifer

The Biscayne Aquifer is unlike most karst aquifers and is one of the most permeable aquifers in the world (Parker et al., 1955). The Biscayne Aquifer has been identified as the aquifer that contains the most permeable sediments of the surficial aquifer system of south Florida, (Cunningham et al., 2009). The rock formations in southern Florida are among the youngest in the country (Hoffmeister, 1974). The Biscayne Aquifer is comprised of Pleistocene shallow-marine platform carbonate rocks that make up the Miami limestone and Fort Thompson formations. The Miami limestone has been divided into the oolitic facies which forms the Atlantic coastal ridge in eastern south Florida, interfingering with the bryozoan facies to the west, under the Everglades
The Biscayne Aquifer is bounded beneath by the upper confining unit of the Floridan Aquifer (Parker et al., 1955). The Biscayne Aquifer extends from Miami-Dade County to Palm Beach County and is an important source of water for the population of south Florida. About 4 million people in the southeastern tip of Florida rely on water supplies from the aquifer (USGS 2010), and with threatening issues like salt water intrusion, effects of urbanization on stream ecosystems, transport of contaminants to public supply wells, and transport and fate of agricultural chemicals, the importance of obtaining accurate aquifer parameter characterization is clear.

1.2.1  Biscayne Aquifer Karst

Parker et al. (1955), list the karst features in the Biscayne Aquifer: sinkholes, vertical solution pipes, jagged rock pinnacles, deep solution passages, a natural limestone bridge and large solution holes connected to the land surface. Carbonate porosity nomenclature for the Biscayne Aquifer rock was obtained from the classifications in Choquette and Pray (1970). 15 basic types of porosity are identified in all sedimentary carbonates. Vug porosity evident in the Biscayne Aquifer rock is labeled as one of the seven most common types and it is further distinguished by other elements of classification including size, giving it its name, “megavug” (megaporous vug carbonate rock). “Megapore” describes pore sizes which are larger than 4 mm in diameter (Choquette and Pray, 1970) and this type of porosity is evident in the Biscayne Aquifer rock.

Biscayne Aquifer rock-porosity classifications were also obtained from Cunningham et al. (2006). The literature describes a large range in hydrologic properties throughout the Biscayne Aquifer karst that is a result in nested pore networks sometimes
referred to as a triple porosity system. This system includes small porous networks referred to as (1) matrix porosity, (2) larger porous networks or megapores referred to as “touching-vug” porosity, which are believed to create stratiform groundwater flow zones (the focus of my study) where permeability values are yet to be confirmed and published, and (3) the less common conduit porosity composed of bedding plane vugs, thin vertical solution pipes and cavernous vugs.

The Biscayne Aquifer is defined by the formation of a connected network of large pores and/or conduits (touching-vug megapores) leading to an extensive range in hydrologic properties throughout the aquifer (Cunningham et al., 2009). Figure 1 below shows the location of the Biscayne Aquifer in south Florida and identifies the locations from which megaporous carbonate samples were used for analysis in the Cunningham et al. study (2009). The study focused on characterizing the megaporosity through a series of methods including cyclostratigraphy, ichnology and borehole imagery. The aim was to indentify flow zones throughout the aquifer.
Figure 1. The Biscayne Aquifer of south Florida. Location of megaporous rock samples used in the Cunningham and Sukop (2011) study. Sample ML-1 is the sample used in this research. (Figure from Cunningham et al., 2009)

The high transmissivity of the Biscayne Aquifer, unlike the more typical karst aquifers I have previously noted, has been determined to be a result of the megaporosity of the carbonate rock. The following research has provided information to better characterize the Biscayne Aquifer heterogeneity and furthermore provide an understanding of these touching-vug rock types. The formation of these megaporous rocks in the Biscayne Aquifer has been described by “eogenetic biogenic processes” (Vacher and Mylorie, 2002; Cunningham et al., 2009). The biogenic megapores observed
in the Biscayne Aquifer are attributed to one or more of the following post-depositional, pre-burial (eogenetic) processes: inter- and intra-burrow megaporosity, inter- and intra-root megaporosity and fossil-moldic megaporosity. The rock fabric evident in the rock sample selected for this current research is a result of burrowing by the Calinassid shrimp, which produces a unique intra-burrow megapore fabric named the Ophiomorpha ichnofabric (Cunningham et al., 2009).

To obtain a better visual understanding of the scale of the megapores that characterize the Biscayne Aquifer Ophiomorpha ichnofabric, the rock sample (ML-01) used for this research is shown in Figure 2. This sample was taken from the Deering Glade outcrop identified in Figure 1. The ML-01 sample portrays the intense Ophiomorpha ichnofabric present in rock of the Biscayne Aquifer and shows the resulting megaporosity. The megapores evident in this sample illustrate the post-depositional complex burrow system produced by the Calinassid shrimp. The resulting Ophiomorpha fabric is defined by well-connected “touching-vug” porosity (Cunningham et al., 2009). This particular sample below is an example of the bioturbated, oolitic grainstone belonging to the Miami Limestone formation. The average pore and pore-throat diameter of this sample is approximately 0.02 m.
The high productivity of the Biscayne Aquifer has been attributed to connected megapore flow zones, but the extreme hydraulic conductivities and potential deviation from Darcian behavior may make it inappropriate to simulate these zones with traditional groundwater flow models (Manda and Gross, 2006). These megapores are partially responsible for the heterogeneity in hydrologic properties that exists throughout the Biscayne Aquifer. The pore network offers a range of permeability values that are at least 13 orders in magnitude (Cunningham et al. 2004; 2006a,b; 2009). Of course these measurements and estimates can depend on sample scale; when the scale is such that large pores dominate the rock, permeabilities approach that of straight pipes of comparable radii, while samples dominated by rock matrix have much lower permeability. The extremely permeable nature of the megaporous carbonate rock in my research also makes it difficult to use common tests for measuring permeability. A
principal reason for this being that it is nearly impossible to maintain Darcian flow regimes under an easily-measurable head difference if water is used as the test fluid.

1.2.2 Previous Biscayne Aquifer Parameter Characterization

The principal aquifer parameter of interest in this research is the hydraulic conductivity $K$, which is closely related to the permeability $k$. Both of these reflect the ease of fluid flow through rock or any other porous medium. The relationship between them is

$$K = k \frac{\rho g}{\mu},$$

where $k$ is the intrinsic permeability, $\rho$ is the density of the fluid, $g$ is the gravitational acceleration, and $\mu$ is the dynamic viscosity of the fluid. The hydraulic conductivity or permeability is used in Darcy’s Law,

$$\mathbf{q} = K \nabla h,$$

in which the hydraulic conductivity tensor, $K$, is the proportionality constant between $\nabla h$, the hydraulic gradient (change in head over a specified length), and $\mathbf{q}$, the Darcy flux, which is the volumetric flow per unit area through the rock.

In previous large-scale aquifer tests done by Fish and Stewart (1991), hydraulic conductivities found in the surficial aquifer system of south Florida ranged $3.5 \times 10^{-8}$ to greater than $0.4$ m/s (from 0.001 ft/day to > 10,000 ft/day) with the higher conductivities > $0.035$ m/s ( > 1,000 ft/day) only occurring in the Biscayne Aquifer. Apparently, the extreme magnitude of high conductivities was not accessible to the techniques employed then, resulting in the ‘greater than’ result. The study by Fish and Stewart (1991)
identified difficulties in estimating the hydraulic conductivity of the aquifer that included, (1) large wells and pumps were needed to adequately stress the highly transmissive aquifer, (2) the aquifer had a layered non-uniform permeability distribution and, (3) small and rapid drawdown may have introduced inertial effects (turbulence) and made early time data difficult to analyze.

Parker et al. (1955), offered analysis of the transmissivity ($T$) and storativity ($S$) coefficients they measured for the Biscayne Aquifer. Via a series of aquifer tests throughout the Biscayne Aquifer, they concluded that values of $T$ obtained from the well tests ranged extensively from 4 mgd per ft to as much as 15 mgd per ft which equated to hydraulic conductivities ranging from 0.06 m/s to 0.23 m/s assuming a 100-foot aquifer thickness. Using this aquifer thickness, they defined a highly productive aquifer as one with a $T$ of 5 mgd per ft which is equivalent to a conductivity $K$ of 0.08 m/s. However they reported some limitations to their measurements. They concluded that the higher values of $T$ obtained from the aquifer tests probably indicated that water moved into the well through solution channels in the limestone that were large enough to produce turbulent flow. Numerous aquifer tests with higher pumping rates were implemented in order to obtain a measurable drawdown in areas of high well productivity, but with little success. No matter what measures were taken, the drawdown was slight and Parker et al. (1955) acknowledged that the coefficient of transmissivity they obtained was not precise.

Specifically important to this research are the results of the study done in 2009, by Cunningham et al. They used a combination of petrophysical laboratory permeability measurements, cyclostratigraphy, ichnology, and borehole geophysical analyses of continuous core holes, tracer test analyses, and Lattice Boltzmann method (LBM) flow
simulations to quantify and identify the type of megaporosity in the Biscayne Aquifer and the corresponding permeabilities. This was done in order to identify zones in the Biscayne Aquifer that were hypothesized to be transmitting most of the flow. However, as a consequence of flow-rate or other limitations of standard petrophysical laboratory apparatus or procedures, the results obtained with LBM identified permeability values that were about five orders of magnitude greater than laboratory measurements.

Standard petrophysical laboratory methods, which use air permeability measurement techniques, delivered a maximum permeability value that was drastically inconsistent with LBM values of intrinsic permeability computed for a portion of the same megaporous rock sample from the Biscayne Aquifer. The results obtained by Cunningham and Sukop (2011) are shown in Figure 3. The permeability results from the simulations were recorded in units of darcies, where 1 darcy (D) is equivalent to $10^{-12}$ m$^2$. The intrinsic permeabilities computed with LBM ranged from $10^{-8}$ to $10^{-5}$ m$^2$ ($10^4$ to $10^7$ D), which were 3 to 6 orders of magnitude higher than values obtained by a petrophysical core laboratory, using air permeability measurement techniques as reported by Cunningham et al. (2009; 2010).

One possible reason for the failure to produce values consistent with the LBM was discovered when laboratory and equipment manufacturer websites reported maximum capabilities of up to 30 D (Core Laboratories, 2013; Coretest Systems Inc., 2013), which corresponds to SI unit permeability of $3 \times 10^{-11}$ m$^2$ and a hydraulic conductivity of about $3 \times 10^{-4}$ m/s. This limit is shown in Figure 3 as the outline of a black plane, and is compared to the permeability of a 2 cm pipe, which has a known permeability of $10^7$ D. The comparison to the 2 cm pipe is considered because the pore-
throat diameters in the megaporous sample ML-01 are on average 2 cm. The ML-01 sample is shown in Figure 3 with only an LBM estimation of intrinsic permeability. The properties and correct permeability of this sample will be investigated in this research.

Possible reasons for the inconsistencies between permeability values obtained from both methods were investigated. For rock-permeability measurement, petrophysical laboratories use methods such as the Hassler-type core holders (Figure 4) that use rubber sleeves to deliver a confining pressure on the rock samples to replicate in-situ conditions. However in order to avoid possible rupturing of the rubber sleeves against the unusually

Figure 3. Bar chart showing the marked difference between petrophysical and LBM permeability test results, permeability of a 0.02 m pipe, and reported limit of air permeability measurements.

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<th>U</th>
<th>Unoriented permeability</th>
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<td>H</td>
<td>Horizontal permeability</td>
<td>Measured air permeability</td>
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<tr>
<td>V</td>
<td>Vertical permeability</td>
<td>Sample volume, in cubic centimeters</td>
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LBM = Lattice Boltzmann methods
large pores of MI-01, the laboratory reported that the pores in this process were stuffed with cotton, which may have ultimately resulted in considerable underestimation of the intrinsic permeability values (Sukop et al., 2013).

Figure 4. ASTM Hassler Sleeve Core Holder - Standard Test Method for Permeability of Rocks by Flowing Air (2001).

The method of measurement was therefore likely inappropriate for the highly megaporous rocks investigated in this research. However, confirmation was needed to determine which method, LBM or petrophysical laboratory, was more appropriate in measuring these megaporous rock samples. Therefore in this research, specialized techniques had to be developed to obtain the permeability for such rocks in an attempt to determine which method was correct.
1.2.3 *Megaporous Rock Modeling Challenges*

As noted before, the hydrologic properties of the carbonate Biscayne Aquifer are highly variable. The karst carbonate rock has measured permeability values that range over more than 13 orders of magnitude (Cunningham and others, 2004; 2006 a, b; 2009) and porosity values of up to 80% (Manda and Gross, 2006). The Biscayne Aquifer rock that is perhaps most important and investigated in this research is characterized by ‘touching-vug megaporosity’, which can create stratiform, areally extensive flowpaths (Cunningham and Sukop, 2011). As discussed in the earlier works including, Parker et al. (1955), Fish and Stewart (1991), and Cunningham et al. (2009), clearly the nature of the megaporous carbonate rock makes it difficult to measure rock conductivities using simple and common field and laboratory tests. Aquifer tests in the Biscayne Aquifer reported difficulties in measuring head differences while maintaining Darcian flow regimes, which are necessary for applying Darcy’s Law to obtain hydrologic parameters like hydraulic conductivity.

Darcian flow is a laminar-type flow that honors Darcy’s Law. Adherence to Darcy’s equation has to be confirmed by one who wishes to use it to estimate permeability or hydraulic conductivity. However, the large interconnected pores of the touching-vug parts of the Biscayne Aquifer make it nearly impossible to maintain Darcian flow regimes and obtain easily measurable head gradients to make calculations if water is the working fluid in both field- and laboratory-scale methods. Given previous results for non-Darcian flow in these rocks (Sukop et al., 2013), the head gradient would have to be below $10^{-6}$ in order to obey the law described above.
As introduced in the previous paragraph, in order to ensure Darcian flow through the megaporous rock, the Reynolds number defined below has to be kept low (Re < 1) (Brinkman, 1949; Brown, 2002). These sources also state that experimental tests show that Re numbers up to 10 may still be Darcian, but for this research, to ensure laminar, steady-state flow, the Re number was kept well below 1. Reynolds number as expressed in equation (3) is a dimensionless number that gives a measure of the ratio of inertial to viscous forces,

\[ Re = \frac{\mu L}{v}, \]  

where \( u \) is the fluid velocity, \( L \) is a characteristic length (usually the pore diameter), and \( v \) is the kinematic viscosity of the fluid. To keep the Reynolds number low in the laboratory measurements part of research, the solution is to use highly viscous fluids. This makes it possible to maintain the laminar flow necessary (Re < 1) to carry out the laboratory experiment to determine the permeability of this touching-vug megaporous carbonate rock.

1.3 Laboratory Permeability Measurement

For laboratory-scaled measurements of the megaporous rock, a high viscosity substance was used to obtain measurable head differences across the rock while maintaining laminar flow. The first of two viscous fluids employed is the ISO 680 grade gear oil. According to engineering data presented by Exxon Mobil Corporation (2010), the mid-point kinematic viscosity of ISO 680 gear oil is 680 centistokes (1 cst = 10^-6 m^2/s) at 40 degrees Celsius, approximately 680 times greater than that of water. The viscosity of this gear oil is given as a range at one specific temperature, so in order to know more
precisely the viscosity at the laboratory temperature, a calibrated viscometer with a suitable range (500 – 2,500 cst) was used to measure the viscosity of the ISO 680 gear oil. At an approximate standard room temperature, the viscosity was measured independently using the viscometer. The second highly viscous fluid was 98% glycerol-aqueous solution. With extensive data published on this simple polyol compound, it was easy to obtain densities and viscosities at the ambient temperature in the laboratory.

A permeability test system was specially designed and constructed from mostly PVC components to measure extremely high rock permeabilities. For the laboratory method, using actual rock samples was unrealistic, simply because the megaporous carbonate rock is too fragile to be cored and cut into shapes appropriate for laboratory experiments. To solve this issue, a facsimile of the rock was built from three-dimensional computer printing techniques. Rather than etching or other subtractive methods to remove material and leave behind the desired shape, 3D printing forms shapes by adding material in designated patterns (Physics Today, 2012).

The physical sample of ML-01 was taken from the outcrop location, and was scanned at the High-Resolution X-ray Computer Tomography (CT) facility at the University of Texas. The X-ray source is a 200kV FeinFocus model FXE200.20, with an image intensifier detector system from which data are captured and digitized by a CCD 1024x 1024 camera. The resulting 2D sliced images were 16- and 8-bit 1024x1024 images which were 1.0 mm thick with a 0.2 mm overlap to produce a 0.8 mm-interslice spacing. The images were provided to the USGS and Dr. Lee Florea of Ball State University was responsible for the manufacture of the 3D printed epoxy-resin core model of the rock.
The following specifics for the creation of the epoxy model were obtained from Florea et al. (2008). For the creation of the epoxy-resin model, the set of slices were reconstructed into a 3-dimensional file that had equal spacing in the x, y and z-directions. For 3-dimensional printing techniques, the best technologies produced by Sanders Prototype Inc. explained below were used in order to reproduce the complexities of the megaporous digital rock-core. The cylindrical core was first cut from the virtual samples of the physical rock with a radius of 0.05 m and the cast of the megaporous rock was built. The technology used for 3-dimensional printing was carefully picked based on its ability to connect pendants or other hanging features in the rock model. Software Avizo (Mercury Computer Systems) was used to read the image slices and interpolate between them to create stereolithography “.stl” files in which “ultraviolet laser radiation is directed onto a vat of polymer resin; fused deposition modeling, which generates molding by extruding a controlled bead of molten polymer through a fine nozzle; and selective laser sintering, in which an object is created by sequentially fusing thin layers of powder with a scanning laser beam” (Florea et al., 2008). The megaporosity of the ML-01 rock-core based on CT data was measured and recorded to be 54% (Sukop et. al, 2009).

1.4 Computational Permeability Measurement

The second permeability measuring procedure was to implement Lattice Boltzmann Methods (LBM) to compute the permeability of the same rock sample (the ML-01 virtual core). The CT images used to print the epoxy-resin core for the laboratory experiment were used to generate the virtual domain in which fluid flow was simulated with LBM. The LBM simulation results are assessed to estimate the permeability. The LBM is a relatively new technique used to simulate flow in complex fluid systems.
(Succi, 2001; Sukop and Thorne, 2006; Mohamad, 2011). Other Computational Fluid Dynamics (CFD) methods use the finite volume and finite element methods to solve the Navier-Stokes equation in terms of macroscopic properties like mass, momentum, and velocity. Contrastingly, the LBM represents discrete fluid particles on a microscopic level by a distribution function that represents a mesoscopic step before the macroscopic properties are calculated. The particulate nature of the LBM makes it a more attractive model for certain types of simulations compared to its more conventional CFD counterparts. The advantage of employing the LBM in this research is its ability to analyze flow in the presence of complex pore-solid boundary conditions (Chen and Doolen, 1998; Zhou, 2004), a characteristic feature of this touching-vug carbonate rock.

2 Aim/Objectives of Research

This research will directly address the questions raised by the inconsistency in Lattice Boltzmann modeling and petrophysical laboratory measurements identified in Cunningham and Sukop’s (2011) study. The ultimate goal of this research is to determine the permeability of the megaporous carbonate networks in the Biscayne Aquifer of south Florida, identified by Cunningham (2004). The hypothesis to be tested is that the LBM is the more appropriate method for obtaining permeability measurements of megaporous rock samples than the air permeameter measurements.

The techniques employed in this research include a laboratory approach to test the physical observed flow through the three-dimensional epoxy-resin cast of the Biscayne Aquifer rock. The second approach is to solve this fluid dynamics problem using numerical methodology. This approach employs the Lattice Boltzmann method to
simulate flow through a virtual domain based on computed tomography images of the same sample of carbonate rock used to make the epoxy-resin core.

Like LBM, 3-D printing is a technology that is relatively new and an integral part of my research. Both the numerical and experimental procedures will test the main hypothesis of this research. Corroboration between the two procedures in this research will provide support for the accuracy of the permeability result. The measured permeability value can then be applied to the megaporous karst that defines the principal hydrostratigraphic pathways in the Biscayne Aquifer.

3 Laboratory Technique

3.1 Background

A fundamental understanding of fluid movement in a pipe is essential to this study. This research focuses on a both a physical laboratory experiment to determine a fluid’s behavior as it moves through a pipe and also quantifies that same behavior using numerical processes for digitized domains that include sinusoidal pipes and a rock-core. The basic understanding herein is that the simplest model of rock pores is straight pipes. Therefore the analytical methods used in this research will be tested using the laws and published results regarding flow through a pipe. As an introduction, a review of viscous, incompressible flow through a pipe is therefore a necessary topic to revisit before this research and its findings are presented.
3.1.1 Navier-Stokes equation

The Navier-Stokes equation is a direct result of applying Newton’s second law,

\[ F = \frac{dp}{dt} = m \frac{dv}{dt} = ma, \quad (4) \]

where \( F \) is force, and \( p \) is momentum of fluid motion. Fluids are governed by the continuity equation (conservation of mass, which simplifies to conservation of volume when density is constant and therefore incompressible).

\[ \nabla \cdot \mathbf{u} = 0 \quad (5) \]

and the Navier-Stokes equation (Bird et al., 1960),

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} \quad (6) \]

where \( \nabla \) is the gradient operator, \( \mathbf{u} \) is the flow velocity, \( P \) is the kinematic pressure \((P/\rho)\), \( t \) is time, and \( \nu \) is the kinematic viscosity. The Navier-Stokes equation is nonlinear in the velocity \( \mathbf{u} \), which prohibits an analytical solution except for a few cases (Wolf-Gladrow, 2000).

3.1.2 Reynolds Number

Flow occurs in many different settings and a key term used to describe flow is the Reynolds number, which was already introduced as equation (3). The Reynolds number is a dimensionless number that relates the inertial to the viscous forces within the flow. It is therefore dependent on the rate of flow, the type of fluid, and the dimensions of the domain (pipe diameter/ rock pore diameter) through which it is flowing. The Reynolds number is telling of the behavior of the flow. With Reynolds number less than 1, laminar,
steady-state flow is expected. For complex pipe and pore geometries, eddies occur in the 
flow regime at values higher than 1 and, even in straight, smooth pipes, turbulent flow 
exists at Reynolds numbers in excess of 2000.

3.1.3 Laminar flow theory: pipe

One solution to the continuity and the Navier-Stokes equations is the Poiseuille-
type flow. According to Tritton (1977), once the Reynolds number is below 30, the 
Poiseuille flow theory always provides an accurate description of viscous, incompressible 
flow in a smooth, straight pipe. The assumption herein is that there is only one non-zero 
component of the velocity and so the continuity equation (5) reduces to

\[
\frac{\partial u}{\partial x} = 0
\]

(7)

and the original Navier-Stokes equation (6) also reduces to

\[
0 = -\frac{\partial p}{\partial x} + \mu \nabla^2 u
\]

(8)

With laminar, Poiseuille type flow in a straight pipe of constant diameter, the 
pressure maintained by a reservoir of fluid at one end will be different from the pressure 
at the other end of the pipe. In this case it is plausible to assume gravitational force on the 
fluid is irrelevant, either because the pipe is horizontal or because the force is small 
compared with the forces associated with the pressure difference (Tritton, 1977). Flow in 
a pipe is described as Hagen-Poiseuille flow. For this type of flow, a no-slip condition for 
the walls is assumed. The velocity profile is a function of the radius of the pipe \( u(r) \) and 
the following derivation for it was obtained from Tritton, (1977).
For this case, the flow direction is normal to the page, in the $x$ direction. We consider an element of fluid shaded in Figure 5 and with a length $\delta x$ in the flow direction. The viscous forces on the two faces of this fluid element parallel to $x$ and perpendicular to $r$ now differ slightly not only because the velocity gradients differ but also because the two faces have different areas.

The force on one face is,

$$\mu \left( \frac{\partial u}{\partial r} \right) r \frac{\partial}{\partial r} \Phi \delta x,$$

(9)

where $\mu$ is the dynamic viscosity of the fluid, $\partial u/\partial r$ is the velocity gradient, $r$ is the radius of the pipe, and $\partial \Phi$ is the change in the angle for the fluid element.

And the net viscous force on the element is

$$\mu \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \frac{\partial}{\partial r} \alpha \Phi \delta x \delta \Phi.$$

(10)

The pressure force on one end of the element is

$$p r \frac{\partial}{\partial r} \Phi,$$

(11)

and on the other end of the element is
\[
\left( p + \frac{\partial p}{\partial x}\right)r\partial r\partial \Phi, \quad (12)
\]

hence, the net pressure force is,
\[
-(\frac{\partial p}{\partial x})r\partial x\partial \Phi\partial r. \quad (13)
\]

and flow through the pipe can be defined as
\[
\mu \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = -Gr. \quad (14)
\]

Integration and the implementation of boundary conditions of \( u=0 \) at \( r=a \) gives,
\[
u = \frac{G}{4\mu} (a^2 - r^2). \quad (15)
\]

The velocity profile is hence a paraboloid with an average speed defined by
\[
u_{avg} = \frac{Gr^2}{8\mu}. \quad (16)
\]

With this average velocity defined for pipes, the Darcy law is now introduced to consider permeabilities of these pipes. In 1856, Henry Darcy published a relationship for the flow rate of water in sand filters and his law was given as
\[
Q = AK \frac{\Delta h}{L}, \quad (17)
\]

\( Q \) is the volume flow rate, \( A \) is the cross-sectional area of porous medium normal to the flow, \( K \) is the hydraulic conductivity, \( \Delta h \) is the change in hydraulic head, and \( L \) is the length of the flow path (Darcy, 1856; Brown, 2002). Darcy velocity \( q \), was defined by dividing both sides of equation (17) by \( A \) and obtaining the following,
\[
q = -K \frac{\Delta h}{L}. \quad (18)
\]
which includes a negative sign because the fluid flows from high hydraulic head to low hydraulic head. Rewriting the Poiseuille equation (16) in terms of Darcy velocity \( q = u_{avg} \), we obtain.

\[
q = \frac{Gr^2}{8\mu} = \frac{\partial P}{8\mu} = \frac{\rho g \partial h / \partial x}{8\mu},
\]

(19)

and

\[
q = K \frac{\partial h}{\partial x} = \frac{k \rho g \partial h}{\mu \partial x}.
\]

(20)

Now, equating equations (19) and (20), the permeability \( k \), can be expressed as

\[
k = \frac{r^2}{8}.
\]

(21)

Hence, for any pipe of known radius \( r \), the permeability \( k \), can be computed directly from equation (21).

The pipe permeabilities are of significant interest to determining and understanding the concept of megaporous rock permeabilities because one can think of the pores in the rock as being composed of singular pipes that run through it. Analysis has been done on understanding the relationship of the intrinsic permeability of the individual pipe/pore \( k_p \), to the adjusted permeability \( k_{adj} \), when modeling the rock as a whole.
Using $A_p$ to define the cross-sectional area of the pore in Figure 6 and the $A_{rock}$ to define the area of a cylindrical core, similar to the one in my research, the volumetric flow, $Q$ is given as

$$Q = K \frac{\partial h}{\partial x} A_p = \frac{k_p g}{\nu} \frac{\partial h}{\partial x} A_p.$$  \hspace{1cm} (22)

Similarly, for the rock, the volumetric flow can be defined in the following manner

$$Q = \frac{k_{adj} g}{\nu} \frac{\partial h}{\partial x} A_{rock}.$$ \hspace{1cm} (23)

Because flow is only occurring in the open pore, the volumetric flow $Q$, in (22) and (23) has to be equal, and we can equate the two equations to obtain,

$$\frac{k_p g}{\nu} \frac{\partial h}{\partial x} A_p = \frac{k_{adj} g}{\nu} \frac{\partial h}{\partial x} A_{rock},$$ \hspace{1cm} (24)

which reduces to

$$k_{adj} = k_p \frac{A_p}{A_{rock}}.$$ \hspace{1cm} (25)
The pore and rock matrix are cylindrical in this research and so the cross-sectional areas $A_p$ and $A_{rock}$ are known for cylinders, and the permeability of the pore is defined using equation (21). Thus

$$k_{adj} = \frac{r_p^2}{8} \frac{\pi r_p^2}{\pi r_{rock}^2}, \quad (26)$$

and hence, the permeability of a rock can be calculated from the pore and core sizes using the following relationship.

$$k_{adj} = \frac{r_p^4}{8r_{rock}^4}, \quad (27)$$

From this relationship, the pipe permeability provides estimates for the measured permeability of the rock. For a 0.1016 m (4-inch) diameter rock fitted with a 2 cm pipe-pore, the estimated permeability is $4.8 \times 10^{-7}$ m$^2$.

Now consider a 0.1016 m (4-inch) circular pipe representative of a rock-core. The pipe is packed with 0.02 m (2 cm) pipes arranged with either square or hexagonal packing representative of the megaporous carbonate rock in the Biscayne Aquifer. Figure 7 uses a rectangular window to display the square and hexagonal packings of the 2 cm pipes within the 0.1016 m pipe (indicative of the rock-core).
Figure 7. Square packing (left) hexagonal and packing (right) arrangements of 2 cm diameter pipes contained in a 4-inch diameter pipe. (http://en.wikipedia.org/wiki/Circle_packing)

Square packing yields a packing density (the ratio of the area of the pore to the area of the rock) of about 0.785 and hexagonal packing a density of approximately 0.9069. Hence, the number of 2-cm pipes $n$ that can be inscribed in the 4-inch diameter face of the core can be obtained from the following relationship between the cross-sectional area of the small 2-cm pipes $A_s$, and the entire area of the 4-inch pipe $A$.

$$\frac{nA_s}{A} = \text{packing density}, \quad (28)$$

From equation (28), $n$ was calculated to be 20 for the square packed pipe and 23 for the hexagonally packed pipe. The $k_{adj}$ for the respective packing was computed from the product of the respective $n$ value and equation (27) to be $9.7 \times 10^{-6}$ m$^2$ and $1.1 \times 10^{-5}$ m$^2$ which provided reference permeabilities for the permeability measurement of the rock-core sample ML-01 from this research.
4 Methods/Experimental Design:

The methods described in this section start off with initial tests to confirm the functioning of the flow system created for the epoxy-resin rock-core analysis. To test the system the first step was to use the apparatus to measure the permeability of a pipe of known diameter and confirm the laboratory value with the expected permeability of the pipe calculated from equation (21). The preliminary pipe test was done using the high viscosity substances. Confirmation of pipe permeabilities and the appropriateness of the selected viscous fluid were required before laboratory permeability measurements of the epoxy-resin rock core model were obtained. The measurements taken for these laboratory analyses are shown in Table 1 along with the precision of the instruments used for each measurement.

Table 1. Laboratory measurements for permeability analysis.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Instrument</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>Graduated Cylinder</td>
<td>+/- 0.5 cm³</td>
</tr>
<tr>
<td>Time</td>
<td>Stopwatch</td>
<td>+/- 0.01 seconds</td>
</tr>
<tr>
<td>Head Difference</td>
<td>Ruler</td>
<td>+/- 0.1 cm</td>
</tr>
<tr>
<td>Temperature</td>
<td>Digital Thermometer</td>
<td>+/- 0.5 °C</td>
</tr>
</tbody>
</table>

4.1 Permeability of 0.0254 m (1-inch) pipe

The first test of the laboratory experimental setup was done using the 0.0254 m pipe shown in Figure 8 below. The 0.0254 m diameter of the pipe was confirmed by measuring the inside diameter with a vernier caliper. The vertical pipe on the left side is
at an elevation of 2 cm higher than on the right. The electrical pump was kept at a constant rate to maintain the hydraulic head on the higher (left) side of the apparatus. The resultant head difference across the pipe was measured from fluid in the tubes connected to the end of the reservoir and compared at the middle where they are elevated with the crane stand. The pump also worked to circulate the ISO 680 gear oil from the bucket on the ground on the right of the table (not in picture) throughout the entire system. The stability of the pipes was maintained with the red harness straps seen in Figure 8 below.

![Figure 8: Initial test with 1" diameter pipe](image)

4.1.1 Expected Results

Initially, the experiment to measure the permeability of the 0.0254 pipe was conducted using the ISO 680 gear oil as the fluid. This was done to test the apparatus by comparing the measured permeability against the analytical, expected value of the permeability of a 0.0254 m pipe. Subsequent agreement of the two permeability values would indicate the functionality of the apparatus and confirm the solution of the fluid. For laminar, incompressible, steady-state flow through a pipe, the permeability of the
cylindrical pipe can be modeled using equation (21) which relates radius of pipe directly to the permeability.

Therefore, an expected permeability for this experiment was calculated from the following,

\[ k = \frac{r^2}{8} = \left( \frac{0.0254}{2} \right)^2 = 2.0 \times 10^{-5} \, m^2, \]  

(29)

and some predictions were made on the expected behavior of the apparatus shown in Table 2. With an expected head difference of 2 cm, estimates of expected discharge were calculated. The viscosity \( \nu \), which was measured independently, is discussed in detail later on. The discharge \( Q \) was calculated using

\[ Q = \frac{q}{A}, \]  

(30)

where \( A \) is the cross-sectional area of the pipe, and \( q \), the Darcy flux, was obtained from

\[ q = K \frac{\partial h}{\partial x} \]  

(31)

where \( dh/dx \) is the head gradient with 2 cm head difference across the 1 m pipe and \( K \) is the hydraulic conductivity,

\[ K = k \frac{g}{\nu}, \]  

(32)

already obtained from the intrinsic permeability \( k \), the gravitational force \( g \), and the kinematic viscosity of the fluid \( \nu \). The results are shown in Table 2.
Table 2: Expected flow and Reynolds number results for 0.0254 m pipe with 2 cm head difference for ISO 680 gear oil.

<table>
<thead>
<tr>
<th>Oil T (°C)</th>
<th>Q (m³/s)</th>
<th>q (m/s)</th>
<th>ν (m²/s)</th>
<th>$k = r^2/8$ (m²)</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.3</td>
<td>8.7E-07</td>
<td>0.0017</td>
<td>0.0023</td>
<td>2.0E-05</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Table 2 provides a basis for comparison to Table 3, which contains actual results obtained and which is discussed later on. However, while running the experiment, some minor problems were observed. The PVC pipes that were initially connected using Vaseline gel began to leak and the experiment had to be aborted to glue the pipes securely with PVC glue.

4.1.2 Experimental Technique

The experiment was repeated; measurements of the total volume of oil $V$, out of the overflow pipe were recorded along with time $t$, to compute the discharge $Q$, from

$$Q = \frac{V}{t}. \quad (33)$$

The head difference across the two center tubes was also recorded at approximately 0.017 m across the 1 m pipe expanse for the experiment shown in Figure 8. In Figure 9 below, the head difference resulting from the difference in elevation across the apparatus is shown. The image on the right in Figure 9 shows a magnification of the difference. This was measured using a ruler.
Figure 9: Magnified head difference

The experimental permeability was calculated from the following equation:

\[ k = \frac{qV}{g \frac{\partial h}{\partial x}} \]  
(34)

where \( q \) is the Darcy flux determined from:

\[ q = \frac{Q}{A}. \]  
(35)

4.1.3 Viscosity of ISO 680 gear oil

An independent, measurement-based verification of the oil kinematic viscosity was determined using a viscometer. The Cannon-Fenske type viscometer is a capillary tube which is calibrated to correspond to the range of the viscosity of the ISO 680 gear oil at room temperature. It was purchased by the FIU Earth and Environment department.

To maintain and control the temperature for recording the viscosity, a water bath with an external basin was constructed. The external basin was constructed because the internal water bath of the temperature control system was not deep enough for the capillary to be fully submerged to the desired depth as shown in Figure 11. With the help
of Tom Beasley and Mike Sukop, a glass coil was fitted to the tubes and inserted into the basin to circulate the controlled-temperature water from the water bath into the external basin filled with its own water. The apparatus is shown below in Figure 10.

Figure 10: Water bath and external basin with viscometer

Figure 11 is a close-up image of the external basin. The efflux time is the time measured with a stopwatch, for the ISO 680 gear oil to drop from the top line to the bottom line markers of the capillary tube. These markers are shown in Figure 11 below. This method of measuring the viscosity is specific to the instrument.
The viscosity of the gear oil was determined for a range of temperatures 18.8-21.4 degrees Celsius and also determined several times at each temperature. As seen in Figure 11 above, all temperatures were recorded using the Digital Multimeter Thermometer, model number 002916 and the appropriate probe sensitive for the range of temperatures in the laboratory, model number 08516-55 manufactured by the Cole-Parmer Instrument Company. The thermometer was consistently used to measure the oil temperature throughout the permeability test of the 0.0254 m pipe and was used in the viscosity measurements on the fluid as well. The thermometer, which uses a probe to measure the temperature, was used to stir the water in the basin before the recordings were taken. The constant mixing was performed to ensure that the temperature of the water and the changes in temperature controlled by the water bath were delivered uniformly throughout the entire external basin. Figure 12 below illustrates the inverse temperatures versus
kinematic viscosities, which were plotted to obtain an equation that could be applied to estimate the viscosity at any temperature encountered during the permeability test.

![Graph showing inverse temperature versus kinematic viscosity for ISO 680]

\[ \nu = 0.08921 \left( \frac{1}{T} \right) - 0.00185 \]

\[ R^2 = 0.97079 \]

**Figure 12. Inverse temperature versus kinematic viscosity for ISO 680**

4.1.4 Permeability results for ISO 680

The results from the preliminary pipe permeability measurements, which include the results from the viscosity measurements, are shown in Table 3 below. This table also includes the percent difference between the measured and expected permeability for the 0.0254 m pipe.
Table 3: Results of ISO 680 gear oil viscosity and permeability measurements including a comparison between expected and measured permeability for the 0.0254 m pipe.

<table>
<thead>
<tr>
<th>Oil $T$ ($^\circ$C)</th>
<th>$Q$ (m$^3$/s)</th>
<th>$q$ (m/s)</th>
<th>$v$ (m$^2$/s)</th>
<th>Measured $k$ (m$^2$)</th>
<th>Expected $k = r^2/8$ (m$^2$)</th>
<th>Re</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.3</td>
<td>8.42E-07</td>
<td>0.0017</td>
<td>0.0025</td>
<td>2.5E-05</td>
<td>2.0E-05</td>
<td>0.017</td>
<td>26</td>
</tr>
<tr>
<td>20.4</td>
<td>7.51E-07</td>
<td>0.0015</td>
<td>0.0025</td>
<td>2.2E-05</td>
<td>2.0E-05</td>
<td>0.015</td>
<td>11</td>
</tr>
<tr>
<td>20.4</td>
<td>7.84E-07</td>
<td>0.0015</td>
<td>0.0025</td>
<td>2.5E-05</td>
<td>2.0E-05</td>
<td>0.016</td>
<td>24</td>
</tr>
<tr>
<td>20.45</td>
<td>8.12E-07</td>
<td>0.0016</td>
<td>0.0025</td>
<td>2.4E-05</td>
<td>2.0E-05</td>
<td>0.016</td>
<td>20</td>
</tr>
<tr>
<td>20.45</td>
<td>7.42E-07</td>
<td>0.0015</td>
<td>0.0025</td>
<td>2.2E-05</td>
<td>2.0E-05</td>
<td>0.015</td>
<td>10</td>
</tr>
<tr>
<td>20.5</td>
<td>8.40E-07</td>
<td>0.0017</td>
<td>0.0025</td>
<td>2.5E-05</td>
<td>2.0E-05</td>
<td>0.017</td>
<td>24</td>
</tr>
<tr>
<td>20.6</td>
<td>8.71E-07</td>
<td>0.0017</td>
<td>0.0025</td>
<td>2.7E-05</td>
<td>2.0E-05</td>
<td>0.018</td>
<td>35</td>
</tr>
<tr>
<td>20.8</td>
<td>8.29E-07</td>
<td>0.0016</td>
<td>0.0024</td>
<td>2.4E-05</td>
<td>2.0E-05</td>
<td>0.017</td>
<td>19</td>
</tr>
<tr>
<td>20.8</td>
<td>8.92E-07</td>
<td>0.0018</td>
<td>0.0024</td>
<td>2.7E-05</td>
<td>2.0E-05</td>
<td>0.018</td>
<td>36</td>
</tr>
<tr>
<td>21</td>
<td>7.99E-07</td>
<td>0.0016</td>
<td>0.0024</td>
<td>2.3E-05</td>
<td>2.0E-05</td>
<td>0.017</td>
<td>13</td>
</tr>
</tbody>
</table>
Table 3 shows that preliminary values are not within desirable agreement with the expected exact analytical value of permeability $2.0 \times 10^{-5} \ m^2$. Moreover, all of the measured values are greater than the expected value, suggesting some systematic bias. The error in the permeability values (column 6) that were obtained experimentally does not relate to changes in temperature (column 1). The errors calculated from the difference between the expected and the measured permeability ranges from 10% to 36%. Discharge here was measured as volume per time and discharge in Table 2 was calculated using hydraulic conductivity, head gradient, and cross-sectional area. The discharge obtained was consistently greater than the expected discharge leading to a higher calculated permeability. Head difference also differs here by 0.3 cm from the expected 2.0 cm head difference used in Table 2.

4.1.5 Non-Newtonian Behavior

Table 3 indicates permeabilities that are considerably higher (an average of 22%) than the expected permeability. While many potential sources of this discrepancy have been considered and dismissed, the main possible reason being considered is that at the temperature in the lab (approximately 20 degrees Celsius), the ISO VG 680 gear oil is behaving as a non-Newtonian fluid, which means that the relationship between shear stress to strain rate is not linear. These gear oils are engineered to function at high temperatures to lubricate and cool axle differentials and other gears. The temperatures at which the dynamic viscosity of this fluid is documented in all of the literature are 40 and 100 degrees centigrade.

Most fluids are non-Newtonian (Chhabra, 2010). The assumption of Newtonian behavior simplifies the mathematics and is widely used, for example in the derivation of
Hagen-Poiseuille flow above in equation (15). Newtonian behavior defines a fluid as one where the relationship between the shear stress and spatial gradient of velocity is linear. If changes in fluid density are unimportant, the constant of proportionality is the dynamic viscosity (Schowalter, 1978),

\[ \tau = \mu \frac{\partial u}{\partial y}, \]

(36)

where \( \tau \) is the shear stress “drag” of the fluid, \( \mu \) is the constant of proportionality known as the shear or dynamic viscosity of the fluid and \( \partial u/\partial y \) is the velocity gradient perpendicular to the direction of shear. Newtonian fluids are modeled using the continuity and momentum Navier-Stokes equations in fluid dynamics. These equations are further simplified for incompressible, steady-state flow systems. One such simplification of the Navier-Stokes equation is the Poiseuille equation that was used in this research to obtain a permeability measurement of the Biscayne Aquifer megaporous carbonate rock.

According to Chhabra (2010), non-Newtonian fluids have been of recent interest to industry primarily because most polymeric melts and solutions do not conform to the linear relationship of shear stress to strain rate. Chhabra (2010) states that so widespread is the non-Newtonian fluid behavior in nature and technology that it would be no exaggeration to say Newtonian fluid behavior is an exception rather than the rule. The desire to avoid research using non-Newtonian fluids is explained by Schowalter (1978). He rationalizes that in academia it will be commonplace to avoid the use of complex fluids because the subject is sufficiently new.
Over the years, many mathematical equations of varying complexity have been published for modeling non-Newtonian fluids, some even explore straightforward attempts at curve fitting the experimental data (Savins, 1969 and Chhabra, 2010). For time-independent flows, three possibilities for modeling the fluid as non-Newtonian are shear-thinning or pseudoplastic behavior, visco-plastic behavior, and shear-thickening or dilatant behavior. Chhabra (2010), points out that using the Power law or Ostwald de Waele equation to define the relationship between shear stress and shear rate on a log-log scale can be represented linearly over an interval of shear rate. Equation (37) gives this relationship in terms of apparent viscosity $\eta$

$$\eta = \mu(\gamma)^{n-1},$$

(37)

where $\mu$ is Newtonian viscosity, $\gamma$ is the strain rate and $n$ is the power-law exponent. Values of $n < 1$ will yield shear thinning behavior, and dilatant and shear-thickening fluids will result from positive powers. The velocity profiles for these fluids are shown in Figure 13.

Figure 13. Shear thinning velocity profile (left), and shear thickening velocity profile (right) compared the Newtonian Poiseuille velocity profile (blue).
Figure 14 shows that when $n=1$, Newtonian behavior will be recovered as the power law collapses to a linear equation (Binous, 2012).

![Figure 14. Newtonian fluid velocity profile recovered at $n=1$.](image)

Experimental data for the ISO 680 grade gear oil has produced consistently higher permeability measurements than expected for the specified pipe diameter. Using the power-law non-Newtonian viscosity model to try to interpret this behavior leads to the understanding that because the experimental gear oil velocity is higher for positive exponents, that will lead to higher measured permeability. Hence, the ISO 680 is likely behaving as a shear-thickening fluid. According to Savins (1969), shear-thickening fluids are the most controversial and least understood. He reports this behavior is mostly seen in highly concentrated polymeric or micellar materials, proteins, and in protoplasm with less than 1% wt solute.

With these uncertainties for the ISO gear oil and their potential contribution to the lack of agreement between the measured and theoretical permeability of the 1-inch pipe, the fluid to be used for experimental analysis was re-considered. It is important for
purposes of this research that the fluid exhibits the correct results for the built apparatus
and, to simplify the analysis, the fluid should exhibit Newtonian behavior for modeling
and applying the standard Darcy and Poiseuille laws.

4.2 Permeability of the 0.0127 m (0.5 inch) pipe

The fluid chosen for further analysis was a glycerol-aqueous solution. This fluid
was used in a pipe of diameter 0.0127 m. The PVC pipe was purchased from the
hardware store, however, the actual pipe diameter was confirmed to be in fact 0.01534 m
(0.604 inches), when measured with a vernier calipers. The expected permeability of this
pipe equates to,

\[ k = \frac{r^2}{8} = \frac{\left(0.01534\text{ in}\right)^2}{8} = 7.355 \times 10^{-6}\text{ m}^2 \]  

(38)

4.2.1 Glycerine 98+ % (C₃H₈O₃)

Because of complications with the ISO 680 grade gear oil, this simple polyol
product was used for further experimental investigation. As documented by the Soap and
Detergent Association (1990) and the Royal Society of Chemistry (2008), glycerine is a
formulation of water and glycerol and applies to purified commercial products containing
95% or more glycerol. The anhydrous form, glycerol, is scientifically known as 1,2,3
propanetriol. Glycerine, which is a well known substance, is used widely in industries
like food, cosmetics, and pharmaceuticals.

Known properties of glycerine which make it an appropriate candidate for this
study include stability under normal storage conditions, harmless to the environment,
odorless, and non-irritating to humans. Conversely, one property that did pose a potential
problem was its hygroscopic nature. Glycerol contains three hydrophilic alcoholic hydroxyl groups, which are responsible for its solubility in water (Royal Society of Chemistry, 2008). To take precaution against this property which could change the viscosity of the glycerol during an experiment and so affect the value of permeability if not accounted for, the apparatus was wrapped in cellophane paper and the fluid at the outlet poured into a sealed container, and most importantly, the entire laboratory procedure was done as quickly as possible.

Measurement of specific gravity is the principal means of determining the glycerol content of distilled glycerine (Soap and Detergent Association, 1990; Forney and Brandl, 1992). In the event of absorption of water from the surrounding humidity in the lab, the % aqueous solution, $G_w$, was confirmed using a calibrated hydrometer appropriate for measuring the specific gravity of a high glycerine weight solution and the following equation (39) obtained from Forney and Brandl (1992),

$$G_w = 383(SG) - 383. \tag{39}$$

With a specific gravity of 1.256, the result confirmed that the glycerol-aqueous solution was in fact 98% weight, and was stabilized at this concentration according to further tests throughout the experiment.

Property data on the glycerol solution were readily available and so inverse temperature versus specific gravity and versus dynamic viscosity graphs were plotted in order to obtain equations to correct for any temperature changes in the laboratory while the experiment progressed. The dynamic viscosity and the density were obtained from data published by the Soap and Detergent Association in 1990 on an overview of the properties of glycerine. The overview gave values of the specific gravity of glycerine at
varying percents glycerol and at the temperatures, 15, 15.5, 20 and 25 degrees Celsius.

With the data provided, a plot of the specific gravity versus the inverse of the temperature was used to obtain an equation for the relationship between the two variables. This plot was used to find the specific gravity of the 98+% glycerine fluid at any given ambient temperature during the experiment.

![Figure 15. Inverse relationship of temperature versus specific gravity for 98% Glycerol solution.](image)

\[
SG = 0.1351(1/T) + 1.2515 \\
R^2 = 0.9959
\]

The specific gravity for 98+ % glycerine at the any temperature was multiplied by the density of water (1000 kg m\(^{-3}\)), to obtain the density of the solution at any temperature. To obtain kinematic viscosity, published data from the Soap and Detergent Association (1990) again were used to obtain an equation for the dynamic viscosity of the fluid at the various temperatures. The dynamic viscosity \(\mu\), was obtained from the following, which relates it to the kinematic viscosity \(\nu\), by the factor of density \(\rho\).
The graph of dynamic viscosity in centipoise (1cP = 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}) versus the inverse of temperature is shown below.

![Graph showing inverse relationship between temperature and dynamic viscosity for 98% Glycerol solution.]

\[ \mu = \rho \nu \tag{40} \]

Knowing both density and dynamic viscosity, the kinematic viscosity was obtained from rearranging equation (40):

\[ \nu = \frac{\mu}{\rho} \tag{41} \]

The volume of outflow \( V \), obtained from the apparatus was recorded along with the collection time \( t \). These values allowed for a calculation of volumetric flow rate \( Q \), and consequently flux \( q \), through the system,
Finally the intrinsic permeability of the pipe was calculated from the Darcy equation using equation (34). The % difference between the experimental and expected permeability was determined for the 10 runs shown in Table 4. The mean difference of -0.1% from the theoretical value indicates a favorable comparison in contrast to previous results obtained for the ISO 680 gear oil. The mean measured value of permeability for the 0.01534 m pipe was determined to be $7.28 \times 10^{-6}$ m$^2$, consistent with the expected permeability of $7.35 \times 10^{-6}$ m$^2$. The experimental data had a standard deviation of $3.35 \times 10^{-7}$ m$^2$.  

\[ q = \frac{Q}{A} = \frac{V}{\pi r^2}. \]  

(42)
Table 4: Permeability Results for 0.0127 m PVC pipe and Glycerine fluid

<table>
<thead>
<tr>
<th>Gly T (°C)</th>
<th>Q (m³/s)</th>
<th>dh/dx</th>
<th>q (m/s)</th>
<th>ν (m²/s)</th>
<th>k (m²)</th>
<th>$k = r^2/8$ (m²)</th>
<th>Re</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4</td>
<td>6.69E-07</td>
<td>0.036</td>
<td>0.00362</td>
<td>0.000658</td>
<td>6.80E-06</td>
<td>7.36E-06</td>
<td>0.084</td>
<td>-7.53</td>
</tr>
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<td>21.7</td>
<td>6.86E-07</td>
<td>0.036</td>
<td>0.00371</td>
<td>0.000643</td>
<td>6.80E-06</td>
<td>7.36E-06</td>
<td>0.089</td>
<td>-7.54</td>
</tr>
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<td>22.0</td>
<td>7.07E-07</td>
<td>0.035</td>
<td>0.00382</td>
<td>0.000627</td>
<td>7.03E-06</td>
<td>7.36E-06</td>
<td>0.094</td>
<td>-4.36</td>
</tr>
<tr>
<td>22.6</td>
<td>7.37E-07</td>
<td>0.033</td>
<td>0.00399</td>
<td>0.000598</td>
<td>7.52E-06</td>
<td>7.36E-06</td>
<td>0.102</td>
<td>2.24</td>
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<td>22.8</td>
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<td>0.033</td>
<td>0.00405</td>
<td>0.000591</td>
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<td>7.36E-06</td>
<td>0.105</td>
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<td>23.0</td>
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<td>0.033</td>
<td>0.00414</td>
<td>0.000579</td>
<td>7.44E-06</td>
<td>7.36E-06</td>
<td>0.110</td>
<td>1.18</td>
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<td>23.1</td>
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<td>0.033</td>
<td>0.00419</td>
<td>0.000575</td>
<td>7.48E-06</td>
<td>7.36E-06</td>
<td>0.112</td>
<td>1.66</td>
</tr>
<tr>
<td>23.2</td>
<td>7.79E-07</td>
<td>0.033</td>
<td>0.00422</td>
<td>0.000570</td>
<td>7.57E-06</td>
<td>7.36E-06</td>
<td>0.113</td>
<td>2.91</td>
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<tr>
<td>23.2</td>
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<td>7.01E-06</td>
<td>7.36E-06</td>
<td>0.089</td>
<td>-4.65</td>
</tr>
</tbody>
</table>
5 Laboratory permeability of ML-01 core

5.1 Epoxy-resin core-pipe model

A cylindrical cut with diameter of 375 pixels (101.8 mm) enclosed in a digital pipe with a thickness of 6 pixels (1.63mm) was created from cropped 8-bit CT images.

Figure 17. Digital image of ML-01 sample before (left) and after (right) the core (green) was emplaced. Images courtesy USGS.

With the core positioned, the cast of the rock core was generated from three-dimensional printing as previously discussed. The final product was a 0.1 m diameter by 0.25 m long solid epoxy-resin core model (Figure 18) that was used for laboratory measurements.
5.2 Laboratory Measurements

The 0.1m diameter core was placed inside a 0.1016 m (4-inch) diameter PVC pipe. An experimental system similar to that for the measurements of pipe permeabilities was built (Figure 8). With the help of Mike Sukop, Tom Beasely and the Earth and Environment Department at FIU, a harness was built to hold the system that was made from individual PVC components that were pieced and glued together. Drill holes on both ends of the core were made to insert nipples and to attach tubes for measuring the head difference ($\Delta h$), across the core. A litre bucket of glycerol was purchased for the measurements.
Figure 19. Laboratory apparatus set up for epoxy core measurements.

Table 5 shows the results for glycerine temperature $T$, kinematic viscosity $\nu$, density $\rho$, volumetric discharge $Q$, and Darcy flux $q$, used to obtain the permeability of the core, $k$. The average hydraulic conductivity $K$, of the megaporous carbonate rock epoxy core cast was calculated to be $13.1$ m/s with a standard deviation of $3.9$ m/s. The Reynolds number was calculated using equation (3) where the fluid velocity, $u$ was defined as,

$$ u = \frac{q}{\eta_c} ,$$

(43)
where the effective porosity $\eta_e$ was 54%, obtained from the GSA poster by Sukop et al. (2009). The megaporosity was measured using a bulk voxel count from imaging software ImageJ which counts the total number of pores and solids that comprise the core. Further analysis on the rock included LBM flow simulations on the CT images obtained from the University of Texas. The cropped images used to build the core were used for fluid dynamic analysis.
Table 5. Results for the megaporous rock permeability

<table>
<thead>
<tr>
<th>Fluid T (°C)</th>
<th>( Q ) (m³/s)</th>
<th>( \frac{dh}{dx} )</th>
<th>( q ) (m/s)</th>
<th>( v ) (m²/s)</th>
<th>( k ) (m²)</th>
<th>Re</th>
<th>( K ) (m/s)</th>
</tr>
</thead>
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<tr>
<td>21.05</td>
<td>1.69E-06</td>
<td>0.002</td>
<td>0.000208</td>
<td>5.51E-04</td>
<td>6.0E-06</td>
<td>0.014</td>
<td>13.3</td>
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<tr>
<td>20.9</td>
<td>1.59E-06</td>
<td>0.002</td>
<td>0.000196</td>
<td>55.7E-04</td>
<td>6.0E-06</td>
<td>0.013</td>
<td>12.6</td>
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<tr>
<td>20.9</td>
<td>1.58E-06</td>
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<td>0.000195</td>
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<td>0.013</td>
<td>12.5</td>
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<tr>
<td>21</td>
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<td>0.000571</td>
<td>5.53E-04</td>
<td>2.9E-06</td>
<td>0.038</td>
<td>6.6</td>
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<td>0.018</td>
<td>0.00152</td>
<td>5.32E-04</td>
<td>4.6E-06</td>
<td>0.11</td>
<td>10.8</td>
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<td>21.5</td>
<td>1.09E-05</td>
<td>0.009</td>
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<td>0.028</td>
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<td>7.6E-06</td>
<td>0.060</td>
<td>18.3</td>
</tr>
</tbody>
</table>
6 Lattice Boltzmann Modeling

6.1 Background

The literature for this method includes textbooks by Wolf-Gladrow (2000), Succi (2001), and Sukop and Thorne (2006). Lattice Boltzmann Methodology (LBM) is a relatively new technique used to simulate flow in complex fluid systems. In contrast to other complex fluid dynamic (CFD) methods, instead of solving the Navier-Stokes equation to obtain macroscopic properties like density, pressure and velocity, LBM implements an intermediate step by representing discrete particles (microscopic) by a distribution function (mesoscopic). The particulate nature of the LBM makes it a more attractive model compared to some of its CFD counterparts such as finite volume, finite difference, and finite element solutions of discretized partial differential equations. The most important feature of the LBM pertaining to my research is its ability to incorporate complex boundary conditions, (Chen and Doolen 1998, Zhou 2004).

LBM numerical analysis has been one of the methods employed to better determine the permeability of the megaporous carbonate prevalent throughout the Biscayne Aquifer (Cunningham et al., 2009; Sukop et al., 2013). The LBM has been validated against analytical, numerical and experimental results for a range of three-dimensional flow geometries (Llewellin, 2010). LBM computes fluid flow in three-dimensional structures. The method uses a lattice to represent a collection of discrete microscopic particles, an equilibrium distribution to calculate the macroscopic flow properties and the kinetic equation otherwise called the Lattice Boltzmann equation (LBE) (Wolf-Gladrow, 2000). The LBE is a reduced-order kinetic model of the Navier-Stokes equation making it relatively simple as compared to the original, nonlinear full
Boltzmann equation, (Chen et al., 1998). The implementation of this model, however advantageous in its simplified manner, requires intensive parallel computing and computing memory for large scale problems.

The LBE was originally derived from Lattice Gas Cellular Automata (LGCA), as a means of improving on its drawbacks. For one, the LGCA included a lot of statistical noise in its implementation to fluid dynamic simulations. The LGCA principles that govern particle evolution on lattice networks are essential and foundational to particle evolution in LBM. As the LBM evolved to what it is today, its simplest implementation was appropriately renamed LBGK given its direct link to the time-honored Bahatnagar-Gross-Krook model (Succi 2001). The LBGK approach provides a whole family of solutions designated by $D^nQ^m$ where $n$ is the number of dimensions $D$, of the lattice model and $m$ is the number of velocities, $Q$. These discrete models are then translated to obtain the main fluid quantities by summation of directional densities and vector addition of the velocities weighted by the distribution function of discrete particle speeds. Hence a simple and pragmatic approach was born to better accommodate fluid dynamics research.

The lattice framework employed for this research on the megaporous rock is the $D3Q19$ shown in Figure 20. The three dimensional framework employs 19 velocity directions (including the directionless zero velocity) along which the particles proceed from one lattice node to another by way of two processes called streaming and collision. The particles move based on the principles of conservation of mass and momentum.
Particle mass is uniform (1 mass unit or mu) for the simplest cases; therefore microscopic velocities ($e$) and momenta ($me$) are always effectively equivalent. Similarly, the lattice unit (lu) can be taken as the fundamental measure of length in the LBM models and the time unit is the time step (ts), providing an internally consistent set of units (Sukop and Thorne, 2006). For the core flow and other simulations, the simplest no-slip bounceback boundary condition is employed in the applied LBM and this proves favorable for domains of irregular geometries.
6.1.1 Lattice Boltzmann method (LBM) applied to fluid dynamics

LBM was born from Lattice Gas Cellular Automata (LGCA) concepts. The fact that microscopic particle interactions can lead to solutions of traditional macroscopic fluid motion equations is the starting point for the development of the LGCA. Conservation of mass and momentum are central to LGCA and LB models. The fundamental concept is that the conservation of mass and momentum is occurring on a microscopic level where a group of particles located on a lattice interacts by means of two processes known as streaming and collision. The microscopic interaction involves the exchange of momentum as particles collide. During these collisions, mass and momentum are conserved. A concise introduction to the equations for streaming and collision processes can be accessed in Sukop and Thorne (2006).

An important feature of the LBM that makes it suitable for determining intrinsic permeability values of irregular media is the boundary conditions. The LBM boundary conditions (BCs) most important to this research are the ‘no-slip’ and periodic. The periodic BC is the most often used and states that particles exiting the domain at one end will re-enter the domain at the opposite end. For example in a horizontal pipe, the fluid leaving the pipe to the east will return into the domain in the next time step on the west side of the pipe. The no-slip boundary condition is also important. The particles that collide with this type of boundary will be turned 180° and sent in the direction they came from in the next propagation step. This type of boundary is called bounce-back in LBM terminology. The temporal/spatial flexibility in applying boundary conditions in LBM makes it easy to incorporate complex solid boundaries and this has made it possible to simulate flows in realistic porous media (Sukop and Thorne, 2006).
Obtained from various sources, Zhou et al. (1995), He et al. (1997), Yu et al. (2003), and Sukop and Thorne (2006), on these lattice frameworks, the single distribution function, $f$, which represents a number of discrete directional densities summed over each node $n$, is used to compute the macroscopic density at each node:

$$\rho = \sum_{0}^{n-1} f_n \cdot$$  \hspace{1cm} (44)

The macroscopic velocity $u$, is an average of the microscopic velocities $e$, weighted by the directional densities:

$$u = \frac{1}{\rho} \sum_{0}^{n-1} f_n e_n \cdot$$  \hspace{1cm} (45)

Fluid kinematic viscosity is given by the following equation,

$$\nu = c_s^2 \left( \tau - \frac{1}{2} \right),$$  \hspace{1cm} (46)

where the speed of sound $c_s$ is defined by,

$$c_s = \frac{1}{\sqrt{3}},$$  \hspace{1cm} (47)

and the dimensionless relaxation time $\tau > \frac{1}{2}$ for positive (physical) viscosity and $\tau = 1$ is the safest and leads to a value of $\nu = 1/6 \text{ } \text{lu}^2\text{ts}^{-1}$ (Sukop and Thorne, 2006).
Figure 21. Illustration of mid-plane bounceback no-slip movement of specific densities as fluid approaches wall (Figure from Sukop and Thorne, 2006).

LBM is dependent on high-performance computational power. The power is necessary to handle large domains and small discretized spaces characteristic to the method. The Panther Cluster at Florida International University (FIU) was recently introduced to the campus community and opened for use in January, 2013. This cluster currently contains 380 cores with a 10 GB high-speed data network and 1 GB high speed interconnect for messaging. All of the following simulations were run on the Panther Cluster at FIU.
6.2 LBM Method Validation

The code used for LBM simulation was the lb3d-prime-dev code available from Google Code and created by Daniel Thorne and Michael Sukop (https://code.google.com/p/lb3d-prime-dev/). Validation of the general LBM was confirmed using the work by Llewelin published in his 2010 paper. He proves the applicability of LB flow simulation by validating it against analytical, numerical, and experimental results for a range of three-dimensional flow geometries. In this research I have worked on reproducing the “Deiber1” results (Deiber et al., 1992) for a specific sinusoidal pipe geometry that was also highlighted in Lewellin (2010). This method of LB code validation was used by Sukop et al. (2013) and those results are also included in this research to demonstrate the suitability of the LBM, and more importantly the lb3d-prime-dev code, for simulating flow through megaporous rock. These flows provide interest to geoscientists because networks of constricted pipes can be used as analogues for porous media (Deiber and Schowalter, 1979, 1981; Lahbabi and Chang, 1986; Payatakes et al., 1973).

6.2.1 Flow in Sinusoidal Pipe

The following example taken from Llewelin (2010) was included and re-simulated to prove the validation of the code at low Re numbers and higher Re numbers which correspond to flows with eddies. The results of Llewelin show the value of f/Re, which contains the ‘friction factor’, f

\[ f = \frac{R_\alpha \Delta P}{\lambda \rho \bar{u}_0^2} \]  

(48)
calculated from the pressure drop $\Delta P$ across one wavelength $\lambda$ of the pipe and

$$\bar{u}_0 = \frac{Q}{\pi R_0^2},$$  \hspace{1cm} (49)

where $R_0$ relates to the actual radius of the pipe $R(x)$, and was previously determined for “Deiber 1” in Deiber et. al (1992). This factor was multiplied by the Reynolds number $Re$,

$$Re = \frac{2 \bar{u}_0 R_0}{\nu}$$  \hspace{1cm} (50)

for various flow speeds. The following figure shows a cross-section of the pipe geometry referred to as “Deiber 1” in Lewellin (2010). The pipe is discretized onto an LB lattice with physical spacing $\Delta x = R_0/ 50$.

Figure 22. Pipe geometry for Deiber 1. (Lewellin, 2010)
Flows driven by varying gravitational forces and varying fluid properties to create different Reynolds number flows were driven down the $z$ axis of the sinusoidal pipe. The results were collected to compare the measured $f_{Re}$ values to previously obtained analytical solutions for scenarios that apply to low Re and to experimental measurement results for model comparison at higher Re. Figure 23 shows the results I obtained with the current lb3d-prime-dev code as compared to low-Re analytical solution, the experimental data, and the 3-D simulations presented by Llewellyn (2010) and Sukop et al. (2013).

Figure 23. Experimental data of Deiber et al. (1992), low-Re analytical solution by Sisavath et al. (2001), simulations of Llewellyn (2010), simulations of Sukop et al. (2013), and current simulations for a sinusoidal pipe defined by "Deiber 1" (Llewellyn, 2010).

Like the results from Sukop et al. (2013), my own values of $f_{Re}$ seemed to compare more favorably to the analytical value at low Re numbers and to the
experimental results at higher Re in contrast to Lewellin’s over-predicted values for the same range. The results here partially demonstrate the capability and suitability of this code to be applied to the flow analysis on the megaporous karst sample ML-01.

7 LBM permeability of ML-01 core

7.1 Digital rock-core dimensions

For the LBM, to ensure that the slices used for analysis directly mirrored the slices cut for the core, all of the 1182 images were compared to the last frame and the first frame used to build the epoxy-resin core. The last frame from the Avizo model (green) is shown below on the left next to the bottom of the epoxy-resin model. The .bmp images were examined and the slice 949 (red) was chosen as it compared most closely with the Avizo images. Small differences between the images are due to small differences in thresholding and interpolation.

Figure 24. Comparison of the last frame in the Avizo model (left) to the epoxy-resin model (middle) and the raw domain for LBM analysis (right).

The same comparison and matching was done for the first frame and from the chosen images the entire raw file format domain (8 bits/256 megabytes) for the LBM
was built using a Java-based image processing program called ImageJ. A total number of
828 slices corresponded to the core length in the z direction. The entire domain was
380x380x828 equal to the length of the domain in the x, y and z directions in lattice units
(pixels) respectively.

7.2 Assessment of necessary computing power

For parallel computing the amount of memory needed to submit a LBM job
depends on the number of processors, their memory, the size of the domain, and the type
of BGK lattice used for the LBM simulation.

For the D3Q19 (3 dimensional, 19 velocities) BGK lattice used for this LBM
simulation, the computing memory needed for each node was calculated. Per node, the
memory includes: (1) a single bit for the is_solid feature which communicates to the
computer if the node is a solid or open pore node of the domain, (2) 19 directional
densities f, (3) 19 equilibrium densities feq, (4) 19 temporary densities ftemp, (5) 1
macroscopic density ρ, and (6) 1 velocity calculated in each of the 3 directions for the 3D
domain (u,v,w). Items (2) through (6) are in double precision (64 bits = 8 bytes) computer
format. The total memory was calculated to be 489 bytes per node. With a domain of the
size 380x380x828 nodes, the necessary computing memory for the job was calculated to
be about 56 GB. Using 23 processors for the parallel computation, the memory utilized
per node was about 3 GB, which needs to be less than available GB per processor.

The total memory capacity of the panther Cluster includes 40 nodes with a total of
584 cores. However, the cores have varying memories depending on the node to which it
belongs. Nodes 01 to 12 have 16 cores with 128 GB of collective memory, nodes 13 to
26 have 12 cores with 64 GB of collective memory and nodes 27 to 40 have 16 cores
with 32 GB of collective memory. With constraints set at only 32 cores per user and with my core number reduced to 23 because of my domain size, in addition to the numerous jobs being sent to the Panther queue, the demand for memory for job completion is high. The large size of my domain required a lot of memory and the exact amount had to be reserved in the submission script to the parallel cluster before submitting the job to avoid subsequent failure due to insufficient memory in the limited number of processors (32) that are available to any one of my jobs using the Panther Cluster at FIU.

7.3 ML-01 core simulation results

7.3.1 Creation of digitized domain

Processing the grey scale images (.tif) of 1024x1024 pixels sampled by the X-ray facility at the University of Texas at Austin and building the raw format imput file was done using an imaging software package called ImageJ. Figure 25 below shows one of the original images provided by the University of Texas at Austin and the subsequent core-cut 2D image (.bmp format) which were cut to the dimensions of 850x382 pixels using a MATLAB script. The images were also converted from grey scale which ranges from 0 to 255, to black and white images by assigning a threshold gray scale value of 75. This allowed for pores to be assigned the white color and solid rock areas to be assigned black as shown in Figure 25. The images had a resolution of 0.271 mm in length per pixel, which is important to the scaling of the LBM permeability to the actual physical permeability of the rock.
The core-cut 2D images were then pieced together to build the 3D domain represents the core-fitted rock. The final 3D image was comprised of 828 individual slices/images. Since the diameter of the core including the solid sheath encasing it was 362 pixels, the size of the constructed core-domain, which comprised of 850x382 pixel-sized images in the x and y directions, as cut down to 380x380 pixels to conserve computer memory for simulation purposes. The final raw image used for LBM simulations is shown in Figure 26 using ParaView visualization software.
Figure 26. Image of core to view structure for LB analysis; red represents solid rock and core surrounding mesh represents pores.

The cylindrical core is fitted to the rectangular prism necessary for LB analysis with lb3d-prime-dev. The solid red represents solid nodes in LBM that will not permit any flow; instead the flow will be constrained to the white matrix that represents open pores through the virtual rock. The long axis in Figure 26 represents the z axis of the domain. As discussed previously, the LB simulation is sent to the Panther Cluster at FIU. The z axis, which comprises of 828 lu was divided amongst 23 processors to work in parallel. Hence each processor had to work to accommodate 36 slices therefore easing up the power and usage necessary to complete the entire LB simulation. To generate flow in the domain, varying values of gravitational acceleration $g$, were applied down the $z$ axis and the resulting intrinsic permeabilities, $k$, in units of lu$^2$ for each simulated case were obtained by evaluating the output.txt file of the simulation once it approached a steady state.
The output.txt and frames files are two outputs of the LB simulations that are important to this software. The lb3d-prime-dev code outputs these files, which include the maximum, average and minimum velocities, $u_{\text{max}}$, $u_{\text{avg}}$ and $u_{\text{min}}$ respectively as well as the densities which were assigned a value of 1 and should remain steady throughout the simulation. An example frames file for one processor in a simulation that has already reached steady state is shown in Table 6.
Table 6. Simulation results from 36 slices taken from the middle processor of the LBM simulation. Table shows convergence of maximum velocities through ML-01domain. Initial gravitational force on fluid is $10^{-6}$ lu per ts$^2$.

<table>
<thead>
<tr>
<th>Time (ts)</th>
<th>$u[x]$ (lu/ts)</th>
<th>$u[y]$ (lu/ts)</th>
<th>$u[z]$ (lu/ts)</th>
<th>min_rho (mu/lu$^3$)</th>
<th>max_rho (mu/lu$^3$)</th>
<th>ave_rho (mu/lu$^3$)</th>
<th>max/ave</th>
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<td>0.00949</td>
<td>0.022705</td>
<td>0.9946407</td>
<td>1.006079</td>
<td>1</td>
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<tr>
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<td>1</td>
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<td>0.022833</td>
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</tr>
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<td>0.022833</td>
<td>0.9946032</td>
<td>1.006081</td>
<td>1</td>
<td>1.006081</td>
</tr>
<tr>
<td>25000</td>
<td>0.01263</td>
<td>0.00955</td>
<td>0.022833</td>
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</tr>
<tr>
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<td>0.00955</td>
<td>0.022833</td>
<td>0.9946032</td>
<td>1.006081</td>
<td>1</td>
<td>1.006081</td>
</tr>
<tr>
<td>35000</td>
<td>0.01263</td>
<td>0.00955</td>
<td>0.022833</td>
<td>0.9946032</td>
<td>1.006081</td>
<td>1</td>
<td>1.006081</td>
</tr>
<tr>
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<td>0.00955</td>
<td>0.022833</td>
<td>0.9946033</td>
<td>1.006081</td>
<td>1</td>
<td>1.006081</td>
</tr>
<tr>
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<td>0.01263</td>
<td>0.00955</td>
<td>0.022833</td>
<td>0.9946033</td>
<td>1.006081</td>
<td>1</td>
<td>1.006081</td>
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<tr>
<td>50000</td>
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<td>0.00955</td>
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<td>1.006081</td>
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<td>1.006081</td>
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<td>1.006081</td>
</tr>
</tbody>
</table>

65
On convergence and thus completion of the LB simulation, various binary .dat files which include density and velocity values for each time step, are output by the lb3d code. The last time steps representative of a steady state solution are imported from the Panther Cluster to the local desktop. Additional convergence tests were also done on the simulations where the discharge output for each frame from the LBM simulation was plotted. Each frame corresponds to 5,000 ts. Below is an example of this plot showing convergence by the second frame (10,000 ts) which reached a steady state discharge value of 6.8698 lu³/ts but was plotted till the eighth frame (40,000 ts).

![Figure 27. Illustration of convergence for one flow simulation for a flow driven by a gravitational acceleration of 5.42 x 10⁻⁷ lu/ts² with a Re = 0.054](image)

The results for the last time step of all simulations are segmented amongst the 23 processors. In order to visualize and process the data, MATLAB coding is employed. The appropriate .dat files are evaluated to calculate a value of discharge for the entire domain in lu³ per ts.
The calculations from the output include an evaluation of the Darcy flux \( q \), which is obtained from dividing the discharge \( Q \) by the cross sectional area \( A \), of the core. In LB units, the corresponding permeability value was determined from rearrangement of the following form of Darcy’s equation which includes gravitational-equivalent pressure gradient, \( G = \rho g \).

\[
q = \frac{k}{\mu} \frac{\Delta P}{\Delta x} = \frac{k}{\mu} G = \frac{k g}{v}.
\] (51)

To obtain a physical value of permeability for each simulation, the following relationship was obtained from Sukop et al. (2008),

\[
k_{\text{physical}} = k_{LBM} \left( \frac{L_{\text{physical}}}{L_{LBM}} \right)^2.
\] (52)

Finally, the physical value of hydraulic conductivity \( K \), was evaluated from the following equation,

\[
K = k \frac{g}{v},
\] (53)

where the physical values of intrinsic permeability \( k \), the gravitational acceleration \( g \), and the kinematic viscosity \( v \), are used.

To obtain a value of physical head gradient in terms of LBM and physical gravity, length, and kinematic viscosity, the following equation based on arguments in Sukop et al. (2013) was used,

\[
\left[ \frac{\partial h}{\partial x} \right]_{\text{physical}} = \left[ \frac{gL^3}{v^2} \right]_{LBM} \left[ \frac{v^2}{gL^3} \right]_{\text{physical}},
\] (54)
where LB kinematic viscosity was calculated from equation (46) and the kinematic viscosity of water at 20°C (∼10⁻⁶ m²/s) was used for the physical value, $L$ is the length over which the head drops and in this case can be used as the ratio of lattice unit to physical spacing (1lu = 0.000271 m) because both physical $L$ and LBM value of $L$ are raised to the same order of magnitude. $g$ is the gravitational acceleration, 9.81 m/s² for the physical case and the LBM values of gravitational acceleration are shown in Table 7. The values of gravitational acceleration were modified to create different flow simulations, higher accelerations resulted in non-Darcian flows (Re >1). The subscripts $LB$ and $physical$ refer to the parameter units used for the two cases.

Table 7. Gravitational accelerations and $\tau$’s used to drive flow through ML-01 sample for LBM analysis.

<table>
<thead>
<tr>
<th>Simulation Number</th>
<th>Gravitational Acceleration (lu/ ts²)</th>
<th>$\tau$ (ts)</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00E-05</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.00E-09</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>3</td>
<td>1.00E-04</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5.42E-07</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>1.00E-04</td>
<td>0.6</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>2.00E-06</td>
<td>0.6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1.00E-05</td>
<td>0.6</td>
<td>20</td>
</tr>
</tbody>
</table>

The relationship between the hydraulic conductivity and the hydraulic gradient for the LBM simulation results on the ML-01 sample are shown in Figure 28. These results are obtained at a range of hydraulic gradients to mimic creeping through inertial flow. Reduction in the apparent hydraulic conductivity is shown as the physical hydraulic gradient values increase.
Figure 28. LBM simulation results. Apparent hydraulic conductivity values obtained at the respective hydraulic gradients.

To compare to these results where the flow was driven by a gravitational force, calculations using a pressure difference across the domain were simulated for previous trials. This additional method of measurement was done to ensure that the periodic boundary conditions applied to the domain for LBM analysis were not causing an underestimation of the calculated $K$ values, due to miss-matches at the pore wall at the boundary. The hydraulic gradients for simulations 1 and 4 in Table 7 were used to compute appropriate values of pressure difference $\Delta P$, from the following equation and $K$ values within 4% of those in Figure 28 were successfully obtained.

$$\left[ \frac{\Delta h}{L} \right]_{\text{physical}} = \left[ \frac{\Delta P L^3}{L \rho v^2} \right]_{\text{LB}} \left[ \frac{v^2}{gL^3} \right]_{\text{physical}}.$$

(55)
7.3.2 Visualization

Visualization of the flow velocities was done using ParaView, version 3.98.1 64-bit software. Of the seven simulations, I chose two flow results from Table 7: simulation number 2 of a Darcian flow (Re= 0.0001) and simulation number 5 of a non-Darcian flow (Re = 90). From the LBM simulations, binary data on the flow velocities in all 3 directions and density differences were obtained. Using Matlab, the dat files were compiled into a .vtk file suitable for the ParaView visualization software. Figure 29 illustrates flow paths through the entire 380x380x828 domain plotted as tube lines in ParaView that are colored by the magnitude of the flow velocity.

![Flow Magnitude results from the LBM simulation of the 2 scenarios. (a) Darcian flow (b) non-Darcian flow.](image)

Figure 29. Flow Magnitude results from the LBM simulation of the 2 scenarios. (a) Darcian flow (b) non-Darcian flow.
In Figure 29 special attention should be drawn to the scale bars in lu/ts units. For the non-
Darcian flow (b), as expected, the flow velocities are much higher and fill the rock
domain more than in (a). The domain here is 380x380x828 and represents the entire rock
core of radius 362 lu with a solid casing to create the rectangular prism shown for LBM
simulation.

The size of the full rock domain LBM results for any one simulation is
approximately 5GB and this makes it difficult to do data analysis, especially comparison
with the simulation results from other flow scenarios, on a 64bit personal laptop. Ideally
these visualizations should be made on a super computer like the Panther Cluster, but this
feature is not yet available. A subset of the results measuring 380x380x263 lu was
extracted in order to avoid visualization problems and to compare vorticity magnitudes
for the LBM simulations for the two scenarios discussed above. The number 263 slice
was chosen as the endpoint because relatively large flow velocities were identified in this
area using the flow tubes plotted in Figure 29.
Figure 30. Vorticity Magnitude results for the LBM simulation of the 2 scenarios. (a) Darcian flow (b) non-Darcian flow. The vorticities are plotted on a single scale for comparison.

The solid blue lines of Figure 30 (a) correspond to a maximum vorticity of about $5.42 \times 10^{-7}$ ts$^{-1}$. As expected, the higher vortices exist in the non-Darcian flow regime shown in
Figure 30 (b). The Reynolds number of this flow is 90 and the higher vortices may well be indicative of eddies in the flow.

8 Discussion

The principal objective of this research was to obtain a permeability measurement for a sample of rock representative of the megaporous rock in the Biscayne Aquifer as well as to test the hypothesis that the LBM was an appropriate method for permeability measurements of such megaporous rock samples. This research directly addressed the disagreement between petrophysical laboratory measurements and LBM estimates of permeability pointed out in a recent publication by Cunningham and Sukop (2011). The inconsistency was attributed to possible limitations of the methods employed by the laboratories to measure such extreme permeabilities. The equipment used in the laboratories had limits that could have lead to an underestimation of the permeability of the megaporous rock. The megaporous nature of the Biscayne Aquifer rocks and resulting extreme permeability makes it an excellent aquifer for water production but makes it a difficult rock for experimental or field analysis to obtain hydrogeologic parameters such as transmissivities, permeabilities, and hydraulic conductivities. The permeability of the megaporous rock simply lies outside the normal range of interest of petrophysical laboratories.

Challenges in field and laboratory scale permeability analyses were taken into consideration with the planning and execution of this research. Problematic in most of the previous works with aquifer and laboratory tests were the small and sometimes immeasurable drawdowns resulting from the high transmissivity of the Biscayne Aquifer and the difficulty in maintaining Darcian flow regimes with water as the primary working
fluid. The solution to this problem for laboratory methods was to use a highly viscous substance and the chosen fluid was 98% weight Glycerol-aqueous solution. The glycerine proved to be the better working fluid against the ISO 680 gear oil. Pipe permeability values measured using the glycerine solution were more accurate and were on average off by -0.1% from the true pipe permeability value. This result was compared to the ISO 680 gear oil that resulted in permeability measurements that were on average off by 22% from the true permeability of the pipe and systematically higher.

While the 98% Glycerol solution proved favorable for rock analysis because of its ability to reproduce the true pipe permeability, the following relationship between permeability and temperature shown in Figure 31 was investigated further because the parameters were strongly correlated ($R^2 = 0.92$) even though provisions for temperature corrections were made to compute both the viscosity and the density of the fluid.

$$k = 5E-07 \ T - 4E-06$$

$$R^2 = 0.9238$$

**Figure 31. Permeability of half-inch PVC versus temperature.**
A statistical $t$ test outlined in Ott (1984) was applied to the data in Figure 31 in order to test the hypothesis that the slope could equate to zero proving no dependence on temperature for permeability. If this failed, a slope not equal to zero would indicate a dependence on temperature. Using 8 degrees of freedom and a two-tailed test, a $t$ value of 1.15 compared to the $t_α$ value 9.85 rejected the hypothesis and confirmed a strong dependence on temperature. The permeability calculations were re-visited but seemed to be worked through accurately. In spite of the temperature dependence, the variation in permeability estimates was small (standard deviation of $3.44 \times 10^{-7}$ m$^2$), and with glycerine, the apparatus was able to compute pipe-permeabilities within favorable degrees of accuracy. As a result, laboratory pipe permeability measurements confirmed the functioning of the experimental system and confirmed the reliability of glycerine as the working fluid to be used for subsequent megaporous rock-permeability assessment.

Laboratory permeability measurements of an epoxy-resin core model of the megaporous carbonate rock were executed. The carbonate rock from the Biscayne Aquifer is highly porous and rock core recovery is problematic. The proposed solution was to create a digital image of a sample of the rock (ML-01) and from 3D printing produce a cast of the megaporous rock. The sizing, which related physical to digital rock domain, 1 pixel to 0.271 mm was recorded. The rock core was fitted to the experimental system and a constant flow of glycerine was pumped through it, a constant head gradient was maintained, and the outflow was measured. The system set-up and measurements were identical to that used in pipe-permeability analysis.

The relationship between permeability of the megaporous rock and the temperature in this case was investigated and is shown in Figure 32.
The temperature versus permeability correlation reflected by the $R^2$ value of 0.14 was small indicating the relationship between permeability and temperature here is negligible. As done before, a statistical $t$ test was now applied to the data in Figure 32 in order to test the hypothesis that the slope could equate to zero proving no dependence on temperature for permeability. If this failed, a slope not equal to zero would indicate a dependence on temperature. Using 8 degrees of freedom and a two-tailed test, a $t$ value of 1.15 compared to the $t_{\alpha}$ value 1.86 failed to reject the hypothesis that there was no relationship between temperature and permeability.

The standard deviation of the permeability results is $1.6 \times 10^{-6}$ m$^2$, which is approximately 10 times larger than the standard deviation obtained for the pipe permeability measurements. The reason for the difference between the $R^2$ values of Figure 32 and Figure 31 is evident in the scale of the $y$ axis. The pipe permeabilities are

Figure 32. Permeability of megaporous rock sample ML-01 versus temperature.

$$k = 2E-06 \ T - 3E-05$$

$R^2 = 0.1437$
more exact and the range is so small that temperature effects are evident. For the permeability of the megaporous rock, the permeability measurements exist over a wider range and so any temperature effects are minimal. The average permeability of the ML-01 sample from laboratory measurement was $5.6 \times 10^{-6}$ m$^2$, comparable to the permeability values of $9.7 \times 10^{-6}$ m$^2$ and $1.1 \times 10^{-5}$ m$^2$ obtained respectively from the simple square and hexagonal pipe packing model defined in Figure 7. The agreement for the ML-01 rock core estimates adds confidence to the permeability result.

With negligible temperature effects, the value of hydraulic conductivity of the ML-01 epoxy core was also determined using an average of the values obtained from experimental data. Under laminar flow conditions ($Re = 0.05$) the hydraulic conductivity of ML-01 was determined to be $13.10$ m/s and this value compared favorably to computational analysis of rock permeability using LBM. For the comparison, the experimental Reynolds number ($Re = 0.05$) was used as the starting point to obtain an appropriate gravitational acceleration to drive the flow through the domain for the LBM simulation. The corresponding LBM simulation is listed as the number 4 simulation in Table 7. For the virtual domain, the same image slices used to produce the epoxy-resin core model were used to build a .raw format image of the domain for LBM analysis. The LBM value of hydraulic conductivity was calculated to be $14.78$ m/s for the ML-01 sample after simulation convergence was reached (Figure 27). This LBM value of $K$ is within the one standard deviation for the laboratory value. The similarity in the $K$ values shown in Figure 33 prove that the laboratory experimental method was in fact successful at determining hydraulic conductivity for the megaporous rock.
The value of hydraulic conductivity for the Biscayne Aquifer megaporous rock determined by this research is well in excess of the measurements obtained from aquifer tests by Parker et al. (1955), and Fish and Stewart (1991) who recorded the highest values of hydraulic conductivity in the Biscayne Aquifer at 0.23 m/s and >0.035 m/s respectively. However, the results of this research compared more favorably to recent research by Alvarez (2007), who recorded LBM results obtained on a different sample of the same megaporous rock from the Biscayne Aquifer. Alvarez (2007) recorded hydraulic conductivities in the range of 13.74 m/s to 34.5 m/s depending on the total size (sample spacing) of the digital rock-domain used for LBM simulation. The first result was obtained for a rock domain 280 cubed and the second for 336 cubed. He also tested the effect of sample spacing when creating the virtual image of the rock from the CT data. The coarser the resolution, the larger the hydraulic conductivities for each sample became.

My own LBM simulation of the ML-01 rock sample was obtained using one sample-spacing (0.000271 m = 1 lu). Different from Alvarez (2007), the accuracy of the LBM value of hydraulic conductivity (14.78 m/s) was confirmed by comparing it to the experimental value of hydraulic conductivity (13.1 m/s) on the same ML-01 core sample. Also noted in this research, the results for \( K \) obtained herein compared favorably to the LBM results recorded in Cunningham and Sukop (2011) and Sukop et al. (2013) thereby solidifying the appropriateness of the LBM for megaporous rock-permeability measurement.
Figure 33. Hydraulic conductivity for ML-01 obtained from LBM and experiment for Darcian flow (Re = 0.054).

Figure 28 shows the LBM simulation results. The chart illustrates all of the hydraulic conductivity values that were estimated for a range of flow regimes, both Darcian and non-Darcian. The results were used to estimate the Darcy-Forchheimer equation parameter for this particular megaporous medium. The flow resistance at these higher gradients is represented by the Darcy and Forchheimer drag components which are respectively linear and quadratic in velocity (Jeong et al., 2006). Factoring the Forchheimer drag into the study of fluid flow yields the following equation obtained from Zeng and Grigg, (2006).

\[
-\nabla P = \frac{\mu}{k} q + \beta \rho q^2,
\]

(56)
where \( \mu \) is the dynamic viscosity of the fluid, \( k \) is the permeability of the porous medium, \( q \) is the darcy flux and, \( \rho \) is the fluid density. The following equation (57), obtained from Balhoff and Wheeler (2009), is used to estimate the Darcy-Forchheimer \( \beta \) parameter from the apparent permeability \( k_{app} \), the low Re permeability \( k \), the Darcy flux and fluid properties.

\[
\frac{1}{k_{app}} = \frac{1}{k} + \beta \left( \frac{\rho q}{\mu} \right). \tag{57}
\]

The parameter \( \beta \) is the Darcy-Forchheimer constant for this particular porous medium and was estimated at 518 m\(^{-1}\) using a least squares fitting procedure. The least squares method minimizes the sum of the residuals. The residuals are the difference between the simulated and fitted values (Ott, 1984). The best fit curve defined by the Darcy-Forchheimer equation is shown in Figure 34 along with the simulation results.
Figure 34. Effect of increasing gradient on apparent hydraulic conductivity from LBM simulations for ML-01 fitted with the Darcy-Forchheimer equation.

The fit allows for the successful prediction of hydraulic conductivities at a wide range of hydraulic gradients. At realistic field gradients in the Biscayne Aquifer, which range from $10^{-6}$ to $10^{-4}$ regionally, the apparent hydraulic conductivity can be determined.
9 Conclusions

The implementation of LBM to determine the intrinsic permeability and, by extension, the hydraulic conductivity of the megaporous rock in the Biscayne Aquifer is in agreement with new laboratory measurements. The hydraulic conductivities for the ML-01 sample measured by the specialized laboratory technique and the LBM have been recorded at 13.1 m/s and 14.78 m/s respectively. These rock samples in the Biscayne Aquifer have long presented problems to the hydrologists who wish to quantify the aquifer parameters. The use of standard petrophysical laboratory equipment commonly used in industry to measure the permeability of rock samples appears to be inappropriate for the Biscayne Aquifer samples, and the LBM has been tested and shown to be the more correct method for permeability analysis. These Biscayne Aquifer rock samples are highly porous with very large pores, substantially different from samples usually measured by the laboratory equipment used in the petroleum industry. As a result, it appears that the methods recorded intrinsic permeability values that grossly underestimated the true permeability, which was obtained by corroborating independent computational and laboratory methods here.

The rocks in Biscayne Aquifer have been described as having an extensive range of hydrologic parameters. The sample used for this research has a porosity of about 54% and this sample lies in a rock with a porosity range of about 20% to 80%. This research has been successful in obtaining an experimental laboratory result for the permeability of these megaporous rocks. The similarity in the result obtained by the numerical method and the experimental method has offered additional validation of the research hypothesis.
and the LBM has been identified as an appropriate method for measuring rock permeability in the presence of such vuggy porosity.

Furthermore, this research presents analysis and measurements of the apparent hydraulic conductivities that exist throughout the Biscayne at realistic regional field gradients of $10^{-6}$ to $10^{-4}$. At these gradients, flows are expected to deviate from the linear gradient-flow relation assumed in Darcy’s law. The Darcy-Forchheimer equation fit presented herein provides predicted values of apparent hydraulic conductivity at these gradients and the Forchheimer $\beta$ factor of 518 m$^{-1}$ for any researcher interested in quantifying flow in the Biscayne Aquifer. The importance of knowledge on aquifer parameters is increasing with concerns about growing populations and changing climates. With better parameter quantification, scientists can better model the movement of saltwater intrusion and contaminants that may affect the Biscayne Aquifer of south Florida in the future.
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