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Behaviorally Enriched Learning Mechanism for Road Network Emergency Restoration After Disasters

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BEHAVIORALLY ENRICHED LEARNING MECHANISM FOR ROAD

NETWORK

EMERGENCY RESTORATION AFTER DISASTERS

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Maryam Babaee

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To: Dean John L. Volakis
College of Engineering and Computing

This thesis, written by Maryam Babaee, and entitled Behaviorally Enriched Learning Mechanism for Road Network Emergency Restoration after Disasters, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this thesis and recommend that it be approved.

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Florida International University, 2022

DEDICATION

I dedicate this thesis to my husband. Without his patience, understanding, support, and most of all love, the completion of this work would not have been possible

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ABSTRACT OF THE THESIS

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Timely restoration of road networks plays a critical role in the response operations after disasters and helps communities turn back to their normal operations soon. Scarcity of restoration resources, uncertainty of recovery times, and behavioral variations of travelers are the major factors that highly complicate road network restoration operations. Here, these challenges are addressed by developing a Behaviorally-enriched Reinforcement Learning Mechanism (BRLM). Considering gradual adaptation of travelers, the mechanism optimizes scheduling and resource allocation decisions in the restoration process to make the highest acceleration in the post-disaster traffic movement. The performance of BRLM is tested on the road network of Sioux Falls in South Dakota for several tornado scenarios. To evaluate the efficiency of BRLM, a heuristic method is developed that ignores post-disaster traffic movement in making restoration decisions. Results show that the advantages of emergency road restoration on the post-disaster traffic flows completely depend on the behavior of travelers.

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1. Introduction

The proper functionality of every society is heavily reliant on transportation systems. Road networks, in particular, play a vital role since they provide access to various parts of the society and enable running services throughout the community (Homeland Security Presidential Directive, 2003). However, they are often subject to different types of disruption due to their wide spatial distribution, high vulnerability, and the increasing number of disruptive events. These events include random or malicious disruptions caused by man-made disasters and localized disruptions caused by natural disasters (Hu et al., 2016).

After disruptive events, the functionality of a road network is critical to appropriately handle the post-disaster traffic, including the remaining portion of pre-disaster daily traffic and the emerging post-disaster traffic to transfer injured people to hospitals and transport emergency goods to affected sites (Faturechi & Miller-hook, 2014; Zou & Chen, 2021; Fan & Liu, 2010; Orabi et al., 2009; Zhang et al., 2019; Rey & Bar-Gera, 2020). However, parts of the road network may be damaged by the event and become inoperative. This deteriorates the traffic pattern in the area and may lead to a huge economic or public health loss. To significantly expedite the recovery process, the timely and efficient restoration of damaged roads is vital (Vodák et al., 2018). Faturechi & Miller-hooks (2015) reviewed more than 200 articles on this topic, ranging from preparedness strategies to recovery enhancement of transportation infrastructure.

Compared to regular road disruptions resulting from accidents or maintenance closures, natural disasters lead to substantial localized damages to roads and long-term block of traffic. For example, in the 1995 Kobe earthquake, considered as one of Japan's worst earthquakes with $100 billion of interruption losses, the majority of the road infrastructure was demolished and it took a long time for the restoration of infrastructure (Bengtsson & Tómasson, 2008). Hurricane Katrina in August 2005 damaged more than 44 road segments/bridges in the Gulf Coast region and the traffic flow in the area was heavily affected, where the overall cost of restoration was over $1 billion (DesRoches, 2006). In December 2004, a powerful earthquake in the Indian Ocean affected numerous nations due to a tsunami. Indonesia was one of the most affected areas, where 19.7% of the total damage belonged to the transportation infrastructure. Following the Haiti earthquake (2010), despite the availability of relief supplies, transferring emergency aids to affected sites were impossible because the road infrastructure was damaged (Pedraza et al., 2012; Hayat & Amaratunga, 2011). Another example is the Great East Japan Earthquake in March 2011. A 9.0 magnitude earthquake hit the east coast of Japan and caused a massive tsunami (Kazama & Noda, 2012). The tsunami made a substantial damage in railways and road networks. Totally, 78 road segments/bridges were damaged that led to the closure of 76% of highways.

Based on the experience from such events, it is essential to have a holistic plan for timely and efficient road network restoration following large-scale disruptive events (Çelik, 2016). Scarcity of recovery resources and unbeknown impact of restoration decisions on traffic flows highly complicate the restoration process (Oruc & Kara, 2018; Hu & Chen, 2021).

The focus of this study is on the post-disaster restoration of a road network in an affected area to accelerate the traffic movement. The behavioral variations of travelers and their gradual adaptation to the road network reformation, scarcity of recovery resources, and uncertainty in the recovery times are considered in the decision-making process. To address the computational complexity of the problem, a Behaviorally-enriched Reinforcement Learning Mechanism (BRLM) is developed in this study. The BRLM includes the following components (see Figure 1):

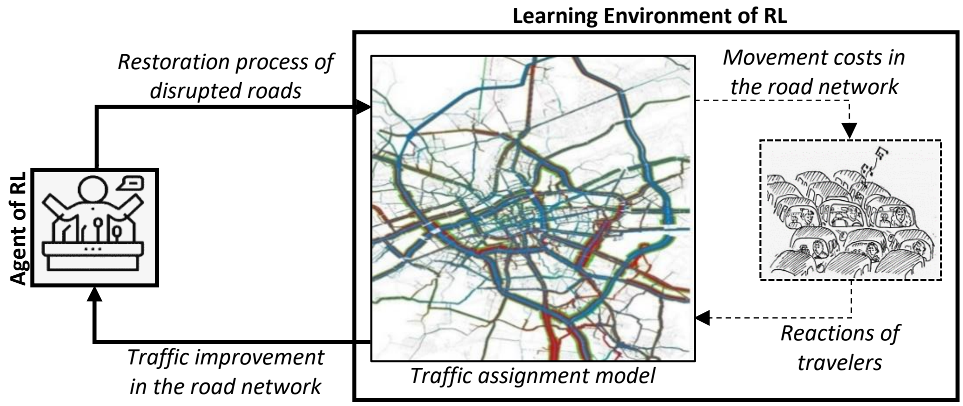

Figure 1.The structure of BRLM

- **Agent of the BRLM:** The decision-making agent of this mechanism represents the Federal Highway Administration (FHWA) that prioritizes road restoration activities after disasters. The agent of the BRLM schedules the sequence of the restoration of damaged roads and assigns recovery crews to the roads selected for recovery at each decision-making step. The agent prioritizes the roads according to the traffic improvement that can be achieved by their recovery in the road network. The consequences of the decisions made by the agent are evaluated in the learning environment of the BRLM.

- **Learning environment of the BRLM:** The learning environment of the BRLM evaluates the agent's decisions through simulating the traffic movements in the road network. Since this study focuses on the short-term road restoration operation, the traffic evolutions after each recovery decision and before reaching the equilibrium are considered in the simulation model. To measure the traffic improvement that each road recovery decision can cause, an inter-periodic/day-to-day traffic modeling approach is applied (He et al., 2010; He & Liu, 2012; Kumar & Peeta, 2015; Yu et al., 2020). This approach helps to capture the traffic evolution rather than the final static equilibrium state (Watling & Hazelton, 2003a). In the post-disaster circumstances that the structure of the road network changes frequently after each road recovery, there is not enough time to reach equilibrium states. Therefore, considering the traffic evolution is vital. This model includes the fact that the route choice decisions of travelers is made daily and may change day-to-day according to traffic variations in the road network. This model captures both the time-minimization behavior of travelers and their inertia to shift to new and uncustomary routes. Considering the behavioral characteristics of travelers makes the traffic model fit better into chaotic and dynamically changing post-disaster circumstances.

Figure 1 displays the general structure of the BRLM. In the next section, a review of the related literature is provided and the contributions of this study is stated. Section 3 gives the problem description, and its modeling structure. In Section 4, the problem is formulated as the BRLM. The environment and decision-making steps of the BRLM are developed in sections 4.1., and 4.2. respectively. Section 5 includes experimentation and computational results. The study is concluded in Section 6.

2. Literature review

The problem of this study is related to four branches of research in the literature: short-term road network restoration, road network design/redesign/expansion, post-disaster traffic management, and uncertainty management in restoration operations. These research branches are respectively discussed in sections 2.1, 2.2, 2.3, and 2.4, and this study's contributions are discussed in section 2.5.

2.1. Short-term Road Network Restoration:

In practice, transportation network restoration is broken into two phases: (1) short-term restoration that may take a few days and focuses on addressing urgent needs of restoring critical components to functional, not necessarily to the pre-disaster, conditions; and (2) long-term restoration that may continue for several months or years and aims to fully restore impacted components back to their pre-disaster conditions. The short-term road network restoration is a very challenging task because it is a time-sensitive project with limited resources and many uncertainties

(Yan & Shih, 2009). Therefore, many studies have focused on the post-disaster short-term road network restoration rather than the long-term one. For example, Aksu and Ozdamar (2014) propose a mathematical model to maximize the road network accessibility by scheduling restoration activities under limited resources. They develop a decomposition-based solution approach to solve real-size models in a reasonable computational time. Similarly, Akbari and Salman (2017) develop a model to dispatch multiple work crews to unblock closed roads in a road network with the objective of maximizing the network connectivity. They assume that the repair time of blocked roads is known and deterministic. In this study, a heuristic approach is developed to solve the model in a timely manner. In another study, Yan and Shih (2009) propose a deterministic multi-objective model that integrates road restoration and relief distribution with the aim of minimizing the completion time of the repair project. Sanci and Daskin (2019) develop a two-stage stochastic model integrating repair facility location and network restoration operations. The objective function is to minimize the total cost (including the costs of operating response facilities, acquiring restoration equipment, distributing relief supplies, and restoring roads). Demands for relief items, damage ratios of response facilities, and repair times of damaged roads are uncertain in the model. They employ the sample average approximation (SAA) method to solve the model. Çelik et al. (2015) address the problem of road debris clearance after disasters by developing a stochastic model. In the model, the sequence of clearing roads is determined while the information on the amount of debris over blocked roads is incomplete and is updated as clearance operations proceed. The objective of the model is to reconnect the supply and demand nodes. Ajam et al. (2019) develop a deterministic model to find the best movement route for recovery crews to minimize the total travel time from a depot to critical nodes affected by a disaster. They employ a heuristic and a metaheuristic approach to solve the model for real-life instances in an acceptable computational time.

While the goal of the short-term road network restoration is to accelerate post-disaster traffic, this performance measure is rarely considered in studies. As seen in the abovementioned literature, researchers used indirect but simpler measures such as maximizing network connectivity/accessibility (Aksu & Ozdamar, 2014; Duque & Sörensen, 2011; Kasaei & Salman, 2016; Taylor & Susilawati, 2012; Yücel et al., 2018; Hu et al., 2016), minimizing the restoration time/cost (El-Anwar et al., 2016; Ajam et al., 2019; Yan & Shih, 2009; Sanci & Daskin, 2019), maximizing network coverage (Chang & Nojima, 2001), maximizing the number of reliable and independent pathways between origin and destination nodes (Zhang & Wang, 2016; Zhang et al., 2017), and maximizing recovered flow or met demand (Alkhaleel et al., 2021; Fang & Sansavini, 2019; Sianca & Nurre, 2021; Çelik et al., 2015).

In the literature, there are few studies considering the impacts of restoration decisions on the post-disaster traffic. For example, Faturechi & Miller-Hooks (2014) propose a two-stage mathematical model for scheduling road restoration operations. By ignoring the traffic evolution in the road network after the recovery of each damaged/blocked road, they only focus on final equilibrium states which is not consistent with the nature of short-term road restoration operations. In a short term, travelers are not expected to reach to an equilibrium and the impacts of transitional stages in the traffic flows are significant. Likewise, Zou & Chen (2021) propose a bilevel model for scheduling recovery efforts in a disrupted transportation network with mixed traffic environment, including connected and autonomuse (CAVs) and human driven (HDVs) vehicles. They measure travel time/cost of CAV and HDV drivers using user equilibrium traffic assignment model. They only consider the final equilibrium state in their model, rather than traffic evolutions. Edrisi and Askari (2019) develop a bi-level model for budget allocation to improve the efficiency of a transportation network after disasters. The performance measure is the total travel time. They enhance the effiency of the network through capacity expansion before disasters and link stabilization after disasters. Similar to other studeis, they employ the user equilibrium traffic assignment method (final equlibrium states) to calculate travel times of roads/links in the network. Also, Rey & Bar-Gera (2020) propose a bilevel optimization model for scheduling reconstruction efforts in a transportation network. The objective of the model is to find the best sequence for recovery tasks under user equilibrium condition. They assume that the restoration time of each road is long enough to let travelers reach the equilibrium state. Therefore, their model is appropiate for long-term road network restoration/reconstruction.

This study fills this gap in the study by developing an integrative mechanism (BRLM) in which road restoration sequence is optimized according to their impacts on the post-disaster traffic. BRLM considers traffic evolutions after each restoration operation, rather than the final equilibrium state. This feature makes BRLM applicable for short-term road restoration operation.

2.2. Road Network Design/Redesign/Expansion:

Due to the growing traffic congestion in cities, road network expansion has become a major concern for city planners. The road network expansion problem requires several environmental, economic, and space considerations which turn it into a challenging task for decision-makers (Marín & Jaramillo, 2008). The traffic planners must decide to either (i) improve the capacity of existing roads, or (ii) construct new roads. In the literature, this type of problems falls into the category of network design problem (NDP). This group of studies is related to this study because they investigate the future impact of their decisions (road construction or expansion) on the traffic improvement. Farahani et al., (2013) provide an overview on the models and solution methods proposed for the NDPs. Mathew & Sharma (2009) propose a bi-level model, where the upper level makes the road capacity expansion decisions, and the lower level formulates the path choice behavior of travelers. Likewise, Karoonsoontawong & Waller (2010) develop a bi-level model that integrates capacity expansion, traffic signal setting, and dynamic traffic assignment problems. Hosseininasab & Shetab-Boushehri (2015) propose three bi-level programming models to concurrently select and schedule projects for NDPs. The upper levels of the models optimize policy makers' objective(s) under budget constraints, and the lower levels calculate the user equilibrium for each policy.

NDPs are the same as this study's problem because in both of them the structure of the road network is changing, and new roads are added to the network over time. In contrast to this study's problem, NDPs are dealing with long-term decisions. Expanding existing or constructing new roads are time-consuming and it is expected that travelers' route choice decisions converge to its equilibrium in the time interval between two successive structure changes. However, the time intervals are much shorter in short-term road restoration problems and the structure of the network may change before reaching to a new user equilibrium. This gap will be filled in this study by considering the traffic evolution in the time interval between two successive restoration activities. The fact that the route choice decisions of travelers are made daily and may change day-to-day according to variations in traffic flows throughout the road network is included. This traffic model captures both the time-minimization behavior of travelers and their inertia to shift to new and uncustomary routes.

2.3. Post-disaster Traffic Management:

Generally, two classes of traffic assignment models exist in the literature: static (traditional) and dynamic models. Static models focus on the final traffic equilibrium in road networks that will be achieved after a long time period. These models don't capture the evolution, day-to-day fluctuations, in the traffic before reaching the final equilibrium. Therefore, they are not useful in operational problems (e.g., short-term road network restoration) where short-run traffic information is needed. Recently, due to growing interests in real-time systems, significant efforts were made to extend static traffic assignment models to models with dynamic settings. As Watling and Hazelton (2003) discuss, dynamic models facilitate the inclusion of different behavioral rules, traffic types, and aggregation levels.

There are different types of dynamic models in the literature: (i) continuous-time approaches, and (ii) discrete-time models. Friesz et al. (1994), Smith (1984), and Zhang & Nagurney (1996) develop three continuous time models in the presence of complete travel cost/time information. In these approaches, travelers are permitted to change their traveling routes any time before reaching the equilibrium. Dynamic discrete-time or day-to-day traffic models have more realistic assumptions in comparison to the continuous-time models. In day-to-day models, travelers choose their traveling routes in a way to decrease their travel costs, and these selections are repeated daily. He et al. (2010) and Nogal et al (2016) propose link-based day-to-day traffic models to quantify traffic evolutions over the links of a road network. All the above-mentioned approaches model the traffic evolution in road networks with fixed topologies. They are not applicable for post-disaster circumstances in which the structure of the road network changes frequently due to the road restoration activities.

There are few studies that study the day-to-day behavior of travelers in post-disruption circumstances. For example, He & Liu (2012) propose a prediction-correction model to quantify traffic evolution after a disruption in a road network. In this model, the predicted flow pattern in each day is regularly modified by real experiences of travelers in previous days. However, there is not any road recovery operations in their model and the structure of the network stays fixed after the disruption. This gap is filled in this study by incorporating the road restoration operations. As the recovery process continues, the traffic flow dynamically evolves over time after terminating the recovery operation of each disrupted road. Quantifying these temporal fluctuations provides a better estimation of traffic patterns in the road network and considering them enhances the efficiency of road restoration operations. While existing studies model the road restoration as a static process with no adaption to traffic evolutions, the restoration process in this problem is continuously updated according to traffic evolutions in a dynamically changing road network structure.

2.4. Uncertainty Management in Restoration Operations:

Due to the lack of reliable data, post-disaster environment is highly stochastic. Therefore, proposing a restoration plan that is robust against uncertainties (e.g., travel demands, number of recovery teams, and the repair time of disrupted roads) is highly important (Dimitriou et al., 2008; Fang & Sansavini, 2019). For example, providing emergency services to the affected population may raise some new travel demands after disasters (Chikaraishi et al., 2020). In addition, the repair time predicted by experts to restore a disrupted road is uncertain and provided in a range. There are few studies that consider uncertainty in the road restoration operations. Sanci & Daskin (2019) propose a two-stage stochastic model for pre-disaster repair facility location and post-disaster road network restoration. The repair time of damaged roads and supply/demand of relief items are stochastic in their model. Alkhaleel et al., (2021) develop a model to schedule repair activities in a disrupted road network with stochastic repair times. In a similar problem, Fang & Sansavini (2019) consider uncertainty in repair times and restoration resources. These studies completely ignore traffic flows in road networks. They are not able to evaluate the impact of the restoration decisions on facilitating traffic flows in road networks. This gap is fulfilled in this study. This study focuses on scheduling post-disaster restoration operations in a road network with stochastic repair times. The restoration decisions are made in a way to make the highest acceleration in traffic flows.

2.5. Contributions of the Research:

The contributions of this study to the road network restoration literature are three-fold:

- Evaluating the impacts of road restoration operations on traffic acceleration: Here, an integrative framework in which the road restoration operations are prioritized according to their impacts on post-disaster traffic flows is proposed.
- Considering traffic evolutions instead of final equilibriums: Due to the short-term nature of restoration operations, the structure of a road network changes frequently during the restoration process and there is not enough time for travelers to reach to final traffic

equilibriums. Therefore, day-to-day traffic evolutions are considered in the proposed framework.

- Making adaptive road restoration decisions: While most of the existing studies model the road restorations as a static process with no adaptation to traffic evolutions, here, the restoration plan is continuously updated according to the expected traffic evolutions in a dynamically changing road network structure.

In this study, a multi-level mathematical framework, with different decision-making techniques and time scales at each level is developed to model the problem (Migdalas et al., 2013; Neumayr et al., 2011). The proposed framework is not a conventional multi-level optimization model. While the outer level of the framework optimizes restoration operations, the inner level includes an iterative algorithm that estimates day-to-day traffic evolutions in the road network. This feature significantly increases the computational complexity of the proposed framework. Thus, in this study, a novel solution approach based on Reinforcement Learning (RL), called BRLM, is developed to tackle its computational complexity.

3. Problem description

The road network is represented as a directed graph, $G(N, L)$, with a set of nodes, N, and a set of links, $L = \{l = (n, m)|\ n, m \in N\}$. In the normal condition (e.g., without any disruption), there are in-equilibrium traffic flows between a set of origin, $O \subset N$, and destination, $E \subset N$, nodes in the network (the notations used in the study are summarized in Appendix A). Considering $M = \{(i, j)|i \in O\ and\ j \in E\}$ as the set of OD pairs, d_{ij} represents the traffic demand between the origin and destination nodes of OD pair $(\overrightarrow{i,j}) \in M$. To calculate the travel cost/time of roads, the cost function of the Bureau of Public Roads is used. Traversing link $l = (\overrightarrow{n,m})$ is associated with a positive cost/time of $c_{\overrightarrow{n,m}}(x_{\overrightarrow{n,m}})$ for travelers which is a function of its traffic flow ($x_{\overrightarrow{n,m}}$), free-flow travel time ($c_{0_{\overrightarrow{n,m}}}$), and nominal capacity ($B_{\overrightarrow{n,m}}$) (Bureau of Public Roads, 1964):

$$c_{\overrightarrow{n,m}}(x_{\overrightarrow{n,m}}) = c_{0_{\overrightarrow{n,m}}}\left[1 + 0.15\left(\frac{x_{\overrightarrow{n,m}}}{B_{\overrightarrow{n,m}}}\right)^4\right] \tag{1}$$

Let's assume that the network is affected by a disaster, which results in disruption and blockage of some links. Disrupted links are completely blocked and inaccessible. Disrupted links are included in set $L' \subset L$. The disruptions significantly reduce the functionality of the network, and its traffic flows would not be in an equilibrium state anymore. Also, after the disaster, the short-term traffic demand for some existing OD pairs may change (e.g., usually reduces) and some new OD pairs may emerge (e.g., to transport casualties from affected sites to hospitals and transport emergency goods from depots to affected sites). Set $\acute{M}$ includes the post-disaster OD pairs and $\acute{d}_{\overrightarrow{\iota,j}}$ shows the traffic demand between the origin and destination nodes of OD pair $(\overrightarrow{\iota,j}) \in \acute{M}$ in a short time interval after the disaster ($\acute{O}$ and $\acute{E}$ show the set of origin and destination nodes after the disaster). Due to the lack of some links and variations in the traffic demands, post-disaster traffic flows in the road network will be in a non-equilibrium state. In a road network with a fixed structure, the non-equilibrium traffic flows evolve through day-to-day adjustments and finally converge to a new equilibrium state. In the problem of this study, the restoration process of disrupted roads is started immediately after the disaster. After the restoration of each disrupted road, the structure of the road network will change. A new structure necessitates a new equilibrium state. Therefore, after each road restoration, traffic evolutions toward a new equilibrium state are initiated in the network. This demonstrates the necessity of considering day-to-day traffic flows (instead of focusing on the final equilibrium state) in scheduling road restoration operations. In this study, the objective is to schedule the road restoration operations in a way to make the highest acceleration in the post-disaster traffic flows in $[0, T]$ time interval (e.g., 2-3 weeks after the disaster) in which the response operations are going on (the disaster happens at time 0 and the response operations continues up to time T).

The general structure of the problem investigated in this study is bi-level programming (Figure 2). In the outer level, restoration decisions are scheduled for the disrupted roads in the presence of limited restoration units, referred as recovery "crews". The inner level measures the impact of restoration decisions on the post-disaster traffic flows in the road network. The objective of the model is to schedule restoration operations in a way to maximize the traffic acceleration after the disaster. The restoration time of each disrupted road is uncertain and depends on the number of crews assigned to that road. The higher the number of crews assigned to each road, the shorter the restoration time of that road.

In each decision-making moment, the outer level determines which disrupted road(s) should be recovered. In the cases in which more than one link is selected for restoration, recovery crews should be assigned appropriately to the selected roads. After the restoration of selected roads, another restoration decision is made, and another sub-set of disrupted roads is selected for recovery. In the inner level, to measure the traffic improvement, a day-to-day traffic assignment model is used. This model estimates traffic evolutions in the road network after each restoration decision. The assessed traffic improvement in the inner level will be sent as feedback to the outer level. The outer level employs these feedbacks to evaluate road restoration schedules, and select the best one

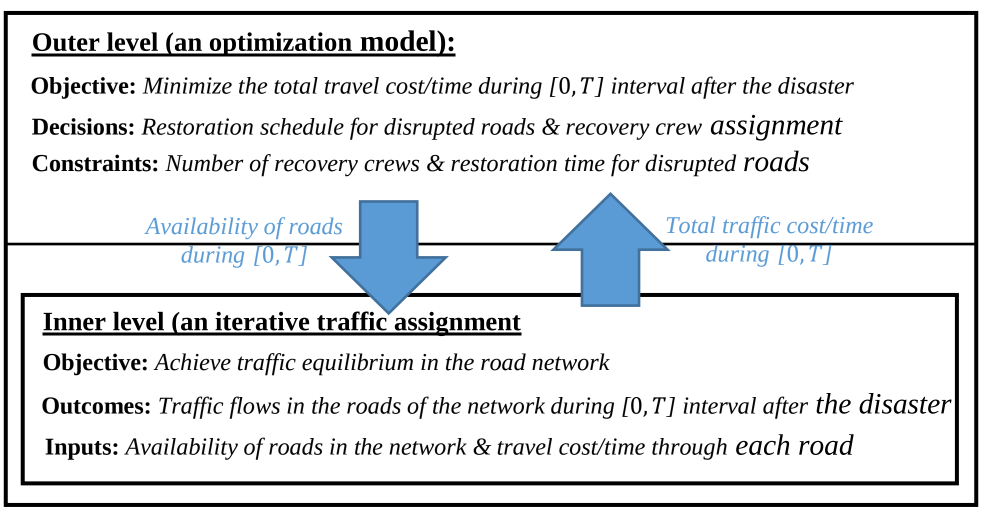

Figure 2.The structure of the bi-level framework

that leads to the highest improvement (e.g., acceleration) in the post-disaster traffic flows. Figure 2 depicts the structure of the bi-level framework.

The bi-level programming represents an interesting and rich field of optimization. Although important progress has been obtained, it is still a fertile area of research (Tuy, H., et al., 1993; Côté, J.-P et al., 2003; Brotcorne, L., et al., 2001). Bi-level models are intrinsically hard to solve because they are neither convex nor continuous (Calvete, H.I. & C. Galé, 2004; Rezapour, S., et al., 2011). Even deterministic and linear bi-level models are shown to be NP-hard (Scheel, H. & S. Scholtes, 2000; Bard, J.F., et al., 2000; Colson, B., et al., 2005). Having an iterative algorithm in the inner level (for day-to-day traffic assignments), and including uncertainty in the outer level (for recovery times) adds more complexity to the framework. Therefore, instead of formulating the problem as a traditional bi-level optimization model, here RL is employed to formulate the problem and make it solvable for large-scale real-size road networks. The structure of the proposed RL mechanism is shown in Figure 1. The outer level of the bi-level framework (in Figure 2) constitutes the decisions made by the agent of the RL (in Figure 1). The agent of the RL schedules the restoration operations for damaged/disrupted roads in the road network in a way to maximize the traffic acceleration after the disaster. The consequences of decisions made by the agent are evaluated in the RL's learning environment. The inner level of the bi-level framework (in Figure 2) constitutes the learning environment of the RL (in Figure 1). The learning environment includes an iterative algorithm that estimates day-to-day traffic evolutions in the road network. This algorithm is behaviorally enriched and captures both the time-minimization behavior of travelers and their inertia to shift to new and uncustomary routes.

4. Mathematical modelling

In section 4.1, the day-to-day traffic assignment algorithm is explained. This algorithm constitutes the learning environment of BRLM in Figure 1. The process of scheduling the restoration of disrupted roads by the agent of BRLM is explained in section 4.2.

4.1. Day-to-Day (DTD) Traffic Assignment Algorithm

During the disaster response phase, damaged/blocked roads will be retrieved according to a restoration schedule (that will be developed in section 4.2) and turn back to the structure of the network. After each restoration activity, the network won't be in the traffic equilibrium condition anymore. This means some travelers find themselves in a costlier path compared to others and their own previous costs. Therefore, they start to shift their travel paths to cheaper ones until the network reaches a new equilibrium. Usually, before reaching the new equilibrium, other damaged/blocked roads are retrieved and added to the network. Thus, considering traffic evolutions (instead of final equilibriums) is necessary to provide an accurate picture of post-disaster traffic during the road restoration process. To estimate traffic evolutions during the restoration horizon ([0,T]), employ a behaviorally-enriched day-to-day traffic assignment algorithm is employed. In this algorithm, the main idea is that on each day, the link flows move from the current state ($\boldsymbol{X} = [x_{\overrightarrow{n,m}}]$) toward a target state ($\boldsymbol{Y} = [y_{\overrightarrow{n,m}}]$) with a certain step size of α (He et al., 2010).

$$\dot{\boldsymbol{Z}} = \alpha(\boldsymbol{Y} - \boldsymbol{X}) \tag{2}$$

Since the current flow pattern ($\boldsymbol{X}$) is known, in order to calculate the flow changes on any day ($\dot{\boldsymbol{Z}}$), the target flow pattern ($\boldsymbol{Y}$) should be determined. The target flow pattern not only minimizes the total travel cost/time in the road network, but also considers the behavioral inertia of travelers. In out-of-equilibrium conditions, while travelers change their paths to minimize their travel cost/time, they tend to avoid making unnecessary changes. This phenomenon is called behavioral inertia. In other words, travelers have no tendency to make any change in their travel paths unless it makes a significant reduction in their travel cost/time. Hence, a combination of travelers' cost minimization and inertia behaviors is used to develop a mathematical model to calculate the target flow pattern (***Y***):

$$\underset{Y=[y_{\overrightarrow{n,m}}]}{\text{MIN}} \ \lambda. \sum_{(\overrightarrow{n,m}) \epsilon L-L'} c_{\overrightarrow{n,m}}\left(x_{\overrightarrow{n,m}}\right). y_{\overrightarrow{n,m}} + (1-\lambda). \sum_{(\overrightarrow{n,m}) \epsilon L-L'} \left|x_{\overrightarrow{n,m}} - y_{\overrightarrow{n,m}}\right| \tag{3}$$

$$\text{S.T} \quad \sum_m y_{\overrightarrow{\iota,m}}^{\overrightarrow{\iota,j}} = \acute{d}_{\overrightarrow{\iota,j}} \qquad \forall(\overrightarrow{\iota,m}) \in L-L', \ \forall(\overrightarrow{\iota,j}) \in \acute{M}, \text{ and } \forall i \in \text{Ó} \tag{4}$$

$$\sum_n y_{\overrightarrow{n,j}}^{\overrightarrow{\iota,j}} = \acute{d}_{\overrightarrow{\iota,j}} \qquad \forall(\overrightarrow{n,j}) \in L-L', \ \forall(\overrightarrow{\iota,j}) \in \acute{M}, \text{ and } \forall j \in \text{É} \tag{5}$$

$$\sum_n y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}} = \sum_k y_{\overrightarrow{m,k}}^{\overrightarrow{\iota,j}} \quad \forall(\overrightarrow{n,m}), \left(\overrightarrow{m,k}\right) \in L-L', \forall(\overrightarrow{\iota,j}) \in \acute{M}, \text{ and } \forall m \in N-\acute{E}-\acute{O} \tag{6}$$

$$y_{\overrightarrow{n,m}} = \sum_{(\overrightarrow{\iota,j})} y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}} \quad \forall(\overrightarrow{n,m}) \in L-L' \text{ and } \forall(\overrightarrow{\iota,j}) \in \acute{M} \tag{7}$$

$$y_{\overrightarrow{n,m}} \text{ and } y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}} \geq 0 \quad \forall(\overrightarrow{n,m}) \in L-L' \text{ and } \forall(\overrightarrow{\iota,j}) \in \acute{M} \tag{8}$$

The first term of the objective function ($\sum_{(\overrightarrow{n,m}) \epsilon L-L'} c_{\overrightarrow{n,m}}\left(x_{\overrightarrow{n,m}}\right). y_{\overrightarrow{n,m}}$) minimizes the total travel cost/time in the road network and the second term ($\sum_{(\overrightarrow{n,m}) \epsilon L-L'} \left|x_{\overrightarrow{n,m}} - y_{\overrightarrow{n,m}}\right|$) reflects travelers' inertia behavior by minimizing the distance between the target and current flow patterns. Objective function (3) is a weighted summation of these two terms. In this model, variable $y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}}$ shows the traffic flow through link $(\overrightarrow{n,m})$ that is related to the traffic demand of OD pair $(\overrightarrow{\iota,j})$. As shown in constraint (7), the summation of these flows on all OD pairs ($\sum_{(\overrightarrow{\iota,j})} y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}}$) is equal to the total flow of link $(\overrightarrow{n,m})$ ($y_{\overrightarrow{n,m}}$). Constrains (4), (5), and (6) ensure the flow balance at the origin (represented by set $\acute{O}$), destination (represented by set $\acute{E}$), and intermediate (represented by set $N - \acute{E} - \acute{O}$) nodes of road network. The process of linearizing and solving Model (3-8) is explained in Appendix B.

In model (3-8), the travel cost/time of each link ($c_{\overrightarrow{n,m}}$) depends on its current traffic flow ($x_{\overrightarrow{n,m}}$), as shown in equation (1). After calculating target flows for links (e.g., using model (3-8)), on each day, the traffic flow of links $(\overrightarrow{n,m})$ on the next day would be as $x_{\overrightarrow{n,m}} + \alpha\left(y_{\overrightarrow{n,m}} - x_{\overrightarrow{n,m}}\right)$. This shows the traffic evolution of link $(\overrightarrow{n,m})$ over two consecutive days. Repeating these steps for all days, traffic evolutions over the entire restoration horizon can be estimated.

After the restoration of each disrupted road, set L' would be updated in model (3-8), and the target values for the links of the network will change. This will put the flow pattern in the road network in an out-of-equilibrium state and initiate other traffic evolutions toward a new equilibrium state.

4.2. Road Restoration Scheduling:

In this section, RL is employed to formulate and solve the problem of scheduling the restoration process of disrupted roads. Section 4.2.1 includes a belief explanation of RL principal. The customized RL for the road restoration scheduling is presented in section 4.2.2.

4.2.1. Principal of RL:

RL is a machine learning approach in which an agent interacts with a learning environment to explore the consequences of its actions/decisions. The feedbacks that are received from the environment will train the agent to revise its actions/decisions in a way to optimize a value function. In other words, the agent tries to optimize its actions/decisions through learning from the experiences it gains while interacting with the environment (Farazi et al., 2021).

To formulate a decision-making process by RL, the following terms should be defined: Set A includes all feasible actions (or decisions) that can be taken (or made) by the agent in each decision-making stage of the RL ($k \in K$). Taking each action places the environment in a specific state. Set S represents all feasible states that the environment can be in. In decision-making stage k, the agent observes the current state of the environment, $s_k \in S$, and takes the most appropriate action of $a_k \in A$ based on its past experiences. This action transits the environment to state s_{k+1} and the agent receives feedback from the environment as a reward, r_k, which represents the goodness of its action. This state-action transition procedure, (s_k, a_k), repeats in RL stages and the agent learns how to enhance its performance by taking a series of actions that guarantees a higher cumulated reward through the decision-making stages of the problem. Generally, the reward is a function of the current state, action, and the next state (Jasmin et al., 2011):

$$r_k = g(s_k, a_k, s_{k+1}) \tag{9}$$

To calculate the total reward, gained rewards from all transitions are cumulated. Future stages' rewards may have lower effect in comparison to the current stage's reward which is reflected in the discount factor of γ $(0 < \gamma \leq 1)$. Therefore, the total reward resulted from actions at stages are calculated as follows:

$$R = \sum_{k=1}^{K-1} \gamma^k g(s_k, a_k, s_{k+1}) \tag{10}$$

The goal of RL is to derive an optimal policy function, $\pi^*: S \to A$, that maps a sequence of space-action that results in the maximum cumulated reward. In other words, an optimal policy (π^*) includes the recommended actions for problem stages in order to gain the maximum cumulated reward:

$$\pi^* = Arg \max_{\pi}\left(E\left[\sum_{k=1}^{K-1} \gamma^k g(s_k, a_k, s_{k+1})\right]\right) \tag{11}$$

To find an optimal policy, Q-learning is one of the most popular and widely used approaches (Watkins, 1989; Sutton & Barto, 1998). This approach employs a utility function called $Q(s_k, a_k)$. Function Q estimates the expected reward of taking an action at a given state. The Q-learning approach calculates a Q-matrix for the problem (Farazi et al., 2021). The Q-matrix follows the shape of $[State, Action]$ and is initiated with values equal to zero that will be updated after each iteration (τ) following the Bellman Equation, where ξ represents the step size of the RL:

$$Q^{\tau+1}(s_k, a_k) = (1 - \xi)Q^{\tau}(s_k, a_k) + \xi\left[g(s_k, a_k, s_{k+1}) + \gamma \max_{a_{k+1}} Q^{\tau}(s_{k+1}, a_{k+1})\right] \tag{12}$$

The main idea behind the RL is that the agent must choose actions that have been previously experienced and found to be effective in producing reward. To find these actions, the agent also needs to try actions that have not been chosen before. Therefore, there is always a challenge for the agent to decide when to exploit what it already knows in order to obtain reward and when to explore in order to find better actions which lead to higher rewards. It's critical that the agent does both exploration and exploitation. In cases of being exclusively used, they both will result in failure. The simplest action selection rule is to select only actions with highest estimated value; however, this method always exploits the existing knowledge to optimize the gained reward. Since there may always be better actions that have not been chosen yet, the agent should sometimes simply select randomly between all actions, independently of their action values. This method is called ε-greedy method (Sutton & Barto, 2018). Using this method, the agent chooses an action randomly with the probability of ε, and chooses the one with the highest reward with the probability of $1-\varepsilon$. The higher the value of ε, the higher the chance to choose an action randomly. To avoid non-convergence (happens for large ε values) and not to slow down the learning process (happens for small ε values), large values are selected for ε in initial iterations. However, it is gradually reduced to smaller values in further iterations. This enables the RL to do a better exploration in initial phases and exploits the goodness of actions in later phases (Jasmin et al., 2011).

4.2.2. RL for road network restoration scheduling:

In this study, the road network restoration problem is formulated as an RL (Q-learning) with the following features: The learning environment of the RL includes the DTD traffic assignment algorithm (explained in section 4.1) formulated for the directed graph of the road network. The road restoration operation starts immediately after the disaster (at time 0) and is scheduled by the agent of the RL. The first decision-making stage happens at time 0 ($k=1$). The agent makes two decisions at each stage: (1) select the best subset of disrupted and unrecovered roads for recovery (${L'}^{k}$); and (2) assign recovery crews (Λ^{k}) to the roads selected for recovery. The recovery time of each road depends on the number of crews assigned to that road. This team allocation is done in a way to minimize the recovery time for selected roads and retrieve them to the road network as fast as possible. Variable $w_{(\overrightarrow{n,m})}$ shows the number of crews assigned to link $(\overrightarrow{n,m}) \in {L'}^k$ and $\bar{\sigma}_{(\overrightarrow{n,m})}$ represents the average restoration time of link $(\overrightarrow{n,m})$ if only one crew is assigned to that link. The optimal team allocation to the links of set ${L'}^k$ can be calculated using the following model:

$$\text{MIN} \quad \vartheta_{{L'}^k} = \underset{\forall(\overrightarrow{n,m})\in {L'}^k}{\text{MAX}} \vartheta^{(\overrightarrow{n,m})} = \left(\frac{\bar{\sigma}_{(\overrightarrow{n,m})}}{w_{(\overrightarrow{n,m})}}\right) \tag{13}$$

$$\text{S.T} \quad w_{(\overrightarrow{n,m})} \leq C_{(\overrightarrow{n,m})} \qquad \forall(\overrightarrow{n,m}) \in {L'}^k \tag{14}$$

$$\sum_{(\overrightarrow{n,m})\in {L'}^k} w_{(\overrightarrow{n,m})} \leq \Lambda^k \tag{15}$$

$$w_{(\overrightarrow{n,m})} \geq 0 \qquad \forall(\overrightarrow{n,m}) \in {L'}^k \tag{16}$$

Objective function (13) minimizes the total time required to recover the roads/links selected for restoration (${L'}^k$). Constraint (14) ensures that the number of recovery crews assigned to each road/link is not higher than the maximum number of crews that can work concurrently on that link ($C_{(\overrightarrow{n,m})}$). According to constraint (15), the total number of crews assigned to the selected links should not be higher that the number of crews available at stage k (Λ^k). The process of linearizing and solving Model (13-16) is explained in Appendix C. Solving this model helps us calculate the best team allocation scheme, $\left\{w^*_{(\overrightarrow{n,m})}\right\}_{{L'}^k}$, and the minimum recovery time, $\vartheta^*_{{L'}^k}$, for each set of links that can be selected by the agent for recovery ($\forall {L'}^k \subset L'$). Therefore, the RL terms in this problem is defined as follows:

- **RL Stages:** The number of RL stages is equal to the number of decision-making moments for its agent. Since there are $|\boldsymbol{L'}|$ number of disrupted roads in the network, the highest number of decision-making happens when the roads are recovered sequentially (there is no concurrent road recovery operation). In this case, the number of decision-making moments would be $|\boldsymbol{L'}|$. Therefore, the road restoration RL includes $|\boldsymbol{L'}|$ stages.

- **RL States:** At each stage or decision-making moment, states of RL display the tentative situations of the road network by representing the sets of disrupted roads that have not been restored by that stage. In the first stage ($\boldsymbol{k} = \mathbf{1}$), there is only one state in which all links/roads of $\boldsymbol{L'}$ are available for restoration. In the other stages, there are $\mathbf{2}^{|\boldsymbol{L'}|}$ number of states.

- **RL Actions:** The action space in state $\boldsymbol{s}$ of stage $\boldsymbol{k}$ consists of all feasible subsets of roads that are available for restoration in that state. In the first stage ($\boldsymbol{k} = \mathbf{1}$), this space includes $\mathbf{2}^{|\boldsymbol{L'}|}$ number of actions at most. Based on the number of recovery crew ($\boldsymbol{\Lambda^k}$), some of the actions can be infeasible. In stage $\boldsymbol{k} = \mathbf{2}$, the set of available roads for restoration reduces to $\boldsymbol{L'} - \boldsymbol{L'}^{\boldsymbol{k=1}}$ and the highest size of the action space decreases to $\mathbf{2}^{\left|\boldsymbol{L'} - \boldsymbol{L'}^{\boldsymbol{k=1}}\right|}$.

- **RL Rewards:** The reward of taking an action is each state (needed to update the $\boldsymbol{Q}$ value) is calculated based on the total reduction that taking this action (restoring its selected roads) causes in the road network. Assume that disrupted roads of set $\boldsymbol{L'}^{\boldsymbol{k}}$ are selected for recovery at the decision-making moment of $\boldsymbol{t^k}$ and the total time needed to restore these roads is $\boldsymbol{\vartheta}_{\boldsymbol{L'}^{\boldsymbol{k}}}$. The reward of taking this action would be equal to the total traffic cost/time in $[\boldsymbol{t^k} + \boldsymbol{\vartheta}_{\boldsymbol{L'}^{\boldsymbol{k}}}, \boldsymbol{T}]$ internal in the presence of $\boldsymbol{L'}^{\boldsymbol{k}}$ roads minus the total traffic cost/time in $[\boldsymbol{t^k} + \boldsymbol{\vartheta}_{\boldsymbol{L'}^{\boldsymbol{k}}}, \boldsymbol{T}]$ internal in the absence of $\boldsymbol{L'}^{\boldsymbol{k}}$ roads.

5. Computational results

5.1. Case Study Problem: Tornado Scenario

One of the common disasters in the U.S. is tornado. On average, 1200 tornadoes happen in the U.S. annually (Perkins, 2002). Severe tornadoes (with high violence levels such as EF4 and

EF5) happen in the U.S. more often than the rest of the world. They are frequent in the central U.S., east side of Rocky Mountains. The term of "Tornado Alley" is usually used to represent the most tornado-prone regions in the U.S. The alley stretches from the northern Texas to Canadian prairies and includes several U.S. states such as Texas, Louisiana, Oklahoma, Kansas, Nebraska, Iowa, and South Dakota (Broyles & Crosbie, 2002). To analyze the performance of the BRLM, the city of Sioux Falls in South Dakota is selected as a study region. The study region and its road network are represented in Figure 3. Nodes of the network represent the city districts and their connecting highways and main roads are represented by links. The OD pairs of the traffic flow in the road network are represented in Appendix D.

In the U.S., tornado forecasts and warnings are only issued by the National Weather Service (NWS) working under The National Oceanic and Atmospheric Administration (NOAA). According to NOAA reports, tornadoes can move in any direction. However, their frequent movement trajectories are from southwest to northeast and from west to east (NOAA, 2021). Most of tornadoes last less than 10 (min). Based on the path length of tornadoes occurred since 1950, the average moving distance of tornadoes is 3.5 (mi).

To generate disaster scenarios, four movement directions (southwest→northeast, west→east, southeast→northwest, & east→west) are considered for the tornado. Each moving direction includes two different starting points and two different moving angles. Three options are considered for the path length (2.5, 3.5, and 4.5 miles) and severity (low, medium, and high) of the tornado. 30%, 60%, and 90% of links/roads located in the moving path of the tornado are disrupted in severity level of low, medium, and high. Three sets of disrupted roads are selected randomly from the set of all roads located in the tornado path. Each problem is solved for 2, 4, 6, and 10 number of recovery teams. The recovery time of each road is selected randomly from $\left[0.9 \times \bar{\sigma}_{\overrightarrow{(n,m)}},\ 1.1 \times \bar{\sigma}_{\overrightarrow{(n,m)}}\right]$ interval. Parameters of the traffic assignment algorithm and RL are set as follows: $\alpha = 0.1, 0.2, 0.4, 0.6$, $\lambda = 100$, $\xi = 0.25$, $\gamma = 1$, $\varepsilon = 0.5$, $T = 18$.

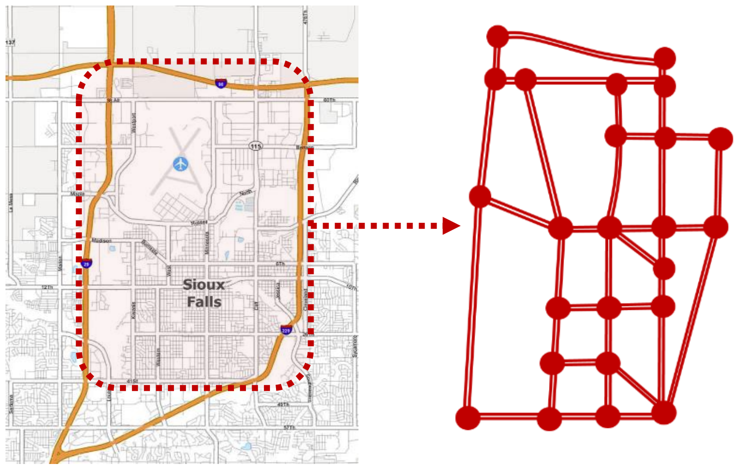

Figure 3. The study region and its road network.

5.2. Sensitivity Analysis: Impact of Travelers' Agility

The results of the BRLM for the case study problem and its tornado scenarios are summarized in Table 1. The cells of the table represent the average reduction in the total traffic time/cost of the road network caused by the road restoration operations during the restoration horizon. The average reduction in each cell is calculated based on all problem instances solved for that cell. Figure 4 shows how the shifts speed of travelers from the current moving paths to the target paths (adjusted by parameter α) impacts the total improvement made by restoration activities that are scheduled by BRLM. Results show that:

- In road networks with ponderous travelers, parameter $\boldsymbol{\alpha}$ takes low values (e.g., $\boldsymbol{\alpha}$ =10% in Figure 4). This means the shift speed of travelers from current moving paths to target paths is low. Therefore, it takes very long time for travelers to reach to a new traffic equilibrium. After each restoration, travelers spend lots of time in non-equilibrium/transitional and costly paths. These long transitions after each restoration lead to negative improvements in the results of BRLM. This means short term road restorations after disasters do not necessarily accelerate the disaster-response traffic flows (e.g., traffic flows in the short response period after disasters) in road networks with ponderous travelers.

- In road networks with very agile travelers, parameter $\boldsymbol{\alpha}$ takes high values (e.g., $\boldsymbol{\alpha}$ =60% in Figure 4). Therefore, after each road restoration, travelers shift quickly from current moving paths to target paths. This leads to a short transition period, but high traffic fluctuations before reaching the new equilibrium. High flow fluctuations correspond to high travel cost/time. These high travel flow fluctuations after each restoration cause a type of chaos in the road network and reduce improvements in the results of BRLM.
- At the intermediate $\boldsymbol{\alpha}$ values (e.g., $\boldsymbol{\alpha}$ =20% and 40% in Figure 4), the length of the transition period is shorter (than road networks with ponderous travelers) and flow fluctuations are smaller (than road networks with very agile travelers). This makes the restoration activities more profitable for road networks. Therefore, improvements in the results of BRLM are higher.

Based on these results, the conclusion is that the advantages of short-term road restoration on the disaster-response traffic flows completely depends on the inertia behavior of travelers. These restorations may even adversely affect the total travel time/cost in road networks with ponderous travelers. The improvements caused by restorations are high in road networks with medium inertia behavior. Increasing the agility of travelers in shifting to shorter/less-costly paths mitigates the advantages of restoration activities. According to Figure 4, the sensitivity of restoration improvement to inertia behavior of travelers alleviates by reducing recovery resources (e.g., number of recovery teams).

Table 1. The numerical results.

		Speed of shifting travelers to target paths (α)			
Number of teams	Approaches	10%	20%	40%	60%
2	*BRLM*	-1,581,787,000,000.00	429,364,300,000.00	619,380,200,000.00	563,292,100,000.00
	Heuristic	-3,072,927,968,718.22	-1,614,846,786,140.51	559,745,161,236.04	124,177,525,972.13
	Over performance	-1,491,140,968,718.22	2,044,211,086,140.51	59,635,038,763.96	439,114,574,027.87
4	*BRLM*	-1,191,120,000,000.00	745,185,400,000.00	1,177,607,000,000.00	700,687,500,000.00
	Heuristic	-3,072,927,968,718.22	-1,614,846,786,140.51	559,745,161,236.04	124,177,525,972.13
	Over performance	1,881,807,968,718.22	2,360,032,186,140.51	617,861,838,763.96	576,509,974,027.87
6	*BRLM*	-654,471,700,000.00	1,418,049,000,000.00	1,194,370,000,000.00	700,687,500,000.00
	Heuristic	-3,072,927,968,718.22	-1,614,846,786,140.51	559,745,161,236.04	124,177,525,972.13
	Over performance	2,418,456,268,718.22	3,032,895,786,140.51	634,624,838,763.96	576,509,974,027.87
10	*BRLM*	-659,001,100,000.00	1,418,049,000,000.00	1,194,370,000,000.00	700,687,500,000.00
	Heuristic	-3,072,927,968,718.22	-1,614,846,786,140.51	559,745,161,236.04	124,177,525,972.13
	Over performance	2,413,926,868,718.22	3,032,895,786,140.51	634,624,838,763.96	576,509,974,027.87

5.3. Benchmark Approach

The main contribution of the BRLM is related to prioritizing restoration activities according to their impacts on post-disaster traffic movements. To evaluate the efficiency of the BRLM, a heuristic approach is designed. In this approach, disrupted roads that are located in the shortest path of a higher number of OD pairs are prioritized for restoration. This restoration criterion is called "betweenness centrality" and is widely used in the literature [Ulusan & Ergun, 2018; Borassi et al., 2019]. Assume that binary parameter $\beta_{(\overrightarrow{n,m})}^{(i,j)}$ is 1 if road $(\overrightarrow{n,m})$ is located in the shortest path of OD $(i,j) \in \acute{M}$, and 0 otherwise. In this case, the restoration criterion would be defined as $\Pi_{(\overrightarrow{n,m})} = \sum_{(i,j)\in \acute{M}} \beta_{(\overrightarrow{n,m})}^{(i,j)}$. The roads with higher $\Pi_{(\overrightarrow{n,m})}$ values are prioritized for recovery. This approach schedules restoration activities without considering their impacts on traffic movements. Table 1 compares the improvements made by the restoration activities of the BRLM and heuristic approach in the post-disaster traffic flows. The over performance of BRLM varies in

[9.6%, 476.1%] interval. It seems that the over performance of the BRLM increases in scenarios with a higher number of recovery teams.

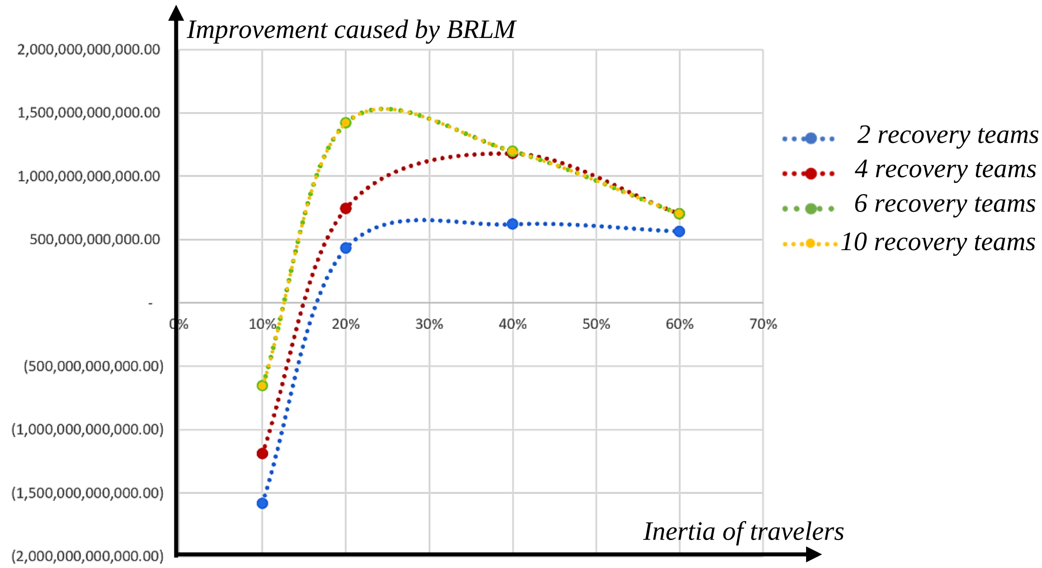

Figure 4. The impact of travelers' agility on the profitability of restoration activities

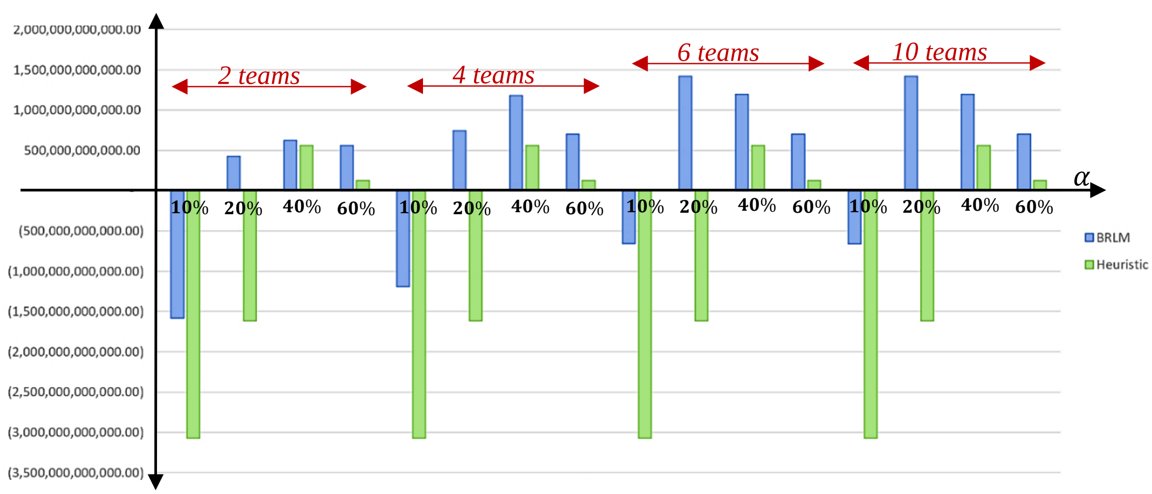

Figure 5. The comparison of improvements made by the BRLM and heuristic

6. Conclusions and Future Work

In this study, a behaviorally-enriched RL mechanism, BRLM, is proposed to schedule post-disaster restoration operations in disrupted road networks. Considering evolutions in the routing behavior of travelers, the objective of the BRLM is to maximize the traffic acceleration after the disaster. The restoration process is conducted under resource limitations (e.g., limited number of recovery crews/equipment) and stochasticity of recovery times (e.g., recovery times of disrupted roads are uncertain).

The proposed BRLM is tested on the road network of Sioux Falls in South Dakota for several tornado scenarios that are developed based on the historical report of National Oceanic and Atmospheric Administration. To evaluate the efficiency of the BRLM, a heuristic method is also developed. In this approach, disrupted roads that are located in the shortest path of a higher number of OD pairs are prioritized for restoration. Computational results show that the advantages of short-term road restoration on the post-disaster traffic flows completely depend on the behavior of travelers. Restorations worsen the total traffic cost/time in road networks with ponderous travelers. Traffic accelerations caused by restoration activities are high in medium inertia behavior of travelers. Increasing the agility of travelers in shifting to shorter/less-costly paths mitigates the advantages of restoration activities.

Suggestions for future research are as follows:

- <u>Restoration under the lack of complete information</u>: In the investigated problem, it is assumed that there is complete information about road network disruptions (e.g., disrupted roads and their recovery times) at the beginning of the planning horizon. Lack of complete information in the chaotic situations after disasters is one of the important post-disaster challenges. Extending the proposed BRLM to include gradual information acquisition would be an interesting future research.
- <u>Interdependent critical infrastructures:</u> In the investigated problem, a road network is considered as an independent critical infrastructure that does not impact and is not impacted by the

performance and restoration process of other critical infrastructures (e.g., electric-power networks and water/wastewater networks). In reality, the performance of critical infrastructures is interdependent (Sharkey et al., 2015). This demonstrates the necessity of having coordinated restoration plans for them. Designing a multi-agent RL for coordinated restoration of interdependent infrastructures would be an interesting extension for this research.

References

A. Garay-Sianca; S. G. N. Pinkley, (2021). Interdependent integrated network design and scheduling problems with movement of machinesEuropean Journal of Operational Research 289, 297-327.

A. Kumar; S. Peeta, (2015). A day-to-day dynamical model for the evolution of path flows under disequilibrium of traffic networks with fixed demand. Transportation Research Part B: Methodological 80, 235-256.

A. Marin; P. Jaramillo, (2008). Urban rapid transit network capacity expansion, European Journal of Operational Research 191, 45-60.

A. Pedraza Martinez; O. Stapleton; L. N. Van Wassenhove, (2010). Using OR to support humanitarian operations: Learning from the Haiti earthquake.

B. A. Alkhaleel; H. Liao; K. M. Sullivan, (2022). Risk and resilience-based optimal post-disruption restoration for critical infrastructures under uncertainty. European Journal of Operational Research. 296, 174-202.

B. E. Oruc; B. Y. Kara, (2018). Post-disaster assessment routing problem. Transportation Research Part B: methodological 116, 76-102.

Bard, J.F., J. Plummer, and J.C. Sourie, (2000). A bilevel programming approach to determining tax credits for biofuel production. European Journal of Operational Research 120, 30-46.

Bengtsson, J., & Tómasson, B., (2008). Vulnerability and risk analysis of road infrastructure in Reykjavik Information. Retrieved from http://www.vegagerdin.is/vefur2.nsf/Files/Vulnerability_riskanalysisPrelimReport/$file/Vulnerability_risk analysis-PrelimReport.pdf

Brotcorne, L., et al., A, (2001). bilevel model for toll optimization on a multicommodity transportation network. Transportation Science 35, 345-358.

Broyles, Chris; C. Crosbie, (2004). Evidence of smaller tornado alleys across the United States based on a long track F3-F5 tornado climatology study from 1880-2003. 22nd Conference on severe local satorms. Hyannis, MA: American Meteorological Society.

Borassi, Michele; Natale, Emanuele, (2019). KADABRA is an ADaptive Algorithm for Betweenness via Random Approximation. ACM Journal of Experimental Algorithmics. 24, 1.2:1–1.2:35

Calvete, H.I. and C. Galé, (2004). Optimality conditions for the linear fractional/quadratic bilevel problem. Monografias del Seminario Matematico Garcia de Galdeano 31, 285-294.

Chikaraishi, M., Garg, P., Varghese, Varun., Yoshizoe, K., Urata, J., Shiomi, Y., Watanabe, R., (2020). On the possibility of short-term traffic prediction during disaster with machine learning approaches: An exploratory analysis. Transport Ppolicy 98, 91-104.

Colson, B., P. Marcotte, and G. Savard, (2005). A trust-region method for nonlinear bilevel programming: algorithm and computational experience. Computational Optimization and Applications 30, 211-227.

Côté, J.-P., P. Marcotte, and G. Savard, (2003). A bilevel modelling approach to pricing and fare optimisation in the airline industry. Journal of Revenue and Pricing Management 2, 23-36.

D. Rey; H. Bar-Gera, 2020. Long-term scheduling for road network disaster recovery. International Journal of Disaster Risk Reduction 42.

D. Sun; R. F. Benekohal; S. T. Waller, (2006). Bi-level programming formulation and heuristic solution approach for dynamic traffic signal optimization. Computer-Aided Civil and Infrastructure Engineering 21, 321-333.

D. T. Aksu; L. Ozdamar, (2014). A mathematical model for post-disaster road restoration: Enabling accessibility and evacuation. Transportation Research Part E: logistics and transportation review 61, 56-67.

D. Watling; M. L. Hazelton, (2003). The dynamics and equilibria of day-to-day assignment models. Networks and Spatial Economics 3, 349-370.

Dimitriou, L., Stathopoulos A., (2008). Reliable Stochastic Design of Road Network Systems. International Journal Systems Engineering 3.

E. Hayat; R. Amaratunga (2011). Road reconstruction in post-disaster recovery; the challenges and obstacles. Road reconstruction in post-disaster recovery; the challenges and obstacles.

E. Sanci; M. S. Daskin, (2019). Integrating location and network restoration decisions in relief networks under uncertainty. European Journal of Operational Research 279, 335-350.

E. Yücel; F. S. Salman; I. Arsik, (2018). Improving post-disaster road network accessibility by strengthening links against failures. European Journal of Operational Research 269, 406-422.

Edrisi, A., Askari, M., (2019). Probabilistic budget allocation for improving efficiency of transportation networks in pre-and post-disaster phases. International Journal of Disaster Risk Reduction 39.

F. Hu; C. H. Yeung; S. Yang; W. Wang; A. Zeng, (2016). Recovery of infrastructure networks after localised attacks. Scientific reports 6, 1-10.

Farazi, P. N., Zou, B., Ahmad T., Barua, L, (2021). Deep reinforcement learning in transportation research: A review. Transportation Research Interdisciplinary Perspectives 11.

Friesz, T. L., Bernstein, D., Mehta, N. J., Tobin, R. L., & Ganjalizadeh, S., (1994). Day-To-Day dynamic network disequilibria and idealized traveler information systems. Operations Research 42, 1120–1136.

Homeland Security Presidential directive, (2003). Critical infrastructure identification, prioritization, and protection.

Hosseininasab, S.-M., & Shetab-Boushehri, S.-N., (2015). Integration of selecting and scheduling urban road construction projects as a time-dependent discrete network design problem. European Journal of Operational Research 246, 762–771.

Jasmin, E.A., Ahamed, Imthias, Jagathy raj, V.P., (2011). Reinforcement learning approaches to economic dispatch problem. Electrical Power and Energy Systems 33, 836-845.

Karoonsoontawong, A., & Waller, S. T., (2010). Integrated network capacity expansion and traffic signal optimization problem: Robust bi-level dynamic formulation. Networks and Spatial Economics 10, 525–550.

L. Özdamar; D. T. Aksu; B. Ergüneş, (2014). Coordinating debris cleanup operations in post disaster road networks. Socio-Economic Planning Sciences 48, 249-262.

M. A. Taylor, (2012). Remoteness and accessibility in the vulnerability analysis of regional road networks. Transportation Research Part A: policy and practice 46, 761-771.

M. Ajam; V. Akbari; F. S. Salman., (2019). Minimizing latency in post-disaster road clearance operations. European Journal of Operational Research 277, 1098-1112.

M. Çelik, (2016). Network restoration and recovery in humanitarian operations: Framework, literature review, and research directions. Surveys in Operations Research and Management Science 21, 47-61.

M. Çelik; Ö. Ergun; P. Keskinocak, (2015). The post-disaster debris clearance problem under incomplete information. Operations Research 63, 65-85.

M. Kasaei; F. S. Salman, (2016). Arc routing problems to restore connectivity of a road network. Transportation Research Part E: Logistics and Transportation Review 95, 177-206.

M. Kazama; T. Noda, (2012). Damage statistics (Summary of the 2011 off the Pacific Coast of Tohoku Earthquake damage). Soils and Foundations 52, 780-792.

M. Nogal; A. O'Connor; B. Caulfield; B. Martinez-Pastor, (2016). Resilience of traffic networks: From perturbation to recovery via a dynamic restricted equilibrium model. Reliability Engineering & System Safety 156, 84-96.

Mathew, T. V., & Sharma, S., (2009). Capacity expansion problem for large urban transportation networks. Journal of Transportation Engineering 135, 406–415.

Migdalas, A., Pardalos, P. M., & Värbrand, P., (2013). Multilevel optimization: algorithms and applications. Springer US. Retrieved from https://books.google.com/books?id=5pXaBwAAQBAJ

Neumayr, B., Schrefl, M., & Thalheim, B. (2011). Modeling techniques for multi-level abstraction.

nn R. Kaschek & L. Delcambre (Eds.), The Evolution of conceptual modeling: From a historical perspective towards the future of conceptual modeling. Berlin, Heidelberg: Springer Berlin Heidelberg, 68–92.

NOAA [https://www.spc.noaa.gov/faq/tornado/]

O. El-Anwar; J. Ye; W. Orabi, (2016). Efficient optimization of post-disaster reconstruction of transportation networks. Journal of Computing in Civil Engineering 30, 04015047.

P. M. Duque; K. Sörensen, (2011). A GRASP metaheuristic to improve accessibility after a disaster. OR spectrum 33, 525-542.

Perkins, Sid, (2002). Tornado Alley, USA. Science News, 296–298.

Q. Zou; S. Chen, (2021). Resilience-based Recovery Scheduling of Transportation Network in Mixed Traffic Environment: A Deep-Ensemble-Assisted Active Learning Approach. Reliability Engineering & System Safety.

R. DesRoches, (2006). Hurricane Katrina: Performance of transportation systems. [Reston, Va.]: American Society of Civil Engineers, 2006.

R. Faturechi; E. Miller-Hooks, (2014). Travel time resilience of roadway networks under disaster. Transportation Research Part B: methodological 70, 47-64.

R. Faturechi; E. Miller-Hooks, (2015). Measuring the performance of transportation infrastructure systems in disasters: A comprehensive review. Journal of Infrastructure Systems 21.

R. Vodák; M. Bíl; Z. Křivánková, (2018). A modified ant colony optimization algorithm to increase the speed of the road network recovery process after disasters. International Journal of Disaster Risk Reduction 31, 1092-1106.

R. Z. Farahani; E. Miandoabchi; W. Y. Szeto; H. Rashidi, (2013). A review of urban transportation network design problems. European Journal of Operational Research 229, 281-302.

Rezapour, S., et al., (2011). Strategic design of competing supply chain networks with foresight. Advances in Engineering Software 42, 130-141.

S. E. Chang; N. Nojima, (2001). Measuring post-disaster transportation system performance: the 1995 Kobe earthquake in comparative perspective. Transportation Research Part A: policy and practice 35, 475-494.

S. Yan; Y.-L. Shih, (2009). Optimal scheduling of emergency roadway repair and subsequent relief distribution. Computers & Operations Research 36, 2049-2065.

Scheel, H. and S. Scholtes, (2000). Mathematical programs with complementarity constraints: Stationarity, optimality, and sensitivity. Mathematics of Operations Research 25, 1-22.

Smith, M., (1984). The stability of a dynamic model of traffic assignment – an application of a method of Lyapunov. Transportation Science 18, 259–304.

Sutton, R. S., & Barto, A. G., (1998). Introduction to reinforcement learning (1st ed.). Cambridge, MA, USA: MIT Press.

Sutton, R. S., & Barto, A. G., (2018). Introduction to Reinforcement Learning (2nd ed.). Cambridge, MA, USA: MIT Press.

Thomas C. Sharkey, Burak Cavdaroglu, Huy Nguyen, Jonathan Holman, John E. Mitchell, William A. Wallace, (2015). Interdependent network restoration: On the value of information-sharing. European Journal of Operational Research 244, 309-321.

Tuy, H., A. Migdalas, and P. Värbrand, (1993). A global optimization approach for the linear two-level program. Journal of Global Optimization 3, 1-23.

Ulusan A, Ergun O., (2018). Restoration of services in disrupted infrastructure systems: A network science approach. PLoS One 13:e0192272.

V. Akbari; F. S. Salman, 2017. Multi-vehicle synchronized arc routing problem to restore post-disaster network connectivity. European Journal of Operational Research 257, 625-640.

W. Orabi; K. El-Rayes; A. B. Senouci; H. Al-Derham, (2009). Optimizing postdisaster reconstruction planning for damaged transportation networks. Journal of Construction Engineering and Management 135, 1039-1048.

W. Zhang; N. Wang, (2016). Resilience-based risk mitigation for road networks. Structural Safety 62, 57-65.

W. Zhang; N. Wang; C. Nicholson, (2017). Resilience-based post-disaster recovery strategies for road-bridge networks. Structure and Infrastructure Engineering 13, 1404-1413.

Watkins, C.J.C.H., (1989). Learning from delayed rewards. PhD Thesis, University of Cambridge, England.

X. He; H. X. Liu, (2012). Modeling the day-to-day traffic evolution process after an unexpected network disruption.Transportation Research Part B: Methodological 46, 50-71.

X. He; X. Guo; H. X. Liu, (2010). A link-based day-to-day traffic assignment model."Transportation Research Part B: Methodological 44, 597-608.

Y. Fan; C. Liu, (2010). Solving stochastic transportation network protection problems using the progressive hedging-based method. Networks and Spatial Economics 10, 193-208.

Y. Yu; K. Han; W. Ochieng, (2020). Day-to-day dynamic traffic assignment with imperfect information, bounded rationality and information sharing. Transportation Research Part C: Emerging Technologies 114, 59-83.

Y.-P. Fang; G. Sansavini, (2019). Optimum post-disruption restoration under uncertainty for enhancing critical infrastructure resilience, Reliability Engineering & System Safety 185, 1-11.

Z. Zhang; B. Wolshon; N. Herrera; S. Parr, (2019). Assessment of post-disaster reentry traffic in megaregions using agent-based simulation. Transportation Research Part D: transport and environment 73, 307-317.

Zhang, D., Nagurney, A., (1996). On the local and global stability of a travel route choice adjustment process. Transportation Research Part B 30, 245–262.

Appendices

Appendix A - Notations:

Table A1. List of notations used in the study.

Sets	
$G(N, L)$:	The road network as a directed graph with a set of nodes, N, and a set of links, $L = \{l = (n, m) \| n, m \in N\}$
O:	The set of origin nodes before the disaster ($O \subset N$)
$\acute{O}$:	The set of origin nodes after the disaster
E:	The set of destination nodes before the disaster ($E \subset N$)
$\acute{E}$:	The set of destination nodes after the disaster
M:	The set of OD pairs before the disaster, $M = \{(i, j) \| i \in O \; and \; j \in E\}$
$\acute{M}$:	The set of OD pairs after the disaster
L':	The set of disrupted links ($L' \subset L$)
S:	The set of states in the RL
A:	The action space of the RL
${L'}^k$:	The set of the links/roads selected for recovery at stage k
$\Phi_{(\overrightarrow{n,m})}$:	The set of all number of teams that can be assigned to link $(\overrightarrow{n,m})$, $\Phi_{(\overrightarrow{n,m})} = \{\theta = 1, 2, \dots, C_{(\overrightarrow{n,m})}\}$
Variables	
$c_{\overrightarrow{n,m}}$:	The positive cost/time of traversing link $(\overrightarrow{n,m})$
$x_{\overrightarrow{n,m}}$:	The traffic flow of link $(\overrightarrow{n,m})$
Y:	The target traffic pattern in the road network
$\dot{Z}$:	The flow variations in the road network
$y_{\overrightarrow{n,m}}^{\overrightarrow{\iota,j}}$:	The traffic flow through link $(\overrightarrow{n,m})$ that is relevant to the traffic demand of OD pair $\overrightarrow{\iota,j}$
R:	The total reward in the RL
r_k:	The received feedback (reward) from the RL's environment at stage k
π^*:	The optimal policy in the RL
Q:	The expected reward matrix of taking actions in the states of the RL
$g(s_k, a_k, s_{k+1})$:	The reward od taking action a_k in state s_k
$w_{(\overrightarrow{n,m})}$:	The number of recovery crews assigned to link $(\overrightarrow{n,m})$
$z_\theta^{(\overrightarrow{n,m})}$:	1 if θ number of crews are assigned to link $(\overrightarrow{n,m})$, and 0 otherwise
$\vartheta_{{L'}^k}$:	The total time needed to recover the links of set ${L'}^k$ selected for recovery
$\vartheta^{(\overrightarrow{n,m})}$:	The time needed to recover link $(\overrightarrow{n,m})$
$v_\theta^{(\overrightarrow{n,m})}$:	This variable substitutes $z_\theta^{(\overrightarrow{n,m})} \times \vartheta_{{L'}^k}$
Parameters	
d_{ij}:	The pre-disaster traffic demand of OD pair $\overrightarrow{\iota,j}$
$\acute{d}_{\overrightarrow{\iota,j}}$:	The post-disaster traffic demand of OD pair $\overrightarrow{\iota,j}$
$c_{0_{\overrightarrow{n,m}}}$:	The free-flow travel time of link $(\overrightarrow{n,m})$

$B_{\overrightarrow{n,m}}$:	The nominal capacity of link $(\overrightarrow{n,m})$
$\boldsymbol{X}$:	The current traffic pattern in the road network
α:	The step size of moving from $\boldsymbol{X}$ to $\boldsymbol{Y}$
λ:	The weight assigned to travelers' cost minimization behavior
$1-\lambda$:	The weight assigned to drivers' inertia behavior
K:	The decision-making stages in the RL
s_k:	The current state of the environment at stage k
a_k:	The chosen action at stage k
T:	The road restoration horizon
γ:	The discount factor of future stages
τ:	The iteration number of the RL
ξ:	The step size of the RL
ε:	The probability for choosing actions randomly
Λ^k:	The number of recovery crews available at stage k
$C_{(\overrightarrow{n,m})}$:	The maximum number of recovery crews that can be assigned to link $(\overrightarrow{n,m})$
$\sigma_{(\overrightarrow{n,m})}$:	The repair time of link $(\overrightarrow{n,m})$ if only one crew is allocated to it

Appendix B

The second term of objective function (3) turns Model (3-8) to a nonlinear programming. To linearize this term, two new non-negative variables such as $v_{\overrightarrow{n,m}}$ and $w_{\overrightarrow{n,m}}$ are defined; and substitute the second term with the following function of $v_{\overrightarrow{n,m}}$ and $w_{\overrightarrow{n,m}}$:

$$\sum_{(\overrightarrow{n,m})\in L-\acute{L}}\left|x_{\overrightarrow{n,m}}-y_{\overrightarrow{n,m}}\right| = \sum_{(\overrightarrow{n,m})\in L-\acute{L}}(v_{\overrightarrow{n,m}}+w_{\overrightarrow{n,m}}) \tag{B1}$$

To ensure the equivalence of $(\sum_{(\overrightarrow{n,m})\in L-\acute{L}}\left|x_{\overrightarrow{n,m}}-y_{\overrightarrow{n,m}}\right|)$ and $(\sum_{(\overrightarrow{n,m})\in L-\acute{L}}(v_{\overrightarrow{n,m}}+w_{\overrightarrow{n,m}}))$ terms, the following constraint should be added to the model:

$$x_{\overrightarrow{n,m}}-y_{\overrightarrow{n,m}} = v_{\overrightarrow{n,m}}+w_{\overrightarrow{n,m}} \qquad \forall(\overrightarrow{n,m})\in L-\acute{L} \tag{B2}$$

After this substitution, the model turns to a linear programming which can be solved using conventional optimization software such as LINGO, GAMS, and GUROBI.

Appendix C

First, objective function (13) is substitute with Min $\vartheta_{L'^k}$ and add constraint (C2), to the model:

$$Min\,\vartheta_{L'^k} \tag{C1}$$

S.T.

$$\vartheta_{L'^k} \geq \frac{\overline{\sigma}_{(\overrightarrow{n,m})}}{w_{(\overrightarrow{n,m})}} \qquad \forall\, (\overrightarrow{n,m}) \in L'^k \tag{C2}$$

$$w_{(\overrightarrow{n,m})} \leq C_{(\overrightarrow{n,m})} \qquad \forall (\overrightarrow{n,m}) \in L'^k \tag{C3}$$

$$\sum_{(\overrightarrow{n,m}) \in L'^k} w_{(\overrightarrow{n,m})} \leq \Lambda^k \tag{C4}$$

$$w_{(\overrightarrow{n,m})} \geq 0 \qquad \forall (\overrightarrow{n,m}) \in L'^k \tag{C5}$$

. Then, to linearize the model, it is assumed that the set of all number of teams that can be assigned to link $(\overrightarrow{n,m})$ is $\Phi_{(\overrightarrow{n,m})} = \{1,2,3, \dots, C_{(\overrightarrow{n,m})}\}$, variable $z_\theta^{(\overrightarrow{n,m})}$ equals to 1 if $\theta \in \Phi_{(\overrightarrow{n,m})}$ number of crews are assigned to link $(\overrightarrow{n,m})$, and 0 otherwise. Therefore, the term $\left(\frac{\overline{\sigma}_{(\overrightarrow{n,m})}}{w_{(\overrightarrow{n,m})}}\right)$ is substituted with $\left(\frac{\overline{\sigma}_{(\overrightarrow{n,m})}}{1z_1^{(\overrightarrow{n,m})}+2z_2^{(\overrightarrow{n,m})}+\cdots+C_{(\overrightarrow{n,m})}z_{C_{(\overrightarrow{n,m})}}^{(\overrightarrow{n,m})}}\right)$, and constraint $\sum_{(\overrightarrow{n,m}) \in L'^k} w_{(\overrightarrow{n,m})} \leq \Lambda^k$ with $\sum_{(\overrightarrow{n,m}) \in L'^k} \sum_{\theta \in \Phi_{(\overrightarrow{n,m})}} \theta\, z_\theta^{(\overrightarrow{n,m})} \leq \Lambda^k$, To ensure that exactly one option is selected as the number of assigned crews for each link, constraint (C9) is added to the model:

$$Min\,\vartheta_{L'^k} \tag{C6}$$

S.T.

$$\vartheta_{L'^k} \geq \frac{\overline{\sigma}_{(\overrightarrow{n,m})}}{1z_1^{(\overrightarrow{n,m})}+2z_2^{(\overrightarrow{n,m})}+\cdots+C_{(\overrightarrow{n,m})}z_{C_{(\overrightarrow{n,m})}}^{(\overrightarrow{n,m})}} \qquad \forall\, (\overrightarrow{n,m}) \in L'^k \tag{C7}$$

$$\sum_{(\overrightarrow{n,m}) \in L'^k} \sum_{\theta \in w_{(\overrightarrow{n,m})}} \theta\, z_\theta^{(\overrightarrow{n,m})} \leq \Lambda^k \tag{C8}$$

$$\sum_{(\overrightarrow{n,m})\in L'^k} z_\theta^{(\overrightarrow{n,m})} = 1 \qquad \forall\, (\overrightarrow{n,m}) \in L'^k \qquad \text{(C9)}$$

The next step is rewriting constraint (C7) as $([1(\vartheta_{L'^k} \times z_1^{(\overrightarrow{n,m})}) + 2\,(\vartheta_{L'^k} \times z_2^{(\overrightarrow{n,m})}) + \cdots + C_{(\overrightarrow{n,m})}(\vartheta_{L'^k} \times z_\theta^{(\overrightarrow{n,m})})] \geq \bar{\sigma}_{(\overrightarrow{n,m})})$, and substituting $\vartheta_{L'^k} \times z_\theta^{(\overrightarrow{n,m})}$ with $v_\theta^{(\overrightarrow{n,m})}$, since $\vartheta_{L'^k}$ is a continuous variable and $z_\theta^{(\overrightarrow{n,m})}$ is a binary variable, $v_\theta^{(\overrightarrow{n,m})}$ should be either equal to zero or $\vartheta_{L'^k}$. To satisfy this, two more constraints (C13) and (C14) are added to the model, The linearized model would be as follows:

$$Min\ \vartheta_{L'^k} \qquad \text{(C10)}$$

S.T.

$$1v_1^{(\overrightarrow{n,m})} + 2v_2^{(\overrightarrow{n,m})} + \cdots + C_{(\overrightarrow{n,m})} v_{C_{(\overrightarrow{n,m})}}^{(\overrightarrow{n,m})} \geq \bar{\sigma}_{(\overrightarrow{n,m})} \qquad \forall\, (\overrightarrow{n,m}) \in L'^k \qquad \text{(C11)}$$

$$v_\theta^{(\overrightarrow{n,m})} \leq M z_\theta^{(\overrightarrow{n,m})} \qquad \forall\, (\overrightarrow{n,m}) \in L'^k,\ \forall\, \theta \in \Phi_{(\overrightarrow{n,m})} \qquad \text{(C12)}$$

$$v_\theta^{(\overrightarrow{n,m})} \leq \vartheta_{L'^k} \qquad \forall\, (\overrightarrow{n,m}) \in L'^k,\ \forall\, \theta \in \Phi_{(\overrightarrow{n,m})} \qquad \text{(C13)}$$

$$v_\theta^{(\overrightarrow{n,m})} \geq \vartheta_{L'^k} - M(1 - z_\theta^{(\overrightarrow{n,m})}) \qquad \forall\, (\overrightarrow{n,m}) \in L'^k,\ \forall\, \theta \in \Phi_{(\overrightarrow{n,m})} \qquad \text{(C14)}$$

$$\sum_{(\overrightarrow{n,m})\in L'^k} z_\theta^{(\overrightarrow{n,m})} = 1 \qquad \forall\, (\overrightarrow{n,m}) \in L'^k \qquad \text{(C15)}$$

$$\sum_{(\overrightarrow{n,m})\in L'^k} \sum_{\theta\in\Phi_{(\overrightarrow{n,m})}} z_\theta^{(\overrightarrow{n,m})} \times \theta \leq \Lambda^k \qquad \text{(C16)}$$

Appendix D

Table D2. The list of the Sioux Falls road network's OD pairs and the demand (population mobility) for each OD pair.

	From node																							
To Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	60	50	300	100	140	250	400	300	800	200	150	300	200	400	200	250	40	150	200	40	250	200	80
2	40	0	70	100	60	100	100	300	160	300	50	50	100	60	80	300	150	0	150	70	0	90	0	0
3	50	30	0	150	60	100	50	150	20	150	200	100	60	50	30	100	30	0	0	0	0	50	20	0
4	200	100	50	0	400	200	200	400	300	500	400	400	100	250	200	200	400	60	150	200	90	200	100	110
5	100	40	40	100	0	50	130	300	500	400	250	100	100	60	50	100	100	0	60	40	60	100	90	0
6	160	300	200	200	150	0	150	500	300	500	100	150	120	50	120	300	400	50	100	150	50	150	50	80
7	250	100	50	200	70	250	0	400	400	900	100	300	200	170	350	700	600	90	100	100	100	200	150	60
8	400	100	50	300	200	300	600	0	300	1000	300	400	400	200	400	1200	900	100	600	600	300	200	100	150
9	200	40	80	400	300	100	200	500	0	1500	400	200	500	200	400	1000	400	100	200	300	200	400	200	100
10	500	300	150	700	600	300	1000	600	1300	0	2500	800	1000	1100	2500	2000	2000	100	1000	500	400	1200	1000	400
11	300	150	100	1000	250	300	400	500	1000	1500	0	1000	700	1000	700	400	800	90	200	100	300	200	1000	200
12	50	50	100	200	100	50	400	200	400	1200	400	0	700	500	300	600	300	100	200	100	200	300	200	400
13	200	200	40	500	100	80	200	200	100	900	300	600	0	350	500	400	300	60	150	400	500	1000	600	0
14	100	40	50	250	40	150	30	200	400	1000	600	200	250	0	700	300	500	0	100	300	300	400	600	300
15	100	20	70	300	150	80	150	200	500	1500	700	400	200	600	0	500	800	50	500	200	400	600	700	200
16	300	100	100	600	400	600	700	1000	400	2400	1000	100	200	400	700	0	1200	100	700	600	500	700	200	100
17	150	50	70	100	100	100	400	500	500	1900	200	300	200	200	700	1600	0	300	1000	500	100	1000	400	100
18	60	0	0	40	0	50	110	200	100	600	10	100	40	100	150	400	300	0	150	300	0	150	50	0
19	150	50	0	50	40	100	300	100	200	800	200	100	150	200	300	600	700	150	0	600	300	1000	100	10
20	100	30	0	100	60	150	400	300	300	2000	500	300	200	200	900	1000	1200	100	600	0	700	900	200	300
21	60	0	0	110	40	50	100	100	100	800	100	100	100	100	400	100	500	100	100	500	0	1400	300	100
22	150	10	50	200	100	50	300	300	300	1400	900	400	300	800	2000	500	700	150	200	1500	400	0	600	1000
23	100	0	80	400	10	50	50	200	300	800	300	500	200	500	200	300	200	50	200	500	400	1500	0	200
24	20	0	0	90	0	20	40	50	100	400	400	100	800	100	200	200	200	0	90	100	400	100	500	0