ESSAYS IN OPEN ECONOMY MACROECONOMICS

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by

Ramón Antonio González Hernández

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To: Dean Kenneth Furton  
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This dissertation, written by Ramón Antonio González Hernández, and entitled Essays in Open Economy Macroeconomics, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

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Florida International University, 2008
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DEDICATION

I dedicate this dissertation to my wife Margarita and my daughters Vera and Miranda. Without their patience, understanding, love and support the completion of this work would not have been possible. I would also like to dedicate this doctoral thesis to my parents, especially to my mother Lourdes who is no longer physically with me, but her love and her examples of hard work and determination are with me forever. Finally, I would like to dedicate this contribution to the economics science to my beloved grandparents Papito and Mamá Mercedes. Papito is the living person on Earth that I admire and respect the most. Without his support I would not have made it.
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My gratitude is extensive to the members of my committee Dr. Peter Thompson and Dr. Mihaela Pintea. They were always there to help me out when I need it. I have a high praise for them, and consider myself very fortunate to have them as friends and colleagues. I am also very appreciative to Dr. Hassan Zahedi for his interest in my research and helpful editorial suggestions.
ABSTRACT OF THE DISSERTATION

ESSAYS IN OPEN ECONOMY MACROECONOMICS

by

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Florida International University, 2008

Miami, Florida

Professor Cem Karayalcin, Major Professor

Research macroeconomists have witnessed remarkable methodological developments in mathematical, statistical, and computational tools during the last two decades. The three essays in this dissertation took advantage of these advances to analyze important macroeconomic issues.

The first essay, “Habit Formation, Adjustments Costs, and International Business Cycle Puzzles” analyzes the extent to which incorporating habit formation and adjustment costs in investment in a one-good two-country general equilibrium model would help overcome some of the international business cycle puzzles. Unlike standard results in the literature, the model generates persistent, cyclical adjustment paths in response to shocks. It also yields positive cross-country correlations in consumption, employment, investment, and output. Cross-country correlations in output are higher than the ones in consumption. This is qualitatively consistent with the stylized facts. These results are particularly striking given the predicted negative correlations in investment, employment, and output that are typically found in the literature.

The second essay, “Comparison Utility, Endogenous Time Preference, and Economic Growth,” uses World War II as a natural experiment to analyze the degree to
which a model where consumers' preferences exhibit comparison-based utility and endogenous discounting is able to improve upon existing models in mimicking the transitional dynamics of an economy after a shock that destroys part of its capital stock. The model outperforms existing ones in replicating the behavior of the saving rate (both on impact and along the transient paths) after this historical event. This result brings additional support to the endogenous rate of time preference being a crucial element in growth models.

The last essay, “Monetary Policy under Fear of Floating: Modeling the Dominican Economy,” presents a small scale macroeconomic model for a country (Dominican Republic) characterized by a strong presence of fear of floating (reluctance to have a flexible exchange rate regime) in the conduct of monetary policy. The dynamic responses of this economy to external shocks that are of interest for monetary policy purposes are analyzed under two alternative interest rate policy rules: One being the standard Taylor rule and another that responds explicitly to deviations of the exchange rate with respect to its long-term trend.
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I. HABIT FORMATION, ADJUSTMENT COSTS, AND INTERNATIONAL BUSINESS CYCLE PUZZLES

I.I. Introduction

As Baxter (1995) points out in her handbook survey, the challenge for an important branch of the open economy macroeconomics literature is to develop a consistent explanation for the fact that business cycles of developed countries tend to move together. Though, the literature has gone a long way in producing models that mimic the observed comovement among some of the macroeconomic variables, it “has proven consistently difficult to generate sufficient comovement across countries in labor input and investment.” ¹

The baseline model adopted in the literature to analyze economic fluctuations has been the simple one-sector growth model where the single final good produced can be used both for consumption and investment.² A simple two-country extension of the baseline model, such as the one considered by Backus et al. (1995, henceforth BKK), generates, however, results that stand in sharp contrast to the international comovements in output, employment, consumption, investment, and productivity observed in the data.³ A recent study by Ambler et al. (2004, henceforth ACZ) examines the empirical findings described in BKK to determine if they could be

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³The sample of ten countries in BKK (1995) includes Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, United Kingdom and the United States. Their study presents the correlations of each country's relevant variables with the same U.S. variable for the period 1970:1-1990:2 and compares it with the correlation predicted by their baseline two-country model.
considered robust stylized facts\(^4\). Their results show that average cross-country correlations in output, employment, consumption, investment, and productivity are indeed all positive but are weaker than the ones suggested by BKK.\(^5\)

As is now well known, a comparison of the predictions of the calibration exercise in BKK with both the observed correlations in their sample and with the more robust cross-country correlations in the ACZ study (See Table I.1.) yields three important puzzles: (i) the observed low positive cross-country correlation in consumption is inconsistent with the large positive correlation predicted by theory\(^6\), (ii) the stylized facts show positive cross-country output correlations that exceed cross-country consumption correlations while the baseline model points in the opposite direction (with the additional flaw that the predicted cross-country correlation in output is negative), and (iii) cross-country correlations of investment, employment and output are positive rather than negative as suggested by the baseline real business cycle model.

A common strategy of the studies that attempt to build models compatible with the observed features of the data is to depart from the baseline international real business cycle model by altering some of its basic assumptions. Such departures so far include the modification of the constraints on trades among agents or countries, or of the specification of agent preferences or of the number of goods.

\(^4\)These authors work with a sample of twenty industrialized countries and consider all pairwise cross-country correlations for the time frame 1960:1-2000:4 (not only the cross-correlations with respect to the U.S. as in BKK).

\(^5\)Nevertheless, when they calculated the cross-country correlations for the exact sample and time period in BKK the results are quite similar (i.e., identical for the cases of consumption and investment and roughly equal for output). The only significant difference is in the correlation in employment, with the magnitude of the coefficient in ACZ being almost twice of the one in BKK.

\(^6\)The results in ACZ suggest that the divergence between the cross-country correlations of consumption predicted by standard models and the one in the data is actually larger than previously thought.
Some prominent attempts worth mentioning would be as follows. Devereaux et al. (1992) develop a two-country model where preferences are not separable between consumption and labor supply and thereby generate lower correlations between consumption levels. Baxter and Crucini (1995) and Kollman (1996) build models with incomplete asset markets and show that while incomplete markets help reduce the cross-country correlations in consumption, cross-country correlations of output, investment and employment remain counterfactually negative. Baxter (1995) presents a model with incomplete markets that can help rationalize output correlations that exceed consumption correlations (both being positive) but investment and labor correlations remain negative. Ricketts and McCurdy (1995) build a two-country model with money, no international trade in investment goods and differing rates of traded-good productivity growth across countries that yields an ordering of cross-country correlations that is similar to the one in BKK data.

Stockman and Tesar (1995) introduce nontraded goods and succeed in lowering the cross-country correlation of consumption while simultaneously raising cross-correlations in output. Boileau (1996) also considers a model with multiple goods, incorporating a nonmarket sector and international externalities in production. The model generates realistic correlations for output and consumption but is less successful in term of the cross-correlations in employment. Canova and Ubide (1998) develop a model with home production that generates correlations of output similar to those of consumption, and positive correlations of investment and employment. More recently Head (2001) constructs a model with differentiated intermediate goods and

---

7 They do not consider cross-country correlations in investment and employment.
monopolistic competition and shows that increasing returns to the variety of intermediate goods can lead to a positive international transmission of the business cycle. Ambler et al. (2002) build a model with a multiple tradable-goods sector that is relatively successful in matching the cross-country correlations in the data with the exception of consumption. Finally, Wen (2007) presents a model in which a combination of restrictions on capital mobility and demand shocks operating through changes in the marginal utility of consumption yield correlations that have the right ranking and signs as in the data. The crucial weaknesses here are that the correlations the model generates remain significantly higher than the ones suggested by stylized facts in most of the cases, and the results depend on demand shocks that are not easy to justify empirically.

In what follows we follow a strategy of deviating from the standard model in a different direction: we introduce habit forming preferences and adjustment costs in investment.

Habit formation (i.e., the existence of `adjacent complementarity" between an individual's current and past consumption or felicity levels) has recently been the subject of intensive investigation by economists. The influence of such behavior on asset pricing was analyzed and empirically tested by, most prominently, Abel (1990) and Constantinides (1990). Ferson and Constantinides (1991), Heien and Durham (1991), and Heaton (1995) test habit persistence using consumption data and found the interaction between these two to have important effects. Carroll and Weil (1994) argue that the observed relationship between growth and saving may be explained by a model
of consumption with habit formation.\textsuperscript{8} Mansoorian (1993, 1996) and Obstfeld (1992) fruitfully apply this idea to the analysis of open economies. Fuhrer and Klein (2006) lends support to the hypothesis that habit formation characterizes consumption behavior among most of the G-7 countries. Sommer (2007) shows that habit formation in consumer preferences can explain two well-known failures of the permanent income hypothesis.

Jermann (1998) develops a closed economy model that uses habit formation and adjustment costs in investment and shows that this combination explains the historical equity premium and the average risk-free return observed in the data, while replicating many of the salient business cycle properties. In addition, the setup also avoids the unappealing feature of having a counterfactually constant value of Q in real business cycles models.

Karayalcin (2003) builds a model that combines habit formation with adjustment costs to analyze the effects of fiscal policy in a small open economy. Here, and elsewhere, the incorporation of adjustment costs in investment tends to lower the degree of consumption smoothing as agents are not longer able to undertake a frictionless adjustment in the capital stock. Used together with habit forming preferences, adjustments costs give rise to cyclical transition paths that are persistent, addressing another shortcoming of standard real business cycle models, which need to rely on ad hoc shocks to generate persistence.

The idea that habit formation might help explain the international consumption correlations puzzle was suggested by Shi (1999), who conjectures that by habit forming

\textsuperscript{8}Campbell (1994) stresses the need to explore models of habit formation.
preferences would help reduce the correlations generated by the standard model in a two-country setup.

In addition to modeling the dynamics of capital accumulation together with consumption, saving, and labor supply decisions, we consider an environment where the rate of time preference is endogenous. This last feature is consistent with the assertion of Hicks (1965), who points out that the independence of consumption levels between successive periods implied by conventional time-additive preferences is counter-intuitive and normally one should expect complementarity between them. Hicks' argument has been corroborated by empirical findings that have generated strong rejections of time-additive preferences.9

The model we present in the next section succeeds in overcoming the puzzling negative correlations found in the literature. The predicted correlations in consumption, investment, employment and output are all positive and therefore in agreement with the empirical evidence. This constitutes a substantial progress in RBC models despite the fact that the predicted correlations remain quantitatively higher than the ones reported in both ACZ and BKK. Another important improvement is that the ratio of home-to-foreign consumption exhibits cyclical adjustment. This stands in sharp contrast with the typical result found in most of the two-country models in the literature where this ratio remains constant along the transition paths.

Moreover, the cross-country consumption correlations we obtain, though higher than the one suggested by stylized facts, are (marginally) lower than the value of 0.88 predicted by the baseline BKK model. The model predicts positive cross-country

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9See Obstfeld (1990) on the relevant literature.
correlations in output higher than those in consumption, thereby tackling the second puzzle mentioned above. In all the simulations the model yields positive cross-country correlations in investment. This result is particularly striking in the light of the typical prediction in the literature of negative cross-country correlations in investment. Finally we also obtain positive cross-country correlations in employment.

The rest of the paper is organized as follows. Section two sets up the model of the integrated world economy consisting of two-large countries with fully integrated stock, output and capital markets. Section three analyzes the international transmission of demand and supply shocks that originate in one of the countries, and presents the international cross-country correlations that arise from the calibration of our model. Section four provides some concluding remarks.

I.II. The Model

Consider and integrated world economy consisting of two large countries, labeled “home” and “foreign”, with both countries having a similar structure in terms of preferences and technology. The number of households is normalized to one in each country. Firms within each country employ capital and labor to generate the same single traded good that can be used for consumption and investment. Governments in both countries collect lump-sum taxes in units of the single good being produced and use the entire tax proceeds to finance their expenditures. The level of government spending in both countries positively affects household utility but does not influence the consumption and labor supply decisions directly. All agents operate under perfect foresight and there is no labor mobility\(^\text{10}\) among the two countries in the setup. The

\(^{10}\)A central assumption in most models of international trade and finance is that labor is much less mobile internationally than either commodities or capital. Language and cultural barriers, family and
world markets for stock, output and capital are fully integrated in the framework. We now proceed to describe the behavior of households and firms in some detail.

**Households**

Households derive utility from consumption of private \((C)\) and public \((G)\) goods, and leisure \((1 - L)\)\(^{11}\). Saving takes the form of accumulation of equities issued by home and foreign firms. Equities from both countries are perfect substitutes in the portfolios of agents, thus they must yield the same rate of return \((R = R^*)\)\(^{12}\). The representative household in the home country maximizes lifetime utility by choosing private consumption \((C)\) and leisure \((1 - L)\) optimally, treating the level of government spending \((G)\) as given at each point in time.

In what follows, we adopt the habit-forming recursive time preference structure proposed by Shi and Epstein (1993), which is a more tractable extension of the utility function considered in Ryder and Heal (1973). In this setting consumption habits are modeled through the representative agent's endogenous rate of time preference rather than through the agent’s instantaneous utility function.

The lifetime utility takes the following form:

\[
\int_0^\infty V(C, L, G) \exp \left( - \int_0^t \beta(Z(\tau))d\tau \right) dt
\]

where \(V(\cdot, \cdot, \cdot)\) is the instantaneous utility function and \(\beta(Z)\) is the endogenous discount rate that depends on an index of past utility (stock of habits) denoted by \(Z\). It is assumed that \(\beta(Z)\) satisfies \(\beta > 0, \beta' > 0, \beta'' = 0\).

---

\(^{11}\)Leisure is defined as total time, \(T\) available to households minus the time dedicated to labor activities \((T - L)\). We further set \(T = 1\), without loss of generality.

\(^{12}\)An asterisk denotes that a variable pertains to the foreign country.
We further set \( V(C,L,G) \equiv -1 \) which yields particularly simple solutions while preserving all the essential properties of a more general function. Now defining \( \alpha(t) \equiv \exp(-\int_0^t \beta(Z(\tau))d\tau) \) the representative household optimization problem can be expressed as follows:

\[
\max \int_0^\infty (-\alpha(t))dt
\]

subject to

\[
\dot{A} = RA + WL - C - T, \quad A_0 > 0 \quad \text{given,} \tag{3}
\]
\[
\dot{Z} = \sigma U(C,L,G) - Z, \quad Z_0 > 0 \quad \text{given,} \tag{4}
\]
\[
\dot{\alpha} = -\beta(Z)\alpha, \quad \alpha_0 = 1 \tag{5}
\]
\[
C, L, G, A \geq 0
\]

where \( \sigma > 0 \) is the speed of adjustment in the stock of “habits” and \( A, W, T \), \( U(C,L,G) \) denote the level of nonhuman (financial) wealth, wage rate, lump-sum tax receipts, and an aggregator function. To see what is involved in the optimization problem, it is useful to integrate (4):

\[
Z(t) = \sigma \int_{-\infty}^t U(C,L,G)\exp[\sigma(\tau - t)]d\tau \tag{6}
\]

showing that \( Z(t) \) is a weighted average of past felicity levels with weights declining exponentially into the past at the rate \( \sigma \). If \( \sigma = 0 \) the conventional time-additive utility function with a constant rate of time preference is obtained. If, on the other hand, \( \sigma = \infty \) we get the Uzawa (1968) endogenous rate of time preference.
We specialize the aggregator function to:

$$U(C, L, G) = u(C, L) + v(G)$$  \(\text{(7)}\)

where

$$u(C, L) = \frac{C^\gamma (1-L)^{(1-\gamma)} - 1}{1-\theta}, \quad \text{for } \theta > 0, \quad \theta \neq 1, \quad \gamma \in (0, 1)$$  \(\text{(7.1)}\)

$$v(G) = aG$$  \(\text{(7.2)}\)

One implication of this specification is that though the consumption of the public good increases household utility, it does not directly affect consumption decisions. Another consequence of (7.1) is that \(u(C, L)\) satisfies \(u_C > 0, u_L < 0, u_{CC} < 0\). It is further assumed that \(\theta > 1\) so that \(u_{CL} > 0, u_{LL} < 0\).

The discounted Hamiltonian for the problem in (2)-(5) is:

$$H = -\alpha + \tilde{M}(RA + WL - C - T) + \Psi \sigma U(C, L, G) - Z - \Phi \beta(Z)$$  \(\text{(8)}\)

with

$$\tilde{M} \equiv \alpha M, \ldots \bar{\Psi} \equiv \alpha \Psi, \quad \bar{\Phi} \equiv \alpha \Phi$$

The standard solution technique yields, in addition to the constraints (3) and (4), the first-order necessary conditions:

$$u_C = M / \sigma \Psi$$  \(\text{(9)}\)

$$-u_L / u_C = W$$  \(\text{(10)}\)

$$\dot{M} = M[\beta(Z) - R]$$  \(\text{(11)}\)

$$\dot{\Psi} = \Psi(\sigma + \beta(Z)) + \Phi \beta'(Z)$$  \(\text{(12)}\)

$$\Phi = 1 + \beta(Z) \Phi$$  \(\text{(13)}\)

where \(\Phi\) is the shadow value of the auxiliary variable \(\alpha\) and \(\Psi\) and \(M\) represent the shadow values of the stock of habits and the nonhuman household wealth, respectively.

Note that \(\Psi_t\) by definition also measures the maximized lifetime utility starting from
an arbitrary point in time $t$. First order condition (9) reveals that an optimizing household will compare the marginal benefit of an extra unit of consumption with the effective or normalized marginal cost of increasing nonhuman wealth. Equation (10) states the well known intratemporal condition that the marginal rate of substitution between consumption and leisure equals the opportunity cost represented by the real wage rate. Conditions 11) to 13) present the optimal rules of motion for the shadow values of nonhuman household wealth, the stock of habits and of the shadow value of the auxiliary variable $\alpha$. In addition the following transversality conditions must be satisfied: 

$$\lim_{t \to \infty} \alpha(t) M(t) A(t) = \lim_{t \to \infty} \alpha(t) \Psi(t) Z(t) = \lim_{t \to \infty} \alpha(t) \Phi(t) = 0.$$ 

It is a straightforward exercise to show, as in Epstein and Shi (1993) that this setup gives rise to an endogenous rate of time preference $\rho = \beta(Z) - [\Psi(\sigma + \beta(Z)) + \Phi \beta'(Z)] / \Psi$. The assumption $\beta'(\cdot) > 0$ implies that as households get wealthier they become more impatient to consume. This notion has struck some as counter intuitive. However, those who defend the notion have offered a number of justifications of such `increasing marginal impatience` (see Lucas and Stokey (1984)). First, such preferences are necessary for dynamic stability. Second, the alternative, $\beta'(\cdot) < 0$, which implies that as households get wealthier their desire to accumulate wealth increases, is also counter intuitive. Third, one could justify increasing impatience as pertaining to economies with relatively higher levels of consumption, while decreasing impatience would apply to low levels of consumption and wealth. 13

Finally, there is recent empirical evidence supporting this last notion in the form of a

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13See Shi and Epstein (1993), Epstein (1987), and Obstfeld (1990) for more on increasing impatience.
non-linear relationship between income levels and savings rates, showing that savings rates first rise and then fall with increases in income levels.\textsuperscript{14}

Moving now to a description of foreign households, it should suffice to point out that equations (9)-(13) have their foreign counterparts, with foreign variables denoted by asterisks.

**Firms**

Identical, competitive firms in each country employ capital, $K$, and labor, $L$, to produce a single good that is used for both consumption and investment under constant returns to scale. Here we formalize firm behavior in the home country and leave it to the reader to extend this to the foreign firms.

We assume that the production function is of the conventional constant elasticity of substitution (CES) type:

\[ Y = F(K, L) = [\Lambda \delta K^{-\eta} + (1 - \delta) L^{-\eta}]^{-1/\eta}. \]

For analytical convenience, the rate of depreciation of capital is set equal to zero. We suppose that there are adjustment costs in investment, so that it takes $I[1 + \Gamma(I/K)]$ units of output to increase the capital stock by $I$ units. The installation cost function is specialized to $\Gamma(I/K) = (1/2\chi)(I/K)$.

Firms choose the time path of investment to maximize the present discounted value of net profits $\Pi = F(K, L) - I(1 + \Gamma) - WL$ subject to the constraint $I = K$.\textsuperscript{15} The solution of this problem yields:

\[ Q = 1 + \Gamma(I/K) + (I/K)\Gamma'(I/K), \quad (14) \]

\[ \dot{Q} = RQ - F_K(K, L) - (I/K)^2\Gamma'(I/K), \quad (15) \]


\textsuperscript{15}For notational convenience it is assumed that investment is exclusively financed by retained earnings.
where (14) implies that the shadow value of investment (Tobin's $Q$) is equal to one plus the marginal cost of investment. The law of motion for $Q$ is given by (15). (16) is the usual intratemporal equilibrium condition of marginal productivity of labor being equal to the wage rate. Equation (14) can be used to express investment as the following function of $Q$:

$$I = \dot{K} = K\varphi(Q), \quad \varphi'(Q) = \chi > 0, \quad \varphi(1) = 0.$$ (17)

That is, investment is an increasing function of the value of capital; and when the value of capital equals its unitary replacement cost ($Q = 1$) investment is zero. Analogous optimality conditions hold in the foreign country.

**The government**

The government is modeled in the most simple way here. As Ricardian Equivalence holds in our setting, we assume that all government spending is financed by lump-sum taxes. Thus, $G = T$ and $G^* = T^*$. Examples of previous studies with household utility depending positively on the level government spending can be found in Bailey (1962) and Baxter and King (1993).

**Markets and prices**

At a given point in time for households to be satisfied with the composition of their portfolios the rates of return on home and foreign equities, $R$ and $R^*$, should be identical

$$R \equiv \left(\frac{\Pi + Q\dot{Q}}{QK}\right) + \frac{\dot{Q}}{Q} = \left(\frac{\Pi^* + Q^*\dot{Q}^*}{Q^*K^*}\right) + \frac{\dot{Q}^*}{Q^*} \equiv R^*$$ (18)

where the terms in parenthesis denote an equity's current yield, and $\frac{\dot{Q}}{Q}$ and $\frac{\dot{Q}^*}{Q^*}$ represent capital gains. The current yield terms consist of `cash` dividends, $\Pi$ and $\Pi^*$,
and equity dividends, \( QI \) and \( Q^*I^* \). For modeling purposes it helps to define \( \Theta \equiv \frac{Q^*}{\bar{Q}} \)
as the relative price of foreign equity, and rewrite condition (18) as follows:

\[
\hat{\Theta} = \left( \frac{\Theta}{\bar{Q}} \right) \left[ \left( \frac{\Pi + QI}{K} \right) - \left( \frac{\Pi^* + \Theta QI^*}{\Theta K^*} \right) \right]
\]

(19)

Clearance of the world output market requires that world supply equal world demand:

\[
F(K, L) + F^*(K^*, L^*) = C + C^* + I(1 + \Gamma(\cdot)) + I^*(1 + \Gamma^*(\cdot)) + G + G^*.
\]

(20)

Finally, world capital market equilibrium determines the rate at which home households accumulate foreign equity. This is the market where flows of equity of uniform yield are traded using current output as the means of payment. Defining the stocks of nonhuman or financial wealth in the home and foreign country as \( A \equiv QK + Q^*B \) and \( A^* \equiv Q^*K^* - Q^*B \), where \( B \) represents the net holdings of foreign equity by domestic households, and using (3), (18) and \( G = \bar{T} \), capital market equilibrium can be represented by the following equation:

\[
\hat{B} = \left( \frac{1}{Q^*} \right) \left[ F(K, L) - I(1 + \Gamma(I/K)) + (\Pi^* + Q^*I^*) \left( \frac{B}{K^*} \right) - C - G \right]
\]

(21)

which yields the accumulation of foreign assets by domestic households.

**Characterization of the equilibrium**

To characterize the equilibrium behavior in this economy we begin by describing its steady state. The long-run relations that must be satisfied come from the market clearance and optimality conditions. It is obvious from Equations (15), (17) and its foreign counterparts, and the definition of \( \Theta \) that at the steady state \( \bar{Q} = 1 \), \( \bar{Q}^* = 1 \), \( \bar{\Theta} = 1 \) and \( \bar{R} = F_K(\bar{K}, \bar{L}) \).\(^{16}\)

\(^{16}\)In what follows steady state values of the variables are represented with an upper bar.
From (11), (12), (13), (18), (19) and (20) it is straightforward to see that:

\[
\tilde{R} = \beta(\tilde{Z}) \equiv \xi \tilde{Z}, \quad \tilde{R}^* = \beta(\tilde{Z}^*) \equiv \xi^* \tilde{Z}^*, \quad \Phi = -\tilde{R}^{-1}, \quad \Phi^* = -\tilde{R}^{*-1},
\]

\[
\hat{\Psi} = \frac{\xi}{\hat{R}(\sigma + \hat{R})}, \quad \hat{\Psi}^* = \frac{\xi^*}{\hat{R}^*(\sigma^* + \hat{R}^*)}
\]  \quad (I)

where we have specialized \( \beta(Z) = \xi Z \). Note that from \( \tilde{R} \equiv \tilde{R}^* \) it follows that \( \Phi = \Phi^* \).

Similarly from (4), (9), (10), (15), (16), (17), and their foreign counterparts the following steady-state conditions are derived:

\[
\tilde{M} = \sigma \hat{\Psi} u_c(\tilde{C}, \tilde{L}), \quad \tilde{M}^* = \sigma^* \hat{\Psi}^* u_c^*(\tilde{C}^*, \tilde{L}^*),
\]

\[
-\frac{u_L(\tilde{C}, \tilde{L})}{u_c(\tilde{C}, \tilde{L})} = \tilde{W} = F_L(\tilde{K}, \tilde{L}), \quad -\frac{u_L^*(\tilde{C}^*, \tilde{L}^*)}{u_c^*(\tilde{C}^*, \tilde{L}^*)} = \tilde{W}^* = F_L^*(\tilde{K}^*, \tilde{L}^*),
\]

\[
U(\tilde{C}, \tilde{L}, G) = \tilde{Z}, \quad U^*(\tilde{C}^*, \tilde{L}^*, G^*) = \tilde{Z}^*,
\]

\[
\tilde{R} = F_K(\tilde{K}, \tilde{L}), \quad \tilde{R}^* = F_K^*(\tilde{K}^*, \tilde{L}^*)
\]  \quad (II)

The steady state values of \((\hat{\Psi}, \hat{\Psi}^*, \Phi, \Phi^*, \tilde{Z}, \tilde{Z}^*, \tilde{M}, \tilde{M}^*, \tilde{C}, \tilde{C}^*, \tilde{L}, \tilde{L}^*, \tilde{K}, \tilde{K}^*)\) are obtained from the fourteen equations shown in blocks (I) and (II) while the steady state value \( \tilde{B} \) is obtained from \( \tilde{R} \tilde{B} + F(\tilde{K}, \tilde{L}) = \tilde{C} + \tilde{G} \), which is the steady state condition associated with equation (21).

With the steady-state values of all the variables at hand, it is convenient to follow Campbell (1994) and loglinearize the optimality conditions and dynamic equations around the steady state. We start by loglinearizing the expressions for consumption and labor (in what follows lowercase letters denote log deviations from

\footnote{Note that here with endogenous rates of time preference; we do not have to impose, as in the standard model, the strong and arbitrary condition that both countries have the same exogenous rate of time preference. One consequence of this is that here the real rate of interest will freely adjust in the long run. Another consequence is that policies will not be hysteretic; on this see Karayalcin (1999).}
the steady state, with, for instance \( c = \ln C - \ln \bar{C} \), denoting the log deviation of consumption:

\[
c = \eta_{c, \psi} \psi - \eta_{c, \mu} \mu + \eta_{c, k} k
\]  

(22)

\[
l = \nu (\epsilon_{w,k} k - c)
\]  

(23)

where

\[
\eta_{c, \psi} = \frac{\sigma_c (\nu_l - \epsilon_{w,l})}{(\epsilon_l^{-1} - \epsilon_{w,l})}, \quad \eta_{c, \mu} = \eta_{c, \psi}, \quad \eta_{c, k} = \frac{\sigma_c (\theta - 1)(1 - \gamma) \nu_l \epsilon_{w,k}}{\epsilon_l^{-1} - \epsilon_{w,l}}
\]

\[
\nu_l \equiv \frac{\bar{L}}{1 - \bar{L}}, \quad \nu \equiv \frac{1}{(\nu_l - \epsilon_{w,l})}, \quad \epsilon_l \equiv \frac{1}{\sigma_c \theta \nu_l}, \quad \sigma_c = \frac{1}{\gamma(\theta - 1) + 1}
\]

Coefficients \( \eta_{c, \psi} \), \( \eta_{c, \mu} \) and \( \eta_{c, k} \) denote the elasticities of consumption with respect to the capital stock and the shadow prices of the stock of habits (\( \psi \)) and wealth (\( \mu \)). The intertemporal elasticity of substitution of consumption is denoted by \( \sigma_c \), while \( \epsilon_l \) stands for the wage elasticity of labor supply. \( \nu_l \) represents the steady-state ratio of labor to leisure, whereas \( \epsilon_{w,l} \) and \( \epsilon_{w,k} \) stand for the elasticity of the wage rate with respect to the labor and capital inputs. Finally, \( \nu \) measures the effect of changes in consumption on labor supply taking into account the negative effect of an increase in employment on the real wage.

The expression for \( \eta_{c, k} \) shows that a rise in the capital stock will increase consumption if \( \theta > 1 \), given the fact \( \epsilon_{w,k} > 0 \) and \( \epsilon_{w,l} < 0 \). Intuitively, an increase in the capital stock raises the real wage rate leading to an increase in labor supply\(^{18}\). Given \( u_{cL} > 0 \) (as would be the case when \( \theta > 1 \)) this pushes consumption up. If preferences are separable in consumption and leisure, as in the logarithmic felicity case with \( \theta = 1 \), accumulation of capital would, ceteris paribus, have no effect on

\(^{18}\) Here we assume that the real wage is not sufficiently high to induce a backward bending section in the labor supply curve.
consumption. An increase in $\mu$ or a decrease in $\psi$ raises the marginal utility of consumption (equation (9)), changing the consumption-leisure trade-off against consumption and in favor of labor services supplied.

The effect of changes in the capital stock and the relevant shadow prices on labor supply can also be seen from the following expression:

$$l = \frac{-\sigma_c}{(\varepsilon_l^1 - \varepsilon_w)} \psi + \frac{\sigma_c}{(\varepsilon_l^1 - \varepsilon_w)} \mu + \frac{\varepsilon_w k}{(\varepsilon_l^1 - \varepsilon_w)} k$$

$$= -\eta_l \psi + \eta_l \mu + \eta_l k$$

(23.1)

As the economy accumulates capital, the real wage rises by a factor of $\varepsilon_{w,k}$, inducing an increase in labor supply. Yet, this accumulation of capital, by stimulating consumption (when $\theta > 1$) also lowers labor supply to a smaller extent.

Loglinearizing (18) and (20), the following expressions for $r = \ln R - \ln R$ and $q = \ln Q - \ln Q$ are obtained:

$$r = \eta_{r,k} k + \eta_{r,l} l + \eta_{r,q} q$$

(24)

$$q = \eta_{q,k} k + \eta_{q,l} l + \eta_{q,l} l^* + \eta_{q,c} c + \eta_{q,\theta} \theta$$

(25)

With these expressions for $r$ and $q$, and using the conditions for optimal behavior of households and firms in both countries, as well as the equilibrium relations from stock, output and capital markets, we obtain a system of twelve differential equations characterizing the evolution of the integrated world economy over time:

$$\dot{x}^T = \Omega x^T$$

(26)

---

19 If $\theta < 1$, increases in income will tend to reduce consumption. This contradicts empirical evidence.

20 Again, the results for the home country apply *mutatis mutandis* to the foreign economy.

21 The interpretation of (22) and (23) and the associated definitions can be found in Karayalcin (2003).
where \( x = (\mu, \mu^*, \psi, \psi^*, \phi, \phi^*, \theta, k, k^*, z, z^*, b) \), \( \dot{x} \) is a vector containing the derivatives of the variables in \( x \) with respect to time, and \( \Omega \) is a \( 12 \times 12 \) Jacobian matrix of partial derivatives with its \( ij \) elements given by \( \eta_{i,j} = \frac{\partial \dot{x}_i}{\partial x_j}, \ \forall \ i \in \dot{x} \) and \( \forall \ j \in x \).\(^{22}\)

Since the system has five predetermined variables \((k, k^*, z, z^*, b)\) and seven jumping variables \((\mu, \mu^*, \psi, \psi^*, \phi, \phi^*, \theta)\), for it to be locally saddlepath stable it must be the case that five of the eigenvalues must be either real and negative or have negative real parts. It is straightforward to show that this is the case here. One can also show that under the plausible benchmark parameters used in the simulations discussed below the variables in the system will exhibit cyclical transient paths.\(^{23}\)

I.III. International comovements: Transmission of shocks in the world economy

In order to study the international comovements of variables, we study the international transmission of two shocks: 1) the effects of a permanent fiscal expansion financed by nondistortionary taxes \( dT = dG > 0 \) in the home country and 2) the effects of a positive domestic productivity shock, \( d\Lambda > 0 \) also in the home country.

To calibrate the model we choose the parameters so that we have initial values of \( L \approx 1/3, \ R \approx 0.01 \) (which is the standard per quarter value in real business cycles models), \( C/Y \approx 0.7, \ G/Y \approx 0.3, \) and \( \frac{R+K}{Y} \approx 1/3 \) in both economies. We also

\(^{22}\)See the appendix for the coefficients that appear in (24), (25) and in \( \Omega \) of the system given by (26).

\(^{23}\)See Shi and Epstein (1993) who also show that cycles are more likely to happen if habits adjust at a pace slower than a critical level, with this critical level increasing with the value of the parameter that governs the response of the endogenous discount rate to changes in the stock of habits (\( \xi \) here).

\(^{24}\)This is consistent with the finding by Ghez and Becker (1975) that households allocate approximately one-third of their productive time to market activities.
assume that $\sigma = 0.15$ in both countries. As stated before, a low value of $\sigma$ (consistent with habits spreading over longer periods) makes cyclical transient paths more likely to arise. Finally, Eichenbaum et al. (1988) present statistical evidence from U.S. time series suggesting that a value of $\theta \in [0.5,3]$ is appropriate, so we restrict our choices to this range. All the parameters are chosen in conformance to the best practice in the literature where available. The initial values of the parameters and variables are displayed at the bottom of Table I.2.

For each of the shocks, we show the effects in the long run in both countries in Table 2 and then illustrate graphically (Figures I.1. and I.2.) the adjustment paths. We explain the underlying economic forces that generate these paths, highlighting in each case both the initial reaction to the shock and the subsequent convergence to the new steady-state. Table I.3. presents the cross-country correlations that summarize the predicted international comovements of our model.

**Increase in government spending: Long run**

**Home economy**

In the long-run the fiscal shock leads to the typical crowding-out effect in private spending. Both consumption and the capital stock fall with respect to the original steady state. The increase in government spending also leads to higher employment levels in the long-run. Intuitively, the higher level of taxes needed to finance the increase in government spending, $G$, lowers household wealth, triggering a fall in both consumption and leisure. The rise in long-run employment (work effort) increases the marginal productivity of capital (the real interest rate) and reduces the

---

25See, for instance, Ahmed (1986) who presents empirical evidence that government spending has a significant crowding-out effect on private spending.
marginal productivity of labor (the wage rate). The fall in the capital stock and the rise in employment have opposing effects on output in the long run. Under the parameters chosen here output rises in the long-run. The increase in home government spending predictably lowers home holdings of net foreign assets.

**Foreign economy**

The crowding-out effect of the domestic fiscal expansion is “exported” to its trading partner: foreign consumption, capital, output, labor and its marginal productivity all decline in the long-run. This is a consequence of the increase in the long-run real interest rate induced by the demand shock in the home country.  

**Increase in government spending: Transitional dynamics**

To see how the home economy adjusts in response to the fiscal shock and how this shock is internationally transmitted observe Figure I.1. The figure shows the adjustment paths of the capital stock, labor, investment, output, marginal productivities of capital and labor, Tobin's Q, world's return on equity, and the stock of net foreign assets in both countries after a 1% increase in government spending in the home economy.

**Home economy.**

Initially, given the predetermined capital stock, increased provision of government services financed by higher taxes has two opposing effects on consumption and leisure. The negative wealth effect of higher taxes, ceteris paribus,

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26 Devereaux and Shi (1991) which consider a two country model with recursive preferences where rates of time preference are endogenous as in here, also find that a higher government spending will reduce consumption in both countries. However, since in their model steady-state consumption and the real interest rate are monotonically related, the reduction in consumption implies lower interest rates and a higher steady-state capital stock in both countries. This stands in contrast with our finding of higher real interest rates and lower steady-state capital stocks in both economies.
would cause consumption and leisure to drop. However, the effect of higher public services on the shadow price of habits is such that households would optimally choose to increase consumption and leisure. This second effect dominates on impact in our simulation. With labor supply (employment) down, on impact output falls as well. Since the wage rate rises and the marginal productivity of capital falls, home firms suffer an immediate decrease in profits. One consequence of this is a decline in domestic equity prices on impact, triggering a period of capital decumulation.

As Figure I.1 shows, as employment gradually starts to recover so does output. Following the decline in the capital stock, the marginal productivity of capital starts rising, gradually increasing equity prices. As the stock market recovers and domestic equity prices rise, $Q$ eventually rises above the critical level of $1$ when firms again find it profitable to undertake investments. This process leads the economy to its new long-run equilibrium. Note that the hump-shaped transient paths in the figures are consistent with the cyclical dynamics generated by the complex characteristic roots of the system.

**Foreign economy.**

The long-run rise in the real rate of interest (triggered by fiscal expansion in its trading partner), leads to a drop in foreign equity prices as well. As the shock is transmitted from the home to the foreign economy, we observe an initial increase in foreign consumption and leisure, which is gradually reversed. As foreign labor supply and employment falls, output, profits and equity prices follow suit on impact. Firms start a process of disinvestment in response to lower equity prices.
As is the case in the home country, capital decumulation gradually raises the marginal productivity of capital, eventually starting a recovery process in the stock market. The convergence to the new equilibrium in the foreign economy is hump-shaped as is the case in its trading partner, with the economy following a cyclical adjustment path.

**Cross-country correlations**

The first point to note here is that, on impact the ratio of home to foreign consumption levels, $C/C^*$, jumps up, then gradually falls, starting a cyclical adjustment path which ends at a lower steady state level (Figure I.3.). Again, this behavior contrasts with the standard result in most of the two-country models in the literature, where the ratio remains constant along the transition path, giving rise to a correlation coefficient of unity. We should note that the cross-country consumption correlation we obtain, though higher than the one suggested by stylized facts, is lower (though marginally) than the value of 0.88 predicted by the baseline BKK model.

Further, the predicted cross-country consumption correlation in our model is lower than the output correlation. This is a significant improvement over the literature which typically obtains not only lower output correlations than consumption, but also gets the wrong (negative) sign for the former.

Given the evolution of labor discussed above, we also obtain positive cross-country employment correlations, albeit higher than the ones suggested by stylized facts. Furthermore, the model here consistently yields relatively high positive correlations in investment for reasons discussed above (see Table I.3.). Finally, given the link between investment and the stock market prices, the model predicts positive
cross-country correlations in equity prices as well. These positive cross-country correlations in employment, investment and output constitute a marked improvement with respect to the standard negative correlations predicted by the BKK model, and that are typically found in the literature.

A positive productivity shock: The long run

Home economy

As expected, in the long run the positive supply shock increases the levels of output, consumption, employment, and the marginal productivities of capital and labor in the home country. The long-run rise in the productivity of capital translates into an increase in the rate of interest across steady states. Faced with such higher interest rates firms choose a lower capital stock in the long run.

Foreign economy

In the long run the rise in home productivity creates an excess stock demand for home equity, the elimination of which requires an decrease in foreign capital as well. By reducing the marginal productivity of foreign labor and, thus, its wage, this calls for a fall in foreign labor supply and employment. Facing lower incomes, foreign households reduce their consumption levels across steady states.

A positive productivity shock: Transitional dynamics

Figure I.2. shows the transitional paths of the relevant variables for both economies as they adjust to a 1% positive productivity shock in the home economy.

Home economy.

Initially, ceteris paribus, the positive supply shock gives rise to a positive wealth effect. As a result both consumption and leisure increase, the latter leading,
thereby, to a fall in labor supply and employment. Since the shock raises long-run world interest rate, we observe a drop in home equity prices on impact. However, with the labor supply rising initially on the adjustment path, wages fall and profits and equity prices start rising, though they remain below the replacement cost of capital for a while. As long as this is the case home firms decumulate capital. Eventually, however, decreases in the capital stock push the productivity of capital up so much that home equity prices exceed the replacement cost of capital, triggering a period of capital accumulation. The net result of the cycles in the long-run is that the home economy ends up with a lower capital stock.

**Foreign economy.**

The positive supply shock that hits the home economy raises rates of return everywhere. On impact, foreign households taking advantage of the opportunity created, increase their consumption levels and leisure, lowering their labor supply. On the other hand, the shock in home productivity, creates an excess stock supply of foreign equity, reducing their price and creating a period of foreign capital decumulation. As income remains low as a result, consumption decrease along the initial phase of the transition. So does leisure with the consequence that foreign labor supply displays an upward movement.

Along the adjustment path, continued capital decumulation gradually raises the marginal productivity of capital and puts upward pressure on equity prices. The foreign economy goes through the by now familiar cyclical adjustment to eventually converge to its new steady state.
Cross-country correlations

The positive supply shock that hits the home economy here gives rise to a cyclical adjustment in the home-to-foreign consumption levels and positive consumption correlations across countries as in the case of the demand shock. Here the model also predicts a positive cross-country output correlation. This is in line with empirical evidence. However, as with the previous shock, the correlation coefficients remain higher than the ones observed in the data. The cross-country consumption correlation is again slightly lower than the one predicted by the baseline BKK model. Furthermore, as in the case of the government shock, the model yields the right ranking with regard to consumption and output cross-country correlations. As far as cross-country investment and employment correlations are concerned, once again the present results show a significant improvement upon the typical results in the literature, yielding positive employment, investment (and stock market) correlations across countries that conform qualitatively to the stylized facts.

I.IV. Conclusions

The paper studies the effects of supply and demand shocks in an integrated world economy and the associated international cross-country correlations. The paper adopts the habit-forming endogenous rate of time preference structure of Shi and Epstein (1993). Firms face adjustment costs in investment. The model generates long-lasting responses to shocks. Consumption levels of households in different countries do not mimic each other and display cyclical adjustment. The predicted cross-country consumption correlations, though higher than the one suggested by stylized facts, are (marginally) lower than the value predicted by BKK (1995). The
model also predicts positive cross-country correlations in output, investment (and the stock market) and employment, a result that stands in sharp contrast to the puzzling negative correlations found in the literature. The positive cross-country correlations generated by the model, albeit high, are qualitatively in line with the international business cycle stylized facts.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample Average&lt;sup&gt;a&lt;/sup&gt;</td>
<td>BKK sample&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Output</td>
<td>0.22 (0.03)</td>
<td>0.59 (0.02)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.14 (0.02)</td>
<td>0.51 (0.02)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.18 (0.04)</td>
<td>0.53 (0.02)</td>
</tr>
<tr>
<td>Employment</td>
<td>0.20 (0.03)</td>
<td>0.61 (0.01)</td>
</tr>
</tbody>
</table>

First line: Cross-country correlation. Second line: Standard deviation.

<sup>a</sup> This column correspond to the averages from 190 pairwise cross-country correlations in ACZ (2004).

<sup>b</sup> This column correspond to the correlation of each variable with respect to the same U.S. variable using the countries and the period considered by BKK (1995).
Notes:

Benchmark initial parameter values in A):

- Production function:
  \( \Lambda \approx 0.15; \delta = 0.34; \eta = 0.005; K = 2.65; L = 0.33 \)
  \( \Lambda^* \approx 0.18; \delta^* = 0.31; \eta^* = 0.005; K^* = 2.74; L^* = 0.33 \)

- Utility function:
  \( \gamma = 1/3; \theta = 3.0; a \approx 142.90; G = 0.033; C \approx 0.07 \)
  \( \gamma^* = 1/3; \theta^* = 1.4; a^* \approx 42.54; G^* = 0.033; C^* \approx 0.08 \)

- Other key parameters:
  \( \sigma = 0.15; \xi = 11; \chi = 0.9 \)
  \( \sigma^* = 0.15; \xi^* = 5; \chi^* = 0.5 \)

This set of parameters is consistent with the following initial values:

- Consumption: \( C / Y \approx 0.67; G / Y \approx 0.33; RK / Y \approx 0.33 \)
- Capital stock: \( FK(K,L) \approx 0.70; FL(K,L) \approx -0.36 \)
- Output: \( R = R^* \approx 0.01 \)

Benchmark initial parameter values in B):

- Production function:
  \( \Lambda \approx 0.12; \delta = 0.58; \eta = 0.50; K = 2.60; L = 0.33 \)
  \( \Lambda^* \approx 0.14; \delta^* = 0.59; \eta^* = 0.57; K^* = 2.75; L^* = 0.33 \)

- Utility function:
  \( \gamma = 1/3; \theta = 3.0; a \approx 139.33; G = 0.033; C \approx 0.07 \)
  \( \gamma^* = 1/3; \theta^* = 1.4; a^* \approx 41.81; G^* = 0.033; C^* \approx 0.08 \)

- Other key parameters:
  \( \sigma = 0.15; \xi = 11; \chi = 0.85 \)
  \( \sigma^* = 0.15; \xi^* = 5; \chi^* = 0.45 \)

This set of parameters is consistent with the following initial values:

- Consumption: \( C / Y \approx 0.68; G / Y \approx 0.33; RK / Y \approx 0.33 \)
- Capital stock: \( FK(K,L) \approx 0.71; FL(K,L) \approx 0.54 \)
- Output: \( R = R^* \approx 0.01 \)

Table I.2. International transmission of shocks.
Long-run elasticities.

<table>
<thead>
<tr>
<th></th>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>-0.69</td>
<td>-0.21</td>
</tr>
<tr>
<td>Labor</td>
<td>0.67</td>
<td>-0.21</td>
</tr>
<tr>
<td>Capital stock</td>
<td>-0.40</td>
<td>-1.22</td>
</tr>
<tr>
<td>Output</td>
<td>0.31</td>
<td>-0.52</td>
</tr>
<tr>
<td>( F_K(K,L) )</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>( F_L(K,L) )</td>
<td>-0.36</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

A) 1% Increase in government spending (Positive demand shock)

B) 1% Increase in productivity (Positive supply shock)
Table I.3. International cross-country correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, C*</td>
<td>0.86</td>
</tr>
<tr>
<td>Y, Y*</td>
<td>0.87</td>
</tr>
<tr>
<td>L, L*</td>
<td>0.86</td>
</tr>
<tr>
<td>I, I*</td>
<td>0.93</td>
</tr>
<tr>
<td>Q, Q*</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Note: The values in the table are the Pearson's correlation coefficients calculated from the values of the variables along the entire transient paths.

A) 1% Increase in government spending (Positive demand shock)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, C*</td>
<td>0.85</td>
</tr>
<tr>
<td>Y, Y*</td>
<td>0.86</td>
</tr>
<tr>
<td>L, L*</td>
<td>0.85</td>
</tr>
<tr>
<td>I, I*</td>
<td>0.93</td>
</tr>
<tr>
<td>Q, Q*</td>
<td>0.74</td>
</tr>
</tbody>
</table>

B) 1% Increase in productivity (Positive supply shock)
Figure I.1 Transient paths of consumption, employment, investment, output, marginal productivities of capital and labor, shadow price of capital, world’s return on equity, and net foreign asset holdings after a 1% increase in government spending in the home economy.
Figure I.2 Transient paths of consumption, employment, investment, output, marginal productivities of capital and labor, shadow price of capital, world’s return on equity, and net foreign asset holdings after a 1% positive productivity shock in the home economy.
Figure I.3. Transient paths of the ratio of home-to-foreign consumption levels.
II. COMPARISON UTILITY, ENDOGENOUS TIME PREFERENCE, AND ECONOMIC GROWTH

II.I. Introduction

This paper exploits a natural experiment, the large-scale destruction of capital during World War II (henceforth, WWII) in Europe and Japan, to analyze how well a model where consumers' preferences exhibit comparison-based utility and endogenous discounting do in mimicking the post-war paths taken by these economies.

The most important papers in the literature studying the transitional dynamics after a shock destroying part of the capital stock or productive capacity have confined their attention to specifications with a constant and exogenous rate of time preference. Carroll et al. (1997) is the first to examine the responses of an economy to a negative shock that reduces its capital stock in a framework of comparison-based utility. Their main goal is to study the different dynamics that can be obtained by introducing time non-separable preferences with respect to the conventional case of time-separable preferences. In order to isolate the role of preferences, they intentionally restrict the production side to the simplest framework of an AK technology. They succeed in introducing sluggishness as the economy approaches its balanced growth equilibrium along a transitional path.

This is in contrast to the result with conventional preferences where the economy is always on its balanced growth path after a shock. Yet, as Alvarez-Cuadrado et al. (2004) show, whether the production function has diminishing rather than constant returns to capital (like in the AK technology) has important consequences. The adjustment paths of the simulations performed under the AK
technology exhibit monotonic behavior, something that is not desirable to help replicate certain observed stylized facts.

Since the evolution of the key economic variables in the countries that experienced the largest destructions in their capital stocks is non-monotonic, the failure of the model in Carroll et al. (1997) to resemble the empirical evidence in these circumstances does not come as a surprise. Alvarez-Cuadrado et al. (2004) show that introducing time non-separable preferences in conjunction with a more flexible neoclassical technology can generate time paths for the growth rate and the savings rate during the early stages of the transition following an initial loss in the capital stock that are qualitatively similar to the ones observed in the data.

In a recent work on the subject Alvarez-Cuadrado (2007) identifies a comprehensive set of empirical regularities (stylized facts) of the post-war transitional dynamics in Europe. Taking the experiences of Austria, France, Germany, Italy and the Netherlands, where between one third and one quarter of the pre-war productive capacity was destroyed during WWII, he finds that the European post-war experience was characterized by high growth rates, non-monotonic adjustment of the saving rate, and a smoothly increasing capital-output ratio and wage share of output. He then uses the stylized facts of the post WWII experience as a benchmark to discriminate among the most popular model specifications found in the growth literature.

His best match to the stylized facts is obtained under a setting that combines time non-separable preferences with a constant elasticity of substitution (CES)

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27The stylized facts in Alvarez-Cuadrado (2007) are to be compared with the predicted transitional dynamics of the models after a few periods after the shock and not with respect to the reaction on impact, since the data pertaining to most of the relevant variables in his sample of European countries is available starting in 1950.
technology (henceforth, NSCES). The stylized facts identified by Alvarez-Cuadrado (2007) are summarized as follows:

- **Stylized fact 1.** The adjustment process is characterized by high and very slowly decreasing growth rates. The growth rate peaks several periods after the end of the conflict.

- **Stylized fact 2.** After 1955 the capital--output ratio smoothly increases.

- **Stylized fact 3.** The saving rate exhibits a characteristic inverted u-shape. During the first years it monotonically increases reaching its maximum after more than a decade, and thereafter slowly decreases\(^28\).

- **Stylized fact 4.** The labor income share of output, adjusted for self-employment, exhibits an upward trend, increasing on average above 10% in the period considered. The average unadjusted wage share increased almost 17%.

It should be noted that stylized fact 3 can be complemented with an observation from the experience of Japan during WWII. According to the data in Maddison (1992), the saving rate dropped on impact and then started to growth, resembling the aforementioned inverted u-shape\(^29\). Therefore, ideally a model should be able to replicate this initial drop in the saving rate. For expositional convenience the long-term trend of saving and growth in Japan and the aforementioned sample of European


\(^{29}\) There is no data for the year 1945 in Japan (the year where the atomic bombs were thrown, the conflict ended and the capital stock is assumed to have reached its minimum level), so I take the change from 1944 and 1946 to represent the initial reaction to the shock.
countries considered by Alvarez-Cuadrado (2007) is shown in Figure II.1. as a way to illustrate graphically stylized facts 1 and 3.

It is the main goal of this paper to analyze the extent to which a setting where the rate of time preference is endogenous provides a better match of these stylized facts. As I mentioned before, the most important papers on the subject have confined their attention to the behavioral assumption of a constant rate of time preference, apparently because this make the models more tractable. Nevertheless, there is no reason to assume a priori that the subjective discount rate is constant. As a matter of fact, the empirical evidence seems to support the existence of a nonlinear saving schedule, which is consistent with a nonlinear time-preference schedule such that people are more time impatient at low levels of income, become more patient as income starts to rise, and then gradually become less patient. Moreover, as noted by Fisher (1930) a person's rate of preference for present over future consumption, given a certain income stream, will be high or low according to the past consumption habits of the individual.

The reason why endogenous rates of time preference might affect transitional dynamics in a growth model is quite simple. We just have to acknowledge the fact that optimal consumption (and therefore saving) is mainly influenced by the behavior of the

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30 The proxy considered in this paper for the long-term trend in the variables is a standard Hodrick-Prescott trend filter. For the case of the saving rate the filter is applied directly to the observed saving rate and for the case of the growth rates the filter was applied to the levels of real output before calculating the growth rates. Alvarez-Cuadrado (2007) reports 5-year moving averages for each country and variable as an alternative way to assess the long-term trend in the data.

31 For a graphical illustration of stylized facts 2 and 4 the reader is encouraged to take a look at Figures 1 and 2 in Alvarez-Cuadrado (2007).

32 See, for instance, Sahota (1993), Masson et al. (1995), and Ogaki et al. (1996).
marginal productivity of capital (real interest rate) *vis-a-vis* the rate of time preference (subjective discount rate). It immediately follows that a model where the rate of time preference is endogenous, in addition to being intuitively more appealing, has the potential to provide new predictions and theoretical insights. We can expect to have much richer dynamics in the transition paths because of the additional role being played by the endogenous rate of time preference in consumption/saving decisions.

This will be the case no matter what specification for the underlying production technology we might consider. Since growth models are routinely used to derive policy conclusions, an improvement upon existing ones is a valuable contribution for policy analysis.

The incorporation of an endogenous time preference formulation in a one-sector growth model with transitional dynamics has been studied before by Uzawa (1968), Epstein and Hynes (1983), Epstein (1987), Obstfeld (1990) and Shi and Epstein (1993). A noticeable difference between the specification that I pursue in this paper and the endogenous time preference literature is that the latter has typically specified the discount rate to be a function of the level of consumption (workable only in the neoclassical model with zero per capita growth in steady state) while the specification I present here allows for positive per capita steady-state growth. As can be seen in the next section, the model formulation is rather involved and guaranteeing the convergence of the endogenous rate time preference to an economically plausible finite value is an important (technical) contribution of this paper. Previous models in the literature aiming at this objective are scarce. In this latter sense the contributions of Zee (1997) and Burgstaller and Karayalcin (1996) are the closest in spirit to the
specification that allows for endogenous discounting with positive per capita steady-state growth.33

The novelty of the paper is the introduction of a rate of time preference that varies with the relationship at each point in time between consumption and a reference stock. Instead of operating directly through the instantaneous utility function as in Carroll et al. (1997), comparison utility operates through the endogenous rate of time preference in our specification. The model is flexible enough to study the implications of different assumptions regarding the evolution of the consumer's reference stock. If the consumer's reference stock is entirely built from his own past consumption we have inward-looking preferences (habit formation), while if the reference stock is built mainly from the consumption of others or from the average consumption in the economy we have outward-looking preferences (the catching-up-with-the-Joneses phenomenon).

Empirical support for the use habit formation in growth models can be found in Deaton and Paxson (1992), Dynan (1993), and Carroll and Weil (1994), which argue that habit formation may be necessary to explain various time-series features of consumption data. Van de Stadt et al. (1985) estimate a model in which both one's own past consumption and the consumption of others influences utility. They find that the weight on the former is roughly twice the weight on the latter. More recently, Grishchenko (2007) show that habit persistence with a sufficiently long history of

33Zee (1997) builds a model with positive steady-state growth where the rate of time preference is endogenous to study the growth effects of an income tax. However, due to the degree of nonlinearities of the key equations in his model, he is forced to rely on graphical analysis to illustrate the growth implications of his exercise and no closed-form solution for the rate of time preference is provided. Burgstaller and Karayalcin (1996) study how the across-household pattern of tastes regarding economic status affects aggregate time preference, the distribution of wealth and the rate of economic growth in an economy with heterogeneous households.
consumption realizations is more consistent with observed aggregate returns properties than catching-up with Joneses; while Ravina (2007) finds the strength of internal habits to be higher than the one of external habits in household consumption choices.

Figure II.2. shows the predictions obtained with the model in this paper. These are qualitatively in line with all the stylized facts listed above. A comparison of these results with the predictions under NSCES of Alvarez-Cuadrado (2007) reveals that the specification in this paper outperforms the latter in capturing the initial drop in the saving rate and also in terms of the number of periods it takes for the saving rate to reach its peak along the transitional dynamics.

Yet, it is worth clarifying that this paper does not intend to provide a full account of the paths taken by Japan and Europe after WWII but rather to take advantage of this historical event to provide further insights about the process of economic growth by comparing the observed evidence with the predictions obtained under a theoretical framework where the rate of time preference is endogenous.

The rest of the paper is organized as follows. Section two sets up the model. Section three analyzes the adjustment paths of saving, growth, the capital-output ratio and the labor income share of output following a shock destroying a significant part of the capital stock. The predicted evolution of these variables is contrasted with the observed stylized facts, and additional insights regarding the process of economic growth are provided. Section four presents some concluding remarks.

II.II. The Model

The model shares the comparison utility framework of Carroll et al. (1997), Alvarez-Cuadrado et al. (2004) and Alvarez-Cuadrado (2007) where consumers' utility
positively depends not only on the level of their consumption but also on how their consumption compares to some reference stock. The new ingredient is that the model considers intertemporal preference dependence à la Epstein and Hynes (1983) and Shi and Epstein (1993) extended to allow for convergence of the endogenous rate of time preference to a finite constant in an environment of positive per capita steady-state growth. The consumers' problem is to maximize the discounted, infinite stream of utility:

$$\max_{\text{max}} \int_0^\infty - \exp \left( - \int_0^t U \left( C_i(\tau), \frac{C_i(\tau)}{Z_i(\tau)} \right) d\tau \right) dt$$  \hspace{1cm} (1)$$

where $C_i$ is consumer $i$'s current consumption level, $\frac{C_i}{Z_i}$ is the ratio of the current consumption level to a reference stock $Z_i$, and the utility function $U(\ldots)$ serves as the representative consumer's variable discount rate. It is assumed that $U(\ldots) > 0$, $U'(\ldots) > 0$, and $U''(\ldots) < 0$. Specifically, I define $U(\ldots)$ as:

$$U(C_i, \frac{C_i}{Z_i}) \equiv (1 - \gamma) \bar{u}(C_i) + \gamma u(\frac{C_i}{Z_i}) > 0$$  \hspace{1cm} (2)$$

where $0 \leq \gamma \leq 1$ reflects how important is the comparison component in the utility function. If $\gamma = 0$, only the absolute level of consumption, $C_i$, is important, while if $\gamma = 1$, consumption relative to the reference stock, $\frac{C_i}{Z_i}$, is all that matters. For values of $\gamma$ between zero and one, both are important in deriving utility.

Following Becker et al. (1989) and Obstfeld (1990), it is assumed that $\bar{u}(C_i)$ has an upper bound $0 < \beta < \infty$; that is, there exists a $\bar{C}_i \in \mathbb{R}_+ \cup \{+\infty\}$ such that $\bar{u}(C_i) \leq \beta$ for $C_i \leq \bar{C}_i$ and $\bar{u}(C_i) = \beta$ for $C_i > \bar{C}_i$. $^{34}$ Consequently, although in a

$^{34}$This assumption is crucial to the model possessing a well-defined and economically interesting steady state.
setting of endogenous growth the level of consumption \( C_l \) will increase without bound, the rate of time preference, as it will be shown, shall not become unbounded if consumption relative to the reference stock \( \frac{C_l}{Z_l} \) stabilizes. Moreover, and merely for analytical simplicity, it is further assumed that the utility functions \( \bar{u} \) and \( u \) take the logarithmic form. Thus, \( U(C_l, \frac{C_l}{Z_l}) \equiv (1 - \gamma)\ln(C_l) + \gamma\ln(\frac{C_l}{Z_l}). \)\(^{35}\)

Turning to the evolution of the reference stock, the present model adopts the specification of Alvarez-Cuadrado et al. (2004) where consumer \( i \)'s reference stock (the standard being used to compare his current level of consumption with) is given by:

\[
\dot{Z}_i = \sigma(C_i^\varepsilon \bar{C}^{1-\varepsilon} - Z_i) \quad \text{with} \quad \bar{C} = \frac{1}{N} \sum_{i=1}^{N} C_i
\]

where \( \bar{C} \) is the average level of consumption of the economy that is assumed to be populated by \( N \) identical and infinitely lived consumers that grows at the exogenous rate \( \frac{\dot{N}}{N} = \dot{n} \). Integration of the previous expression from \( -\infty \) to time \( t \) yields:

\[
Z(t) = \sigma \int_{-\infty}^{t} (C_i(\tau)^\varepsilon \bar{C}(\tau)^{1-\varepsilon}) \exp(\sigma(\tau - t)) d\tau
\]

so the reference stock at a point in time is just an exponentially declining weighted average of past consumption levels of a particular consumer and the economy-wide average level of consumption. The speed of adjustment, \( \sigma \), parameterizes the relative importance of the recent past in determining the reference stock. The larger is \( \sigma \) the more important is consumption in the recent past. If

\(^{35}\)Using a utility function more general than the logarithmic one--say, an iso-elastic version has no effect on the qualitative results of the paper. Burgstaller and Karayalcin (1996) use the same logarithmic specification working in a setting with heterogeneous, rather than homogeneous agents, and where the ratio serving as argument in \( u(\cdot) \) has a different meaning.
$\sigma = 0.1$; for example, then consumption over the last ten years receives 63% of the weight in determining the reference stock. If $\sigma = 0.3$; then consumption over the last ten years receives 95% of the total weight\textsuperscript{36}. Exclusively inward-looking preferences are obtained by considering $\varepsilon = 1$; a purely outward looking benchmark is obtained by setting $\varepsilon = 0$.

Regarding the production side of this economy, the setting in this paper considers that output generated by consumer $i$, $Y_i$, is determined by that consumer's capital stock, $K_i$, and his level of inelastically supplied labor, $L_i$. In the context of this paper $L_i$ is set to 1 without loss of generality. Labor productivity is assumed to grow at the exogenous constant rate, $\frac{\dot{A}}{A} \equiv g$. The production function takes the CES form as in Alvarez-Cuadrado (2007):

$$Y_i = \Lambda[\eta K_i^{-b} + (1 - \eta)(AL_i)^{-b}]^{-\frac{1}{b}} \quad b > -1$$

(5)

where $\Lambda$, might reflect any institutional factors that affect the level of output, $\eta$ determines the functional distribution of income, and $1/(1 + b)$ is the elasticity of substitution between capital and augmented labor\textsuperscript{37}. It is evident that the production technology exhibits diminishing marginal product to each individual private factor and constant returns to scale.

Final output can either be consumed currently, or saved and transformed into additional capital to yield future consumption. Assuming that the existing aggregate

\textsuperscript{36}See Carroll et al. (1997).

\textsuperscript{37}The use of a CES production technology parameterized to yield an elasticity of substitution between inputs below unity is necessary to be able to resemble stylized fact 4. Under Cobb–Douglas technology (elasticity of substitution between inputs of unity), a shock destroying part of the capital stock leads to an exactly offsetting increase in its rate of return, and consequently constant factor shares characterize the adjustment paths of any simulation performed under this technology.
capital stock in the economy depreciates at a rate $\delta$, consumer $i$'s capital stock evolves according to:

$$\dot{K}_i = Y_i - C_i - (n + \delta)K_i = (R_i - n - \delta)K_i + W_i - C_i \quad (6)$$

where in the last equality we have used the fact that (given the nature of the production function) $Y_i = R_iK_i + W_i$, with $R_i, W_i$ being the marginal productivities of capital and labor, respectively:

$$R_i = \frac{\partial Y_i}{\partial K_i} \quad (7)$$

$$W_i = Y_i - \frac{\partial Y_i}{\partial K_i}K_i \quad (8)$$

Finally, in order to make the model more tractable I follow Shi and Epstein (1993) and define an auxiliary state variable $\theta_i(t) \equiv \exp \left(-\int_0^t U \left(C_i(\tau), \frac{C_i(\tau)}{Z_i(\tau)} \right))d\tau.\right.$

Thus, the optimization problem of the representative consumer can be re-written as:

$$\max \int_0^\infty (-\theta_i(t))dt \quad (9)$$

subject to:

$$\dot{\theta}_i = -U(C_i, \frac{C_i}{Z_i})\theta_i, \quad \theta_i(0) = 1, \quad (10)$$

$$\dot{Z}_i = \sigma(C_i^{\epsilon}C_i^{1-\epsilon} - Z_i), \quad Z_i(0) > 0 \text{given}, \quad (11)$$

$$\dot{K}_i = (R_i - n - \delta)K_i + W_i - C_i, \quad K_i(0) \text{given}, \quad (12)$$

$$C_i, K_i \geq 0$$

---

$^{38}$In the closed-economy setting of this paper, consumer $i$'s level of nonhuman wealth coincides with $K_i$. The equality arises because the price of a unit of capital is normalized to 1 and there are no bond holdings.
The associated discounted Hamiltonian for the problem defined in (9)-(12) is:

\[
H = -\theta - \Phi_i U(c_i) + \tilde{\Phi}_i \sigma (C_i^{1-\varepsilon} - Z_i) + \\
\tilde{M}_i ((R_i - n - \delta) K_i + W_i - C_i)
\]

with

\[
\Phi_i \equiv \theta_i \Phi, \quad \Psi_i \equiv \theta_i \Psi, \quad \tilde{M}_i \equiv \theta_i M_i
\]

yielding, in addition to constraints (11) and (12), which are the rules of motion governing the evolution of the reference stock and of capital, the following first-order necessary conditions:

\[
\dot{M}_i = \Psi_i \sigma \left( \frac{c_i}{C_i} \right)^{1-\varepsilon} - \Phi_i U_c(C_i, \frac{C_i}{Z_i})
\]

\[
\dot{\Phi}_i = 1 + U(C_i, \frac{C_i}{Z_i}) \Phi_i
\]

\[
\Psi_i = \left[ U(C_i, \frac{C_i}{Z_i}) + \sigma \right] \Psi_i + \Phi_i U_Z(C_i, \frac{C_i}{Z_i})
\]

\[
\dot{\tilde{M}}_i = \left[ U(C_i, \frac{C_i}{Z_i}) - (R_i - n - \delta) \right] M_i
\]

where \( \Phi_i \) is the shadow price of the auxiliary variable \( \theta_i \) that serves as the discount factor in the model, and \( \Psi_i \) and \( M_i \), represent the current value shadow prices of the reference stock of consumption, and of capital (nonhuman wealth), respectively.

First order condition (15) reveals that an optimizing agent seeks to equalize the marginal utility of an extra unit of consumption to the shadow value of capital taking into consideration the overall effect of the additional unit of consumption on the reference stock and its shadow value.
Conditions 16) to 18) present optimal rules of motion for the shadow values of the auxiliary variable $\theta_i$, the reference stock and capital, respectively. In addition, the following transversality conditions must be satisfied:

$$\lim_{t \to \infty} \theta_i(t)M_i(t)K_i(t) = \lim_{t \to \infty} \theta_i(t)\Psi_i(t)Z_i(t) = \lim_{t \to \infty} \theta_i(t)\Phi_i(t) = 0$$

(19)

II. III. Macroeconomic equilibrium and transitional dynamics

I now derive the macroeconomic equilibrium. With the economy populated by N identical individuals, aggregate capital ($K$), aggregate output ($Y$), aggregate consumption ($C$), and the aggregate reference consumption stock ($Z$), are given by $K \equiv NK_t$, $Y \equiv NY_t$, $C \equiv NC_t$, and $Z \equiv NZ_t$, respectively. In addition, it is useful to notice that, in equilibrium, the average levels of the variables coincide with the actual levels of the same variables for the representative consumer (all individuals are identical in this framework).

The aggregate production function in this setting is given by:

$$Y = \Lambda[\eta K^{-b} + (1 - \eta)(AN)^{1-b}]^{1/b}$$

(20)

which yields the standard outcome of identical steady-state growth rates of capital and output ($\bar{K}^*$ and $\bar{Y}^*$):

$$\bar{K}^* = \bar{Y}^* = n + g$$

(21)

That is, along a balanced growth path, aggregate capital and aggregate output both grow at a rate equal to the population growth rate plus the exogenous growth rate in labor productivity. For analytical convenience I write the system describing the behavior of the economy in terms of variables that remain constant in steady-state

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39 Similar expressions follow for the case of the shadow prices of the capital stock, the reference consumption stock and the auxiliary state variable representing the discount factor in the model.
equilibrium. As is customary this is achieved by expressing all variables in terms of units of effective labor. I will denote the variables in terms of units of effective labor by lowercase letters. For example, \( k \equiv \frac{K}{AN} \equiv \frac{K_i}{A} \) is aggregate capital per unit of effective labor.

**Optimality conditions and steady-state equilibria**

Using the specific functional form for \( U(C_t, \frac{c_t}{z_t}) \equiv U(Ac, \frac{c}{z}) \) and incorporating the equilibrium condition \( C_t = C \), the first order necessary conditions are re-written in terms of units of effective labor as:

\[
\begin{align*}
\dot{z} &= \sigma(c - z) - zg \\
\dot{k} &= y - c - (n + g + \delta)k \\
\mu &= \sigma\epsilon\psi - \Phi\frac{1}{c} \\
\dot{\phi} &= \frac{1+U(Ac, \frac{c}{z})\phi}{A} - \phi g \\
\dot{\psi} &= \left[U(Ac, \frac{c}{z}) - g\right] \psi - \Phi\frac{1}{z} \\
\dot{\mu} &= [U(Ac, \frac{c}{z}) + n + \delta - (g + r)]\mu
\end{align*}
\]

where \( y = \Lambda[\eta k^{-b} + (1 - \eta)]^{-\frac{1}{b}} \) and \( r = \frac{\partial y}{\partial k} = \Lambda\eta((1 - \eta)\eta k^b + \eta)^{-\frac{(1+b)}{b}} \).

Imposing the stationary conditions, \( \dot{z} = \dot{k} = \dot{\phi} = \dot{\psi} = \dot{\mu} = 0 \), and using (24), the steady state values \((z^*, k^*, c^*, \mu^*, \phi^*, \psi^*)\) are determined in the following recursive manner\(^{40}\). First, (27) is solved for \( k^* \) directly. Second, (23) is solved for consumption \((c^*) \) as a function of capital \((k^*)\). Third, (22) yields the steady-state consumption to

\(^{40}\)Steady-state values are denoted with asterisks.
reference stock ratio \( \left( \frac{c^*}{z^*} \right) \) as a function of the exogenous parameters \( \sigma \) and \( g \), which in turn solves for \( z^* \).

Finally, after some algebraic manipulations using (24) to (26) and the previous results we get the steady-state values of \( \mu^* \), \( \phi^* \) and \( \psi^* \). Notice that it is necessary to assume the existence of an upper bound in the component of the utility function that depends on the level of consumption, in order to be able to obtain a finite value for the steady-state rate of time preference \( (\rho^*) \). In this setting the steady-state value of the rate of time preference is given by \( \rho^* = (1 - \gamma)\beta + \gamma\ln \left( \frac{c^*}{z^*} \right) \).

The steady-state values are as follows:

\[
k^* = \left[ \frac{\eta - (L^*-g+n+\delta) - b}{\lambda\eta} \right]^{\frac{1}{b}}
\]

\[
c^* = \Lambda\eta k^* - b + (1 - \eta) \frac{1}{\eta} - (n + g + \delta)k^*
\]

\[
\frac{c^*}{z^*} = \frac{\sigma + g}{\sigma}
\]

\[
\phi^* = -\frac{1}{\rho^*-g}
\]

\[
\frac{\mu^*}{\psi^*} = \Omega = \sigma\varepsilon - \frac{\sigma}{\gamma(\sigma+g)}[\rho^* + \sigma - g]
\]

\[
\frac{\phi^*}{\psi^*} = c^*[\sigma\varepsilon - \Omega]
\]

**Optimality conditions in log-deviations from the steady-state**

In order to tackle the nonlinearities inherent in the model, it is useful to follow Cambell (1994) and seek approximate analytical solutions by log-linearizing the optimality conditions around the steady state. In terms of notation, variables with a tilde denote natural log-deviations of that variable from its steady state. For example,
the log-deviation of aggregate capital per unit of effective labor from its steady state value is denoted by $\bar{K} = \ln k - \ln k^*$. The log-linearized first order necessary conditions of the model are given by:

$$\ddot{\mathcal{Z}} = \sigma \left(\frac{e^*}{z^*}\right) \ddot{c} - (\sigma + g) \ddot{z} \tag{34}$$

$$\dot{\bar{K}} = [r^* - (n + g + \delta)] \bar{K} - \left(\frac{e^*}{K^*}\right) \ddot{c} \tag{35}$$

$$\ddot{\mathcal{C}} = \ddot{\Phi} - \sigma e c^* \frac{\psi'}{\psi} \ddot{\psi} + c^* \frac{\mu'}{\mu} \ddot{\mu} \tag{36}$$

$$\dot{\bar{\Phi}} = \ddot{c} - \gamma \ddot{z} + (\rho^* - g) \ddot{\Phi} \tag{37}$$

$$\ddot{\mathcal{Y}} = \ddot{c} + \gamma \left(\frac{\phi^{*1}}{\psi' z^*} - 1\right) \ddot{z} - \gamma \left(\frac{\phi^{*1}}{\psi' z^*}\right) \ddot{\Phi} + (\rho^* + \sigma - g) \ddot{\psi} \tag{38}$$

$$\ddot{\mathcal{U}} = \ddot{c} - \gamma \ddot{z} + \left(\frac{(1+b)(1-\eta)r^* b k^* b}{b(1-\eta) b k^* b + \eta}\right) \ddot{K} + (\rho^* - (r^* - n - \delta) - g) \ddot{\mu} \tag{39}$$

With these expressions at hand, a system of five differential equations characterizing the evolution of the economy over time is obtained:

$$\dot{x} = \Delta x \tag{40}$$

where $x = (\bar{K}, \ddot{z}, \ddot{\Phi}, \ddot{\psi}, \ddot{\mu})$, $\dot{x}$ is a vector containing the derivatives of the variables in $x$ with respect to time, and $\Delta$ is a $5 \times 5$ Jacobian matrix of partial derivatives with its $ij$ elements given by $\frac{\partial \dot{x}_i}{\partial x_j}$, $\forall \ i \in \dot{x}$ and $\forall \ j \in \ddot{x}$.\(^{41}\)

Since the system has two predetermined variables ($\bar{K}, \ddot{z}$) and three jumping variables ($\ddot{\Phi}, \ddot{\psi}, \ddot{\mu}$), for it to be locally saddlepath stable it must be the case that two

\(^{41}\)See the appendix for the coefficients that appear in $\Delta$ of the system given by (40).
of the eigenvalues must be either real and negative or have negative real parts. It is straightforward to show that this is the case here.

**The Destruction of capital: Initial reaction and transitional dynamics**

As in the paper by Carroll et al. (1997) I perform simulations considering different values of the speed of adjustment of the reference stock, \( \sigma \). Yet, the simulations in this paper provide additional insights regarding the determinants of the steady-state saving rate and the process of economic growth. In this setting the speed at which the reference stock adjusts directly affects the steady state value of the saving rate, while in Carroll et al. (1997) the capital stock does not appear in the equations describing the dynamics of consumption and the reference stock. In Carroll et al (1997) the steady-state saving rate is the same for all possible values of \( \sigma \).

In the present setting, \( \sigma \) is a fundamental determinant of both the steady-state values of the rate of time preference and of the capital stock. Thus, considering different values of this crucial parameter allows us to analyze how the equilibrium values of the gross return on capital, rate of time preference, saving rate, capital-output ratio, labor income share of output and the asymptotic speed of convergence change as different values of \( \sigma \) are considered. In addition I analyze how the initial responses and the transitional dynamics of these variables is affected by the shock destroying a significant part of the capital stock.

I follow Alvarez-Cuadrado (2007) and perform the calibration exercises choosing as the initial value of the capital stock 50% of its steady state value. The available evidence on capital stocks suggests that by the end of the conflict the
economies had lost more than 30% of their pre-war capital stock. On the other hand, output figures suggest a loss close to 90% of the pre-war stock of capital.

**Steady-State values under alternative parameterizations**

Unlike the paper by Alvarez-Cuadrado (2007) that uses outward-looking preferences as his benchmark case, the simulations presented in this paper are performed under a parameterization consistent with inward-looking preferences ($\varepsilon = 1$). As previously stated, habit formation with a sufficiently long history of consumption realizations is more consistent with observed aggregate returns properties than catching-up with Joneses and the strength of internal habits has been found to be higher than the one of external habits in household in explaining consumption behavior.

The fact that the present specification does a better job than the one of Alvarez-Cuadrado (2007) in capturing the behavior of the saving rate confers additional support to habit formation models, which have become increasingly successful and important in explaining a variety of macroeconomic issues, such as the equity premium puzzle\textsuperscript{43}, output persistence\textsuperscript{44}, the relationship between savings and growth\textsuperscript{45}, and the response of consumption to monetary shocks\textsuperscript{46}.

\textsuperscript{42}The qualitative responses that arise from the present framework under catching up with the Joneses appear to be observationally equivalent to those of a model with conventional time additive preferences and therefore, inconsistent with the stylized facts.

\textsuperscript{43}See, for example, Constantinides (1990), Campbell and Cochrane (1999), and Abel (1990)

\textsuperscript{44}See Boldrin, Christiano, and Fisher (2001).

\textsuperscript{45}See Carroll, Overland, and Weil (2000)

\textsuperscript{46}See Fuhrer (2000)
The transitional dynamics of output, the saving rate, the capital-output ratio, and the labor share of output predicted by the model are qualitatively in line with all the stylized facts listed above.

I now describe the rest of the parameter values upon which the baseline scenario of this paper is based. $\Lambda$ is normalized to 1. $\beta$ is set to 0.04 which is the typical value used in the literature for the rate of time preference when assumed to be constant and exogenous. The depreciation rate $\delta = 0.05$, the rate of population growth $n = 0.015$, and the rate of exogenous technological change, $g = 0.02$ are standard and require no further explanation.

The parameter that controls the importance of the reference stock, $\gamma$, is set to 0.95 consistent with Van de Stadt et al. (1985) who found evidence supporting the hypothesis that utility is almost completely relative. This high value for $\gamma$ is also consistent with the range of estimates provided by Fuhrer (2000) and Ravina (2007). In the baseline scenario I assume a value of 0.35 for the speed of adjustment of the reference stock, $\sigma$, and a value of $b = 3/7$ (elasticity of substitution between inputs of 0.7) as in Alvarez-Cuadrado (2007).

Finally, the distribution parameter, $\eta$, is set to 0.55 in order to match a labor income share of output of around 2/3 (also in the baseline scenario) as the empirical evidence overwhelmingly suggests. Gollin (2002) finds labor shares for most countries in the range 0.65 – 0.80 once the proper adjustments to account for the income of the self-employed and proprietors are made. The adjusted labor shares of Japan, France, Italy and the Netherlands are all in the neighborhood of 2/3.
Table II.1 presents the steady-state values of the most relevant variables for four different values of the speed of adjustment of the reference stock \((\sigma = 0.20, \sigma = 0.35 \text{ (baseline)}, \sigma = 0.50, \text{ and } \sigma = 0.65)\). Consistent with the previous parameterization all the scenarios display an equilibrium growth rate in output per capita of 2%. The equilibrium saving rate, capital-output ratio, labor income share and the asymptotic speed of convergence increase as higher values of \(\sigma\) are considered. The opposite happens with the gross return on capital and the rate of time preference. The equilibrium values of all the variables are within an economically plausible range that is consistent with the evidence for OECD countries, and the speeds of convergence are consistent with the estimates reported by Islam (1995) and Caselli et al. (1996).

**Initial responses to the destruction of capital**

The initial reaction of the main variables to the shock destroying 50% of the capital stock is summarized in Table II.2. Output decreases between 19-22% while consumption declines fluctuate in a wider range (12-26%). A particularly striking result in the paper is that if the speed of adjustment of the reference stock is rather low, like in the case of \(\sigma = 0.20\), then saving is expected to rise on impact. For the remaining scenarios the initial drop on the saving rate fluctuates in the range of 6-17%. This decline is still very low compared with the actual experience of Japan as reported in Maddison (1992) that shows the saving rate falling 33% from 1944-1946. Yet, it is certainly closer to the empirical evidence than the one predicted under the NSCES specification of Alvarez-Cuadrado (2007) which yields a reduction of the saving rate on impact of barely 1%.

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47This paper corroborates the finding in Carroll et. al. (1997) that higher values of \(\sigma\) both reduces the initial drop in income and increases the speed with which the economy returns to its steady state level.
Another result worth mentioning is that it seems to be case that high values of the speed of adjustment in the reference stock of habits are consistent with a decline in the rate of time preference on impact, after a negative transitory supply shock.

**Transitional Dynamics after the shock**

The adjustment paths in growth models based on a comparison utility framework are typically driven by the interaction of two forces; the "rate of return effect" and the "status effect". When capital is rather low, its rate of return is high, and therefore the relative price of current relative to future consumption is high and the present value of human wealth is low. These factors contribute to the "rate of return effect" that reduces consumption increasing savings. On the other hand, the comparison with a reference stock or "status effect" induces agents to minimize the potential deviations of consumption from a benchmark level predetermined by their habits works in the opposite direction. In the context of this paper these effects operate through the rate of time preference rather than directly through the instantaneous utility function.

The predicted transient paths are shown in Figure II.2. Another improvement with respect to the NSCES specification pertains to the number of periods it takes for the saving rate to reach its peak during the transition. The saving rate peaks far too early after about only 5 periods under the NSCES specification, while in this model the peak

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48 Since the scenario with \( \sigma = 0.20 \) translates into the saving rate jumping on impact contradicting the experience of Japan, and the one with \( \sigma = 0.65 \) is qualitatively similar to the the ones in the middle range, I choose to plot only the transitional dynamics from scenarios \( \sigma = 0.35 \ and \sigma = 0.50 \).
occurs about 10 years after the shock. The rate of time preference exhibits the same inverted u-shape as the saving rate along the transient paths.

In terms of replicating the post WWII growth rates in the countries most severely affected by the conflict, the model successfully captures the average downward trend beginning in 1950. Regarding the capital-output ratio and the labor share, the model does a good job in resembling the upward trend that was observed during the post-war in Europe. Finally, the rate of return to capital peaks on impact and then monotonically decreases.

II.IV. Conclusions

This paper studies the implications for the process of economic growth of a model in which agents have comparison utility embedded in the rate of time preference. A striking feature of the proposed endogenous discounting framework is that it is compatible with positive per capita steady-state growth, standing in sharp contrast with the typical result in the literature where endogenous rate of time preference is only workable in environments with zero per capita steady-state growth. A closed-form analytical solution for the steady-state rate of time preference is derived; and simulations considering different values of the speed of adjustment of the reference stock of habits (one of its key determinants) are performed providing additional insights for the process of economic growth. The equilibrium saving rate, capital-output ratio, labor income share and the asymptotic speed of convergence increase as higher values of speed of adjustment of the reference stock are considered. The opposite happens with the gross return on capital and the rate of time preference.

49The evidence in Figure 1 shows the peak occurring around 1960 for Germany, Italy and the Netherlands, that is about 15 years after the massive destruction of capital. For the cases of France, Austria and Japan the peak takes places substantially later.
The model improves upon the existing literature in mimicking the behavior of the saving rate, in the light of the experience of Japan and Europe after the large destruction of capital that took place during WWII.
Table II.1  
Steady-state values of the most relevant variables for different values of the speed of adjustment of the reference stock

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma = 0.20$</th>
<th>$\sigma = 0.35^*$</th>
<th>$\sigma = 0.50$</th>
<th>$\sigma = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saving rate</td>
<td>0.22</td>
<td>0.28</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>Capital-Output ratio</td>
<td>2.64</td>
<td>3.30</td>
<td>3.72</td>
<td>4.00</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0.64</td>
<td>0.67</td>
<td>0.69</td>
<td>0.70</td>
</tr>
<tr>
<td>Growth output per capita</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Gross return on capital</td>
<td>15.8%</td>
<td>12.0%</td>
<td>10.4%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>9.3%</td>
<td>5.5%</td>
<td>3.9%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Asymptotic convergence speed**</td>
<td>6.3%</td>
<td>8.1%</td>
<td>10.6%</td>
<td>12.3%</td>
</tr>
</tbody>
</table>

$^*$Baseline scenario  
$^{**}$Corresponds to the smallest eigenvalue in absolute terms.

Table II.2  
Initial responses after the shock as a percentage (%) of the steady-state values

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma = 0.20$</th>
<th>$\sigma = 0.35^*$</th>
<th>$\sigma = 0.50$</th>
<th>$\sigma = 0.65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital**</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
</tr>
<tr>
<td>Output</td>
<td>-22</td>
<td>-20</td>
<td>-20</td>
<td>-19</td>
</tr>
<tr>
<td>Consumption</td>
<td>-26</td>
<td>-19</td>
<td>-15</td>
<td>-12</td>
</tr>
<tr>
<td>Saving rate</td>
<td>+15</td>
<td>-6</td>
<td>-13</td>
<td>-17</td>
</tr>
<tr>
<td>Labor income share</td>
<td>-10</td>
<td>-9</td>
<td>-9</td>
<td>-8</td>
</tr>
<tr>
<td>Gross return on capital</td>
<td>+71</td>
<td>+73</td>
<td>+74</td>
<td>+74</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>+50</td>
<td>+15</td>
<td>-25</td>
<td>-74</td>
</tr>
</tbody>
</table>

$^*$Baseline scenario  
$^{**}$Corresponds to the shock that is being considered.
Figure II.1. Evolution of Saving and Growth after World War II.

* Calculated using data from Maddison (2006). The output series was smoothed using a Hodrick-Prescott filter before calculating the growth rate.

** Hodrick-Prescott trend filters of the observed series. The data is from Maddison (1992) except Italy and Austria that are from Heston et al. (2006).
Figure II.2. Transitional dynamics generated by the model.
III. MONETARY POLICY UNDER FEAR OF FLOATING: MODELING THE DOMINICAN ECONOMY

III.I. Introduction

As Taylor (2001) points out for a country that chooses not to “permanently” fix its exchange rate through a currency board, or a common currency, or some kind of dollarization, the only alternative monetary policy that can work well in the long run is based on the trinity of (1) a flexible exchange rate, (2) an inflation target, and (3) a monetary policy rule. He further argues that an important and still unsettled issue for monetary policy in open economies is how much of an interest rate reaction there should be to the exchange rate in a monetary regime based on the aforesaid trinity.

Moreover, as Svensson (2000) asserts, all real-world inflation-targeting economies are open economies with free capital mobility, where shocks originating in the rest of the world are important, and where the exchange rate plays a prominent role in the transmission of monetary policy. He further argues that, in general, including the exchange rate in the discussion of inflation targeting has at least three important consequences. First, the exchange rate allows additional channels for the transmission of monetary policy. Second, as an asset price, the exchange rate is inherently a forward-looking and expectations-driven variable. This contributes to making forward-looking behavior and the role of expectations essential in monetary policy.

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50 Inflation target is the inflation rate around which the central bank would like the actual inflation rate to fluctuate. A monetary policy rule is a contingency plan or strategy that specifies how the central bank should adjust the instruments of monetary policy (the interest rate or a monetary aggregate) in order to meet its inflation and other targets.

51 Taylor (1993) presents a multi-country model that confers an important role for the exchange rate. Simulations using this model show, however, that if the central bank reacted strongly to the exchange rate then macroeconomic performance would worsen. That was why Taylor omitted the exchange rate in the celebrated 1993 rule for the Fed. But it is not clear that the same conclusion would hold for other countries, as Taylor himself recognizes.
Third, some foreign disturbances will be transmitted through the exchange rate, for instance, changes in foreign inflation, foreign interest rates and foreign investors' foreign-exchange risk premium. It follows that disturbances to foreign demand for domestic goods will directly affect aggregate demand for domestic goods.

This paper analyses the performance in stabilizing inflation and real output of a policy rule that reacts explicitly to deviations of the exchange rate with respect to its long-term trend, and to the stance of the economy, represented by the output gap, in the context of a small scale macroeconomic model. The exercise is carried out taking the experience of the Dominican Republic (DR) as a benchmark. This is the case mainly for two reasons. First, this country has a strong fear of floating tradition which makes it an ideal yardstick to gauge the responses of monetary policy in an environment where the exchange rate has been widely used as a nominal anchor to fight inflation. Second, in the recent past DR has taken clear steps towards strengthening its monetary policy framework and it is foreseeable that the country will eventually adopt an inflation targeting scheme.

The performance of the Dominican economy under the above policy rule is then compared to the one we would have in the presence of a standard Taylor rule where interest rates react to the output gap and to deviations of inflation from its target\textsuperscript{52}. The simulations consider the response of the key economic variables to external shocks that are of particular interest for monetary policy purposes in a highly integrated small open economy. Two shocks are considered, an economic slowdown abroad and an increase in the relevant foreign nominal interest rate.

\textsuperscript{52} In the context of this paper the inflation target is represented by the inflation rate associated with the long-term trend in the consumer price index.
The Dominican economy: A brief background

Throughout most of the second half of the last century the Dominican economy operated under a fixed exchange rate regime of DR$1 per US$\textsuperscript{53}. At the same time interest rates were fixed by resolutions of the “Monetary Board”, the organism in charge of conducting monetary policy. The fixed exchange rate regime collapsed in 1985. Interest rates began to be market-determined in 1991, and local banking operations denominated in foreign currency (deposits and loans) were allowed starting in 1996.

“Fear of floating” has characterized monetary and exchange rate policies in Dominican Republic (DR) for more than two decades now. This fear to commit to a floating exchange rate regime takes the form of frequent sterilized foreign exchange market interventions and the promotion of higher interest rates whenever the nominal exchange rate shows signs of “instability”.\textsuperscript{54} In other words the exchange rate has served as the nominal anchor in the Dominican economy monetary policy.

Figure III.1. illustrates the “de facto” swings in exchange rate policy in DR for the period 1980-2004. As it can be seen dirty floatation has been the predominant \textit{de facto} exchange rate regime since 1985. Even during the period from 1994-1998 which according to Levy-Yeyati and Sturzenegger's (2003,2005) classification can be considered as one of free floatation, fear of floating was present in the form of interest rate movements aimed at smoothing the exchange rate path.

\textsuperscript{53}The Central Bank of the Dominican Republic was created and the "Dominican peso" started to circulate as the official currency in 1947.

\textsuperscript{54}Instability is here understood as the exchange rate rising above its long-term trend.
More recently, the combination of fraudulent activities undertaken by some private banks for more than a decade, poor regulation of the financial system and the implicit guarantee of a stable currency (from the monetary policy) created the conditions for the collapse of macroeconomic stability starting in August 2002 when the Central Bank of the Dominican Republic (CBDR) begun providing liquidity assistance to the commercial bank that started the 2003-2004 banking crisis. Several months later the monetary authorities decided to go for a massive rescue of depositors to avoid a widespread collapse of the financial system. Nevertheless, two other banks that were "weak" and that participated in similar unethical activities also went broke and the magnitude of the rescue amounted to more than 20% of GDP.

Within this scenario the authorities initiated conversations with the International Monetary Fund (IMF) and signed a Stand-By Agreement. The fact that the president at the time was running for reelection together with a lack of integration within the government's economic team led to a violation of the initial agreement with the IMF. The exchange rate rose from DR$18.81 per US$ at the end of September 2002 to DR$48.62 per US$ at the end of June 2004. A new government was elected, the Stand-By Agreement was put back on track and macroeconomic stability was
restored\textsuperscript{55}. The average exchange rate at the end of December 2007 was DR$33.67 per US$\textsuperscript{56}.

As a result of the aforementioned Stand-By agreement, nowadays the CBDR exhibits a higher degree of transparency and accountability and is starting to commit more seriously to an explicit quantitative inflation target, which are key pre-conditions for an inflation targeting scheme\textsuperscript{57}. Yet, the CBDR still lacks, or at least has not made public, a coherent framework for policy decisions that adopts an internal conditional inflation forecast as an intermediate target variable. In my view, the fear of floating is so embedded in policymakers that it would take some time to substitute an inflation target anchor for the accustomed nominal exchange rate anchor.

**Small scale Models: Useful tools for policy analysis**

The bottom-line from the above exposition is that an eventual inflation targeting framework (consistent with free flotation) brings challenges for the central bank in its role of controlling inflation, specially in an environment of fear of floating. Among other things, the development of models and indicators to help in the decision making process of the monetary authorities are a must-have component.

\textsuperscript{55}It is important to mention that the counterpart of this stability has been a huge deterioration in the balance sheet of the CBDR and a substantial reduction in the degrees of freedom to conduct monetary policy through open market operations in a sustainable basis. As of December 2007, the value of standing CBDR's securities was 4.4 times the amount of domestic currency in circulation in the economy. This same ratio was barely 0.2 at the end of 2001. Unless there is a substantial change of economic policy aiming to capitalize the CBDR, it is reasonable to expect further instability in the future.

\textsuperscript{56}See Sanchez-Fung (2005) for more details on the recent economic history of the Dominican Republic and the sequence of events that lead to the 2003-2004 crisis.

\textsuperscript{57}See Svensson (1999) for more on the characteristics of an inflation targeting regime.
As Corbo and Tessada (2003) assert, the available analytical options to guide policy makers range from simple one-equation models of the most relevant variables to elaborated micro-founded models with rational expectations and a large number of relations, estimated or calibrated, that incorporate uncertainty in the solution, obtaining not only a point forecast but also a range for the key endogenous variables with a probability distribution.\textsuperscript{58}

Among the available alternatives, small scale models have gained popularity in macroeconomic analysis and policy design. Although these models are not immune to the Lucas' critique\textsuperscript{59}, they are robust enough to deal with monetary policy changes and other shocks related to it when the focus is on short-term forecasts and the qualitative impacts of policies. In addition to being easier and faster to solve, an important advantage of a small model is its ability to describe, clearly and simply, the interrelation between the main variables that are related to the transmission mechanism of the monetary policy, while maintaining theoretical coherence.\textsuperscript{60} Yet, the simplicity has the cost of leaving outside the analysis important considerations, such as the degrees of freedom to conduct monetary policy at a moment in time.\textsuperscript{61}

The rest of the paper is organized as follows. Section two presents the model and discusses its structure. To the extent that is possible the reduced-form behavioral

\textsuperscript{58}The models used in the elaboration of the monetary policy reports by most central banks that operate under an inflation targeting framework are a good example of these models. See for example Bank of England (1999) for a description of the wide range of models they use.

\textsuperscript{59}See Lucas (1976).

\textsuperscript{60}See Argov et al. (2007) for more on the advantages of small-scale macroeconomic models.

\textsuperscript{61}A relevant example that applies to the case of the DR is a rather high level of Central Bank securities in circulation and the associated quasi-fiscal (and monetizing) losses. A high level of Central Bank debt could reduce the monetary policy effectiveness when a monetary contraction is called for.
equations in the model are both firmly grounded in economic theory and able to resemble the actual evolution of the Dominican economy. Section three analyzes the response of the key economic variables to some shocks that are of particular interest for monetary policy purposes. Finally, section four provides some concluding remarks.

III.II. The Model

This paper follows the standard modeling strategy for small-scale macroeconomic models in the literature. A non-exhaustive list of papers similar in structure to the one I present in this paper are Batini and Haldane (1999), Clarida et al. (1999), Gali(2000), Svensson (2000), Kotlan (2002), Gomez (2002), Martinez, Messmacher and Werner (2002), Corbo and Tessada (2003), Arreaza et al. (2004), Golinelli and Rovelli (2005), Levin (2004), and Agov et al. (2007).

The model includes four structural equations: i) Aggregate demand (AD), normalized in the output gap, ii) aggregate supply (AS), normalized in the rate of inflation, iii) uncovered interest parity (UIP), normalized in the expected nominal exchange rate, and iv) a policy rule(PR) for setting the domestic interest rate.

The structural behavioral relations are estimated using quarterly data for the period 1992-2007. Whenever stability tests exhibit evidence of a structural break (coming from the 2003-2004 financial crisis) a reduced sample period 1992-2002 or a dummy variable to control for the peak of the crisis are considered.

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63 Chow Breakpoint, CUSUM, CUSUM of Squares and Recursive Coefficients tests.
The reduced-form equations in the model are forward looking in nature whenever accepted by the data. This is consistent with the specifications laid out in the highly influential works of Clarida et al. (1999) and Svensson (2000).

**Aggregate Demand**

The first equation in the model is a simple version of an open economy IS relating domestic output gap, \( y_t - y_t^{trend} \), to the foreign output gap, \( y_t^{US} - y_t^{US,trend} \), the real exchange rate gap, \( q_t - q_t^{trend} \), and the domestic real interest rate, \( r_t \). The foreign output gap corresponds to the one of the United States (US), which is by far the DR's most important trading partner. Output is represented by the real gross domestic product in each case. The nominal interest rate, \( i_t \), corresponds to the commercial lending rate. The real exchange rate is defined as \( q_t = e_t + p_t^{US} - p_t \), where \( e_t \) is the nominal exchange rate expressed in DR$ per US$, and \( p_t \), \( p_t^{US} \) are the consumer price indexes of the DR and the US respectively. All variables except for the case of the interest rate are expressed in natural logs.

Finally, the domestic real interest rate is derived from the Fisher equation by subtracting expected future inflation from the nominal interest rate, \( r_t = i_t - E_t \Delta p_{t+1} \), where future inflation is defined as \( \Delta p_{t+1} = p_{t+1} - p_t \). The model

---

64For the case of the aggregate demand equation a neo-Keynesian specification of the IS equation including the expected future level of the output gap as an explanatory variable --condition that is derived from intertemporal optimization--was not satisfactory in overall terms. The standard ad hoc, yet intuitively appealing specification used in this paper yields much better and robust results.

65Their results come from dynamic neokeynesian models. They incorporate rational expectations and, in general, also emphasize the explicit role of forward looking variables. Other important papers dealing with neokeynesian models are Rotemberg and Woodford (1997) and McCallum and Nelson (1999a).

66The trends, proxies of the long-term "equilibrium" values of the variables, are obtained applying a Hodrick-Prescott filter to each of the relevant variables.
specification is consistent with the assumptions that agents form their expectations in a rational way and that there is perfect foresight, \( E_t \Delta p_{t+1} = \Delta p_{t+1} \).

Before proceeding to the formal econometric exercises the order of integration of the time series at hand was investigated. The application of the standard Augmented Dickey-Fuller (ADF) test revealed that the levels of the variables \( y_t - y_t^{trend} \), \( q_t - q_t^{trend} \), and \( r_t \) are integrated of order zero, \( I(0) \), while \( y_t^{us} - y_t^{us,trend} \) is integrated of order one, \( I(1) \).

The econometric estimation is performed in two steps. In the first step a specification à la Pesaran, Shin and Smith (2001) is pursued. This procedure is robust against the series being \( I(0) \) or \( I(1) \) when estimating the long-run relationships among the variables. Having estimated the relevant long-run relationships, the short-run dynamics are investigated using a procedure analogous to the second step of the popular Engle-Granger\(^67\) two-step method.

An important consideration in any regression is whether the variance of the error term is constant with time. It is well-known that heteroskedasticity itself does not lead to biases. Yet, the efficiency of the estimator is affected in its presence. For sake of robustness either the Newey-West or White estimation approaches are used.

Moreover, the estimations of the short-run dynamics (second step) are performed using the generalized method of moments (GMM) given the fact that there is a forward-looking component (expected inflation) in the real interest rate.

\(^67\) A thorough analysis of Co-integration analysis in econometrics can be found in Banerjee et al. (1993).
The parametric representation of the AD equation is given by:

\[
\Delta(y_t - y_t^{trend}) = \alpha_0 [(y_{t-1} - y_{t-1}^{trend}) + \alpha_1 (y_{t-1}^{us} - y_{t-1}^{us^{trend}})]
\]

\[ + \alpha_2 \Delta(r_t) + \alpha_3 \Delta(q_t - q_t^{trend}) + \epsilon_t^{AD} \quad (1)
\]

where \(\epsilon_t^{AD} \sim N(0, \sigma_{AD}^2)\) is assumed to be a conventional white noise disturbance term with mean zero and finite variance.

Equation (1') shows the results:\n
\[
\Delta(y_t - y_t^{trend}) = -0.41^{**} (y_{t-1} - y_{t-1}^{trend}) - 1.00^{**} (y_{t-1}^{us} - y_{t-1}^{us^{trend}})
\]

\[ - 0.21^{*} \Delta(r_t) + 0.10^{*} \Delta(q_t - q_t^{trend}) \quad (1')
\]

As can be seen from \(\alpha_1 = 1.00\) there is a one-to-one correspondence between the output gaps of DR and the US in the long-run. Also \(\alpha_0 = -0.41\) means that about 40% of the deviations from this long-term relationship is corrected within one quarter. The value of the t-statistic (-4.30) associated with the error correction coefficient \(\alpha_0\) indicates that co-integration between these variables is robust.

In line with basic economic intuition, a more restrictive monetary policy has contractionary effects in output in the short-run (\(\alpha_2 < 0\)) and real exchange rate depreciations have short-term expansionary effects (\(\alpha_3 > 0\)).

**Aggregate Supply**

I now turn to the AS equation. The specification is forward-looking in conformity with the most recent advances in monetary policy modeling.

\[68\text{The operator } \Delta \text{ stands for the first difference of a variable. } * \text{and } ** \text{ denote significance at the 5% and 1% level respectively. Standard errors (in parenthesis) and t-statistics are shown below each relevant estimated coefficient. A Jarque-Bera test can not reject the null hypothesis of normality of the residuals. Also a LM test shows that the residuals are not autocorrelated up to 4 lags.}\]
The estimation follows closely the two-step procedure in Gali (2000) and Arreaza et al. (2004) in order to avoid potential biases. First, the inflation lags and leads coefficients are estimated by GMM, and then these coefficients are imposed in an OLS regression (second step) to get the coefficients of the rest of the relevant variables in the specification. The microfoundations for the specification are New-Keynesian in essence.69

The parametric representation of the AS equation is given by:

\[
\Delta (p_t) = \beta_0 E_t \Delta (p_{t+1}) + \beta_1 \Delta (p_{t-1}) + \beta_2 \Delta (p_{t-2}) + \beta_3 (y_{t-1} - y_{t-1}^{trend}) + \beta_4 \Delta (e_t - e_t^{trend}) + \epsilon_t^{AS}
\]

where \( \epsilon_t^{AS} \sim N(0, \sigma_{AS}^2) \) is as in the previous AD case assumed to be a white noise disturbance term with mean zero and finite variance.

Equation (2') presents the estimated dynamic price equation (AS)70:

\[
\Delta (p_t) = 0.25^{**} \Delta (p_{t+1}) + 0.19^* \Delta (p_{t-1}) + 0.69^{**} \Delta (p_{t-2}) + 0.32^† (y_{t-1} - y_{t-1}^{trend}) + 0.21^* \Delta (e_t - e_t^{trend})
\]

(2')

In the short-run current inflation is positively influenced by the expected future inflation, two lags of inflation (that play the crucial role of avoiding autocorrelation of the residuals), the output gap lagged one period (captures inflation pressures when output is transitorily above its potential) and the depreciation of the nominal exchange rate above its long-term trend. The presence of this last variable in the short-run

69See MacCallum and Nelson (1999b) and Clarida, Gali and Gertler (1999)

70† Denotes significance at the 10% level. A Jarque-Bera test can not reject the null hypothesis of normality of the residuals. Also a LM test shows that the residuals are not autocorrelated up to 4 lags.
dynamics means that, ceteris paribus, preventing the nominal exchange rate to rise above its trend reduces inflation pressures in the economy.

**Uncovered Interest Parity**

Regarding the exchange rate expectation formation mechanism, the model assumes that the nominal exchange rate is governed by uncovered interest parity (UIP) as a benchmark case. In addition to being the standard assumption in small-scale macro models, this is in harmony with Sanchez-Fung and Prazmowski's (2004) finding that the uncovered interest parity is in fact the main driver of exchange rate expectations in the Dominican Republic.

Accordingly, the short run dynamics of the nominal exchange rate follows the following forward-looking and risk-adjusted uncovered interest rate parity specification:

\[
E_t e_{t+1} = e_t + (i_t - i_t^{US}) + u_{t+1}
\]

(3)

where \( u_{t+1} \) is a random error term usually interpreted in the literature as a risk premium\(^{71}\). Equation (3) states that the expected risk-adjusted returns\(^{72}\) from assets in different currencies should be equal. The risk premium is assumed to be exogenous for the purposes of the simulations performed in this paper. In reality the perception of risk is related to the intertemporal solvency of the public sector, the soundness of the financial system, the institutional framework of the country and other fundamentals.

\(^{71}\)See, for example, Svensson (1992) and Isard (1995).

\(^{72}\)The analysis is performed from a lender's perspective. Thus, the interest rates correspond to lending rates both in DR and in the US. Lending rates are selected for convenience since the specification for the AD includes the real lending interest rate as an explanatory variable. As long as lending and deposits rates are proportional (as it seems to be the case) this do not raise any concerns.
Monetary Policy: Transmission Mechanism

Before turning to the specification of the policy rules it is convenient to describe the simple underlying monetary policy transmission mechanism in this paper. By controlling the evolution of nominal interest rates (for given foreign rates) the central bank also influences the path of the nominal exchange rate and may therefore reduce inflation pressures in the economy by trying to minimize deviations of the exchange rate from a long-term trend perceived as "normal" by the monetary authority.\footnote{This "normal" trend of the nominal exchange rate is proxied by a Hodrick-Precott filter of the actual exchange rate in this paper.} This policy translates into high and persistent real interest rates. As a result, taking into account the lag structure with which these effects take place in practice, a policy of persistent high real interest rates leads to a process of gradual disinflation whenever the economy is experiencing "higher than normal"\footnote{As in the case of the nominal exchange rate path, "higher than normal" inflation means deviations of inflation from the inflation level calculated from the long-term trend (Hodrick-Precott filter) of the CPI index.} levels of inflation. So inflation goes down because there are reduced pressures coming form the domestic price of tradables and also form the effect of the higher real interest rates on aggregate demand.

Policy Rules

The two alternative policy rules being compared in this paper are estimated using GMM given the complication that arises from having two explanatory variables that depend on the observed values of the interest rate, as explained in Clarida et al. (1998) for the case of developed countries and in Corbo (2002) for the case of Latin American countries.
The final, preferred, reduced empirical specifications for the two policy rules being considered are as follows:

\[
\Delta(i_t - i^*_{t}) = 0.14^{**}(\Delta p_{t+1} - \Delta p_{t+1}^{trend}) + 0.10^{**}(y_t - y_t^{trend}) \\
\Delta(i_t - i^*_{t}) = 0.02^{**}(e_t - e_t^{trend}) + 0.12^{**}(y_t - y_t^{trend}) + 0.33^{***}\Delta(i_{t-1} - i^*_{t-1})
\]

(4a) (4b)

The first thing to notice is that these are balanced\(^75\) dynamic equations without an error correction term\(^76\). The coefficients are robust and statistically significant, yet the values are rather low. These low values are due to the persistently high interest rate differential that have characterized the Dominican economy in the recent past. The typical Taylor rule specification usually show interest rates reacting to inflation expectations with a noteworthy order of magnitude. However, if interest rates have been at a high level relative to the rate of inflation for a rather long period, like in the case of DR, then it is plausible to expect a much lower reaction to changes in the rate of inflation or in the inflation pressures represented in the nominal exchange rate gap.\(^77\)

**III.III. Simulation exercises**

To simulate the model I assume the economy is at the steady state with initial values of all variables set to zero. To take advantage of the forward looking nature of

\(^75\) The term balanced refers to the variables in the specification having the same order of integration, \(I(0)\).

\(^76\) A wide variety of specifications were attempted and the error correction term was never significant.

\(^77\) A similar argument, applied to a set of three transition economies can be found in Golinelli and Rovelli (2005).
the model's specification I consider fully anticipated and transitory shocks and evaluate both the pre and post-shock responses of the key macroeconomic variables.

**An economic slowdown abroad**

This shock takes the form of a 1% decrease in the US output gap. A key issue in policy debates in the Dominican Republic (and in most small open economies as well) pertains to the impact the external cycle in the most important trading partners has on domestic output. Nevertheless, a formal characterization of the dynamic effects of this shock has not been addressed for the case of the Dominican Republic.

Figure III.2. shows the evolution of the inflation rate, the output gap, the nominal exchange rate and the nominal interest rate (policy rate) under the two alternative policy rules. First, I will describe the sequence of events associated with the traditional Taylor rule.

Monetary policy endogenously responds to the anticipated slowdown abroad and there is a reduction in the interest rate before the shock actually happens\(^7\). As economic agents perceive the economic slowdown abroad as imminent, they expect the same to happen domestically, given the strong relationship between the two output gaps. Domestic output gap bottoms one quarter before the slowdown abroad. A policy response reducing the domestic interest rate is obliging. This foreseeable policy reaction increases the interest rate differential between the domestic and the foreign interest rate which translates into a jump in the expected nominal exchange rate (actual jump, because of the perfect foresight nature of the present model). The jump in the

\[^7\text{Here I set the shock to take place in quarter 5. That is a year ahead from the starting quarter 1.}\]
nominal exchange rate is reflected vis-a-vis into a jump in the inflation rate two-quarters before the actual shock reducing output abroad takes place.

The interest rate reach a bottom at the same time the shock abroad actually happens, and then it starts to go up again towards its initial level, since the shock is transitory and only lasts one quarter. A similar mechanism to the one just explained operates in the opposite direction with the nominal exchange rate an the inflation rate going down and output going up. Domestic output gap peaks three quarters after the shock. That is two quarters after the foreign output gap is already back at its original "equilibrium". Domestic output is again within the ballpark of the original normalized level of zero in quarter ten. Thus, most of the dynamics in this simulation is confined to a period of two and half years.

I now turn to the evolution of the variables under the alternative policy rule. The trajectory of the domestic output gap is qualitatively similar to the previous case. Yet, domestic output gap's reaction is substantially less prominent. This result does not come as a surprise because explicit forward looking elements are absent in this alternative policy rule. This absence explains why both the nominal exchange rate and inflation (practically) do not react to the anticipated shock before it happens. Interest rate fall slightly the period before the shock to partially offset the contractionary impact on the shock abroad. The upward pressures of the slowdown abroad on the nominal exchange rate (and therefore in inflation) operate with a lag of two periods. Most these pressures are neutralized by a transitory increase in the interest rate, that contrary to what happens under the Taylor rule, rises above the initial normalized level of zero right after the shock. This alternative policy rule, consistent with higher and more
volatile interest rates, seems to fit more closely than a standard Taylor rule the evidence of the DR. In this small open economy, higher and more volatile interest rates are promoted in exchange for having a more stable path of the nominal exchange rate (and inflation). This is an excellent illustration of Calvo and Reinhart's (2002) fear of floating behavior.

The desirability of such policies is an aspect that I do not intent to address formally in this paper. Though, it is reasonable to argue that consistently high and volatile interest rates are not in the best interest of a sound financial system. High interest rates tend to be associated with more credit defaults, and, in general, policies aiming to avoid an otherwise equilibrium adjustment in the exchange rate only serve to postpone the adjustment, making it more marked than it needed to be in the first place.

**An increase in the foreign interest rate**

This shock takes the form of a 1% fully anticipated increase in the interest rates in the US that only last for one quarter. The upward adjustment is made in quarter 5 in order to be able to analyze, as in the case of the previous shock, the anticipated responses that arise from the forward-looking components in the model.

Figure III.3. shows the evolution of the inflation rate, the output gap, the nominal exchange rate and the nominal interest rate (policy rate) under the two alternative policy rules. As before, I will start describing the sequence of events associated with the traditional Taylor rule.

Monetary policy endogenously responds to the anticipated increase in the foreign interest abroad. Since the relevant endogenous variable in the policy rule
specification is the interest rate differential, an increase in the domestic interest rate to match the increase abroad is also expected to occur at quarter 5.

In order to partially mitigate the temporary decline in the domestic output gap the interest rate drops slightly the quarter preceding the shock. Since this is fully expected, the nominal exchange rate jumps up in anticipation by virtue of the forward looking UIP specification in the model. At period 5 the domestic interest rate rises surpassing slightly the increase in foreign interest rates. 79 This expected "overshooting" of the interest rate--that comes as a direct result of the estimated parameters--leads to a wider interest rate differential and to the nominal exchange rate rising even further. The expected (actual) inflation rate jumps accordingly. Because the exchange rate stabilizes at a higher level relatively fast, and the interest rate shock is transitory, inflation drops back to its normalized level of zero by quarter 9.

The expected increase in inflation decreases the real interest rate substantially in the previous period, which in turn generates an increase in output via an increase in aggregate demand. The short-run expansionary effects of a real exchange rate depreciation also contributes to explain the peak in the output gap that takes place in quarter 5 together with the foreign interest rate shock. As before, the same line of reasoning applied in the opposite direction explain the return of both output and inflation to their normalized "equilibrium" levels after the reversion of the transitory shock.

The trajectories of inflation and the domestic output gap are qualitatively similar under the alternative policy rule. The explanation follows the same logic.

79 This "overshooting" in the interest rate is the direct result of the estimated parameters in the dynamic equations.
Again, the lack of explicit forward looking elements in the alternative policy rule explains why output and inflation fluctuations are less pronounced under the alternative policy rule. As in the case of the previous shock, the alternative policy rule is consistent with a more stable nominal exchange rate. Fear of floating is again manifested in the form of higher volatility of the interest rate. The interest rate falls below its normalized level beginning in quarter 7 until quarter 14, reducing the normal spread between the interest rates. Given the forward looking nature embedded in the UIP this induces an appreciation in the nominal exchange rate. The alternative policy rule seems to fit more closely than a standard Taylor rule the evidence of the DR under the foreign interest rate shock as well.

III.IV. Conclusions

This paper builds a small macroeconomic model for the Dominican economy, a country heavily characterized by fear of floating in conducting monetary policy. Simulations are performed under two alternative interest rate policy rules: One standard Taylor rule, and another that responds explicitly to deviations of the exchange rate with respect to its long-term trend.

Two main findings emerge from the simulations. First, output and inflation exhibit higher volatility under the the standard Taylor rule. This result comes from the predominance of a forward looking component (expected inflation) in the specification for this policy rule. Second, the alternative policy rule leads to higher stability of the nominal exchange rate at the expense of a higher and more volatile interest rate.
Figure III.1. Exchange rate policy in Dominican Republic (De facto classification)

Data from Levy-Yeyati and Sturzenegger (2003, 2005)
Figure III.2. Responses of inflation, the output gap, the nominal exchange rate, and the interest rate in response to a 1% decrease in the foreign output gap.
Figure III.3. Responses of inflation, the output gap, the nominal exchange rate, and the interest rate in response to a 1% increase in the foreign interest rate.


http://www.bankofengland.co.uk/publications/other/beqm/modcobook.htm


Masson, P., Bayoumi, T. & Samiei, H., 1995. “Saving behavior in industrial and developing countries”. International Monetary Fund, Staff Studies for the World Economic Outlook, September.


Appendix 1: Coefficients Chapter I.

\[ \eta_{r,k} = \left( -\frac{L F_{KL}}{R} \right) < 0, \quad \eta_{r,l} = \left( -\frac{L^2 F_{LL}}{R K} \right) > 0, \quad \eta_{r,q} = -1 < 0, \]

\[ \eta_{q,k} = \frac{R K}{(K^* \chi + K^* \chi')} > 0, \quad \eta_{q,l} = \frac{\bar{W} L}{(K^* \chi + K^* \chi')} > 0, \]

\[ \eta_{q,k^*} = \frac{\bar{R}^* K^*}{(K^* \chi + K^* \chi')} > 0, \quad \eta_{q,l^*} = \frac{\bar{W}^* \bar{L}^*}{(K^* \chi + K^* \chi')} > 0, \]

\[ \eta_{q,c} = \left( \frac{-\tilde{C}}{(K^* \chi + K^* \chi')} \right) < 0, \quad \eta_{q,c^*} = \left( \frac{-\tilde{C}^*}{(K^* \chi + K^* \chi')} \right) < 0, \]

\[ \eta_{q,\theta} = \left( \frac{-\bar{R}^* \chi^*}{(K^* \chi + K^* \chi')} \right) < 0. \]

The matrix \( \Omega \) in (26) is the following:

\[
\Omega = \begin{bmatrix}
\eta_{\mu,\mu} & \eta_{\mu,\mu^*} & \eta_{\mu,\psi} & \eta_{\mu,\psi^*} & 0 & 0 & \eta_{\mu,\theta} & \eta_{\mu,k} & \eta_{\mu,k^*} & \eta_{\mu,z} & 0 & 0 \\
\eta_{\mu^*,\mu} & \eta_{\mu^*,\mu^*} & \eta_{\mu^*,\psi} & \eta_{\mu^*,\psi^*} & 0 & 0 & \eta_{\mu^*,\theta} & \eta_{\mu^*,k} & \eta_{\mu^*,k^*} & \eta_{\mu^*,z} & 0 & 0 \\
0 & 0 & \eta_{\psi,\psi} & \eta_{\psi,\psi^*} & 0 & 0 & \eta_{\psi,\phi} & \eta_{\psi,\phi^*} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \eta_{\phi,\phi} & 0 & 0 & 0 & 0 & 0 & 0 & \eta_{\phi,z} & 0 \\
0 & 0 & 0 & 0 & \eta_{\phi^*,\phi^*} & 0 & 0 & 0 & 0 & 0 & \eta_{\phi^*,\phi^*} & 0 \\
\eta_{\theta,\mu} & \eta_{\theta,\mu^*} & \eta_{\theta,\psi} & \eta_{\theta,\psi^*} & 0 & 0 & \eta_{\theta,\theta} & \eta_{\theta,k} & \eta_{\theta,k^*} & \eta_{\theta,z} & 0 & 0 \\
\eta_{k,\mu} & \eta_{k,\mu^*} & \eta_{k,\psi} & \eta_{k,\psi^*} & 0 & 0 & \eta_{k,k} & \eta_{k,k^*} & \eta_{k,z} & 0 & 0 & 0 \\
\eta_{k^*,\mu} & \eta_{k^*,\mu^*} & \eta_{k^*,\psi} & \eta_{k^*,\psi^*} & 0 & 0 & \eta_{k^*,k} & \eta_{k^*,k^*} & \eta_{k^*,z} & 0 & 0 & 0 \\
\eta_{z,\mu} & 0 & \eta_{z,\psi} & 0 & 0 & 0 & \eta_{z,k} & 0 & \eta_{z,k^*} & 0 & 0 & 0 \\
0 & \eta_{x^*,\mu} & 0 & \eta_{x^*,\psi} & 0 & 0 & 0 & 0 & \eta_{x^*,k} & 0 & \eta_{x^*,k^*} & 0 \\
\eta_{b,\mu} & \eta_{b,\mu^*} & \eta_{b,\psi} & \eta_{b,\psi^*} & 0 & 0 & \eta_{b,\theta} & \eta_{b,k} & \eta_{b,k^*} & 0 & 0 & \eta_{b,b} \\
\end{bmatrix}
\]

\[ \eta_{k,\mu} = -\eta_{k,\psi}, \quad \eta_{k,\mu^*} = -\eta_{k,\psi^*}, \]

\[ \eta_{k,\psi} = -\frac{\chi(\bar{C}^* + \nu \bar{W} L) \eta_{c,\psi}}{(K^* \chi + K^* \chi')}, \quad \eta_{k,\psi^*} = -\frac{\chi(\bar{C}^* + \nu \bar{W}^* \bar{L}^*) \eta_{c,\psi^*}}{(K^* \chi + K^* \chi')}, \]

\[ \eta_{k,\theta} = -\frac{\chi X^* \bar{K}^*}{(K^* \chi + K^* \chi')}, \quad \eta_{k,k} = \frac{\chi (R K + \nu \bar{W} L (\partial_x c_k - \eta_{c,k}) - c n_{c,k})}{(K^* \chi + K^* \chi')}, \]

90
\[ \eta_{k,k^*} = \frac{\chi (\tilde{R}^{*} + \nu \tilde{w}^{*} L (\epsilon_{w,k^*} - \eta_{c,k^*}^{*}) - \tilde{c}^{*} \eta_{c,k}^{*})}{(\tilde{K}_{X} \tilde{R}^{*} \chi^{*})} \]

\[ \eta_{k^*,\mu} = \frac{\chi^* \eta_{k^*,\mu}}{\chi}, \quad \eta_{k^*,\mu} = \frac{\chi^* \eta_{k^*,\mu}}{\chi}, \quad \eta_{k^*,\psi} = \frac{\chi^* \eta_{k^*,\psi}}{\chi^{*}}, \quad \eta_{k^*,\psi^*} = \frac{\chi^* \eta_{k^*,\psi^*}}{\chi^{*}} \]

\[ \eta_{k^*,\theta} = \frac{\chi^* (\chi + \eta_{k^*,\theta})}{\chi}, \quad \eta_{k^*,k} = \frac{\chi^* \eta_{k^*,k}}{\chi^{*}}, \quad \eta_{k^*,k^*} = \frac{\chi^* \eta_{k^*,k^*}}{\chi^{*}} \]

\[ \eta_{\psi,\psi} = (\sigma + \bar{R}), \quad \eta_{\psi,\phi} = -(\sigma + \bar{R}), \quad \eta_{\psi,z} = \bar{R}, \]

\[ \eta_{\psi^*,\psi^*} = (\sigma^* + \bar{R}^{*}), \quad \eta_{\psi^*,\phi^*} = -(\sigma^* + \bar{R}^{*}), \quad \eta_{\psi^*,z^*} = \bar{R}^{*}, \]

\[ \eta_{\mu,\psi} = -\eta_{\mu,\psi}, \quad \eta_{\mu,\mu^*} = -\eta_{\mu,\psi^*}, \]

\[ \eta_{\mu,\mu} = \frac{-\bar{R}}{(\tilde{K}_{X} \tilde{R}^{*} \chi^{*})} \] if \( \bar{R} = \bar{R}^{*} \) is true, we have

\[ \eta_{\mu^*,\mu} = \eta_{\mu,\mu}, \quad \eta_{\mu^*,\mu^*} = \eta_{\mu,\mu^*}, \quad \eta_{\mu^*,\psi} = \eta_{\mu,\psi}, \quad \eta_{\mu^*,\psi^*} = \eta_{\mu,\psi^*}, \]

\[ \eta_{\mu^*,\theta} = \eta_{\mu,\theta}, \quad \eta_{\mu^*,k} = \eta_{\mu,k}, \quad \eta_{\mu^*,k^*} = \eta_{\mu,k^*}, \quad \eta_{\mu^*,z^*} = \eta_{\mu,z} \]

\[ \eta_{\phi,\phi} = \bar{R}, \quad \eta_{\phi,z} = \bar{R}, \]

\[ \eta_{\phi^*,\phi^*} = \eta_{\phi,\phi^*}, \quad \eta_{\phi^*,z^*} = \eta_{\phi,z^*}, \]

\[ \eta_{\theta,\mu} = -\eta_{\theta,\psi}, \quad \eta_{\theta^*,\mu^*} = -\eta_{\theta,\psi^*}, \quad \eta_{\theta,\psi^*} = \nu \bar{L} F_{KL} \eta_{c,\psi^*}, \]

\[ \eta_{\theta^*,\phi} = \nu^* \bar{L}^* F_{KL} \eta_{c^*,\phi^*}, \]

\[ \eta_{\theta,\theta} = \bar{R}, \quad \eta_{\theta,k} = F_{KK} \bar{R} + \nu \bar{L} F_{KL} (\epsilon_{w,k} - \eta_{c,k}). \]
\[ \eta_{\theta,k^*} = -F_{\tilde{R}^*}\tilde{R}^* - v^*\tilde{L}^*F_{\tilde{R}^*}\tilde{L}^* (\varepsilon_{w^*,k^*} - \eta_{c^*,k^*}) \]

\[ \eta_{z,\mu} = -\eta_{z,\psi}, \quad \eta_{z,\psi} = \frac{\tilde{M}}{\psi}\left(\tilde{C} + v\tilde{W}\tilde{L}\right)\eta_{c,\psi} \]

\[ \eta_{z,k} = \frac{\tilde{M}}{\psi}\left(\tilde{C}\eta_{c,k} - v\tilde{W}\eta_{c,k}\right), \quad \eta_{z,z} = -\sigma \]

\[ \eta_{2^*,\mu^*} = -\eta_{2^*,\psi^*}, \quad \eta_{2^*,\psi^*} = \frac{\tilde{M}^*}{\psi^*}\left(\tilde{C}^* + v^*\tilde{W}^*\tilde{L}^*\right)\eta_{c^*,\psi^*} \]

\[ \eta_{2^*,k^*} = \frac{\tilde{M}^*}{\psi^*}\left(\tilde{C}^*\eta_{c^*,k^*} - v^*\tilde{W}^*\eta_{c^*,k^*}\right), \quad \eta_{2^*,z^*} = -\sigma^* \]

\[ \eta_{b,\mu} = -\eta_{b,\psi}, \quad \eta_{b,\mu^*} = -\eta_{b,\psi^*} \]

\[ \eta_{b,\psi} = -\frac{\tilde{R}^*\tilde{L}^*\left(\tilde{C} + v\tilde{W}\tilde{L}\right)\eta_{c,\psi}}{\tilde{B}(\tilde{K}\tilde{R} + \tilde{R}\tilde{R}^*)} \]

\[ \eta_{b,\psi^*} = \frac{\tilde{K}\tilde{R}\left(\tilde{C}^* + v^*\tilde{W}^*\tilde{L}^*\right)\eta_{c^*,\psi^*}}{\tilde{B}(\tilde{K}\tilde{R} + \tilde{R}\tilde{R}^*)} - \eta_{\theta,\psi^*} \]

\[ \eta_{b,\theta} = \frac{\tilde{K}\tilde{R}^*\tilde{L}^*}{\tilde{B}(\tilde{K}\tilde{R} + \tilde{R}\tilde{R}^*)} \]

\[ \eta_{b,k} = \frac{\tilde{X}^*\tilde{R}^*}{\tilde{X} \tilde{B}} \eta_{k,k} \]

\[ \eta_{b,k^*} = -\frac{\tilde{K}}{\tilde{B}}\eta_{k^*,k^*} - \eta_{\theta,k^*} \]

\[ \eta_{b,b} = \tilde{R} \]
Appendix 2: Coefficients Chapter II

The Jacobian matrix $\Delta$ in (40) is the following:

$$
\Delta = \begin{bmatrix}
\frac{\partial k}{\partial k} & \frac{\partial k}{\partial z} & \frac{\partial k}{\partial \phi} & \frac{\partial k}{\partial \mu} \\
\frac{\partial z}{\partial k} & \frac{\partial z}{\partial z} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \mu} \\
\frac{\partial \phi}{\partial k} & \frac{\partial \phi}{\partial z} & \frac{\partial \phi}{\partial \phi} & \frac{\partial \phi}{\partial \mu} \\
\frac{\partial \mu}{\partial k} & \frac{\partial \mu}{\partial z} & \frac{\partial \mu}{\partial \phi} & \frac{\partial \mu}{\partial \mu}
\end{bmatrix}
$$

where:

$$
\frac{\partial k}{\partial k} = r^* - (n + g + \delta); \quad \frac{\partial k}{\partial z} = 0; \quad \frac{\partial k}{\partial \phi} = -c^*; \quad \frac{\partial k}{\partial \mu} = 0;
$$

$$
\frac{\partial z}{\partial k} = \frac{\sigma c^* \psi^*}{k^* \phi^*}; \quad \frac{\partial z}{\partial z} = -\mu^* \frac{c^*}{k^* \phi^*}; \quad \frac{\partial z}{\partial \phi} = 0; \quad \frac{\partial z}{\partial \mu} = -(\sigma + g);
$$

$$
\frac{\partial \phi}{\partial k} = 0; \quad \frac{\partial \phi}{\partial z} = -\gamma; \quad \frac{\partial \phi}{\partial \phi} = 1 + (\rho^* - g); \quad \frac{\partial \phi}{\partial \mu} = -\frac{\sigma c^* \psi^*}{\phi^*};
$$

$$
\frac{\partial \mu}{\partial k} = 0; \quad \frac{\partial \mu}{\partial z} = 0; \quad \frac{\partial \mu}{\partial \phi} = \gamma \left( \frac{\phi^*}{z^* \psi^*} - 1 \right); \quad \frac{\partial \mu}{\partial \mu} = 1 - \frac{\gamma \phi^*}{z^* \psi^*};
$$

$$
\frac{\partial \psi}{\partial k} = (\rho^* - g) + \sigma - \frac{\sigma c^* \psi^*}{\phi^*} \frac{\partial \phi}{\partial k}; \quad \frac{\partial \psi}{\partial z} = \frac{c^* \mu^*}{\phi^*}; \quad \frac{\partial \psi}{\partial \phi} = \frac{c^* \mu^*}{\phi^*}; \quad \frac{\partial \psi}{\partial \mu} = \frac{(1+b)(1-\eta)r^*k^{-b}}{(1-\eta)k^{-b} + \eta}.
$$

$$
\frac{\partial \phi}{\partial \phi} = 0; \quad \frac{\partial \phi}{\partial \mu} = 1; \quad \frac{\partial \mu}{\partial z} = -\frac{\sigma c^* \psi^*}{\phi^*}; \quad \frac{\partial \mu}{\partial \phi} = (\rho^* + n + \delta) - (r^* + g) + \frac{c^* \mu^*}{\phi^*}.
$$
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