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FLORIDA INTERNATIONAL UNIVERSITY

Miami, Florida

THREE ESSAYS ON TRADE POLICY IN A MULTI-COUNTRY WORLD

A dissertation submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

 in

ECONOMICS

by

Enrique Valdes

2022

To: Dean John F. Stack, Jr. Steven J. Green School of International and Public Affairs

This dissertation, written by Enrique Valdes, and entitled Three Essays on Trade Policy in a Multi-Country World, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

Cem Karayalcin

Norihiko Matsuda

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Kaz Miyagiwa, Major Professor

Date of Defense: June 27, 2022

The dissertation of Enrique Valdes is approved.

Dean John F. Stack, Jr. Steven J. Green School of International and Public Affairs

Andrés G. Gil Vice President for Research and Economic Development and Dean of the University Graduate School

Florida International University, 2022

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DEDICATION

To my family and friends who have been a source of strength and joy. Especially my mother, Ines de Leon, for always encouraging me to do my best and always supporting me in all my endeavors. She is the kindest person I know and has sacrificed so much for so many people. I aspire to be like her. To my late father, Jorge Valdes, whose unconditional love I know I could always count on regardless of what I did in life. To my brothers, Jorge Valdes, Jorge Luis Valdes, and Gabriel Caro, who have always been supportive of me. To my nephew, Jorge Enrique Valdes, who motivates me to be the best version of myself. To the Perez family, spending time with them has always been a source of laughter and happiness for me, especially my uncle, Isidro Perez, and my cousin, Isidro Perez. To all of my friends who supported me throughout this process, especially Stan Kent Tucker Jr., who I served with in the Army and became a big brother to me, his encouragement and guidance have been invaluable in my life.

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ABSTRACT OF THE DISSERTATION THREE ESSAYS ON TRADE POLICY IN A MULTI-COUNTRY WORLD

by

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This dissertation involves analyzing welfare in developing countries due to a variety of trade policies. Because all three chapters focus on developing countries and their interactions with more developed countries, we need a model that accounts for differences in income between countries, easily extending to more than two countries. For our analysis, we draw on the Ricardian model developed by Matsuyama (2000).

The first chapter investigates the effects tariff wars have on developing countries. We study the effect of a tariff war between two richer countries on a non-participant low-income developing country. Surprisingly, we find that the low-income country can benefit from tariff war and even be the sole beneficiary of the tariff war. Lastly, we find that the low-income country can also influence the outcome of a tariff war. For instance, its growth in size exacerbates the structural disadvantage suffered by the middle-income country and helps the high-income country win the tariff war.

In the second chapter, we ask if developing countries are better off forming customs unions or signing free trade agreements. We focus on developing countries that have identical production technologies. Utilizing a numerical analysis, we find that if the developing countries are of equal population, they will be better off forming a customs union than signing a free trade agreement. Additionally, when developing countries have asymmetric populations, the country with the larger population will always prefer a custom union, while the smaller country may prefer a free trade agreement.

In the third chapter, we assume that the world is divided into three countries with different income levels. Moreover, we assume the richest country is embargoing goods from the middle-income country. We ask whether a low-income country could improve its welfare by joining the high-income country in embargoing goods from a middle-income country. We find that the lower-income country will always be worse off due to restricting trade with the middle-income country. Surprisingly, for a partial embargo, the welfare of the middle-income country could rise.

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ABBREVIATIONS AND ACRONYMS

CES	Constant Elasticity of Substitution
CET	Common External Tariff
CU	Customs Union
FTA	Free Trade Agreement
PTA	Preferential Trade Agreement
RoO	Rules of Origin
SOC	Second Order Condition
WTO	World Trade Organization

CHAPTER 1

INTRODUCTION

This dissertation consists of three separate chapters on trade policy. While each chapter analyzes a different trade policy issue, all three chapters are related in that they all revolve around how trade policy affects developing countries. Moreover, all three chapters use a general equilibrium framework with the nonhomothetic preferences to account for how demand differs at different levels of income. These differences in income can have important implications on international trade and welfare. The general structure of the models discussed in this dissertation is as follows. The model consists of three countries distinguished by their labor productivity differences. Productivity differences give rise to differences in per capita income. There is a continuum of indivisible goods, and consumers have nonhomothetic preferences, consuming at most one unit of each good subject to their budget Moreover, goods are ordered in descending order of the marginal constraints. utility of money so that lower-indexed goods are consumed in all countries. In contrast, higher-indexed goods are consumed only by the countries rich enough to afford them, i.e., wealthier consumers consume a wider variety of goods. On the production side, goods are produced competitively, and poorer countries have a comparative advantage in producing lower-indexed goods while higher-income countries have a comparative advantage in producing higher-indexed goods. Unlike standard economic models with homothetic preferences, here, changes in income lead to the reduction or expansion of consumption of higher-indexed goods produced in the richer countries. Furthermore, in each of the chapters, along with the analytical analysis, we provide a numerical analysis to understand the results better and make sharper comparisons among different equilibriums.

The first chapter studies the effects of a bilateral tariff war between two richer countries on a developing economy. Surprisingly, we find that the low-income country can benefit from a tariff war and even be the sole beneficiary of a tariff war. The mechanism here is that as two richer countries place tariffs on each other, they depress the other's wages and make the prices of each other's goods fall. Developing countries can benefit from this and consume more as a result. However, due to the structure of the model, if the richest country "wins" the tariff war, they cause their wages to rise, making their goods more expensive. This hurts welfare in the low-income country. However, if the middle-income country wins the tariff war, depressing wages in the richest country, then the welfare in the lower-income country will be higher.

In the second chapter, we study whether developing countries with identical technologies should enter into a free trade agreement or a customs union with each other. When countries sign free trade agreements, they can independently set their tariff rates. However, if countries form a customs union, they give up that ability because members of a customs union must set a common external tariff (CET). We find that it would be best for developing countries to enter into a customs union if they have symmetric populations; they will set a higher tariff and enjoy higher welfare in a customs union setting than they would in a free trade agreement. When populations are asymmetric, countries with a larger population would prefer to form a customs union to set higher tariffs. Still, the smaller country would prefer a free trade agreement and set a lower tariff than its larger counterpart would have preferred.

In the third chapter, we assume that the world is divided into three countries with different levels of income. Moreover, we assume the richest country is embargoing goods from the middle-income country. The research question we ask is whether a low-income country could improve its welfare by joining the high-income country in embargoing goods from a middle-income country. Additionally, we look at how welfare changes for all countries in the model. In our analysis, we find that the lower-income country will make itself worse off due to restricting trade with the middle-income country and, at the same time, improving the welfare in the highincome country. Surprisingly, if a low-income country places a partial embargo on the middle-income country, the welfare of the middle-income country could rise.

CHAPTER 2

DO POOR COUNTRIES GAIN FROM RICH COUNTRIES' TARIFF WAR?

2.1 Introduction

America's recent volte-face from free trade and globalization has raised the specter of a trade war among powerful economies and resurrected the classical question in international trade theory: can a country win a trade war?

There is already a vast literature that has addressed this question in diverse settings, establishing what we call the classical proposition; namely, when the belligerent countries are similar in size, they are both harmed by a trade war, but when they are lopsided in size, the larger country wins a trade war. However, this literature's exclusive use of two-country trade models has left unexamined the question of how trade war affects non-belligerent countries. This is a curious lacuna in the literature because, in the real world, most countries are neutral, and yet their economies can be seriously affected by trade wars through international trade. In this paper, we aim to fill this lacuna. More specifically, our objectives are twofold: to examine how a trade war between rich countries affects the economies of developing countries and how the developing countries can shape the outcome of a trade war between rich countries.

Given our objectives, our model must possess the following two features. First, it must have at least three countries, representing two belligerents and one bystander country. Second, our model must be able to distinguish between developed and developing countries. There are trade models designed to highlight the differences. For example, in the North-South model of vertical quality differentiation by Flam and Helpman (1987), workers in the North are endowed with a greater amount of effective labor and hence richer than their counterparts in the South. Although we need this sort of feature in our analysis, their two-country two-good framework cannot readily be extended into a multi-country setting. In this paper, we draw on the Ricardian model developed by Matsuyama (2000), which enables us straightforwardly to incorporate the above two requirements in our analysis.

We now outline our model. It has three countries, distinguished by their labor productivity differences.¹Productivity differences give rise to differences in per capita income, and we study the impact of a tariff war between the two richest countries on the welfare of the lower-income countries. There is a continuum of goods, and consumers have non-homothetic preferences, consuming at most one unit of each good subject to their budget constraints. Goods are ordered in descending order of the marginal utility of money so that lower-indexed goods are consumed in all countries while higher-indexed goods are consumed only by the countries rich enough to afford them.

As for trade patterns, technologies are such that the low-income country has comparative advantages in lower-indexed goods while the high-income country has a comparative advantage in higher-indexed goods. As a consequence, our model gives rise to two trade patterns, depending on whether the low-income country imports goods exclusively from the middle-income country or from both the middle-income and the high-income country. It is found that if the low-income country imports exclusively from the middle-income country, its welfare is completely impervious to the rich countries' tariff war. For this reason, we focus our analysis on the case in which the low-income country has imports from both the middle-income and the high-income country.

¹An extension to more than three countries does not affect the qualitative results; see the appendix.

We now state our main results. First, the low-income country's welfare is closely aligned with that of the middle-income country and opposite to that of the highincome country. Second, and more interestingly, the low-income country generally "fares" better than the middle-income country in a tariff war in the sense that whenever the middle-income country wins the tariff war, the low-income country also benefits from it, but the converse is not true. Third, there are cases in which the lowincome country is the sole beneficiary of a trade war. Fourth, our model replicates the classical proposition. When the middle-income and the high-income countries are similar in size, both are harmed by a tariff war, but when they are lopsided, the larger country wins the tariff war, and the smaller country loses. However, there is an extra wrinkle. The low-income country can influence the outcome of a trade war. In particular, its growth in size exacerbates the structural disadvantage suffered by the middle-income country and tilts the scale in favor of the high-income country.

The remainder of this paper is organized into five sections. Section 2.2 reviews relevant literature. Section 2.3 describes the model. Section 2.4 investigates the effect of a trade war. Section 2.5 provides further results numerically. Lastly, section 2.6 concludes.

2.2 The review of literature

The modern literature on trade war dates back to Johnson (1953–1954), who has analyzed trade war in terms of the offer curves and has shown that trade wars need not end up in a prisoners' dilemma as previously conjectured by de Scitovszky (1942). Gorman (1958) has extended the Johnson results by clarifying the range of elasticity of the offer curves for winning a trade war. Rodriguez (1974) and Tower (1975) have found that a trade war with import quotas completely eliminates trade and harms both countries.²These papers, however, have failed to link their results to more fundament economic parameters.

With advances in game theory, subsequent research has expanded beyond analysis based on offer curves. For example, Otani (1980) has found that if countries are sufficiently symmetrical, a tariff war leads to a prisoner's dilemma situation. A first complete proof of the classical proposition has been provided by Kennan and Riezman (1988). Studying the Nash equilibrium of tariff warfare in a two-country two-good pure exchange model, Kennan and Riezman (1988) have demonstrated that when two countries are sufficiently lopsided in endowments, the large country wins while the smaller one loses.³ Syropoulos (2002) has verified a similar result in the Heckscher-Ohlin model, while Opp (2010) has done the same for the continuumof-goods Ricardian model developed by Dornbusch, Fischer, and Samuelson (1977).

The literature has also been extended to new trade models. Gros (1987) has found that under monopolistic competition, optimal uniform ad valorem tariffs exist even for a small economy and that the optimal tariff is an increasing function of the size of the economy. Felbermayr et al. (2013) have confirmed that Gros's result extends to the Melitz (2003) model and that optimal tariffs are higher when the degree of dispersion in the firm-level productivity distribution is higher. Interestingly, however, their calibrated model has been unable to find cases in which trade war improves either country's national welfare, reminiscent of the de Scitovszky conjecture.

All the above studies have examined trade wars in two-country models. An exception is Harrison and Rutström (1991), who have applied the computable general

²Gorman (1958) finds that a country may gain from a tariff war even if its demand for imports is highly elastic compared with the demand for its own exports.

³In a follow-up paper Kennan and Riezman (1990) study optimal tariffs and customs unions. They show that the formation of a customs union can basically lead to the formation of a larger trading partner, which would make it more likely to gain from a tariff war.

equilibrium models developed by Whalley (1985,1986). Simulating a trilateral trade war between the U.S., E.U., and Japan, they have found that the U.S. and the E.U. improve welfare while Japan and all the other countries are harmed by the trade war. They have also simulated a bilateral trade war between the U.S. and Canada to obtain a surprising result; every country in the world loses. However, these simulation results remain yet to be related to more basic parameters of the model, such as endowments.

Our paper aims to contribute to this rich line of research. The Ricardo-Matsuyama (2000) model is capable of accounting for trade among multiple countries that are at different stages of economic development. It thus allows us to study the effect on the welfare of developing countries from a trade war between two richer countries, e.g., the U.S. and China. We believe, to the best of our knowledge, that this is the first formal analysis of bilateral trade war in a multi-country setting.

2.3 The model

Consider a world with three countries labeled by i = H, M, L. Country L represents a multitude of developing low-income countries, each of which is too small to influence the terms of trade,⁴ whereas countries H and M are the world's high-income countries, with country H being the richer of the two. Country i is endowed with N_i units of labor, the only factor of production. Labor is freely mobile within each country but is immobile across countries.

All three countries can produce a continuum of goods $z \in [0, \infty)$ using distinct technologies. Let $a_i(z)$ denote the unit labor requirement (labor-output ratio) for good z production in country *i*.

⁴It is shown in the appendix that our results hold qualitatively in the presence of more than one developing country.

Assumption 1: For all $z \in [0, \infty)$,

- (a) $a_H(z) \leq a_M(z) \leq a_L(z)$ with the equalities holding possibly at z = 0 only.
- (b) $a_i(z)$ is continuously differentiable and monotone-increasing.

Assumption 1 gives country H the absolute advantage and country L the absolute disadvantage, in the production of all goods, with country M's advantage in between. Additionally, we define the labor-requirement ratio functions $A(z) \equiv a_M(z) / a_H(z)$ and $B(z) \equiv a_L(z) / a_M(z)$.

Assumption 2: A(z) and B(z) are (twice) continuously differentiable and monotoneincreasing in $z \in (0, \infty)$.

Assumption 2 below is analogous to the assumption found in Dornbusch, Fischer, and Samuelson (1977) and says that the comparative advantages are more accentuated at higher-indexed goods. All goods are produced competitively. Free entry fixes the unit production cost of good z in country i at $w_i a_i(z)$, where w_i is the wage in country i. Assumption 1 implies

Lemma 1: $w_H > w_M > w_L$.

We set $w_M = 1$.

All consumers supply one unit of labor supply inelastically, so the wage w_i is their income. They have identical preferences and buy at most one unit of each good, deriving utility u(z) > 0 from consumption of one unit of z and zero otherwise. Assume that the ratios $u(z)/a_i(z)$ are strictly decreasing. If we let p(z) denote the price of good z, it follows that the marginal utility of money

$$u(z)/p(z) = u(z)/\min\{w_H a_H(z), w_M a_M(z), w_L a_L(z)\}$$

is strictly decreasing. Hence, consumers buy one unit of good 0 and keep buying higher indexed goods until their budget is exhausted. As a result, consumers in country *i* consume every good in the range $[0, c_i]$, where c_i denotes the highestnumbered good they buy. Since they consume one unit of every good in that range, consumer welfare increases monotonically as the consumption range expands. This establishes a one-to-one correspondence between consumer welfare and c_i and enables us to use c_i as the welfare measure of consumers in country *i*.

Lemma 1 implies that

$$[0,c_L] \subset [0,c_M] \subset [0,c_H]$$

that is, country L consumes the smallest number (measure) of goods available while country H consumes all the goods that are produced in the world. This consumption pattern is shown in figure 2.1 below, where the horizontal line extends from 0 to infinity, representing z. Note that c_H is also the highest indexed good produced in the world.

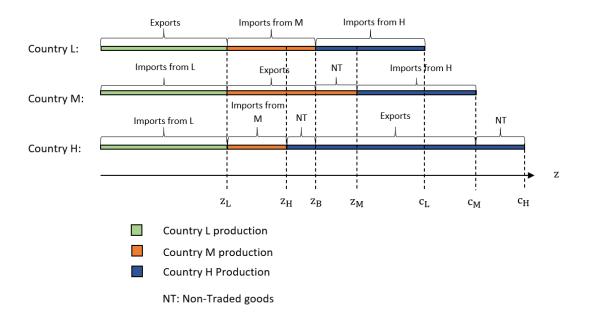


Figure 2.1: Production and Trade Patterns

Suppose that countries H and M are embroiled in a bilateral tariff war. Let t_i denote the ad valorem tariff rate imposed by country i(=H, M) on all goods imported from country $j = H, M; i \neq j$. Denote $T_i = 1 + t_i$. Both countries maintain free trade mutually with country L.

Assumption 1 and lemma 1 imply that country L specializes in lower-indexed goods $[0, z_L]$, country M in the middle range $[z_L, z_M]$, and country H in the high range $[z_H, c_H]$; see figure 2.1. In the absence of tariff war, we have that $z_M = z_H$, and hence three countries produce the distinct sets of goods according to their comparative advantage (with the exception of the two boundary goods z_L and $z_M = z_H$, which are produced by two adjacent countries). With trade war, however, $z_H < z_M$, as shown in figure 2.1, implying that the goods in the range $[z_H, z_M]$ are produced by both countries and hence are not traded between them. The emergence of a range of non-traded goods under tariffs is familiar from Dornbusch, Fischer, and Samuelson (1977). Though not traded between the belligerent countries, some of those goods are exported to country L. Since country M has comparative advantages in lower-indexed goods vis-à-vis country H, there is a good z_b for which their comparative advantages are equalized with respect to exports to country L. Then, country M exports goods indexed below z_b while country H exports the range of goods from z_b to c_L country L.

If $c_L < z_b$, however, country L has no imports from country H. Since country Hstill imports low-indexed goods from country L, we call this trade pattern *asymmetric*. On the other hand, if $c_L \ge z_b$, country L imports goods from both belligerent countries, giving rise to bilateral trade among all three countries. We call this *symmetric* trade. Figure 2.1 illustrates a typical case of symmetric trade.

Tariff war between two rich countries affects their comparative advantages and alters z_b , the boundary of their comparative advantage vis-à-vis country L. In symmetric trade, a change in z_b prompts country L to switch suppliers, adjusting c_L . By contrast, under asymmetric trade, country L's consumption range does not reach z_b , making country L's welfare completely independent of a trade war between two richer countries. Since we are primarily interested in the question of how a trade war between rich countries affects developing countries, we focus our analysis on symmetric trade.⁵

We now describe the model in detail. Begin by writing the conditions to be satisfied by four boundary commodities, z_L, z_H, z_M , and z_b in equilibrium. Recall that free trade prevails between country L and country M so that they produce goods in the continuous non-overlapping ranges $[0, z_L]$ and $[z_L, z_M]$, respectively. Thus, the unit costs of the boundary good z_L must be equal in both countries:

$$w_L a_L \left(z_L \right) = a_M \left(z_L \right). \tag{2.1}$$

Country M protects its domestic industries against exports from country H with tariffs, so the unit cost of its upper boundary good z_M must equal the unit cost of that good in country H plus the tariff rate by country M; i.e.,

$$a_M(z_M) = w_H a_H(z_M) T_M \tag{2.2}$$

Similarly, as country H produces the range of goods in $[z_H, c_H]$, the unit cost of its lower-boundary good z_H must equal the unit cost of that good in country M plus its tariff rate; thus

$$T_H a_M(z_H) = w_H a_H(z_H) \tag{2.3}$$

⁵It is possible that the trade pattern switches from symmetric to asymmetric, depending on tariff rates. However, when it occurs, the low-income country's welfare is the same as if there was no tariff war.

Because A(z) is monotone-increasing (2.2) and (2.3) imply that $z_H < z_M$, our earlier claim that goods in the range $[z_H, z_M]$ are non-traded between country M and country H. Finally, country L practices free trade with the belligerent countries, so the unit costs of z_b , the boundary of their comparative advantage, must be equalized:

$$a_M(z_b) = w_H a_H(z_b); \qquad (2.4)$$

Equations (2.2) - (2.4) imply that

$$z_H < z_b < z_M.$$

as in figure $2.1.^6$

On the consumption side, consumers in country L satisfy their demands in $[0, c_L]$ by procuring goods in $[0, z_L]$ from domestic suppliers at price $w_L a_L(z)$, and importing goods in $[z_L, z_b]$ from country M at price $a_M(z)$ and goods in $[z_b, c_L]$ from country H at price $w_H a_H(z)$. With an income w_L , a typical consumer in country L faces the budget constraint

$$w_L \int_0^{z_L} a_L(z) \, dz + \int_{z_L}^{z_{b,}} a_M(z) \, dz + w_H \int_{z_{b,}}^{c_L} a_H(z) \, dz = w_L.^7 \tag{2.5}$$

Consumers in country M import goods in $[0, z_L]$ from country L and goods in $[z_M, c_M]$ from country H while procuring goods in $[z_L, z_M]$ from domestic suppliers. For imports from country H, consumers pay the tariff distorted prices $(1+t_M)w_Ha_H(z)$, but their government rebates the collected tariff revenue to each consumer in a lump-

⁶We do not allow tariff evasion through smuggling or transhipments to country L.

⁷Although figure 2.1 shows that $c_L < z_M$, a quick reflection verifies that (2.5) is valid even if $c_L \ge z_M$.

sum fashion. This rebate equals $t_M w_H \int_{z_M}^{c_M} a_H(z) dz$, which exactly offsets the extra duty a consumer pays. Thus, in net, a consumer in country M faces the undistorted budget constraint

$$w_L \int_0^{z_L} a_L(z) \, dz + \int_{z_L}^{z_M} a_M(z) \, dz + w_H \int_{z_M}^{c_M} a_H(z) \, dz = 1, \qquad (2.6)$$

Lastly, consumers in country H import goods in $[0, z_L]$ from country L and goods in $[z_L, z_H]$ from country M while satisfying demands in $[z_H, c_H]$ domestically. The tariff revenue rebated by the government completely offsets the duty consumers pay for imports. Thus, they face the budget constraint

$$w_L \int_0^{z_L} a_L(z) \, dz + \int_{z_L}^{z_H} a_M(z) \, dz + w_H \int_{z_H}^{c_H} a_H(z) \, dz = w_H.$$
(2.7)

To close the model, we impose resource constraints for each country. Since all three countries buy the goods in the range $[0, z_L]$ from country L, the total demand for labor equals $N \int_0^{z_L} a_L(z) dz$, where N is the world population. In equilibrium, this labor demand equals the country L's labor supply N_L ;

$$N \int_{0}^{z_{L}} a_{L}(z) dz = N_{L}.$$
(2.8)

Country M produces goods in the range $[z_L, z_M]$ and exports goods in $[z_L, z_b]$ to country L and goods in $[z_L, z_H]$ to country H. Thus, the country faces the resource constraint

$$N_M \int_{z_L}^{z_M} a_M(z) \, dz + N_L \int_{z_L}^{z_b} a_M(z) \, dz + N_H \int_{z_L}^{z_H} a_M(z) \, dz = N_M.$$
(2.9)

This completes the description of the model. There are nine unknowns: w_H , w_L , z_L , z_H , z_b z_M , c_L , c_M , and c_H , and nine equations (2.1) – (2.9). Country *H*'s resource constraint holds in equilibrium by Walras' Law.

The model can be solved semi-recursively. Equations (2.8) determines the equilibrium z_L . Substituting it into (2.1) pins down the equilibrium w_L . Substituting those values into (2.2) – (2.4) and (2.9), we can solve the resulting four-equation system for four unknowns: w_H , z_H , z_b , z_M . These values are substituted into (2.5), (2.6), and (2.7) to determine c_L , c_M , and c_H , respectively.

2.4 Tariff war

The solution procedure described above indicates that w_L and z_L are obtained from equations (2.8) and (2.1) alone. Since these equations do not contain the T_i ', the determination of equilibrium w_L and z_L is independent of the tariff rates. However, country L's welfare is not impervious to trade war because its welfare index c_L is endogenous to the block of the equations (2.2) – (2.4) and (2.9).

Let a circumflex over a variable denote a percentage change of its value; e.g., $\hat{x} \equiv dx/x$. Differentiation of (2.2) – (2.4) yields

$$\varepsilon_M \hat{z}_M = \hat{w}_H + \hat{T}_M \tag{2.10}$$

$$\hat{T}_H + \varepsilon_H \hat{z}_H = \hat{w}_H \tag{2.11}$$

$$\varepsilon_b \hat{z}_b = \hat{w}_H \tag{2.12}$$

where

$$\varepsilon_j \equiv z_j A'(z_j) / A(z_j)$$

is the elasticity of $A(z_j) = a_M(z_j)/a_H(z_j)$. Total differentiation of (2.9) entails

$$z_{h}\lambda_{H}a_{M}\left(z_{H}\right)\hat{z}_{H}+z_{b}\lambda_{L}a_{M}\left(z_{b}\right)\hat{z}_{b}+z_{M}\lambda_{M}a_{M}\left(z_{M}\right)\hat{z}_{M}=0$$

where $\lambda_i \equiv N_i/N$ denotes country *i*'s share of the world's labor endowment N. Substituting from (2.10) – (2.12) converts the last equation to

$$\frac{z_H \lambda_H a_M \left(z_H \right) \left(\hat{w}_H - \hat{T}_H \right)}{\varepsilon_H} + \frac{z_b \lambda_L a_M \left(z_b \right) \hat{w}_H}{\varepsilon_b} + \frac{z_M \lambda_M a_M \left(z_M \right) \left(\hat{T}_M + \hat{w}_H \right)}{\varepsilon_M} = 0.$$
(2.13)

which can be solved for

$$\hat{w}_H = \phi_H \hat{T}_H - \phi_M \hat{T}_M \tag{2.14}$$

with

$$\phi_i \equiv \frac{\frac{z_i \lambda_i a_M(z_i)}{\varepsilon_i}}{\frac{z_H \lambda_H a_M(z_H)}{\varepsilon_H} + \frac{z_b \lambda_L a_M(z_b)}{\varepsilon_b} + \frac{z_M \lambda_M a_M(z_M)}{\varepsilon_M}}$$
(2.15)

Substitute for \hat{w}_H from (2.14) into (2.10) – (2.12) yields the changes in the three boundary goods:

$$\hat{z}_M = \frac{\hat{w}_H + \hat{T}_M}{\varepsilon_M} = \frac{\phi_H \hat{T}_H + (1 - \phi_M) \hat{T}_M}{\varepsilon_M}$$
(2.16)

$$\hat{z}_H = \frac{\hat{w}_H - \hat{T}_H}{\varepsilon_H} = -\frac{(1 - \phi_H)\hat{T}_H + \phi_M\hat{T}_M}{\varepsilon_H}$$
(2.17)

$$\hat{z}_b = \frac{\hat{w}_H}{\varepsilon_b} = \frac{\phi_H \hat{T}_H - \phi_M \hat{T}_M}{\varepsilon_b}$$
(2.18)

(2.16) and (2.17) say that the range of non-traded goods $[z_H, z_M]$ expands when either country raises its tariff; a result familiar from Dornbusch, Fischer, and Samuelson (1977). In contrast, z_b depends positively on the effect on w_H . Naturally, the greater a wage hike in country H, the more competitive country M becomes in exports to country L. As a result, it expands the range of exports to country L at the expense of country H, raising z_b .

Now we are ready to examine the welfare effect of a trade war. Differentiating (2.5), we find

$$w_{H}c_{L}a_{H}(c_{L})\hat{c}_{L} = -\left(\int_{z_{b}}^{c_{L}} w_{H}a_{H}(z)\,dz\right)\hat{w}_{H}.$$
(2.19)

The integral in parentheses represents the cost of imports from country H. A wage hike in country H makes imports from that country more expensive and causes country L to cut back on their consumption range, lowering country L's welfare. Combining equations (2.19) and (2.14), it is straightforward to show that country L welfare improves when country M raises its tariffs and deteriorates when country H raises its tariffs.

Turning to the other countries, differentiation of (2.6) and (2.7), respectively, yields

$$w_H c_M a_H(c_M) \, \hat{c}_M = -\left(\int_{z_M}^{c_M} w_H a_H(z) \, dz\right) \hat{w}_H - w_H z_M a_H(z_M) \, (T_M - 1) \hat{z}_M \quad (2.20)$$

and

$$w_H c_H a_H(c_H) \, \hat{c}_H = \left(w_H - \int_{z_H}^{c_H} w_H a_H(z) \, dz \right) \hat{w}_H + z_H a_M(z_H) \, (T_H - 1) \hat{z}_H. \tag{2.21}$$

The integral in (2.20) measures the costs of imports to country M from country H. A higher wage in country H raises these import costs and causes consumption in country M to decline. The integral in (2.21) measures the value of goods country H consumers purchase from local suppliers. Hence the expression in parentheses equals the total costs of imports to country H from both country M and country L. For country H, a higher wage makes its imports from both countries cheaper. By contrast, for country M and country L, a lower wage in country H makes only the imports from that country cheaper. This structural asymmetry will later prove to be a key to understanding the influence country L exerts on the outcome of the tariff war.

In addition to the terms-of-trade effects, tariff war expands the range of nontraded goods $[z_H, z_M]$ between the belligerent countries, giving rise to the effect captured by the second term in (2.20) and (2.21). In (2.20), the tariffs induce consumers in country M to substitute towards domestic suppliers at the margin. The domestic price $a_M(z_M)$ of the boundary good z_M , however, is greater than its import price $w_H a_H(z_M)$ net of the tariff because $a_M(z_M) - w_H a_H(z_M) = w_H a_H(z_M)(T_M -$ 1) > 0 in view of (2.2). Therefore, an expansion of the non-traded goods harms consumers in country M. A similar phenomenon occurs in country H at the boundary good z_H as its consumers substitute imports for domestic production. Let us call these deleterious consequences of tariffs the *trade diversion effects* in the tradition of the customs union literature.

Equations (2.19) – (2.21) accord us two additional observations. First, country L's welfare is more closely linked to the welfare of country M than that of country H. This follows from the fact that country L and country M import their respective highest-indexed goods c_L and c_M from country H and hence are affected in similar ways by a change in terms of trade. Second, the trade diversion effect has an unfavorable impact on country M, but country L is spared that effect because it practices free trade. Thus, country L fares better than country M in the sense that it may benefit from a trade war even if country M is harmed by it.

The next proposition summarizes the preceding discussion.

Proposition 1:

(a) Country L benefits from tariff war if and only if the wage in country H falls.(b) Whenever country H wins a tariff war, country L and country M are harmed

(c) Whenever country M wins a tariff war, country L also benefits from it.

We have seen that trade diversion casts a deleterious effect on the belligerent countries but spares country L. This raises an interesting question in light of proposition 1: can country L benefit from a tariff war while the belligerent countries are harmed by it? We pursue this issue in the next section.

2.5 Nash equilibrium tariff war

by it.

In this section, we characterize a tariff war as a game in which countries M and H choose tariff rates simultaneously to maximize their respective welfare measure $(c_i$'s), given the other country's tariff rate. To derive country M's best response function, we substitute for \hat{z}_M from (2.10) and \hat{w}_H from (2.14) into (2.20), and then we evaluate the resulting equation at $\hat{T}_H = 0$ (since country H's tariffs are taken as given). The first-order condition is

$$-\left(\int_{z_M}^{c_M} a_H(z) dz\right) \phi_M - z_M a_H(z_M) \left(T_M - 1\right) \frac{(1 - \phi_M)}{\varepsilon_M} = 0,$$

which can be rearranged to yield country M's best response function

$$(T_M - 1) = \frac{\phi_M \varepsilon_M}{(1 - \phi_M) z_M a_H(z_M)} \left(\int_{z_M}^{c_M} a_H(z) \, dz \right) \tag{2.22}$$

An analogous procedure applied to (2.21) yields the first-order condition for country *H*'s tariffs

$$w_H \left(1 - \int_{z_H}^{c_H} a_H(z) dz\right) \phi_H - z_H a_H(z_H) \left(T_H - 1\right) \frac{(1 - \phi_H)}{\varepsilon_H} = 0$$

which defines country H's best-response function.

$$(T_H - 1) = \frac{w_H \phi_H \varepsilon_H}{(1 - \phi_H) z_H a_H (z_H)} \left(1 - \int_{z_H}^{c_H} a_H (z) \, dz \right)$$
(2.23)

Recall that the integral in (2.22) represents the value of goods country M imports from country H while the expression in parentheses in (2.23) equals the total value of goods country H imports from the other two countries. This implies that country M competes with country L in exports to country H, but that country H faces no such competition exporting to country M. We show that this structural difference handicaps country M in tariff war vis-a-vis country H.

The analytical complexity of these equations prevents further analysis, compelling us to turn to numerical analysis for sharper results. We begin by specifying the functional forms.

Assumption 3: $a_L(z) = e^{kz}$, $a_M(z) = e^z$ and $a_H(z) = 1$, with k > 1.

Then $A(z) = a_M(z) / a_H(z) = e^z$ and $A'(z) = e^z$. Hence $\varepsilon_z = z$. The boundary goods z_H, z_b , and z_M satisfy

$$e^{z_M} = w_H T_M \tag{2.24}$$

$$T_H e^{z_H} = w_H \tag{2.25}$$

$$e^{z_b} = w_H \tag{2.26}$$

Substituting these relations into (2.15) yields

$$\phi_M = \frac{\lambda_M T_M}{\lambda_M T_M + \lambda_L + \lambda_H / T_H}; \phi_H = \frac{\lambda_H / T_H}{\lambda_M T_M + \lambda_L + \lambda_H / T_H}.$$
 (2.27)

Evaluating the integrals in (2.9), we obtain

$$\lambda_M e^{z_M} + \lambda_L e^{z_b} + \lambda_H e^{z_H} - e^{z_L} = \lambda_M. \tag{2.28}$$

Solving (2.8) yields

$$e^{kz_L} = k\lambda_L + 1 \equiv \theta. \tag{2.29}$$

Substituting this into (2.1) yields $w_L = \theta^{-1+1/k}$. Using these results along with equations (2.24) – (2.26), we can rewrite (2.28) as $\left(\lambda_M T_M + \lambda_L + \frac{\lambda_H}{T_H}\right) w_H = \theta^{1/k} + \lambda_M$, which gives us the equilibrium wage

$$w_H = \frac{\theta^{1/k} + \lambda_M}{\lambda_M T_M + \lambda_L + \frac{\lambda_H}{T_H}}.$$
(2.30)

From (2.6), we obtain

$$w_H \int_{z_M}^{c_M} a_H(z) dz = 1 - w_L \int_0^{z_L} a_L(z) dz - \int_{z_L}^{z_M} a_M(z) dz$$
$$= 1 - \lambda_L \theta^{-1 + \frac{1}{k}} - w_H T_M + \theta^{\frac{1}{k}}.$$
(2.31)

Substituting these results into (2.22) gives us country M's best response function

$$T_M - 1 = \frac{\lambda_M T_M}{\lambda_H / T_H + \lambda_L} \left[\frac{\left(1 - \lambda_L \theta^{-1 + 1/k} + \theta^{1/k}\right) \left(\lambda_M T_M + \lambda_L + \frac{\lambda_H}{T_H}\right)}{\theta^{1/k} + \lambda_M} - T_M \right]$$

From (2.7), we have that

$$w_{H}(1 - \int_{z_{H}}^{c_{H}} a_{H}(z) dz) = w_{L} \int_{0}^{z_{L}} a_{L}(z) dz + \int_{z_{L}}^{z_{H}} a_{M}(z) dz$$
$$= \lambda_{L} \theta^{-1+1/k} + w_{H}/T_{H} - \theta^{1/k}.$$
(2.32)

Substituting the last expression into (2.23) yields country *H*'s best response function

$$(T_H - 1) = \frac{\lambda_H / T_H}{\lambda_M T_M + \lambda_L} \left(\lambda_L \theta^{-1 + 1/k} - \theta^{1/k} + \frac{\theta^{1/k} + \lambda_M}{T_H (\lambda_M T_M + \lambda_L) + \lambda_H} \right)$$

We can show that the tariffs are strategic substitutes.

Figure 2.2 displays the two countries' best response functions, where we set k = 2, $\lambda_L = 0.5$ and $\lambda_M = \lambda_H = 0.25$. Given these parameters, the Nash equilibrium tariff rates are approximately 6.17 percent for country M and 9.21 percent for country H.

In figure 2.3, we present the welfare effect of a tariff war under the parameter values $\lambda_L = 0.5$ and k = 2. To construct the figure, we first compute the equilibrium welfare levels (c_i) under free trade and under Nash equilibrium tariffs at various λ_M values and plot the percentage change in the c_i 's on the vertical axis. The curves take positive values when countries gain from tariff war and negative values when they are harmed by it. Note that these curves are drawn only for $\lambda_M \leq 0.35$.

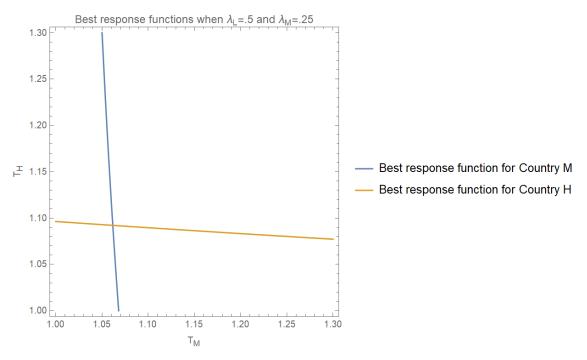


Figure 2.2: Tariff War Best Response Functions

At higher values of λ_M , trade ceases to be symmetric, so that country *L*'s welfare becomes independent of tariff rates and remains at the free-trade (pre-tariff war) level.⁸

Three things stand out in figure 2.3. First, country L has smaller welfare losses from a tariff war compared with country M. This is just a verification of the statement that country L fares better than country M in a trade war. Second, figure 2.3 confirms our conjecture given at the end of the last section; namely, when λ_M is between 0.28 and 0.33, country L is the sole beneficiary of a tariff war while the belligerents are both harmed by it.

The third observation from figure 2.3 is that our model also yields the classical proposition; when the belligerent countries are similar in size, their welfare is lower than under free trade, but when they are lopsided, the larger country's welfare is

⁸This statement holds if there is a Nash equilibrium for asymmetric trade. However, we have been unable to find any trade war equilibrium under asymmetric trade.

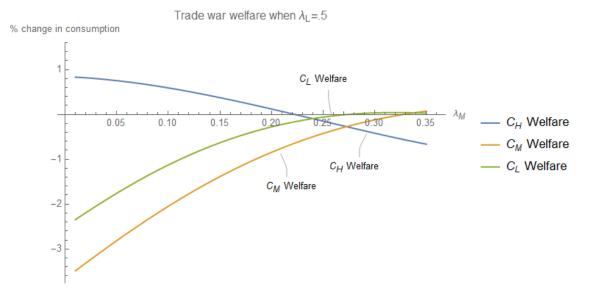


Figure 2.3: Percentage Change in Welfare Due to a Tariff War, $\lambda_L = 0.5$

greater, and the smaller country's welfare is lower compared with the free-trade levels. More specifically, in figure 2.3, country H wins the tariff war when it has 28% of the world's endowment of labor (and hence country M has 22% of it.) Thus, the threshold for country H to win the trade war equals $\frac{\lambda_H}{\lambda_M} = \frac{0.28}{0.22} = 1.27$. On the other hand, to win the tariff war, country M must have 33% of the world's endowment (leaving 17% to country H) or the threshold of $\frac{\lambda_M}{\lambda_H} = \frac{0.33}{0.17} = 1.94$. Thus, country H wins a trade war only if it is 27% bigger than country M, whereas the latter must be almost twice as big as the former to win a trade war.

What causes such lopsidedness in thresholds? To investigate this question, we turn to figure 2.4, which is similar to figure 2.3 but where we give country L only 10% of the world's endowment ($\lambda_L = 0.1$). The two belligerent countries are now harmed by tariff war when λ_M lies between 0.34 and 0.53.⁹ As in figure 2.3, country L fares better than country M and is the sole beneficiary of tariff war when λ_M lies between 0.43 and 0.53.

⁹When λ_M is below 0.34, country H benefits from tariff war while if λ_M is above 0.53 country M benefits from tariff war.

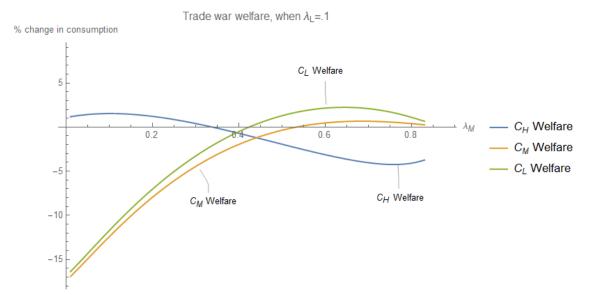


Figure 2.4: Percentage Change in Welfare Due to a Tariff War, $\lambda_L = 0.1$

Although the two figures are similar, in figure 2.4, the thresholds for winning war are $\frac{\lambda_M}{\lambda_H} = \frac{.54}{.36} = 1.5$ for country M and $\frac{\lambda_H}{\lambda_M} = \frac{0.57}{0.33} = 1.72$ for country H. The difference is narrower than in figure 2.3. Changing the parameter λ_L we obtain similar results; the larger the size of country L, the more difficult for country M to win the tariff war, given its country size relative to that of country H.¹⁰This again results from the structural disadvantage alluded to earlier. To see why differentiate (2.31) with respect to λ_L while holding the tariffs and w_H and obtain that

$$\frac{w_H d\left(\int_{z_M}^{c_M} a_H\left(z\right) dz\right)}{d\lambda_L} = (k-1)\,\lambda_L \theta^{-2+\frac{1}{k}} > 0$$

¹⁰When $\lambda_L = .3$ the threshold for winning the tariff war is $\lambda_M/\lambda_H = .46/.24 = 1.91$ for country M and $\lambda_H/\lambda_M = 0.41/0.29 = 1.41$ for country H. When $\lambda_L = .7$ the thresholds for winning the tariff war is $\lambda_H/\lambda_M = 0.165/0.135 = 1.22$ for country H. Country M is unable to win a tariff war when $\lambda_L = .7$.

In view of (2.20), this implies that an increase in λ_L intensifies the terms-of-trade effect for country M. For country H, we have the opposite result because differentiation of (2.32) yields

$$\frac{d\left(w_{H}(1-\int_{z_{H}}^{c_{H}}a_{H}(z)\,dz)\right)}{d\lambda_{L}} = -\left(k-1\right)\lambda_{L}\theta^{-2+\frac{1}{k}} < 0$$

implying that an increase in λ_L mitigates the terms-of-trade effect for country H. It follows that at larger λ_L values, country H can harm country M with a small tariff rate increase, whereas for country M, harming country H requires a greater tariff rate. However, since higher tariffs exacerbate the trade diversion effect, country M is restrained from raising its tariffs so aggressively. This structural disadvantage can be countervailed only by the advantage in size. Thus, country M must be substantially bigger than country H to win the tariff war when λ_L is larger.

2.6 Concluding remarks

Recent experiences demonstrate that a full-scale trade war is not a thing of the past. However, a trade war is usually engaged bilaterally, leaving other countries in neutral positions. Yet, the economies of neutral countries can still be disrupted by a tariff war through trade with the belligerent countries. The question of how trade war affects neutral economies has never received attention in the literature. In this paper, we have attempted to fill this lacuna, paying particular attention to the impact of a trade war between richer countries on the economies of developing countries. To address this issue, we have utilized the three-country Ricardo-Matsuyama (2000) model, where countries are ranked in terms of income per capita.

Our main findings are as follows. First, the low-income country's welfare is closely linked to that of the middle-income country but is better protected from the ravaging effect of a tariff war than the middle-income country. As a consequence, when the belligerent countries are similar in endowments, they both lose, but the low-income country can still benefit from the tariff war. Our model also replicates the standard result from the two-country models. When the belligerent countries are lopsided in size, the large country wins the tariff war. However, the threshold for either country to win a trade war is sensitive to the size of the low-income country. We have shown numerically that the size of the low-income country favors the high-income country in winning a tariff war.

2.7 Appendix

We show that with asymmetric trade country L's welfare is independent of the belligerents' tariffs. Under asymmetric trade, its budget constraint is written

$$\int_{0}^{z_{L}} w_{L} a_{L}(z) \, dz + \int_{z_{L}}^{c_{L}} a_{M}(z) \, dz = w_{L}.$$

Its resource constraint

$$\int_{0}^{z_{L}} a_{L}\left(z\right) dz = \lambda_{L}$$

first determines z_L . Then the equality $w_L a_L(z_L) = a_M(z_L)$ determines the wage w_L as in symmetric trade. Substitution of these equilibrium values into the budget constraint yields the equilibrium c_L value, which is independent of the tariffs. This conclusion holds true even if we have more than one developing country as long as none imports from country H. For example, suppose there are 4 countries, labeled by H, M, L, and 0. Country 0's productivity is even lower than country L's. As

countries L and 0 practice free trade, country 0 produces the goods in the range $[0, z_0]$, and country L produces goods in the range $[z_0, z_L]$. Hence, $w_0a_0(z_0) = w_La_L(z_0)$ and $w_La_L(z_L) = a_M(z_L)$, where country 0 has wage w_0 . The resource constraint for country 0 is

$$\int_{0}^{z_{0}} a_{0}\left(z\right) dz = \lambda_{0}$$

and its budget constraint is

$$\int_{0}^{z_{0}} w_{0}a_{0}(z) dz + w_{L} \int_{z_{0}}^{c_{0}} a_{L}(z) dz = w_{0},$$

assuming it does not import from country M. Country L now has the resource constraint

$$(1 - \lambda_0) \int_{z_0}^{z_L} a_L(z) \, dz + \lambda_0 \int_{z_0}^{c_0} a_L(z) \, dz = \lambda_L.$$

These five equations can be solved to determine c_0 , the welfare of country 0. Substituting the resulting values into country L's budget constraint, now written

$$\int_{0}^{z_{0}} w_{0}a_{0}(z) dz + w_{L} \int_{z_{0}}^{z_{L}} a_{L}(z) dz + \int_{z_{L}}^{c_{L}} a_{L}(z) dz = 1,$$

determines the c_L value. Both these welfare values are independent of the tariff rates.

If country 0 imports from country M, its budget constraint is

$$\int_{0}^{z_{0}} w_{0}a_{0}(z) dz + w_{L} \int_{z_{0}}^{z_{L}} a_{L}(z) dz + \int_{z_{L}}^{c_{0}} a_{M}(z) dz = w_{0}$$

Country L has the resource constraint

$$\int_{z_0}^{z_L} a_L(z) \, dz = \lambda_L.$$

Here it is even more simple to determine c_0 and c_L . To see this, notice that z_0 is determined by the resource constraint in country 0, then because country 0 imports from country M, z_L is determined in country L's resource constraint. Next, w_L is determined by $w_L a_L (z_L) = a_M (z_L)$. Lastly, c_0 is determined by the budget constraint of country 0 and c_L from the budget constraint of L. In this case, too, c_0 and c_L are independent of the tariffs. With more than two developing countries, computation gets more tedious and is not pursued here.

CHAPTER 3

ARE POOR COUNTRIES BETTER OFF FORMING CUSTOMS UNIONS OR SIGNING FREE TRADE AGREEMENTS?

3.1 Introduction

Are developing countries better off forming a customs union (CU) or signing a free trade agreement (FTA) with each other? This question has been studied scantly in the literature. While there is a vast literature examining the welfare effects of CUs that dates back to the seminal work of Jacob Viner's 1950 book *The Custom Union Issue*; few studies have gone in-depth as to whether a developing country is better off signing an FTA or forming a CU. The main purpose of this article is to better understand the nationalistic motivations behind developing countries forming a CU or signing an FTA. Specifically, we focus on how the welfare of less developed countries will be affected by entering into a trade agreement leaving the developed rest of the world as a nonmember.

Due to the nature of the research question, the model used to study this question must have two characteristics. One, it is easily extendable to three countries, two countries entering into a trade agreement and one nonmember. Two, the model should be able to differentiate between developed countries and developing countries. For these reasons, we present a Ricardian model with nonhomothetic preferences. We use the model developed by Matsuyama (2000) for our study. This model is a North-South trade model with indivisible goods in which consumers have hierarchic preferences. This model is ideal because it reflects the thorough empirical evidence that richer households have more diverse consumption baskets than poorer households.¹

We give a brief outline of the model. The model is a Ricardian model with three countries, two developing countries that have identical technologies, collectively labeled "South", and a developed country labeled North. Goods in the model are produced competitively, and trade is a result of differences in productivity with trading partners. Labor is the only factor of production, and technologies are more advanced in the North, i.e., labor is more productive in the North. Accordingly, productivity differences give rise to differences in per capita income. Following Matsuvama (2000), we assume there is a continuum of $goods^2$, goods are indivisible, and consumers consume at most one unit of each good subject to their budget constraint. These simple assumptions provide a tractable way to model differences in the consumption bundles of consumers with different incomes, i.e., consumers are assumed to have non-homothetic preferences. Goods are ordered along the continuum by priority; lower-indexed goods provide the highest utility per dollar and are therefore consumed first. Higher-indexed goods can only be consumed if a consumer's budget constraint allows for it; therefore, all consumers consume lowerindexed goods, but only richer ones can afford higher-indexed goods.³

We focus on two types of trade agreements, CUs and FTAs. In both cases, we assume that the developed North trades freely with both developing countries. In our model, developing countries signing an FTA allows each member country to

 $^{^1\}mathrm{See}$ Clements et al. (2006), Falkinger and Zweimüller (1996), Grigg (1994), Jackson (1984)

²Matsuyama (2000) follows Dornbusch, Fischer, and Samuelson (1977) in assuming a continuum of goods.

³Unlike Dornbusch, Fischer, and Samuelson (1977) in which each country consumes every good along the continuum, in this setting, richer countries consume a more extensive variety of goods.

place tariffs independently against the developed North, while in the case of a CU, both countries set a common external tariff (CET).

The main results of the paper are as follows. Whether a developing country benefits from an FTA or a CU is subject to the population in the developing country relative to the other trade agreement member. When the two developing countries have symmetric populations, they will set higher tariffs and enjoy higher welfare in a CU setting than they would in an FTA. When developing countries have asymmetric populations, the member of an FTA with the larger population will set higher tariffs. Moreover, our numerical analysis shows that when members of a trade agreement have asymmetric populations, the larger country will benefit more from a CU since they would be able to set a higher tariff. However, if the member country with a smaller population is small enough, then it would prefer to enter into an FTA. Lastly, the more populated country in an FTA will have lower welfare than the other trade agreement member.

The remainder of this paper is organized as follows. Section 3.2 reviews the relevant literature. Section 3.3 describes the model in a setting where there is global free trade and outlines how consumption and trade patterns differ between the North and the South. In section 3.4, we derive the common external tariff assuming that the two developing countries form a CU. In section 3.5, we compare the tariffs set by the developing countries in a CU with tariffs they would set if they formed an FTA. Additionally, we describe how welfare differs in each setting. In section 3.6, we provide a more in-depth analysis of when developing countries are better off in either setting; this is done using a numerical analysis. Section 3.7 concludes, and section 3.8 provides an appendix. The appendix shows that when developing countries are identical, the tariffs obtained in the CU are the same as

if the developing countries signed an FTA and maximized their joint welfare rather than maximizing independent welfare.

3.2 The review of literature

As noted above, the literature on CUs dates back to Viner's (1950) The Customs Union Issue. Viner proposed that the formation of a CU could be welfare improving if, as a result of tariffs decreasing, a country imported from a lower-cost producer instead of producing domestically, which he termed trade creation. Additionally, the formation of a CU could be welfare deteriorating if, as a result of tariffs decreasing, trade with a foreign producer was simply replaced with trade within the CU, which he termed term diversion. Subsequent to Viner's contribution, there has been much debate over the original terminology and whether trade diversion could be welfare improving.⁴

Following Viner's seminal work, there is a large body of literature analyzing whether preferential trade agreements (PTA's), including FTAs and CUs, are globally welfare-enhancing or welfare diminishing. An important result is what is known as the Kemp-Wan-Ohyama-Vanek⁵ Theorem (Kemp and Wan, 1976; Ohyama, 1972; Vanek, 1965), which states an "appropriate" CU could be welfare improving for members and nonmembers if the CET were set such that trade with the rest of the world remained the same and lump-sum transfers between member countries were allowed. Along the same lines of showing how CUs could be globally welfareenhancing, one of the few studies on CUs and developing countries is by Krishna and Bhagwati (1997), proves what they term the Cooper-Massell-Johnson-Bhagwati

 $^{^{4}}$ It was later shown by Lipsey (1957) and Gehrels (1956) that trade diversion could be welfare improving. While Gehrels (1956) and Kowalczyk (1990) proposed terminology alternative to trade creation and trade diversion.

⁵This Theorem is commonly referred to as the Kemp-Wan theorem.

conjecture.⁶ Essentially, they prove that developing countries can form a CU that is welfare improving globally while maintaining a certain level of industrialization in each member country⁷.

There have been several papers that discuss the welfare implications of a CU vs. FTA. One of the earlier and more cited works is Kennan and Riezman (1990), who use a three-country three good pure exchange model to analyze the formation of CUs when all tariffs are set optimally. They note that the motivation to form a CU comes from forming a large trading bloc with more market power and from internalizing the tariff externality. The tariff externality exists because when one country places a tariff on imports, the world prices fall for these imports; therefore, all other countries which import these goods do so at a lower price. In a CU, member countries can coordinate tariffs jointly, leading to the internalization of the tariff externality. In like manner, Mukunoki (2004) also compares FTAs and CUs, and the paper finds that a CU provides higher welfare than an FTA for member countries while making nonmembers worse off; this paper shows this using a three-country oligopoly model with symmetric countries. Chang and Xiao (2015) extend this analysis to include market size asymmetry between PTA members. They find that it could be the case that a CU is preferred over an FTA when member countries have asymmetric market sizes, but if the market size is sufficiently different, the country with the large market will be better off under a CU. In comparison, the country with a smaller market will be better off under an FTA.

 $^{^{6}\}mathrm{This}$ conjecture is based on Cooper and Massell (1965), Johnson (1960), and Bhagwati (1968)

 $^{^{7}}$ A synthesis of the theoretical literature on preferential trade liberalization is provided by Panagariya (2000)

In contrast, Krueger (1997) is one of the few papers that finds CUs to be Pareto superior to FTA's ⁸. Two key reasons were given in support of this argument; the first is that the World Trade Organization (WTO) stipulates in article XXIV that the CET must be a weighted average of previous tariffs. The second is that, unlike an FTA, a CU does not have Rules of Origins (RoO) requirements that act as a trade barrier. The papers above do not account for countries at different stages of economic development.

Most standard trade models assume preferences that are identical and homothetic; however, the importance of the role that a country's demand plays in its exports is not novel and has been noted by Linder (1961) and Grubel (1970); both of which note how countries tend to export goods which they demand. Recently it has become increasingly popular to use models with nonhomothetic preferences to answer questions about the quality of goods Fajgelbaum et al. (2011); Jaimovich and Merella (2012); Schott (2001) and how prices of tradeable goods are related to percapita income (Bekkers et al. (2012); Hunter (1991); Hunter and Markusen (1988); Markusen (1986); Simonovska (2015). Moreover, Markusen (2013) has shown that the inclusion of per capita income can explain some of the common puzzles in international trade.

The current paper most resembles Stibora and de Vaal (2015) in that they both study PTA's in developing countries using the Matsuyama (2000) model that includes nonhomothetic preferences. This paper differentiates itself from Stibora and de Vaal (2015) in that the latter focuses on how small unilateral tariff changes affect welfare, given an initial uniform tariff, whereas this paper focuses on the comparison of welfare in a customs union setting versus an FTA. Additionally, the

⁸Along with Mukunoki (2004) and Chang and Xiao (2015),Kose and Riezman (2000) and Yi (2000) find that FTA are better for global welfare than CU.

current paper focuses on countries setting tariffs optimally against the richer rest of the world. Lastly, the current paper provides a numerical analysis of the differences in welfare between developing countries forming a customs union and signing an FTA.

3.3 A model of free trade

We develop a model of North-South trade based on Matsuyama (2000). Consider a world with three countries: one in the North and two in the South. There is a continuum of goods indexed by a real number $z \ge 0$. All goods are produced with the composite factor called "labor". The North is endowed with N units of labor, while countries i in the South are endowed with S_i units of labor. Let $S_1 + S_2 = S$. Define the relative endowments by

$$\lambda_i = S_i/(N+S)$$
 and $\lambda_S = S/(N+S)$.

The Southern countries are assumed technologically symmetric and capable of producing each unit of good z with a(z) units of labor, while the North can do so with b(z) units of labor. The North is assumed uniformly superior to the South in technology; that is, we assume a(z)/b(z) > 1. As a result, the wage w in the South is lower than that in the North. Using the latter as the numeraire, we observe

Turning to consumer preferences, we follow Matsuyama in assuming every consumer consumes at most one unit of each good in the markets, which implies non-homothetic preferences. Furthermore, we order the goods in decreasing order of marginal utility, so consumers first buy a unit of good 0 and demand higher-indexed goods until they exhaust their incomes. That is, consumers in the South buy goods in the interval $[0, c_S]$, where c_S denotes the highest indexed good they buy, while the similar interval in the North is $[0, c_N]$, where c_N denotes the highest indexed good consumed there. The fact that w < 1 implies $c_S < c_N$.

Further, we assume that the ratio a(z)/b(z) is increasing so that the South has a comparative advantage in lower indexed goods. Moreover, the gap between technologies in each country, i.e. $g(z) = a_s(z) - a_n(z) > 0$ be increasing at an increasing rate, i.e. g''(z) > 0. With the endowment S_i , country *i* produces the goods in the range $[0, z_i]$ such that we observe

$$S_i = (x_i N + S_i) \int_0^{z_i} a(z) dz,$$

where x_i is the fraction of Northern consumers served by country *i*. Because technologies in the Southern countries are the same, any difference in wages will cause the Southern country with the higher wage not to export anything because all the goods it produces will be priced higher, while the other Southern country should be able to export all its goods to the North, because all of the goods it produces will be cheaper. However, in the general equilibrium setting presented here, the increase in exports from one Southern country to the North should cause the wage in said Southern country to rise as demand for its goods rise. Consequently, the North would shift to importing from the other Southern country where both the wages and prices were lower. In equilibrium, the wages in the South should be identical. There is no trade between the Southern countries with equal wages in the South. All Southern exports are to the North. Thus, the ranges of goods exported by the two countries are identical. Thus, we write $z_i = z_s$. Therefore, the South as a whole faces the resource constraint

$$S = S_1 + S_2 = (N+S) \int_0^{z_S} a(z) dz$$
(3.1)

All goods are produced under perfect competition so that the prices equal the unit production costs. A typical consumer in the South spends a fraction of their income on the Southern goods in the range $[0, z_S]$ and the remaining income on imports from the North. Thus, the budget constraint is given by

$$w = \int_{0}^{z_{S}} wa(z) dz + \int_{z_{S}}^{c_{S}} b(z) dz.$$
 (3.2)

The unit cost of producing the borderline good z_S is equalized between the North and the South,

$$wa\left(z_{S}\right) = b\left(z_{S}\right). \tag{3.3}$$

The North produces all the goods in the range $[z_S, c_N]$ and exports those in the range $[z_S, c_S]$ to the South. Its resource constraint is thus satisfied when

$$N = (N+S) \int_{z_S}^{c_S} b(z) \, dz + N \int_{c_S}^{c_N} b(z) \, dz; \qquad (3.4)$$

The North's budget constraint is satisfied if

$$1 = \int_{0}^{z_{s}} wa(z) dz + \int_{z_{s}}^{c_{N}} b(z) dz$$
(3.5)

The model can be solved for w, z_S , c_S , c_N from Equations (3.1) – (3.5), one of which is redundant. Given our preference structure, consumers' welfare increases monotonically when they consume a greater range of goods. This observation allows us to use the highest-indexed goods c_S and c_N of the consumption baskets as the welfare measures for Southern and Northern consumers. Therefore, throughout the paper, we use the terms consumption and welfare will be used interchangeably.

3.4 A model of a customs union

We now suppose the two Southern countries form a CU, whereby they erect a common external tariff against Northern goods while maintaining free trade between them. In order to get concrete results and to facilitate comparisons later, in this section, we specify the unit labor requirements as follows.

Assumption 1. $a(z) = \exp(z); b(z) = 1.$

Assume that the South imposes a common external tariff on all the North's exports at the rate $e^t - 1$, where t > 0 is our measure of trade protection (t = 0 implies free trade). Since the North imposes no trade restrictions, the South exports all the goods $[0, \underline{z}]$ where \underline{z} satisfies

$$we^{\underline{z}} = 1. \tag{3.6}$$

While the North exports all goods indexed \overline{z} and above to the South, define \overline{z} by

$$we^{\overline{z}} = e^t$$

which implies,

$$w = e^{t - \overline{z}}.\tag{3.7}$$

As is well known, the imposition of a tariff turns the goods in the range $[\underline{z}, \overline{z}]$ into non-traded goods. (3.6) and (3.7) also imply that

$$t = \overline{z} - \underline{z}.$$

The Southern countries face the resource constraint

$$S_1 = (xN + S_1) \int_0^{\underline{z}} dz + S_1 \int_{\underline{z}}^{\overline{z}} dz$$
$$S_2 = ((1-x)N + S_2) \int_0^{\underline{z}} dz + S_2 \int_{\underline{z}}^{\overline{z}} dz$$

Adding them, we get

$$S = (N+S) \int_{0}^{\overline{z}} e^{z} dz + S \int_{\underline{z}}^{\overline{z}} e^{z} dz$$

$$= (N+S) (e^{z} - 1) + S (e^{\overline{z}} - e^{\underline{z}}) = N (e^{\underline{z}} - 1) + S (e^{\overline{z}} - e^{\underline{z}})$$
(3.8)

Rearranging (3.6) and (3.7), we get

$$e^{\underline{z}} = \frac{1}{w}$$
$$e^{\overline{z}} = \frac{T}{w}$$

Where $T = e^t$. Substituting into (3.8), we have

$$S = N\left(\frac{1}{w} - 1\right) + S\left(\frac{T}{w} - 1\right) \tag{3.9}$$

Collecting terms, we obtain

$$w = \frac{N + ST}{2S + N} = \frac{\lambda_N + \lambda_S T}{2\lambda_S + \lambda_N} = \frac{1 + \lambda_S (T - 1)}{1 + \lambda_S}.$$
(3.10)

Since w < 1, (3.10) requires that T < 2.

We next rewrite the budget constraints of a typical consumer in the South. In doing so, we assume that the tariff revenue is collected by the Southern governments and rebated to each consumer in a lump-sum fashion. Our non-homothetic preferences rule out substitutions, implying that the rebated tariff revenue exactly offsets the price increases due to the tariff. As a result, the budget constraint in the South is given by

$$w = w \int_0^{\overline{z}} a(z) dz + \int_{\overline{z}}^{c_S} b(z) dz = w \int_0^{\overline{z}} e^z dz + \int_{\overline{z}}^{c_S} dz.$$

Integrating we get

$$w = w\left(e^{\overline{z}} - 1\right) + c_S - \overline{z} = w\left(\frac{T}{w} - 1\right) + c_S - \ln\frac{T}{w}.$$

Therefore,

$$c_S = 2w + lnT - lnw - T.$$

Substituting for lnw from (3.10),

$$c_S = 2\left(\frac{1+\lambda_S\left(T-1\right)}{1+\lambda_S}\right) + \ln T - \ln\left(\frac{1+\lambda_S\left(T-1\right)}{1+\lambda_S}\right) - T$$
(3.11)

The CU chooses t to maximize c_S . The first-order condition is

$$2\left(\frac{\lambda_S T}{1+\lambda_S}\right) + 1 - \frac{\lambda_S T}{1+\lambda_S \left(T-1\right)} - T = 0 \tag{3.12}$$

Further differentiation of (3.12) shows the second-order condition is satisfied at the optimum⁹ and $\lambda_N = 1 - \lambda_S$.

$$2\left(\frac{\lambda_S T}{1+\lambda_S}\right) - \frac{\lambda_S T \left(1-\lambda_S\right)}{\left(1+\lambda_S \left(T-1\right)\right)^2} - T < 0$$

holds at $t = t^{CU}$. Using the quadratic formula we can solve for the optimal t in equation (3.12)

$$t = \ln\left(\frac{-1 + \lambda_S + \sqrt{5\lambda_S^2 + 2\lambda_S + 1}}{2\lambda_S}\right) \text{ or } t = \ln\left(\frac{-1 + \lambda_S - \sqrt{5\lambda_S^2 + 2\lambda_S + 1}}{2\lambda_S}\right)$$

Given that the second solution would make the term inside parentheses negative, we use the first solution

$$t = \ln\left(\frac{-1 + \lambda_S + \sqrt{5\lambda_S^2 + 2\lambda_S + 1}}{2\lambda_S}\right). \tag{3.13}$$

Plugging the above equation into equation (3.11) yields the average welfare in the developing country.

Lastly, we rewrite the budget constraint for the typical consumer in the North. In doing so, we are able to calculate the consumption of the typical consumer in the North. The north budget constraint is

$$1 = \int_0^{\underline{z}} w \exp\left(z\right) \, dz + \int_{\underline{z}}^{c_N} dz$$

Integrating and solving for c_N we find the typical consumption in the North to be

$$c_N = 1 + \underline{z} - w \left(e^{\underline{z}} - 1 \right)$$

We can use (3.6) to simplify the above as

 $^{^{9}}$ For a brief description and discussion on some of the applications of the Lambert W function see Corless et al. (1996).

$$c_N = w - lnw$$

Lastly, substituting for $w = \frac{1+\lambda_S(T-1)}{1+\lambda_S}$ from (3.10), we find

$$c_{N} = \frac{1 + \lambda_{S} \left(T - 1\right)}{1 + \lambda_{S}} - ln \left(\frac{1 + \lambda_{S} \left(T - 1\right)}{1 + \lambda_{S}}\right).$$

Analogous to the developing country, plugging the optimal common external tariff from equation (3.13) into the above equation yields the average welfare in the developed country.

3.5 Free Trade Agreements

If the South forms a free trade area, each member can choose its own tariff against Northern exports while maintaining free trade between themselves. Let t_i denote country *i*'s tariff measure. The South exports their products in the range $[0, z_0]$ to the North, where z_0 satisfies

$$we^{z_0} = 1.$$
 (3.14)

On the other hand, if the two members of the FTA impose tariffs at different rates, the ranges of non-traded goods differ between them. Let

$$we^{z_1} = e^{t_1}$$
 (3.15)

$$we^{z_2} = e^{t_2}$$
 (3.16)

so that $[z_0, z_i]$ denotes the range of non-traded goods against the North in member *i*. Rearranging the above equation.

$$e^{z_0} = \frac{1}{w}$$
$$e^{z_1} = \frac{e^{t_1}}{w}$$
$$e^{z_2} = \frac{e^{t_2}}{w}$$

In the analysis below, we assume without loss of generality

Assumption 2: $t_2 \ge t_1$.

This assumption implies that $z_2 \ge z_1$ such that the members' resource constraints are written

$$S_{1} = (x_{1}N + S_{1}) \int_{0}^{z_{0}} e^{z} dz + S_{1} \int_{z_{0}}^{z_{1}} e^{z} dz$$
$$S_{2} = ((1 - x_{1})N + S_{2}) \int_{0}^{z_{0}} e^{z} dz + S_{2} \int_{z_{0}}^{z_{2}} e^{z} dz$$

Adding both sides, we get

$$S = (N+S) \int_0^{z_0} e^z dz + S \int_{z_0}^{z_1} e^z dz + S_2 \int_{z_1}^{z_2} e^z dz$$
$$S = N (e^{z_0} - 1) + S_1 (e^{z_1} - 1) + S_2 (e^{z_2} - 1).$$

Substituting for the e^{z_i} 's from the above, we can rewrite the last equation as

$$S = N\left(\frac{1}{w} - 1\right) + S_1\left(\frac{e^{t_1}}{w} - 1\right) + S_2\left(\frac{e^{t_2}}{w} - 1\right).$$

which allows us to solve for w

$$w = \frac{1 + \lambda_1 \left(e^{t_1} - 1 \right) + \lambda_2 \left(e^{t_2} - 1 \right)}{\left(1 + \lambda_1 + \lambda_2 \right)}.$$
(3.17)

A consumer in Country 2 faces the budget constraint

$$w = w \int_0^{z_2} e^z dz + \int_{z_2}^{c_2} dz = w \left(e^{z_2} - 1 \right) + c_2 - z_2,$$

where c_2 denotes the highest-indexed good consumed by a consumer in Country 2. Substituting for z_2 , we can rewrite the above equation as

$$w = w\left(\frac{e^{t_2}}{w} - 1\right) + c_2 - (t_2 - lnw)$$

Rearranging, we get

$$c_2 = 2w - e^{t_2} + t_2 - lnw (3.18)$$

$$= 2\left(\frac{1+\lambda_1\left(e^{t_1}-1\right)+\lambda_2\left(e^{t_2}-1\right)}{(1+\lambda_1+\lambda_2)}\right) - e^{t_2} + t_2 - ln\left(\frac{1+\lambda_1\left(e^{t_1}-1\right)+\lambda_2\left(e^{t_2}-1\right)}{(1+\lambda_1+\lambda_2)}\right)$$

where $\lambda_n = 1 - \lambda_s = (1 - \lambda_2 - \lambda_1)$ is the relative endowment of the North.

We assume that the government of country 2 chooses t_2 to maximize the welfare measure c_2 of the typical consumer, taking t_1 as given. The first-order condition is

$$\frac{\partial(c_2)}{\partial t_2} = \left(\frac{e^{t_2} \left(\lambda_2 - \lambda_1 - 1\right)}{\left(1 + \lambda_1 + \lambda_2\right)}\right) + \left(\frac{\lambda_1 \left(e^{t_1} - 1\right) + 1 - \lambda_2}{1 + \lambda_1 \left(e^{t_1} - 1\right) + \lambda_2 \left(e^{t_2} - 1\right)}\right) = 0 \quad (3.19)$$

This equation implicitly defines the best-response tariff measure $t_2 = b_2(t_1)$ for country 2. Using the quadratic equation, we can solve for the optimal t_2 in equation (3.19)

$$t_{2} = \ln\left(\frac{-(1+\lambda_{1}e^{t_{1}}-\lambda_{S})+\sqrt{(1-\lambda_{S}+\lambda_{1}e^{t_{1}})^{2}-4\lambda_{2}\frac{(1+\lambda_{S})(1-\lambda_{S}+\lambda_{1}e^{t_{1}})}{(\lambda_{2}-\lambda_{1}-1)}}}{2\lambda_{2}}\right)$$

or

$$t_{2} = \ln\left(\frac{-\left(1 + \lambda_{1}e^{t_{1}} - \lambda_{S}\right) - \sqrt{\left(1 - \lambda_{S} + \lambda_{1}e^{t_{1}}\right)^{2} - 4\lambda_{2}\frac{(1 + \lambda_{S})(1 - \lambda_{S} + \lambda_{1}e^{t_{1}})}{(\lambda_{2} - \lambda_{1} - 1)}}}{2\lambda_{2}}\right)$$

Given that the second solution would make the term inside parentheses negative, therefore we use the first solution

$$t_{2} = \ln\left(\frac{-\left(1 + \lambda_{1}e^{t_{1}} - \lambda_{S}\right) + \sqrt{\left(1 - \lambda_{S} + \lambda_{1}e^{t_{1}}\right)^{2} - 4\lambda_{2}\frac{(1 + \lambda_{S})(1 - \lambda_{S} + \lambda_{1}e^{t_{1}})}{(\lambda_{2} - \lambda_{1} - 1)}}}{2\lambda_{2}}\right).$$
 (3.20)

The second-order condition is satisfied if $\frac{\partial^2(c_2)}{\partial(t_2)^2} < 0$ at $t_2 = b_2(t_1)$. Algebra shows that this condition is equivalent to

$$\frac{\partial^2(c_2)}{\partial t_2^2} = \left(\frac{e^{t_2} \left(\lambda_2 - \lambda_1 - 1\right)}{\left(1 + \lambda_1 + \lambda_2\right)}\right) - \left(\frac{\left(\lambda_1 \left(e^{t_1} - 1\right) + 1 - \lambda_2\right) \lambda_2 e^{t_2}}{\left(1 + \lambda_1 \left(e^{t_1} - 1\right) + \lambda_2 \left(e^{t_2} - 1\right)\right)^2}\right) < 0$$

Further, differentiating (3.19) with respect to t_1 yields

$$\frac{\partial(c_2)}{\partial t_2 \partial t_1} = \left(\frac{(\lambda_1 e^{t_1}) (\lambda_2 e^{t_2})}{(1 + \lambda_1 (e^{t_1} - 1) + \lambda_2 (e^{t_2} - 1))^2}\right) > 0$$

Hence,

$$\frac{dt_2}{dt_1} = -\frac{\frac{\partial^2(c_2)}{\partial t_2 \partial t_1}}{SOC} > 0$$

indicating that the tariffs are strategic complements in the neighborhood of the optimum.

A consumer in country 1 faces the analogous budget constraint

$$w = w \int_0^{z_1} e^z dz + \int_{z_1}^{c_1} dz = w \left(e^{z_1} - 1 \right) + c_1 - z_1.$$

It is obvious that all the preceding results apply to country 1 with the country labels interchanged. Denoting country 1's best response tariff measure by $t_1 = b_1(t_2)$, we can define the equilibrium tariff measures as the pair (t_1^{FTA}, t_2^{FTA}) , which simultaneously satisfies both best-response functions. Moreover, we could plug these tariff measures into the consumption function of each country to find the average welfare in each country.

Now, focus we on the symmetric case in which $\lambda_1 = \lambda_2 = \lambda_S/2$ and a symmetric equilibrium. Letting $t_1 = t_2 = t$ in the left-hand expression of (3.19), we have

$$LHS = \left(\frac{e^t \left(\frac{\lambda_S}{2} - \frac{\lambda_S}{2} - 1\right)}{\left(1 + \frac{\lambda_S}{2} + \frac{\lambda_S}{2}\right)}\right) + \left(\frac{\frac{\lambda_S}{2} \left(e^t - 1\right) + 1 - \frac{\lambda_S}{2}}{1 + \frac{\lambda_S}{2} \left(e^t - 1\right) + \frac{\lambda_S}{2} \left(e^t - 1\right)}\right)$$
$$= \frac{\frac{\lambda_S}{2} e^t + 1 - \lambda_S}{1 + \lambda_S \left(e^t - 1\right)} - \frac{e^t}{\left(1 + \lambda_S\right)}.$$

When evaluated at $t = t^{CU}$, this becomes

$$LHS = \frac{\frac{\lambda_S}{2}e^{t^{CU}} + 1 - \lambda_S}{1 + \lambda_S (e^{t^{CU}} - 1)} - \frac{e^{t^{CU}}}{(1 + \lambda_S)}$$
$$= \frac{\frac{\lambda_S}{2}e^{t^{CU}}}{1 + \lambda_S (e^{t^{CU}} - 1)} - 2\left(\frac{\lambda_S e^{t^{CU}}}{1 + \lambda_S}\right) + e^{t^{CU}} - \frac{e^{t^{CU}}}{(1 + \lambda_S)}$$
$$= \frac{e^t \lambda_S}{2(1 + \lambda_S (e^t - 1))} - \frac{e^t \lambda_S}{1 + \lambda_S} < 0$$

where use is made of Eq. (3.12). The last expression is clearly negative, and hence when $t_1 = t^{CU}$, country 2 responds by setting $t_2 = b_2 (t^{CU}) < t^{CU}$. This implies that the South sets higher tariffs when forming a CU than when forming an FTA. Figure 3.1 illustrates this result.

Further, we show, in the appendix, that in the symmetric case, the CU results are obtained if the FTA members maximize the joint welfare $(c_1 + c_2)$. A fortiori that implies that each country's welfare is greater with the formation of a CU than with that of an FTA. Thus,

Proposition 1. In a symmetric case:

- (A) The tariffs are lower with an FTA than with a CU.
- (B) Each member country's welfare is lower with an FTA than with a CU.

We explore the asymmetric case. In doing so, we hold N constant and move a small fraction of labor from country 1 to country 2 such that $d\lambda_2 = -d\lambda_1$. Differentiating (3.19) with respect to λ_2 and evaluating the result at the symmetric equilibrium, we get

$$\frac{\partial^2(c_2)}{\partial t_2 \partial \lambda_2} = \frac{e^t}{\frac{1}{2}\left(1+2\lambda\right)} - \frac{e^t}{\left(1+2\lambda\left(e^t-1\right)\right)} > 0.$$

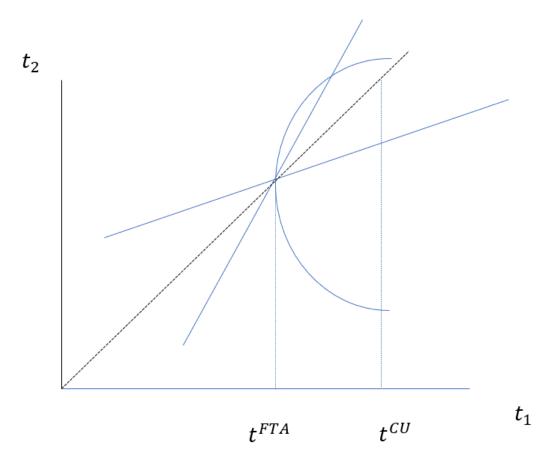


Figure 3.1: Best Response Functions of Developing Countries

In the symmetric equilibrium $\lambda \epsilon(0, .5)$, therefore $\frac{\partial^2(c_2)}{\partial t_2 \partial \lambda_2} > 0$. Hence

$$\frac{dt_2}{d\lambda_2} = -\frac{\frac{\partial^2(c_2)}{\partial t_2 \partial \lambda_2}}{SOC} > 0$$

The larger country in the South tends to set a tariff higher than the smaller country, at least around the symmetric equilibrium.

Proposition 2: With an FTA, the larger member country tends to set its tariff higher than the smaller one.

Furthermore, by differentiating (3.18), we get

$$\frac{dc_2}{d\lambda_2} = 2\left(\frac{\left(-\left(e^{t_1}-1\right)+\lambda_1 e^{t_1} \frac{dt_1}{d\lambda_2}+\left(e^{t_2}-1\right)\right)}{\left(1+\lambda_1+\lambda_2\right)}\right) - \left(\frac{\left(-\left(e^{t_1}-1\right)+\lambda_1 e^{t_1} \frac{dt_1}{d\lambda_2}+\left(e^{t_2}-1\right)\right)}{1+\lambda_1\left(e^{t_1}-1\right)+\lambda_2\left(e^{t_2}-1\right)}\right)$$

$$= \left(-\left(e^{t_1} - 1\right) + \lambda_1 e^{t_1} \frac{dt_1}{d\lambda_2} + \left(e^{t_2} - 1\right) \right) \\ \times \left(\frac{2}{\left(1 + \lambda_1 + \lambda_2\right)} - \frac{1}{1 + \lambda_1 \left(e^{t_1} - 1\right) + \lambda_2 \left(e^{t_2} - 1\right)} \right)$$

Evaluated at the symmetric equilibrium, the last expression can be written as

$$\left(-\left(e^{t}-1\right)+\lambda e^{t}\frac{dt_{1}}{d\lambda_{2}}+\left(e^{t}-1\right)\right)\left(\frac{2}{\left(1+2\lambda\right)}-\frac{1}{1+2\lambda\left(e^{t}-1\right)}\right)$$
$$=\left(\lambda e^{t}\frac{dt_{1}}{d\lambda_{2}}\right)\left(\frac{1}{\frac{1}{2}\left(1+2\lambda\right)}-\frac{1}{1+2\lambda\left(e^{t}-1\right)}\right)<0,$$

where the inequality follows because $\frac{dt_1}{d\lambda_2} = -\frac{dt_1}{d\lambda_1} < 0$ and because in the symmetric equilibrium $\lambda = 0.5$. Thus, in an FTA, the larger country tends to have lower welfare than the smaller country, in particular, around the symmetric equilibrium.

Proposition 3: In an FTA, the larger country tends to have lower welfare than the smaller country, in particular, around the symmetric equilibrium.

Proposition 3 raises the question of whether the larger country may prefer the formation of a CU to that of an FTA. In the next section, section 6, we address this and related questions.

Lastly, we rewrite the budget constraint for the typical consumer in the North. In doing so, we are able to calculate the consumption of the typical consumer in the North. The north budget constraint is

$$1 = \int_0^{z_0} w e^z dz + \int_{z_0}^{c_N} dz$$

Integrating and solving for c_N , we find the typical consumption in the North to be

$$c_N = 1 + z_0 - w \left(e^{z_0} - 1 \right)$$

We can use (3.14) to simplify the above as

$$c_N = w - lnw$$

Lastly, substituting for $w = \frac{1+\lambda_1(e^{t_1}-1)+\lambda_2(e^{t_2}-1)}{(1+\lambda_1+\lambda_2)}$ from (3.17), we find

$$c_N = \frac{1 + \lambda_1 \left(e^{t_1} - 1\right) + \lambda_2 \left(e^{t_2} - 1\right)}{\left(1 + \lambda_1 + \lambda_2\right)} - ln \left(\frac{1 + \lambda_1 \left(e^{t_1} - 1\right) + \lambda_2 \left(e^{t_2} - 1\right)}{\left(1 + \lambda_1 + \lambda_2\right)}\right).$$

Plugging the optimal FTA tariffs yields the average welfare in the developed country. However, due to the analytical complexity, we forgo showing this and instead compare welfare numerically in the following section.

3.6 Welfare comparison

In the CU scenario, we were able to analytically solve for the optimal CET tariff. In the FTA scenario, we were unable to analytically solve for the optimal tariffs that each developing Southern country would place on the developed North. Therefore, we cannot analytically compare the welfare of the developing countries in each scenario. However, optimal tariffs can be solved numerically in both a

CU and an FTA. Doing so allows us to plug these optimal tariffs back into the consumption/welfare function and compare the welfare of the countries in an FTA and a CU. We provide the results of the numerical analysis in the next subsection.

3.6.1 Numerical results

We provide a numerical analysis to further illustrate all the results above. In the figures below, we provide 2-dimensional graphs to illustrate our results. In doing so, we must assume that either the population in the Southern countries is fixed, e.g., $\lambda_s = .5$, or that the populations in the Southern countries are symmetric. However, the results we illustrate in these graphs extend to the cases where the population in the Southern countries is not fixed or symmetric. In the appendix, we provide 3-dimensional graphs, which further confirm our results.

Figure 3.2 below numerically compares the welfare in a CU setup and the FTA. To make this comparison, we calculate the percentage change in consumption when a country changes from a CU to an FTA. We assume half the world population is in the South, i.e., $\lambda_S = .5$. The vertical axis is the percentage change in consumption as a result of going from a CU to an FTA. The horizontal axis is the population in country 1. The population in country 2 is $\lambda_2 = .5 - \lambda_1$. Where the percentage change in consumption is negative indicates that a CU provides more welfare than an FTA. We see that when the populations in each of the Southern countries are equal, in figure $3.2 \lambda_1 = \lambda_2 = .25$, then both countries have higher welfare in a CU than in an FTA, confirming proposition 1 above. The point in figure 3.2 where both curves intersect is the point where $\lambda_1 = \lambda_2 = .25$; because this point is below the horizontal axis, the countries have higher welfare in the CU.

Furthermore, we see that the larger a developing country's population is, the more likely it would benefit from the formation of a CU. A Southern country prefers an FTA when its population is small relative to the other Southern country. In figure 3.2, as we move along the horizontal axis and country 1 gets larger, it is better off in a CU than it would be in an FTA. The same observation can be made of country 2; as its population increases, it is better off in a CU. Lastly, the points that lie above the horizontal axis are points where the corresponding country is better off in an FTA. In this case, either developing country is better off in an FTA when its population is $\lambda_i < .14$, meaning the other developing country would have a population .36 < λ_j < .50. Although the results in figure 3.2 are for the case when $\lambda_S = .5$, the general results extend to all values of λ_S . Mainly, when the population in the developing countries are symmetric, the developing countries are better off in a CU than an FTA, and a developing country is better off in an FTA only when its population is small relative to the other developing country. Figure 3.8, in the appendix, e further illustrates these results by providing a 3-dimensional graph where λ_S is not fixed.

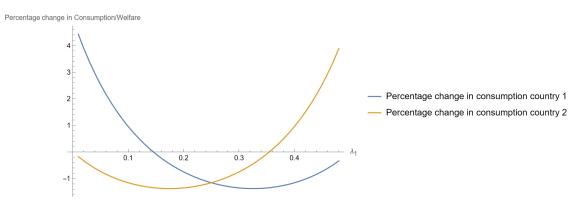
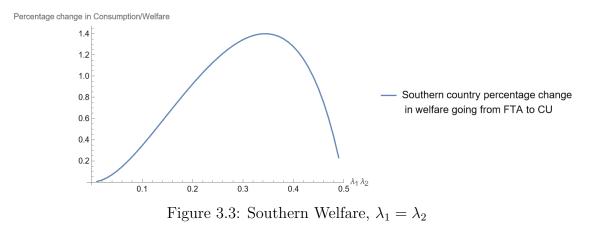


Figure 3.2: Southern Welfare, $\lambda_S = 0.5$

Contrary to figure 3.2, figure 3.3 graphs the percentage change in welfare moving from an FTA to a CU, assuming populations are symmetric in the Southern countries.

Notice that the percentage change in welfare is always positive, meaning that the consumption/welfare in a CU setting is higher than in an FTA when populations are symmetric; this falls in line with proposition 1 mentioned above.



Staying with the symmetric case, we illustrate how tariffs are lower in FTAs than in a CU, a point made in proposition 1. In figure 3.4, we graph the tariff rates in each type of trade agreement. Again, our numerical analysis confirms proposition 1, showing that in an FTA, the tariffs are lower than in a CU when Southern countries are symmetric.

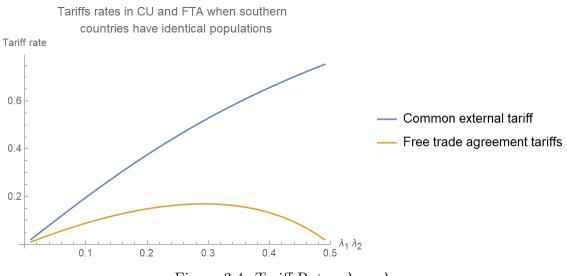


Figure 3.4: Tariff Rates, $\lambda_1 = \lambda_2$

Figures 3.5 and 3.6 help visualize tariff rates and welfare regardless of whether the Southern countries have symmetric populations or not. Figure 3.5 shows the tariff rates under an FTA when the population in the Southern countries is fixed, i.e., $\lambda_s = 0.5$. Similar to figure 3.2, the horizontal axis is the population in country 1 given that the population in the South is half the world population; therefore, the population in country 2 is equal to $\lambda_2 = 0.5 - \lambda_1$. When populations are symmetric, $\lambda_1 = \lambda_2$, we see that each country is setting tariffs equally; additionally, as a country's population increases, the tariff level increases. Figure 3.5 shows Proposition 2 above, which states that the FTA member with the larger population tends to set higher tariffs. In the appendix figure 3.8, illustrates how these results extend to all values of λ_s by providing a 3-dimensional graph where λ_s is not fixed.

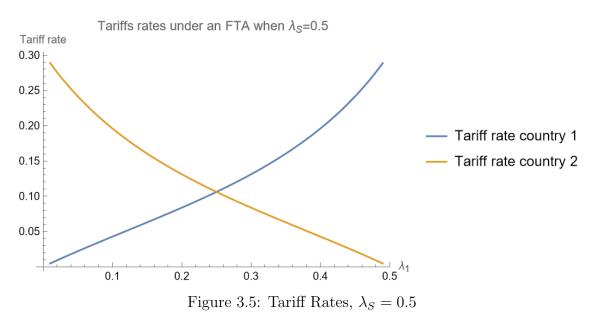


Figure 3.6 shows the welfare in Southern countries when they are in an FTA. Again, we fix the population in the Southern countries to half the world population, $\lambda_S = 0.5$. We see that in the symmetric case, when $\lambda_1 = \lambda_2 = 0.25$, welfare is the same in both countries. Additionally, as the population of country 1 rises higher than 25% of the world population, its welfare will be both lower and less than the other developing country. The same point holds true if the population in country 2 increases. Hence, given that we fix the population in the Southern countries to half the world population, any point where $\lambda_1 > \lambda_2$ ($\lambda_1 < \lambda_2$), the welfare is larger in country 2 (country 1). This figure confirms proposition 3, which states that the country with the larger population has lower welfare in an FTA. Again, these results extend to all values λ_s . To show this, in the appendix, we provide a comparable 3-dimensional graph, figure 3.10, where we do not fix the population in the South.

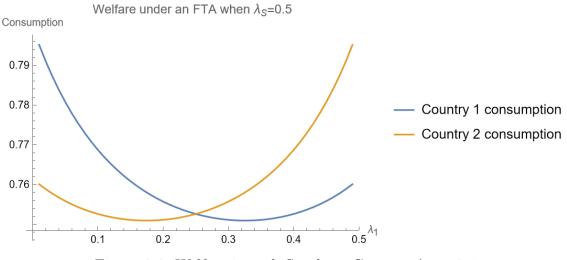


Figure 3.6: Welfare in each Southern Country, $\lambda_S = 0.5$

Lastly, in figure 3.7, we analyze the welfare in the Northern country as a result of the Southern countries going from an FTA to a CU. The welfare in the North is higher when the Southern countries form an FTA instead of a CU when the Southern countries are symmetric in size. Again, these results extend to all values λ_s . To show this, in the appendix, we provide a comparable 3- dimensional graph figure 3.11 where we do not assume the population in the Southern countries are symmetric.

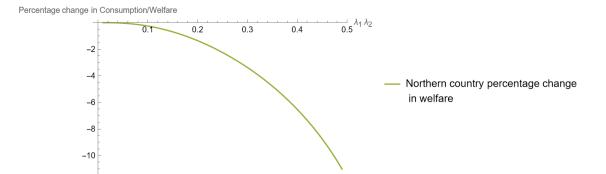


Figure 3.7: Northern Percentage Change in Welfare Going from CU to FTA, $\lambda_1 = \lambda_2$

3.7 Concluding remarks

Few papers have examined regional economic integration under the more realistic assumption of preferences being nonhomothetic; however, there are some limits to our analysis. Because the empirical evidence points to the fact that the variety of goods in a consumer's consumption basket differs with income, it is important to account for this when studying the welfare effects of trade agreements. The Matsuyama (2000) model allows us to include these more realistic assumptions. Moreover, unlike some previous studies, we focus on the nationalistic motivations for entering into a trade agreement. To this end, we assume tariffs are set optimally. However, considering both of these assumptions limit the analytical analysis that can be performed due to the algebraic complexity. For this reason, we specify the unit labor requirements and provide a numerical analysis to gain further insight into the question.

The main question in this paper is whether developing countries are better off signing FTAs or forming CUs. Essentially the answer to this question is that a developing country should form a CU unless it is significantly smaller in size relative to the other potential trade agreement member. Given that these developing countries share the same technologies, the more similar the trade agreement members are in population implies that they will import a comparably diverse set of goods from the high-income country. This implies a larger externality from setting tariffs independently, i.e., when the developing countries have similar import baskets, the benefits from internalizing the externality of tariffs are larger. Similarly, if the CU members are importing similar goods, they can coordinate a common external tariff that is mutually beneficial. Hence, forming a CU can be welfare improving relative to an FTA if the tariff externality is significantly internalized, this result is similar to Kennan and Riezman (1990). Finally, our results are similar to the results of Mukunoki (2004) and Chang and Xiao (2015); the nonmember country, in our paper, the developed rest of the world, is better off when the members of the trade agreement form an FTA instead of a CU.

3.8 Appendix

3.8.1 Cooperative Free Trade Agreement

The following section shows that when developing countries are identical, the tariffs obtained in the CU are the same as if the developing countries signed an FTA and maximized their joint welfare rather than maximizing independent welfare. The analysis is analogous to section 3.5 on Free Trade Agreements, except when the members of the FTA maximize joint welfare.

Again, we assume there are three countries, a developed North and two developing countries in the South. South *i* has a tariff t_i . Moreover, we define the boundary goods the same as equation (3.14)-(3.16) above. From the resource constraint, we can derive the same equation for wage as we have in (3.17). Furthermore, we can use the budget constraints to derive the consumption of each developing country; the consumption in country 2 would be given by equation (3.18) above,

$$c_2 = 2w - e^{t_2} + t_2 - lnw.$$

Plugging in the wage from equation (3.17) yields the consumption of country 2

$$c_{2} = 2\left(\frac{1+\lambda_{1}\left(e^{t_{1}}-1\right)+\lambda_{2}\left(e^{t_{2}}-1\right)}{\left(1+\lambda_{1}+\lambda_{2}\right)}\right)-e^{t_{2}}+t_{2}-ln\left(\frac{1+\lambda_{1}\left(e^{t_{1}}-1\right)+\lambda_{2}\left(e^{t_{2}}-1\right)}{\left(1+\lambda_{1}+\lambda_{2}\right)}\right)$$

Using the budget constraint for country 1 we can derive the consumption in country 1 to be

$$c_1 = 2w - e^{t_1} + t_1 - \ln w.$$

Plugging in the wage from equation (3.17) yields the consumption of country 1 $c_1 = 2\left(\frac{1+\lambda_1(e^{t_1}-1)+\lambda_2(e^{t_2}-1)}{(1+\lambda_1+\lambda_2)}\right) - e^{t_1} + t_1 - ln\left(\frac{1+\lambda_1(e^{t_1}-1)+\lambda_2(e^{t_2}-1)}{(1+\lambda_1+\lambda_2)}\right).$ Given the above consumption functions, we assume the members of the FTA are setting tariffs to maximize the following joint welfare objective function instead of maximizing the individual welfare in each country, respectively,

Joint welfare =
$$\lambda_1(c_1) + \lambda_2(c_2)$$
.

Each Southern country sets its tariff individually to maximize the joint welfare between countries. For South 1, differentiating the joint welfare function

$$\frac{d(joint \ welfare)}{dt_1} = \lambda_1 \left(\left(\frac{2\lambda_1 e^{t_1}}{(1+\lambda_1+\lambda_2)} \right) - e^{t_1} + 1 - \left(\frac{\lambda_1 e^{t_1}}{1+\lambda_1 (e^{t_1}-1)+\lambda_2 (e^{t_2}-1)} \right) \right) + \lambda_2 \lambda_1 e^{t_1} \left(\left(\frac{2}{(1+\lambda_1+\lambda_2)} \right) - \left(\frac{1}{1+\lambda_1 (e^{t_1}-1)+\lambda_2 (e^{t_2}-1)} \right) \right) = 0 \quad (3.21)$$

For South 2.

$$\frac{d(joint \ welfare)}{dt_2} = \lambda_1 \lambda_2 e^{t_2} \left(\left(\frac{2\lambda_2 e^{t_2}}{(1+\lambda_1+\lambda_2)} \right) - \left(\frac{\lambda_2 e^{t_2}}{1+\lambda_1(e^{t_1}-1)+\lambda_2(e^{t_2}-1)} \right) \right) + \lambda_2 \left(\left(\frac{2\lambda_2 e^{t_2}}{(1+\lambda_1+\lambda_2)} \right) - e^{t_2} + 1 - \left(\frac{\lambda_2 e^{t_2}}{1+\lambda_1(e^{t_1}-1)+\lambda_2(e^{t_2}-1)} \right) \right) = 0$$

Differentiating (3.21) with respect to t_1 and adding like terms, we get the SOC

$$\frac{d^2 (joint \ welfare)}{dt_1^2} = \left(\frac{e^{t_1}\lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) - \left(\frac{\lambda_2\lambda_1 e^{t_1}}{1 + \lambda_1 (e^{t_1} - 1) + \lambda_2 (e^{t_2} - 1)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) - \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) - \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_2)}{(1 + \lambda_1 + \lambda_2)}\right) + \frac{1}{2} \left(\frac{1 + \lambda_1 (\lambda_1 - 1 + \lambda_$$

$$\left(\frac{\lambda_1^2 e^{t_1} (\lambda_2 - 1 + \lambda_1)}{(1 + \lambda_1 (e^{t_1} - 1) + \lambda_2 (e^{t_2} - 1))^2}\right) < 0$$

The above equation is negative; therefore, the second-order condition is satisfied.

By symmetry, we have $\lambda_1 = \lambda_2 = \lambda_s/2$

Hence, the LHS of the FOC in (3a) is written

$$\left(\frac{\lambda_S^2 e^t}{2(1+\lambda_S)}\right) - \frac{\lambda_S}{2} \left(e^t - 1\right) - \left(\frac{\lambda_S^2 e^t}{4(1+\lambda_S (e^t - 1))}\right) + \left(\frac{\lambda_S^2 e^t}{2(1+\lambda_S)}\right) - \left(\frac{\lambda_S^2 e^t}{4(1+\lambda_S (e^t - 1))}\right)$$

Which simplifies to

$$\left(\frac{\lambda_S^2 e^t}{(1+\lambda_S)}\right) - \frac{\lambda_S}{2} \left(e^t - 1\right) - \left(\frac{\lambda_S^2 e^t}{2\left(1+\lambda_S \left(e^t - 1\right)\right)}\right)$$

Evaluating the Cooperative FTA at the CU CET t^{CU}

$$\left(\frac{\lambda_S^2 e^{t^{CU}}}{(1+\lambda_S)}\right) - \frac{\lambda_S}{2} \left(e^{t^{CU}} - 1\right) - \left(\frac{\lambda_S^2 e^{t^{CU}}}{2\left(1+\lambda_S \left(e^{t^{CU}} - 1\right)\right)}\right)$$
(3.22)

From (3.12)

$$\frac{2\lambda_S e^{t^{CU}}}{1+\lambda_S} + 1 - e^{t^{CU}} = \frac{\lambda_S e^{t^{CU}}}{1+\lambda_S \left(e^{t^{CU}} - 1\right)}$$

Plugging the above equation into 3.22 yields

$$\left(\frac{\lambda_S^2 e^{t^{CU}}}{(1+\lambda_S)}\right) - \frac{\lambda_S}{2} \left(e^{t^{CU}} - 1\right) - \frac{\lambda_S}{2} \left(\frac{2\lambda_S e^{t^{CU}}}{1+\lambda_S} - \left(e^{t^{CU}} - 1\right)\right) = 0$$

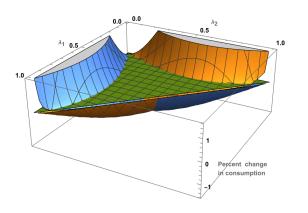
This implies that the optimal CU tariff is the same as the symmetric cooperative FTA tariff.

3.8.2 Additional Figures

Below are additional figures to further illustrate our results. The figures below are all 3-dimensional, and the x and y-axis of all the figures are the percentage of the population in one of the Southern developing countries, denoted by λ_1 or λ_2 .

The axes in figure 3.8 represent the population in each of the Southern countries labeled λ_1 and λ_2 and the percentage change in consumption. The flat green plane is shown for illustration purposes; it represents where the level of consumption is the same in either setup, FTA, or CU. Therefore, the points above the green plane show where a country is better off setting tariffs independently by entering into an FTA. The points below the green plane are points where the country is better off forming a CU.

Percentage change in welfare going from CU to FTA



Percentage change in consumption country 1
 Percentage change in consumption country 2

Figure 3.8: Percentage Change in Welfare Going from CU to FTA

As noted in proposition 1, when developing countries have a symmetric population, they are better off forming a CU. In figure 3.8, when $\lambda_1 = \lambda_2$, we see that all of those points lie below the green plane, which implies that both Southern countries would be better off forming a CU when their populations are the same. Equally important, the larger a developing country's population is, the more likely it would benefit from the formation of a CU. Clearly, whenever one of the developing countries has a population greater than the other, they always prefer a CU. A Southern country prefers a free trade area when its population is small relative to the other Southern country. This can be seen in figure 3.8; the points above the green plane are points where the Southern country has a smaller population relative to the other trade agreement member.

In figure 3.9, we show the tariff rates in an FTA. We see that along the diagonal where $\lambda_1 = \lambda_2$, the tariffs would be the same. Additionally, we see that when the population is greater in either developing country, i.e., to the right or left of the

diagonal. The tariff is higher in a country with a larger population. This further confirms proposition 2.

Tariffs rates in a free trade agreement set-up

Figure 3.9: Tariffs in an FTA

Figure 3.10 shows the welfare of the developing countries when they sign an FTA. Notice how in figure 3.10, along the diagonal line where $\lambda_1 = \lambda_2$, the welfare is the same. Two things stand out in this figure; the first is how when the population in the South rises, the welfare in either country decreases. Moreover, we see the larger country in an FTA has lower welfare than the smaller country.

Lastly, in figure 3.11, we show the percentage change in welfare in the Northern country as a result of the Southern countries going from a CU to an FTA. The axes in figure 3.11 represent the population in each of the Southern countries labeled λ_1 and λ_2 and the percentage change in consumption. All of the points on the plane lie above the zero planes; therefore, regardless of the population in any of the three countries, the welfare is always higher in an FTA set up for the Northern country.

Welfare in Southern countries under an FTA

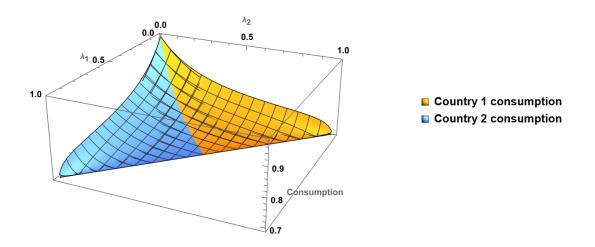


Figure 3.10: Welfare for each Southern Country in an FTA setup

Percentage change in welfare going from CU to FTA

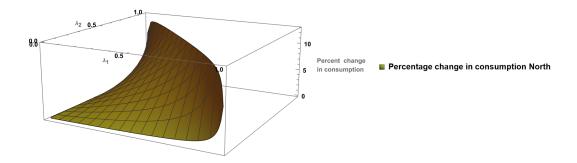


Figure 3.11: Percentage Change in Northern Welfare Going from CU to FTA

CHAPTER 4

EMBARGOES AND DEVELOPING COUNTRIES

4.1 Introduction

A complete prohibition of trade, an embargo, is a rarely used policy tool for extreme cases. Embargoes can potentially cause severe harm to both their targets and their senders. The recent Russian invasion of Ukraine has led the United States and the European Union to impose sanctions on Russia. The western world aims to inflict an economic cost on Russia in hopes of them ending their conflict with Ukraine. Given recent events, an interesting policy question is whether a developing nation should consider joining the western world in imposing sanctions on Russia. While political economy¹ motivations may be involved in joining such a coalition, we are interested in a welfare analysis of such a policy decision.

Policymakers in developing countries have long been concerned about the Prebisch-Singer hypothesis. This hypothesis states that as the world economy grows, demand increases more for manufactured goods produced in developed countries than commodities produced in developing countries; therefore, the terms of trade deteriorate for the developing countries (Prebisch, 1962; Singer, 1950). Restricting trade from a wealthier country allows a developing country to expand domestic production into manufactured goods with higher income elasticities than commodities. Thus, we ask if a developing country could improve its production and wages enough to offset the associated higher prices related to limiting trade. Consequently, it is unclear if a developing nation has enough of an incentive to restrict trade with Russia.

¹For a survey on the political economy of economic sanctions see Ka empfer and Lowenberg (2007).

We use the Ricardian model with nonhomothetic preferences outlined in Matsuyama (2000) to study this question. This framework allows us to model countries with different income levels and, therefore, different consumption patterns in a simple tractable way. Moreover, given the nature of the research question, we need a model that is easily extendable to at least three countries, which this framework is.

We outline the model. It has three countries, distinguished by their labor productivity differences. Productivity differences give rise to differences in per capita income. There is a continuum of indivisible goods, and consumers have non-homothetic preferences, consuming at most one unit of each good subject to their budget constraints. Goods are ordered in descending order of the marginal utility of money so that lower-indexed goods are consumed in all countries while higher-indexed goods are consumed only by the countries rich enough to afford them. Motivated by the recent experience, we consider the case in which a rich country imposes an embargo on a middle-income country, e.g., our recent example of Russia. We further assume that the rich government is sanctioning the middleincome country by either placing a ban on exports, imports, or both. Given the actions of the rich country, we focus on two different settings, one where the lowincome country is trading freely with the middle-income country and the other when the low-income country joins the rich country by also restricting trade with the middle-income country. We study the impact of a low-income county joining a high-income country in an embargo of goods from a middle-income country on welfare in all three countries.

We state our main results; our numerical analysis finds a prohibition of trade is usually deemed welfare diminishing. While we find that prohibiting trade can be welfare deteriorating, there is a case where a low-income country sanctions a middle-income country by not exporting to them, which, surprisingly, improves the welfare in the middle-income country while reducing the welfare in the low-income country, we call this the "embargo paradox". For this to hold, given the assumptions we made in the model, the population in the low-income country would have to be a small percentage of the world's population.

For each of the scenarios analyzed, we found that the lower-income country was always better off trading freely with the middle-income country. However, if the low-income country wanted to impose a trade sanction against the middle-income country, it would do the most harm to the middle-income country and the minimum damage to itself if the population in its country was a large percentage of the world population and the population in the middle-income country is a small percentage of the world's population. The high-income country is always better off due to the lowincome country setting a sanction against the middle-income country. Restrictions placed by the low-income country on the middle-income have the effect of raising wages in the high-income country, causing consumption in the high-income country to rise.

Our paper is organized as follows. In section 2, we review the relevant literature. Section 3 describes the model based on Matsuyama (2000) in detail for different policy scenarios. We focus on the case where the high-income country sets a complete embargo on imports and exports to and from the middle-income country; then, the low-income country must decide between joining the embargo or trading freely with the middle-income country. Section 4 assumes specific unit labor requirements to present the numerical results and graphs. Section 5 is analogous to section 3; it describes the model where the high-income country does not import from the middleincome country; then, the low-income country must decide between ban exports to the middle-income country or continuing to trade freely with them. In section 6, we provide the numerical results from the previous section. Section 7 concludes. In the appendix, we analyze another policy scenario where the high-income country sets an export embargo on the middle-income country. The low-income developing country either places an import embargo on the middle-income country or trades freely with them.

4.2 The review of literature

The motivation for the current paper is a result of the recent conflict between Russia and Ukraine. However, this is not the first time these two countries have had a dispute that led to economic sanctions. In 2014 Russian and Ukraine had a similar conflict which prompted the European Union countries, the United States, and Japan to impose sanctions on Russia. Crozet and Hinz (2020) quantify the losses resulting from these sanctions on both Russia and the western countries imposing the sanctions by conducting a general equilibrium counterfactual analysis. The losses were estimated to be 7.4% of Russian exports and 0.3% of the total exports of western sanctioning countries. While this study does provide a counterfactual analysis of the effect of sanctions on both the target economy and sending economy, it does not provide insight into the potential policy decisions of developing nations.

Studies involving full trade embargoes between countries with large economies are difficult to find in the literature. Because complete embargoes are uncommon, and if it does happen, it is often difficult to find data on such an event. The most mentioned study on embargoes is by Irwin (2005). He uses a general equilibrium analysis to study the effect of the embargo that the United States placed on Britain and France, which lasted between late 1807 and early 1809. The United States Congress imposed an embargo on exports, which led to a significant decrease in imports, leaving the United States in autarky; Irwin (2005) finds that the embargo led to an estimated loss of about 5 percent of GNP^2 in the United States. The literature mostly looks at how successful a sanction was in imposing some economic cost or its ability to coerce some outcome. Both Crozet and Hinz (2020) and Irwin (2005) look at the effect of such sanctions from the perspective of the country imposing the sanction.

While Irwin (2005) provides a rare empirical analysis of a change from free trade to an autarky equilibrium, Bernhofen and Brown (2005) provide another study analyzing welfare gains from moving from autarky to a free trade equilibrium. They examine the impact of Japan opening up its economy after more than 200 years of enforcing an isolation policy; essentially, the country was in autarky for more than 200 years. Their results are surprisingly similar to that of Irwin (2005) in that there was an estimated 8 to 9 percent increase in Japan's GDP as a result of opening up their economy.

Given that data on prices when an economy is in a state of autarky are difficult to find in practice, the methods used to calculate gains from trade in Bernhofen and Brown (2005) and Irwin (2005) become difficult. However, Arkolakis, Costinot, and Rodríguez-Clare (2012) derive a simple formula to study the gains from trade, which simply relies on the trade elasticity and share of expenditure on domestic goods. They show that a change in income is simply

$$\hat{W} = \hat{\lambda}^{\frac{1}{\varepsilon}}$$

where \hat{W} denotes the change in income, $\hat{\lambda}$ denotes the share of expenditure spent on domestic goods, and ε denotes the trade elasticity. The full benefits from trade, \hat{W} ,

 $^{^{2}}$ This was done to hurt both the British and French who were at war with each other and interfering with American vessels, see Irwin (2005) and Frankel (1982).

could be calculated by calculating $\hat{\lambda}$ and estimating ε from an observed equilibrium to autarky³. They show that this simple equation holds for a large class of models, which they refer to as "quantitative trade models". Given the simple equation above, a restriction on trade should lead to welfare deterioration in a given country. However, Stibora and de Vaal (2015) present a model that accounts for differences in income and nonhomothetic preferences; they provide an example where only having an estimate of the trade elasticity and share of expenditure on domestic goods is not enough to calculate welfare from preferential trade agreements.

The model that we use to analyze the effects of an embargo in this paper is the model presented by Matsuyama (2000). Other models extend the Ricardian model with a continuum of goods in Dornbusch, Fischer, and Samuelson (1977) to study issues of North-South Trade, e.g., Flam and Helpman (1987) and Stokey (1991). Both Flam and Helpman (1987) and have many similar features to Matsuyama (2000); the former two papers are more focused on how product quality affects trade patterns and matters of intraindustry trade. Moreover, Matsuyama (2000) can be tractably extended to include three countries which is necessary for our analysis.

Key papers that have analyzed trade policy under nonhomothetic preferences using the Matsuyama (2000) model are Stibora and de Vaal (2007, 2015). While they focus on the effects of trade liberalization when accounting for the distribution of income within a country and the effects of preferential trade agreements on welfare. However, their analysis in both papers does not focus on the possible welfare effects of developing countries joining a richer nation in sanctioning goods from another country.

³Because $\lambda = 1$ in autarky, a change in income takes the even simpler form of $\hat{W} = 1 - \lambda^{-\frac{1}{\varepsilon}}$

While the effect of sanctions has been studied extensively, a welfare analysis of whether developing countries should join a coalition of countries imposing harsh economic sanctions is absent from the literature. Our paper aims to contribute to this line of research.

4.3 General setup

We consider a three-country world, each with a different level of income; as far as their trade patterns, we consider two different scenarios. In the first scenario, we assume the high-income country, labeled H, to have embargoed all goods to and from the middle-income country, labeled M. In this scenario, the low-income country, labeled L, trades freely with the middle-income country. In the second scenario, both countries H and L embargo goods to and from country M. As a result, country M is in autarky. Lastly, we compare the welfare of each country in the first scenario with the second scenario.

Before we describe each scenario in detail, we discuss the general setup of the model that applies in both scenarios listed above. Three countries can produce a continuum of goods z, where $z \in [0, \infty)$, using distinct technologies. Households are homogenous, and each household supplies one unit of labor; each country has N_i households. Industries are competitive, and labor is the only factor of production. Labor is freely mobile across industries but immobile across countries. Let $a_i(z)$ denote the unit labor requirement (labor-output ratio) for good z's production in country i = H, M, L.

Assumption 1: For all $z \in [0, \infty)$,

- (a) $a_H(z) \leq a_M(z) \leq a_L(z)$ with the equalities holding possibly at z = 0 only.
- (b) $a_i(z)$ is continuously differentiable and monotone-increasing.

Assumption 1 says that it takes less labor to produce goods in country H than in country M or L. Moreover, it requires less labor to produce in country M than in country L. Thus, country H has an absolute advantage in the production of all goods over both M and L, while country M has an absolute advantage in the production of all goods over country L. Lastly, as we move along the continuum and the amount of labor necessary to produce good z is non-decreasing.

Assumption 2. The labor-requirement ratio functions $A(z) \equiv \frac{a_M(z)}{a_H(z)}$, $B(z) \equiv \frac{a_L(z)}{a_M(z)}$, and $C(z) \equiv \frac{a_L(z)}{a_H(z)}$ are (twice) continuously differentiable and monotone-increasing in $z \in (0, \infty)$.

Similar to Dornbusch et al. (1977), the labor-requirement ratio functions being monotone increasing implies that country H has a comparative advantage in the production of higher indexed goods over country M. Similarly, country M has a comparative advantage in the production of higher indexed goods over country L. Therefore, along the continuum, country H has a comparative advantage in a higher spectrum of goods, country L in the lowest spectrum, and country M in the middle spectrum of goods. Free entry fixes the unit production cost of good z in country iat $w_i a_i(z)$, where w_i is the wage in country i, i.e., the cost of making a good equals the amount of labor necessary to make it multiplied by the wage.

Populations in each country are homogeneous, and consumers supply one unit of labor supply inelastically, so the wage w_i is their income. Consumers have identical preferences and consume goods in discrete quantities. Each consumer gets zero utility from not consuming a good and are satiated after consuming one unit of a good, deriving utility u(z) > 0. Consumers continue to consume goods along the continuum until they exhaust their income. More formally, households seek to maximize

$$V = \int_0^\infty u\left(z\right) x(z) dz$$

where u(z) is the utility weight for good z while x(z) is an indicator function equal to 1 when good z is consumed and 0 otherwise. Consumers in country *i* are subject to the budget constraint

$$\int_0^\infty p(z) x(z) dz \le w_i,$$

where p(z) denotes the price of good z.

To model some goods having priority over others, we make the following assumption Assumption 3. The ratios $u(z)/a_i(z)$ are strictly decreasing.

Perfect competition in production and free entry ensures the price of a good equals the cost of production $w_i a_i(z)$. Furthermore, the marginal utility of money for country L when it is engaging in free trade with the two other countries is

$$u(z)/p(z) = u(z)/\min\{w_H a_H(z), w_M a_M(z), w_L a_L(z)\}.$$

Thus, given assumption 3, consumers will consume lower-indexed goods first before consuming higher-indexed goods. Moreover, given assumption 2 and its implications on comparative advantage, consumers in country L, when it is engaging in free trade with the two other countries, consume domestically produced lower-indexed goods first. They then import goods from country M before importing higher-indexed goods from country H. For this reason, goods are consumed in an order according to the comparative advantage given by assumption 2.

We denote the last good that is consumed in country i as c_i this good has important welfare implications. Because consumers consume only one unit of each good along the spectrum of goods and because preferences are identical, there is a one-to-one correspondence between a consumer's utility and the number of goods they consume, c_i . Therefore, c_i is a measure of welfare for consumers in country *i*. Essentially, consumer welfare increases as the consumption bundle's diversity expands. Throughout the paper, we look at how c_i changes as a result of a policy change.

Now that we have given a general setup of the model, we describe two trade policy scenarios in the following two sections. First, country H prohibits all imports and exports from country M while country L trades freely with country M. Second, countries H and L embargo all imports and exports from country M.

4.4 Model of a full embargo

4.4.1 Developing country practicing free trade

We start with a country H embargoing exports and imports from country Mwhile country L trades freely with country M. Given the assumption that the labor-requirement ratio functions $A(z) \equiv a_M(z)/a_H(z)$, $B(z) \equiv a_L(z)/a_M(z)$, and $C(z) \equiv a_L(z)/a_H(z)$ are continuously differentiable, and monotone-increasing in $z \in (0, \infty)$, assumption 2, country L has the comparative advantage in producing lower-indexed goods, country H has the comparative advantage in producing higherindexed goods, and country M has a comparative advantage in producing goods in the middle. There exist a marginal good where the comparative advantage changes from country L to country M. We denote this marginal good z_L . The price of this marginal good must be the same in country L and M, taking the wage in the low-income country as the numeraire, $w_L = 1$,

$$a_L(z_L) = w_M a_M(z_L). \tag{4.1}$$

Therefore, the wage in country M is greater than in country L, $w_M > w_L = 1$, because country M has an absolute advantage in producing all goods over country L, assumption 1. Similarly, there exists a marginal good where the comparative advantage changes from country M to country H. We denote this marginal good z_M . The price of this marginal good must be the same in country M and H

$$w_M a_M(z_M) = w_H a_H(z_M). \tag{4.2}$$

To repeat, by assumption 1, country H has an absolute advantage in the production of all goods; therefore, the wage in country H must be greater than in country M, $w_H > w_M > w_L = 1$. Lastly, because country H sets an embargo against exports and imports from country M, country H first imports goods from country L before consuming the goods produced domestically. Thus, there exists a marginal good where the comparative advantage changes from country L to country H. We denote that marginal good z_H ; the price of z_H must be equal in both countries L and H,

$$a_L(z_H) = w_H a_H(z_H). \tag{4.3}$$

Consumers in country *i* satisfy their demands in the range $[0, c_i]$ by buying from the cheapest supplier except when it has barred trade via an embargo, e.g., it may be more affordable for *M* to import higher-indexed goods from country *H*, but the latter country has ban trade with the former country. Therefore, this forces country *M* to consume higher-indexed goods produced domestically, which are more expensive.

Country L trades freely with both higher-income countries. They produce and consume goods along the range $[0, z_L]$ at price $w_L a_L(z)$, import goods from the range $[z_L, z_M]$ at price $w_M a_M(z)$ from country M, and import goods in the range $[z_M, c_L]$ from country H.

With an income w_L , a typical consumer in country L faces the budget constraint

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{z_{M}} a_{M}(z) dz + w_{H} \int_{z_{M}}^{c_{L}} a_{H}(z) dz = 1.$$
(4.4)

Country M is barred from trading with country H but trades freely with country L. Country M imports goods in the range $[0, z_L]$ from country L and consumes domestically produced goods in the range $[z_L, c_M]$. With an income w_M , a typical consumer in country M faces the budget constraint

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{c_{M}} a_{M}(z) dz = w_{M}.$$
(4.5)

Finally, country H imports goods in the range $[0, z_H]$ from country L and consumes domestically produced goods in the range $[z_H, c_H]$. With an income w_H , a typical consumer in country H faces the budget constraint

$$\int_{0}^{z_{H}} a_{L}(z) dz + w_{H} \int_{z_{H}}^{c_{H}} a_{H}(z) dz = w_{H}.$$
(4.6)

To close the model, we impose resource constraints for each country. Country L produces goods in the range $[0, z_L]$ for consumers in countries L and M. They also produce the range of goods between $[0, z_H]$ strictly for consumers in country H. The resource constraint for country L is

$$(N_L + N_M) \int_0^{z_L} a_L(z) \, dz + N_H \int_0^{z_H} a_L(z) \, dz = N_L. \tag{4.7}$$

Country M produces domestic goods in the range $[z_L, c_M]$ and exports goods in $[z_L, z_M]$ to country L. Thus, it faces the resource constraint

$$N_M \int_{z_L}^{c_M} a_M(z) \, dz + N_L \int_{z_L}^{z_M} a_M(z) \, dz = N_M.$$
(4.8)

There are eight unknowns: $w_H, w_M, z_L, z_H, z_M, c_L, c_M$ and c_H and eight equations (4.1) - (4.7). Country *H*'s resource constraint holds in equilibrium by Walras' Law. The model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation $(4.1), z_M$ in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.2). Then equations (4.5), (4.5), and (4.7) create a system of three questions and three unknowns. As a result, we can solve for w_H, w_M, c_M . We substitute w_M and w_H into equations (4.1) and (4.3) to find z_L and z_H , respectively. Next, we plug in w_M and w_H into (4.2) to find z_M . Lastly, we plug all the known variables into (4.4) and (4.5) to find c_L and c_H , respectively.

Figure 4.1 provides a graphical illustration of the described trade and consumption patterns. The figure is drawn such that the consumption in country L, c_L , is greater than the marginal good z_M . We define this as symmetric trade, i.e., country L is rich enough to import goods from the high-income country. It could be the case that country L is so poor that c_L is less than the marginal good z_M . We define this case as asymmetric trade. In the asymmetric case, the budget constraint for country L becomes $\int_0^{z_L} a_L(z) dz + w_M \int_{z_L}^{c_L} a_M(z) dz = 1$, and the resource constraint for country M becomes $N_M \int_{z_L}^{c_M} a_M(z) dz + N_L \int_{z_L}^{c_L} a_M(z) dz = N_M$. The model is solved in the same way as described above except by substituting the two previous equations for equations (4.4) and (4.7), respectively.

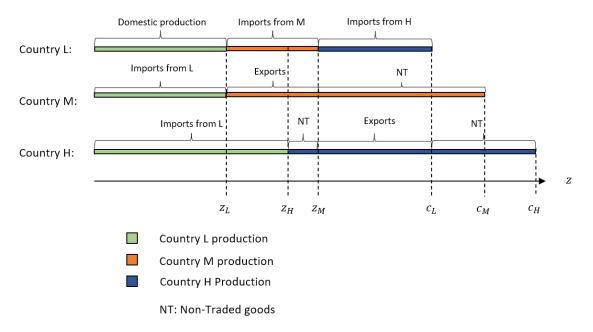


Figure 4.1: Trade and Consumption Patterns, H Setting Full Embargo

4.4.2 Developing country imposing a full embargo

We describe the model when country L joins country H in embargoing exports and imports from country M. This means that country M is isolated from the rest of the world and is in autarky. The rest of the world, countries L and H, trade freely. Because country M is in autarky, they produce and consume all goods domestically, i.e., they produce and consume goods in the range $[0, c_M]$. As previously noted, Country H has a comparative advantage in higher-indexed goods while country L has a comparative advantage in lower-indexed goods; therefore, there exists a marginal good, denoted z_H , where the comparative advantage switches from country L to country M. The price of this marginal good must be the same in countries Land H,

$$a_L(z_H) = w_H a_H(z_H). \tag{4.9}$$

Again because of assumption 1, $w_H > w_L = 1$. Given that country L and country H face the same prices for the goods they consume along the continuum and because

 $w_H > w_L$, this implies that country H consumes all of the goods that country L consumes plus some additional goods which country L cannot afford, i.e., $[0, c_L] \subset [0, c_H]$.

Country L consumes domestically produced goods in the range $[0, z_H]$ and imports goods in the range $[z_H, c_L]$ from country H. With an income $w_L = 1$, a typical consumer in country L faces the budget constraint

$$\int_{0}^{z_{H}} a_{L}(z) dz + w_{H} \int_{z_{H}}^{c_{L}} a_{H}(z) dz = 1.$$
(4.10)

Similarly, country H imports goods in the range $[0, z_L]$ from country L and consumes domestically produced goods in the range $[z_H, c_H]$. With an income w_H , a typical consumer in country H faces the budget constraint

$$\int_{0}^{z_{H}} a_{L}(z) dz + w_{H} \int_{z_{H}}^{c_{H}} a_{H}(z) dz = w_{H}.$$
(4.11)

Lastly, country M consumes domestically produced goods in the range $[0, c_M]$. With an income w_M , a typical consumer in country M faces the budget constraint

$$w_M \int_0^{c_M} a_M(z) \, dz = w_M. \tag{4.12}$$

To close the model, we impose resource constraints for country L

$$(N_L + N_H) \int_0^{z_H} a_L(z) \, dz = N_L. \tag{4.13}$$

There are five unknowns: w_H , z_H , c_L , c_M , and c_H , and five equations (4.9) – (4.13). The model can be solved semi-recursively. First, from equation (4.13), we can solve for z_H , then plugging z_H into equation (4.9), we solve w_H . Subsequently, we can plug in z_H and w_H to solve for c_L and c_H in equations (4.10) and (4.11), respectively. Lastly, we can solve for c_M from country M's budget constraint, equation (4.12). In this scenario, because country M is in autarky, w_M can take any value, which determines the level of consumption, c_M , in country M is the unit labor requirement $a_M(z)$.

In this case, if we have asymmetric trade, i.e., $c_L < z_H$, then country L is in autarky. Because country L will only consume the goods it produces, it will have no incentive to export to country H because it will not get any imports in return from country H. In this case, the entire world is in autarky, and each country's welfare can be determined by its resource constraint.

Transfer payment

Because populations in each country are homogeneous, the analysis of a transfer payment is straightforward. We will focus on the case where country H makes transfer payments to country L to compensate it for joining the complete embargo against country M. First, if there is asymmetric trade, then country L is in autarky, and their production is limited by labor, so an income transfer would not affect their welfare and would only cause welfare to fall in country H. If there is symmetric trade, we are essentially in the two-country case presented in Matsuyama (2000). To be specific, transfer payments between country H to country L do not affect the marginal good z_H or the wage w_H . Formally we show the effect of a transfer on welfare; first, we rewrite the budget constraint in country L and H with a lump-sum tax in the country H and a lump-sum payment in country L

$$\int_{0}^{z_{H}} a_{L}(z) dz + w_{H} \int_{z_{H}}^{c_{H}} a_{H}(z) dz = w_{H} - T$$

$$\int_{0}^{z_{H}} a_{L}(z) dz + w_{H} \int_{z_{H}}^{c_{L}} a_{H}(z) dz = 1 + \frac{N_{H}}{N_{L}}T$$

where T is the transfer payment. Differentiating the above equations with respect to the T, we find that the change in consumption is positive in the low-income and negative in the high-income country

$$\frac{dc_H}{dT} = \frac{-1}{w_H a_H \left(c_H\right)} < 0$$

$$\frac{dc_L}{dT} = \frac{N_H}{N_L} \frac{1}{w_H a_H \left(c_H\right)} > 0$$

4.4.3 Numerical results and graphs for a full embargo

To illustrate and simplify the comparison of welfare between the different policy scenarios, we define functional forms for the unit labor requirement in each country. Assumption 4 $a_H(z) = 1$; $a_M(z) = e^z$; $a_L(z) = e^{2z}$.

Given the above functional forms, we can easily solve for all of the unknowns in the model given some exogenous population levels in each country.

We compare the welfare of each country in the scenario where only the highincome country embargoes goods from the middle-income country to the scenario where both the high-income and low-income country place an embargo on the middle-income country. Given the functional forms, we can derive the marginal good form equation (4.1)-(4.3) as

$$\mathbf{z}_L = \ln\left(\frac{\mathbf{w}_M}{\mathbf{w}_L}\right)$$

$$\mathbf{z}_M = \ln\left(\frac{\mathbf{w}_H}{\mathbf{w}_M}\right)$$

$$\mathbf{z}_H = \frac{1}{2} \ln \left(\frac{\mathbf{w}_H}{\mathbf{w}_L} \right)$$

Moreover, we can derive equations for the different levels of consumption in country i by solving for c_i in the corresponding budget constraint and plugging in the corresponding marginal good. For example, in-country L, we solve for the consumption when they are setting an embargo,

$$\mathbf{c}_{L}' = \frac{\frac{1}{2}(3 - \mathbf{w}_{H}' + \mathbf{w}_{H}' \ln(\mathbf{w}_{H}'))}{\mathbf{w}_{H}'},$$

we denote variables with a prime symbol to denote the variable's value when the country L is setting an embargo. When country L is trading freely with country M, then their welfare can be derived as

$$\mathbf{c}_{L} = \frac{\frac{1}{2} \left(3 - \mathbf{w}_{M}^{2} + 2\mathbf{w}_{M}^{2} - 2\mathbf{w}_{H} + 2\mathbf{w}_{H} ln\left(\frac{\mathbf{w}_{H}}{\mathbf{w}_{M}}\right) \right)}{\mathbf{w}_{H}}.$$

The wages w_M and w_H are solved in the same manner they are solved for in the previous section. Lastly, because we are interested in whether the change in welfare, again, the value of c_L' in comparison to c_L , we calculate the percentage change from the setup where country L is trading freely with country M to the setup where country L is trading freely. We calculate the percentage as such

$$\frac{\mathbf{c}_L' - \mathbf{c}_L}{\mathbf{c}_L} x 100.$$

We repeat the same exercise for consumption levels in each setup for both country M and country H. Our numerical analysis entails solving the above equation for all combinations of the population parameters in each country⁴.

We briefly describe the results before showing them graphically. First, the highincome country is always better off due to the low-income country setting an embargo on the middle-income country, leaving it in autarky. The rationale for this is that when the low-income country embargoes goods from the middle-income country, the low-income country must replace some of the imports it receives from the middleincome country with imports from the high-income country. Accordingly, wages rise in the high-income country, which in turn leads to an increase in consumption c_{H} . Second, the middle-income must be worse off due to the low-income country embargoing goods from them. The intuition is straightforward because the middleincome country no longer exports or imports goods from the low-income country; it must domestically produce the lower-indexed goods it previously imported from the low-income country at a lower price. Purchasing these goods at higher prices than it could have otherwise imported them leads to consumption and, therefore, welfare falling in the middle-income country. Lastly, we find that welfare in the low-income country falls. When the low-income country embargoes goods from the middle-income country, it must replace those imports with more expensive domestic production and imports from the high-income country. Moreover, as wages rise in the high-income, the low-income country will import at higher prices than when trading freely. These two influences force consumption to fall in the lower-income country.

⁴We use the Mathematica technical computing software to calculate all of the variables of the model for different population levels.

We compare the welfare of each country in a setting where only country His setting an embargo on country M to a setting where both the country H and country L imposed an embargo on country M. Figures 4.2, 4.3, and 4.4 below show the percentage change in welfare of each country when country L joins country Hin setting a complete embargo on country M, leaving it in autarky. Specifically, in figure 4.2, we show that the welfare of country L is always lower when they go along with country H in setting an embargo on the middle-income country. Moreover, the larger the population in country M is, the more the welfare falls for the lowincome country. Similarly, in figure 4.3, we show that the welfare of country Mis always lower when country L goes along with country H in setting an embargo on country M. Moreover, the larger the population in the country L is, the lower the welfare will be for country M. Finally, in figure 4.4, we show that the welfare of country H is always higher when both country H and country L are setting an embargo on country M. Moreover, the larger the population in country L are setting an embargo on country M. Moreover, the larger the population in country L are setting an embargo on country M. Moreover, the larger the population in country L are setting an embargo on country M. Moreover, the larger the population in country L are setting an embargo on country M. Moreover, the larger the population in country M is, the more significant the change in welfare is in country H.

Because country L is always worse off due to entering into an embargo and country H always benefits, country H could help mitigate the welfare loss in country L by offering a transfer payment. As we showed in section 4.4.2, a transfer payment would improve welfare in the lower-income country. To get a clearer view of how successful country H could be at boosting the welfare in country L so they are not as worse off, we calculate and graph the equation

$$|w_{H}(c_{H}'-c_{H})| - |w_{H}(c_{L}'-c_{L})|.$$

The value of the absolute difference between the value of consumption gained in country H and the value of consumption lost in country L. If this number is positive,

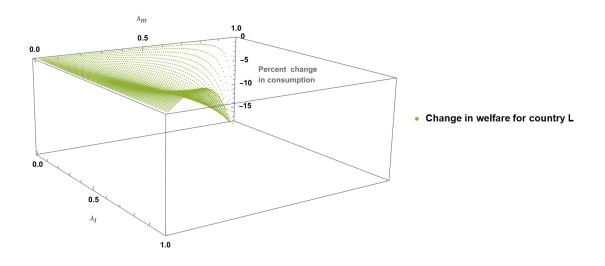


Figure 4.2: Country L Change in Welfare from Placing a Full Embargo on the Country ${\cal M}$

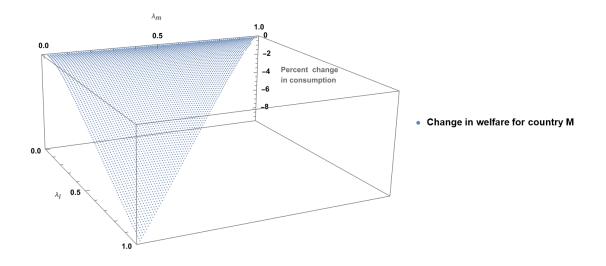


Figure 4.3: Country M Change in Welfare from Country L Placing a Full Embargo on Country M

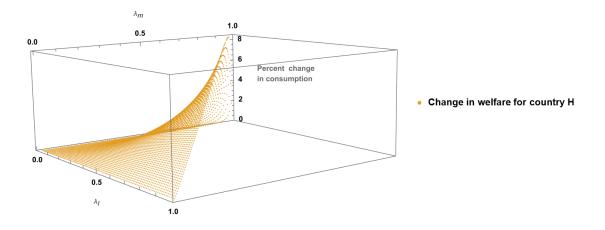


Figure 4.4: Country ${\cal H}$ Change in Welfare from Country L Placing a Full Embargo on Country ${\cal M}$

then there exists a transfer payment such that country L could be just as well off had they practiced free trade with country M.

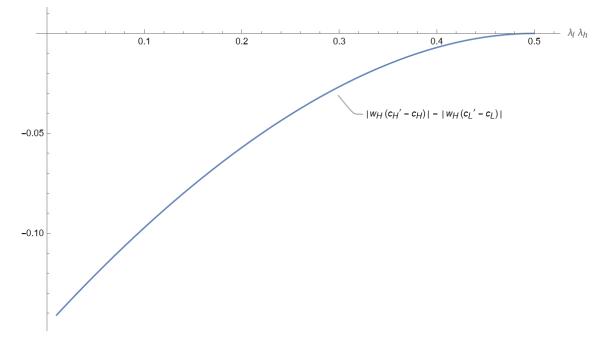


Figure 4.5: Measure of Difference Between the Value of the Consumption Gained by Country ${\cal H}$ and Lost by Country L

Figure 4.5 shows that the value of consumption lost by country L from joining the embargo is larger than the value of consumption gained by country H, when both countries have the same size populations. Mainly, country H could offset some of the welfare losses for country L, but they cannot provide a transfer large enough to completely replace the value of all the consumption lost by country L.

4.5 Model of a partial embargo

We describe a scenario where the high-income country imposes a partial embargo on the middle-income country. The partial embargo means that it either stops exporting or importing from the middle-income country. We assume that the lowincome country trades freely with the middle-income country. We analyze how welfare in each country changes as a result of the low-income country restricting trade to or from the middle-income country. For trade balance not to be violated, the low-income country could only take the opposite position that the high-income country is taking against the middle-income country. For example, if the highincome country completely restricts exports to the middle-income country, further sanctioning the middle-income country by placing some kind of embargo would require the low-income country to restrict imports from the middle-income country. In contrast, if the high-income country embargoed all imports from the middleincome country, the low-income country could restrict all exports to the middleincome country. See figure 4.6 for a graphical depiction.

In the following subsection, we assume that country H does not import from country M and study how welfare changes in all three countries when country L decides to impose an export embargo on country M. In the appendix, we present the opposite case where country H does not export to country M.

4.5.1 Developing country practicing free trade

In this setting, country H does not import from country M while country L trades freely with country M. Again, because country L has a comparative advantage in producing lower index goods, it specializes in making them. They consume these goods domestically and export them to both country H and country M. In this setting, the prices of the boundary goods are the same as in equations (4.1)-(4.3), and the budget constraints for countries H and L are the same as in equations (4.6) and (4.4), respectively. However, because the only sanction against country M is that country H is not importing from them, country M can buy goods

from the country producing at the lowest price. Therefore, the budget constraint in the country M is

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{z_{M}} a_{M}(z) dz + w_{H} \int_{z_{M}}^{c_{M}} a_{H}(z) dz = w_{M}.$$
(4.14)

Because country L produces the same range of goods as in section 4.4.1, the resource constraint in country L is the same as equation (4.7). However, the resource constraint in country M is different because it imports higher-indexed goods from country H. Thus, country M produces goods in the range $[z_L, z_M]$ for domestic consumption and export to country L. Country M faces the resource constraint

$$(N_L + N_M) \int_{z_L}^{z_M} a_M(z) \, dz = N_M. \tag{4.15}$$

There are eight unknowns: $w_H, w_M, z_L, z_H, z_M, c_L, c_M$, and c_H , and eight equations (4.1) - (4.3), (4.4), (4.6), (4.7), (4.14), and (4.15). Country *H*'s resource constraint holds in equilibrium by Walras' Law. The model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation (4.1), z_M in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.3). Then, we solve for w_M and w_H using a system of equations (4.5) and (4.15). Plugging in w_M and w_H into (4.1)-(4.3) to find z_L, z_M , and z_H , respectively. Lastly, we plug in the know variables into w_H, w_M, z_L, z_H , and z_M into equations (4.4), (4.14), and (4.6) to find c_L, c_M , and c_H , respectively. Figure 4.6 illustrates this setup.

In the asymmetric case, the budget constraint for country L becomes

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{c_{L}} a_{M}(z) dz = 1$$
(4.16)

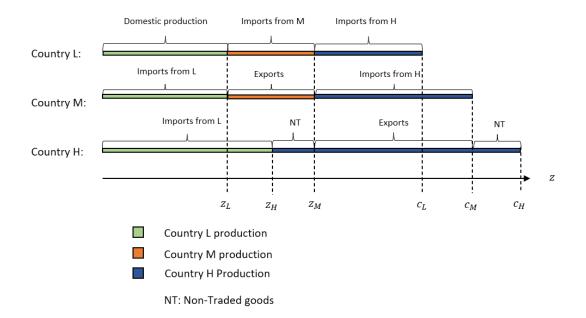


Figure 4.6: Trade and Consumption Patterns, H Setting Import Embargo

and the resource constraint for country M becomes

$$N_{M} \int_{z_{L}}^{z_{M}} a_{M}(z) dz + N_{L} \int_{z_{L}}^{c_{L}} a_{M}(z) dz = N_{M}.$$
(4.17)

Similar to the symmetric case, the model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation (4.1), z_M in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.3). Then, we solve for w_M , w_H , and c_L using a system of equations (4.7), (4.16), and (4.17). Plugging in w_M and w_H into (4.1)-(4.3) to find z_L , z_M , and z_H , respectively. Lastly, we plug in the know variables into w_H, w_M , z_L , z_H , and z_M into equations (4.14) and (4.6) to find c_L and c_H , respectively.

4.5.2 Developing country imposing an export embargo

We describe a setting where country H is not importing from country M and country L is deciding whether to join country H in restricting trade with country M. When considering trade policies such as embargoes, given that country H is not importing goods from the country M, the only embargo policy that country L can adopt while not causing country M to go into autarky is to restrict exports to country M. For example, if country L sets an import embargo or an embargo on imports and exports to and from country M, then for trade balance to be maintained, country M will be in autarky. Country M will not be able to export to either country, so its imports would also have to be zero for trade to be balanced. This would be identical to the setting described in section 4.4.2. In this setting, the prices of the boundary goods are the same as equations (4.1)-(4.3). Country L still buys goods from the country that produces at the lowest cost; therefore, their budget constraint is identical to equation (4.4). Additionally, country H does not import from country M, so its budget constraint is the same as equation (4.6) when country H sets a full embargo on country M. Because country M does not import goods from country L, they consume domestically produced goods in the range $[0, z_M]$ and import goods in the range $[z_M, c_M]$ from country H, the budget constraint for a typical consumer in country M is

$$\int_{0}^{z_{M}} a_{M}(z) dz + w_{H} \int_{z_{M}}^{c_{M}} a_{H}(z) dz = w_{M}.$$
(4.18)

Country L produces goods in the range $[0, z_L]$ for domestic consumption and exports the goods in the range $[0, z_H]$ to country H. The resource constraint in country L is

$$N_L \int_0^{z_L} a_L(z) \, dz + N_H \int_0^{z_H} a_L(z) \, dz = N_L. \tag{4.19}$$

Lastly, country M produces the goods in the range $[0, z_M]$ for itself while exporting the goods in the range $[z_L, z_M]$ to country L. The resource constraint in country Mis

$$N_M \int_0^{z_M} a_M(z) \, dz + N_L \int_{z_L}^{z_M} a_M(z) \, dz = N_M \tag{4.20}$$

There are eight unknowns: $w_H, w_M, z_L, z_H, z_M, c_L, c_M$, and c_H , and eight equations (4.1) - (4.3), (4.4), (4.6), (4.18), (4.19), and (4.20). Country *H*'s resource constraint holds in equilibrium by Walras' Law.

The model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation (4.1), z_M in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.3). Then, we solve for w_M and w_H using a system of equations (4.19) and (4.20). Plugging in w_M and w_H into (4.1)-(4.3) to find z_L , z_M , and z_H , respectively. Lastly, we plug in the know variables into w_H, w_M, z_L, z_H , and z_M into equations (4.4), (4.18), and (4.6) to find c_L , c_M , and c_H , respectively. In the asymmetric case, the budget constraint in country L is

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{c_{L}} a_{M}(z) dz = 1$$
(4.21)

and the resource constraint in country M is

$$N_M \int_0^{z_M} a_M(z) \, dz + N_L \int_{z_L}^{c_L} a_M(z) \, dz = N_M. \tag{4.22}$$

We can solve for z_L in terms of w_M in equation (4.1), z_M in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.3). Then, we solve for w_M , w_H , and c_L using a system of equations (4.19), (4.21), and (4.22). Plugging in w_M and w_H into (4.1)-(4.3) to find z_L , z_M , and z_H , respectively. Lastly, we plug in the know variables into w_H, w_M, z_L, z_H , and z_M into equations (4.18) and (4.6) to find c_M and c_H , respectively.

4.5.3 Numerical results and graph for a partial embargo

We follow the same procedure as in section 4.4.3. We assume the unit labor requirements have the same functional forms as those given by assumption 4. We numerically solve for the consumption in each country and compare how welfare changes as the country L bans exports to country M.

The figures below show the percentage change in welfare going from the scenario where only country H imposes an import embargo on country M to the second scenario where country H imposes an import embargo on country M and country L imposes an export embargo on country M.

In figure 4.7, we show the percentage change in welfare in country L when they join country H in restricting trade from country M. Country L will be worse off by setting an export embargo on goods to country M.

In figure 4.8, we show the percentage change in welfare in country M when country H sets an import embargo and country L sets an export embargo on country M. The larger the population in country L, the worse-off country M will be. Surprisingly, if the population is low enough in country L, country M can be better off if country L prohibits exports to country M. We call this the "embargo paradox". In this case, the increase in demand for country M's goods must be

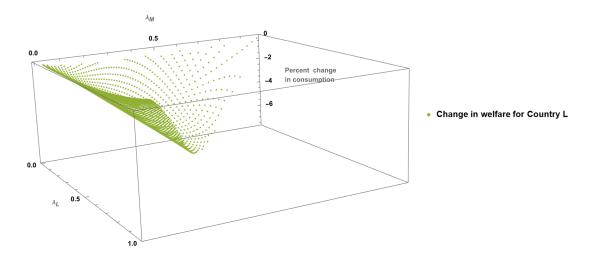


Figure 4.7: Country L Change in Welfare from Country L Placing an Export Embargo on Country M

enough to offset the higher prices they pay for lower-indexed goods after country L sets an export embargo on them and the higher prices for country H's imports.

In figure 4.9, we show the percentage change in welfare in country H it sets an import embargo, and country L sets an export embargo on country M. The larger the population in country L, the better-off country H will be. Country H is always better off when country L joins them in restricting trade with country M. The intuition here is that when country L restricts exports to country M, they have to produce more goods domestically, causing their wages to rise.

4.6 Concluding remarks

Prohibition of trade is usually deemed welfare diminishing. We show different policy scenarios in a setting where preferences are nonhomothetic and countries have varying levels of income. While we find that prohibiting trade can be welfare deteriorating, there is a case where a low-income country sanctions a middle-income country by not exporting to them, and welfare could improve in the middle-income

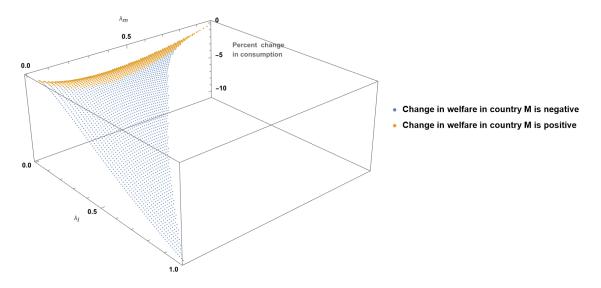


Figure 4.8: Country M Change in Welfare from Country L Placing an Export Embargo on Country M

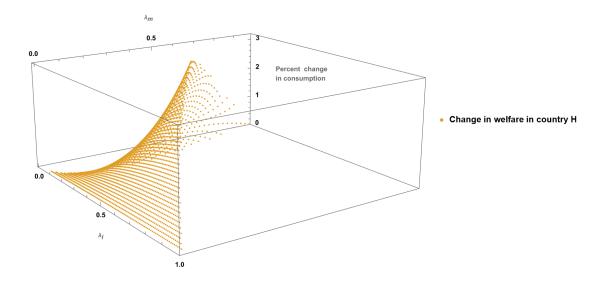


Figure 4.9: Country ${\cal H}$ Change in Welfare from Country L Placing an Export Embargo on Country ${\cal M}$

country while reducing the welfare in their own country. For this to hold, given the assumptions we made in the model, the population in the low-income country would have to be a small percentage of the world's population.

For each of the analyzed scenarios, we found that the lower-income country was always better off trading freely with middle-income countries. However, if the low-income country wanted to impose a trade sanction against the middle-income country; it would do the most harm to the middle-income country and the least damage to itself if the population in its country was a large percentage of the world population and the population in the middle-income country is a small percentage of the world's population. The high-income country is always better off due to the lowincome country setting a sanction against the middle-income country. Restrictions placed by the low-income country on the middle-income have the effect of raising wages in the high-income country, causing consumption in the high-income country to rise. Furthermore, we explore the possibility of the high-income country making a transfer payment to the low-income country to diminish some of the loss in welfare from restricting trade with the middle-income country. While we find that it can mitigate some of the losses of the low-income country, we do not find evidence that it can completely offset the welfare loss of the low-income country.

A further avenue of research we look to pursue is to investigate further what we call the "embargo paradox". Informally, the idea stems from the fact that when the population is small in the developing world, the price of their goods are relatively expensive. Although they still hold a comparative advantage in producing those goods, when the middle-income country switches from importing the lower-indexed goods to producing them domestically, the increase in wages as a result of the rise in demand is enough to offset the loss of having to buy lower-indexed goods at higher

prices. We need to explore formally when exactly this paradox holds with respect to the different parameters of the model.

4.7 Appendix

Partial embargo: High-income country does not export to the middleincome country, while the low-income country trades freely with the middle-income country.

In this setting, country H does export to country M while country L trades freely with country M, see figure. The setup is the same as in the previous scenarios previously described. We highlight the main differences. There are only two boundary goods, z_L and z_M , and their prices are the same as equations (4.1)-(4.3). The budget constraints for countries L and M are the same as in equations (4.4) and (4.5), respectively. However, because the only sanction against country M is that country H is not exporting to them, country H can buy goods from the country producing at the lowest price. Therefore, the budget constraint in the country H is

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{z_{M}} a_{M}(z) dz + w_{H} \int_{z_{M}}^{c_{H}} a_{H}(z) dz = w_{H}.$$
(4.23)

Because country L produces the goods in the range $[0, z_L]$ for domestic consumption and for export to countries M and H, the resource constraint in country L is

$$(N_L + N_M + N_H) \int_0^{z_L} a_L(z) \, dz = N_L. \tag{4.24}$$

Country M produces goods in the range $[z_L, c_M]$ for domestic consumption, and goods in the range $[z_L, z_M]$ export to country L and H. Country M faces the resource constraint

$$(N_L + N_H) \int_{z_L}^{z_M} a_M(z) \, dz + (N_M) \int_{z_L}^{c_M} a_M(z) \, dz = N_M. \tag{4.25}$$

There are seven unknowns: $w_H, w_M, z_L, z_M, c_L, c_M$, and c_H , and seven equations (4.1), (4.2), (4.4), (4.5), (4.23), (4.24) and (4.25). Country *H*'s resource constraint holds in equilibrium by Walras' Law. The model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation (4.1) and z_M in terms of w_M and w_H in (4.2). Then, we solve for w_M in equation (4.24). Next, we plug w_M back into (4.1) to solve for z_L . Plugging z_L and w_M into (4.5), we solve for c_M . Plugging z_L , c_M , and z_M in terms of w_H into (4.25), we solve for w_H , and knowing w_H we can solve for z_M in (4.2). Lastly, we plug in the know variables into w_H, w_M, z_L , and z_M into equations (4.4) and (4.6) to find c_L and c_H , respectively. Figure 4.10 shows a graphical illustration of this case.

In the asymmetric case country L does not have enough income to import goods from the high-income country, i.e., $c_L < z_H$. In this case, the budget constraint for country L becomes

$$\int_{0}^{z_{L}} a_{L}(z) dz + w_{M} \int_{z_{L}}^{c_{L}} a_{M}(z) dz = 1.$$
(4.26)

and the resource constraint for country M becomes

$$N_L \int_{z_L}^{c_L} a_M(z) \, dz + (N_M) \int_{z_L}^{c_M} a_M(z) \, dz + N_H \int_{z_L}^{z_M} a_M(z) \, dz = N_M. \tag{4.27}$$

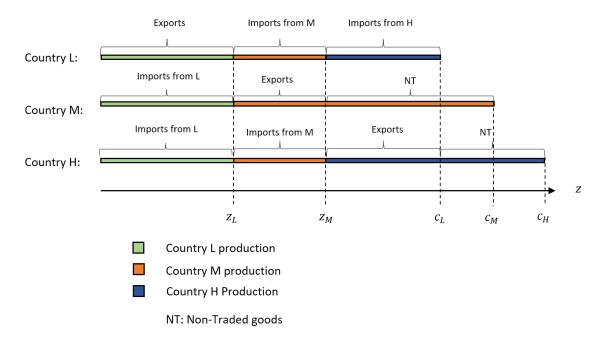


Figure 4.10: Trade and Consumption Patterns, H Setting an Export Embargo

In the asymmetric case, we can solve for z_L in terms of w_M in equation (4.1) and z_M in terms of w_M and w_H (4.2). Then, we solve for w_M in equation 4.24. Next, we plug w_M back into (4.1) to solve for z_L . We can solve for c_L plugging z_L in (4.26). Plugging z_L and w_M into (4.5), we solve for c_M . Plugging z_L , c_L , c_M , and z_M in terms of w_H into (4.27), we solve for w_H and knowing w_H we can solve for z_M in (4.2). Lastly, we plug in the know variables into w_H, w_M, z_L , and z_M into (4.23) to find c_H .

Partial embargo: High-income country does not export to the middleincome country, while the low-income country does not import to the middle-income country

We describe a setting when country L joins country H in restricting trade with country M by not importing goods from country M. In this setting, the prices of the boundary goods are the same as equations (4.1)-(4.3). Country L only consumes domestic goods and imports from country H. Thus, country L's budget constraint would be the same as equation (4.10). Because country M only consumes domestic goods and imports from country L, their budget constraint is the same as equation (4.5). Because country H can import from any country, it can buy goods from the country producing at the lowest price; therefore, the budget constraint for country H is equal to equation (4.23). Country L produces goods in the range $[0, z_H]$ for itself while exporting the goods in the range $[0, z_L]$ to countries M and H. The resource constraint in country L is

$$(N_M + N_H) \int_0^{z_L} a_L(z) \, dz + N_L \int_0^{z_H} a_L(z) \, dz = N_L \tag{4.28}$$

Country M domestic produces goods in the range $[z_L, c_M]$ and exports goods in $[z_L, z_M]$ to country H. Thus, it faces the resource constraint

$$N_M \int_{z_L}^{c_M} a_M(z) \, dz + N_H \int_{z_L}^{z_M} a_M(z) \, dz = N_M.$$
(4.29)

There are eight unknowns: $w_H, w_M, z_L, z_H, z_M, c_L, c_M$, and c_H , and eight equations (4.1) - (4.3), (4.10), (4.5), (4.23), (4.28), and (4.29). Country *H*'s resource constraint holds in equilibrium by Walras' Law. The model can be solved semi-recursively. We can solve for z_L in terms of w_M in equation (4.1), z_M in terms of w_M and w_H (4.2), and z_H in terms of w_H in equation (4.3). Then, we solve for w_M, w_H , and c_M using a system of equations (4.5), (4.28), and (4.29). Plugging in w_M and w_H into (4.1)-(4.2) to find z_L, z_M , and z_H , respectively. Lastly, we plug in the know variables into w_H, w_M, z_L, z_H , and z_M into equations (4.10) and (4.23) to find c_L and c_H , respectively.

In the asymmetric case, i.e., $c_L < z_H$, country L would consume only goods which they produce domestically. Given that it does not import any goods, it will not export; therefore, it will be in autarky. If country L does not export, then country M will also be in autarky because it will only consume domestically produced goods which means that they have no incentive to export. Subsequently, country H will also be in autarky, and there will be no trade between countries.

Numerical results: High-income country does not export to the middleincome country

Again we follow the same procedure as in section 4.4.3. We assume the unit labor requirements have the same functional forms as those given by assumption 4. We numerically solve for the consumption in each country and compare how welfare changes as the country L bans imports from country M. In this scenario, we compare the welfare of each country in a setting where country H is setting an export embargo on country M to a scenario where country H is setting an export embargo, and country L sets an import embargo on goods from country M. The figures below show the percentage change in welfare going from the first scenario, only country H imposing an export embargo on country M, to the second scenario, country H imposing an export embargo on country M and country L imposes an import embargo on country M.

In figure 4.11, we show the percentage change in welfare in country L when they join country H in restricting trade from country M. Country L will be worse off placing an import embargo on goods from country M. Figure 4.12 shows that the change in welfare for country M is always negative. Therefore, they are better off when only country H sets an export embargo, instead of country H setting an export embargo and L an import embargo. Figure 4.13 shows the change in welfare for country H is always positive.

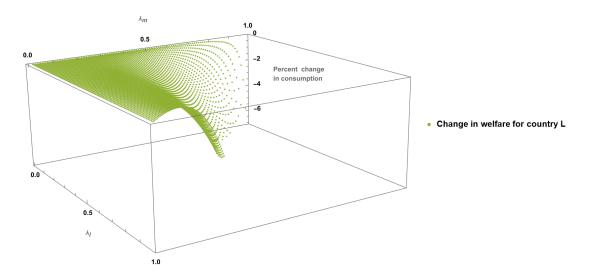


Figure 4.11: Country L Change in Welfare from Country L Placing an Import Embargo on Country M

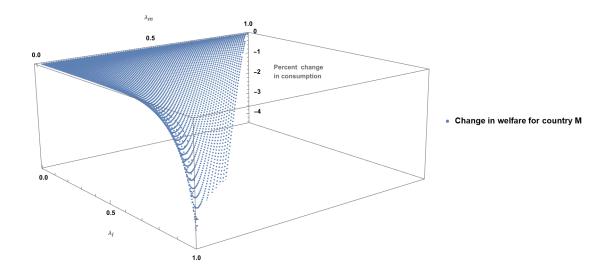


Figure 4.12: Country M Change in Welfare from Country L Placing an Import Embargo on Country M

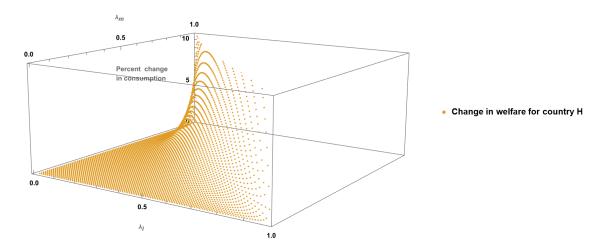


Figure 4.13: Country ${\cal H}$ Change in Welfare from Country L Placing an Import Embargo on Country ${\cal M}$

REFERENCES

- Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare (2012), "New trade models, same old gains?" American Economic Review, 102, 94–130.
- Bekkers, Eddy, Joseph Francois, and Miriam Manchin (2012), "Import prices, income, and inequality." *European Economic Review*, 56, 848–869.
- Bernhofen, Daniel M and John C Brown (2005), "An empirical assessment of the comparative advantage gains from trade: evidence from Japan." *American Economic Review*, 95, 208–225.
- Bhagwati, Jagdish (1968), "Trade liberalization among LDCs, trade theory, and GATT rules." Value, Capital, and Growth: Essays in honour of Sir John Hicks, 21–43.
- Chang, Yang Ming and Renfeng Xiao (2015), "Preferential trade agreements between asymmetric countries: Free trade areas (with rules of origin) vs. customs unions." Japan and the World Economy, 33, 28–43.
- Clements, Kenneth W, Yanrui Wu, and Jing Zhang (2006), "Comparing international consumption patterns." *Empirical Economics*, 31, 1–30.
- Cooper, Charles A and Benton F Massell (1965), "Toward a general theory of customs unions for developing countries." *Journal of Political Economy*, 73, 461—-476.
- Corless, R. M., G. H. Gonnet, D. E.G. Hare, D. J. Jeffrey, and D. E. Knuth (1996), "On the Lambert W function." *Advances in Computational Mathematics*, 5, 329–359.
- Crozet, Matthieu and Julian Hinz (2020), "Friendly fire: The trade impact of the Russia sanctions and counter-sanctions." *Economic Policy*, 35, 97—146.
- de Scitovszky, Tibor (1942), "A Reconsideration of the Theory of Tariffs." *Review of Economic Studies*, 9, 89–110.
- Dornbusch, Rudiger, Stanley Fischer, and Paul Anthony Samuelson (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods." *The American Economic Review*, 67, 823–839.
- Fajgelbaum, Pablo, Gene M. Grossman, and Elhanan Helpman (2011), "Income distribution, product quality, and international trade." *Journal of Political Economy*, 119, 721–765.
- Falkinger, Josef and Josef Zweimüller (1996), "The cross-country Engel curve for product diversification." *Structural Change and Economic Dynamics*, 7, 79–97.

- Felbermayr, Gabriel, Benjamin Jung, and Mario Larch (2013), "Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model." *Journal of International Economics*, 89, 13–25.
- Flam, Harry and Elhanan Helpman (1987), "Vertical Product Differentiation and North-South Trade." *The American Economic Review*, 77, 810–822.
- Frankel, Jeffrey A. (1982), "The 1807-1809 Embargo Against Great Britain." 42, 291–308.
- Gehrels, Franz (1956), "Customs union from a single-country viewpoint." *Review of Economic Studies*, 24, 61–64.
- Gorman, William M (1958), "Tariffs, Retaliation, and the Elasticity of Demand for Imports." *The Review of Economic Studies*, 25, 133.
- Grigg, David (1994), "Food expenditure and economic development." *GeoJournal*, 33, 377–382.
- Gros, Daniel (1987), "A note on the optimal tariff, retaliation and the welfare loss from tariff wars in a framework with intra-industry trade." *Journal of International Economics*, 23, 357–367.
- Grubel, Herbert G. (1970), "The Theory of Intra-Industry Trade." In *Studies in International Economics* (Ian A. McDougall and Rober Hugh Snape, eds.), North-Holland Publishing Company, Amsterdam.
- Harrison, Glenn W and E Elisabet Rutström (1991), "Trade Wars, Trade Negotiations and Applied Game Theory." *The Economic Journal*, 101, 420–435.
- Hunter, Linda (1991), "The contribution of nonhomothetic preferences to trade." Journal of International Economics, 30, 345–358.
- Hunter, Linda and James Markusen (1988), "Per-Capita Income as a Determinant of Trade." In *Empirical Methods for International Trade* (Robert Feenstra, ed.), MIT Press, Cambridge.
- Irwin, Douglas A. (2005), "The welfare cost of autarky: Evidence from the jeffersonian trade embargo, 1807-09." *Review of International Economics*, 13, 631–645.
- Jackson, Laurence Fraser (1984), "Hierarchic demand and the Engel curve for variety." The Review of Economics and Statistics, 8–15.
- Jaimovich, Esteban and Vincenzo Merella (2012), "Quality ladders in a ricardian model of trade with nonhomothetic preferences." Journal of the European Economic Association, 10, 908–937.

- Johnson, Harry G. (1953–1954), "Optimum Tariffs and Retaliation." The Reiew of Economic Studies, 21, 142–153.
- Johnson, Harry G (1960), "The economic theory of customs union." *Pakistan Economic Journal*, 10, 14–32.
- Kaempfer, William H. and Anton D. Lowenberg (2007), "The Political Economy of Economic Sanctions." *Handbook of Defense Economics*, 2, 867–911.
- Kemp, Murray C and Henry Y Jr. Wan (1976), "An elementary proposition concerning the formation of customs unions." *Journal of International Economics*, 6, 95–97.
- Kennan, John and Raymond Riezman (1988), "Do Big Countries Win Tariff Wars?" International Economic Review, 29, 81–85.
- Kennan, John and Raymond Riezman (1990), "Optimal Tariff Equilibria with Customs Unions." *The Canadian Journal of Economics*, 23, 70–83.
- Kose, M Ayhan and Raymond Riezman (2000), "Understanding the welfare implications of preferential trade agreements. In International Trade Agreements and Political Economy." *Review of International Economics*, 8, 619–633.
- Kowalczyk, Carsten (1990), "Welfare and customs unions." NBER working paper.
- Krishna, Pravin and Jagdish Bhagwati (1997), "Necessarily welfare-enhancing customs unions with industrialization constraints: The Cooper-Massell-Johnson-Bhagwati conjecture." Japan and the World Economy, 9, 441–446.
- Krueger, Anne O. (1997), "Free trade agreements versus customs unions." *Journal* of Development Economics, 54, 169–187.
- Linder, Staffan Burenstam (1961), An Essay on Trade and Transformation. Almqvist & Wiksell, Stockholm.
- Lipsey, Richard G (1957), "The Theory of Customs Unions: Trade Diversion and Welfare." *Economica*, 24, 40–46.
- Markusen, James R (1986), "Explaining the Volume of Trade : An Eclectic Approach." *The American Economic Review*, 76, 1002–1011.
- Markusen, James R. (2013), "Putting per-capita income back into trade theory." Journal of International Economics, 90, 255–265.
- Matsuyama, Kiminori (2000), "A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade." *Journal of Political Economy*, 108, 1093– 1120.

- Melitz, Marc (2003), "The impact of trade on aggregate industry productivity and intra-industry reallocations." *Econometrica*, 71, 1695–1725.
- Mukunoki, Hiroshi (2004), "On the optimal external tariff of a free trade area with internal market integration." Japan and the World Economy, 16, 431–448.
- Ohyama, Michihiro (1972), "Trade and welfare in general equilibrium." *Keio Economic Studies*, 9, 37–73.
- Opp, Marcus M. (2010), "Tariff wars in the Ricardian Model with a continuum of goods." *Journal of International Economics*, 80, 212–225.
- Otani, Yoshihiko (1980), "Strategic Equilibrium of Tariffs and General Equilibrium." *Econometrica*, 48, 643–662.
- Panagariya, Arvind (2000), "Preferential trade liberalization: The traditional theory and new developments." *Journal of Economic Literature*, 38, 287–331.
- Prebisch, Raul (1962), "The economic development of Latin America and its principal problems." *Economic Bulletin for Latin America*.
- Rodriguez, Carlos Alfredo (1974), "The non-equivalence of tariffs and quotas under retaliation." *Journal of International Economics*, 4, 295–298.
- Schott, Peter K (2001), "Do Rich and Poor Countries Specialize in a Different Mix of Goods? Evidence from Product-Level US Trade Data." NBER Working Paper No.8492, 1–28.
- Simonovska, Ina (2015), "Income differences and prices of tradables: Insights from an online retailer." *Review of Economic Studies*, 82, 1612–1656.
- Singer, Hans W (1950), "The distribution of gains between borrowing and investing countries." American Economic Review, 40, 473–485.
- Stibora, Joachim and Albert de Vaal (2007), "Trade policy in a Ricardian model with a continuum of goods under nonhomothetic preferences." *Journal of Development Economics*, 84, 350–377.
- Stibora, Joachim and Albert de Vaal (2015), "Does preferential trade benefit poor countries? A general equilibrium assessment with nonhomothetic preferences." *Review of International Economics*, 23, 239–270.
- Stokey, Nancy L. (1991), "The volume and composition of trade between rich and poor countries." *Review of Economic Studies*, 58, 63–80.
- Syropoulos, Constantinos (2002), "Optimum tariffs and retaliation revisited: How country size matters." *Review of Economic Studies*, 69, 707–727.

- Tower, Edward (1975), "The Optimum Quota and Retaliation." The Review of Economic Studies, 42, 623.
- Vanek, Jaroslav (1965), General equilibrium of international discrimination: The case of customs unions. Harvard University Press, Cambridge.
- Viner, Jacob (1950), *The Customs Union Issue*. Carnegie Endowment for International Peace, New York.
- Whalley, John (1985), Trade liberalization among major world trading areas. MIT Press.
- Whalley, John (1986), "Impacts of a 50% tariff reduction in an eight-region global trade model." In *General equilibrium trade policy modeling* (Thirukodikaval N. Srinivasan and John Whalley, eds.), chapter 3, MIT Press.
- Yi, Sang Seung (2000), "Free-trade areas and welfare: An equilibrium analysis." *Review of International Economics*, 8, 336–347.

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