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On the Performance of some Poisson Ridge Regression Estimators

Cynthia Zaldivar
Florida International University, czald001@fiu.edu

FLORIDA INTERNATIONAL UNIVERSITY

Miami, Florida

ON THE PERFORMANCE OF SOME POISSON RIDGE REGRESSION ESTIMATORS

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Cynthia Zaldivar

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To: Dean Michael R. Heithaus
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This thesis, written by Cynthia Zaldivar, and entitled On the Performance of Some Poisson Ridge Regression Estimators, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this thesis and recommend that it be approved.

Wensong Wu

Florence George

B. M. Golam Kibria, Major Professor

Date of Defense: March 28, 2018

The thesis of Cynthia Zaldivar is approved.

Dean Michael R. Heithaus
College of Arts, Sciences and Education

Andrés G. Gil
Vice President for Research and Economic Development
and Dean of the University Graduate School

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ABSTRACT OF THE THESIS

On the Performance of Some Poisson Ridge Regression Estimators

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Cynthia Zaldivar

Florida International University, 2018

Miami, Florida

Professor B. M. Golam Kibria, Major Professor

Multiple regression models play an important role in analyzing and making predictions about data. Prediction accuracy becomes lower when two or more explanatory variables in the model are highly correlated. One solution is to use ridge regression. The purpose of this thesis is to study the performance of available ridge regression estimators for Poisson regression models in the presence of moderately to highly correlated variables.

As performance criteria, we use mean square error (MSE), mean absolute percentage error (MAPE), and percentage of times the maximum likelihood (ML) estimator produces a higher MSE than the ridge regression estimator.

A Monte Carlo simulation study was conducted to compare performance of the estimators under three experimental conditions: correlation, sample size, and intercept. It is evident from simulation results that all ridge estimators performed better than the ML estimator. We proposed new estimators based on the results, which performed very well compared to the original estimators. Finally, the estimators are illustrated using data on recreational habits.

TABLE OF CONTENTS

LIST OF TABLES

TABLE PAGE

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Literature Review

When any pair of independent variables are highly correlated, the corresponding regression coefficients will have large standard errors, which means that the regression coefficients are unstable. When two independent variables are highly correlated, they are not orthogonal, that is the regressors have a near linear relation. In such a situation, the coefficients are less likely to be statistically significant. Inferences made using such a model will not be correct. This type of situation is referred to as multicollinearity. (Allen, 1997)

Two methods of dealing with multicollinearity are collecting more data and model respecification. While collecting more data may be the best way to correct multicollinearity, it is not always possible to collect additional data. There are often financial constraints which render the solution impossible. Finances aside, the process, individuals, or items being studied may no longer be accessible for sampling. Collecting more data may not solve the issue of multicollinearity if the problem arises from limitations of the model or population. If multicollinearity is caused by some characteristic of the model, such as the presence of two highly correlated variables, model respecification may help alleviate the issue. The discovery and implementation of a simple function, such as $x = x1/x2$ or $x = x1x2$ may aid the issue while preserving the information gained from the regressors. One effective method of dealing with multicollinearity is elimination of any highly correlated variables. While this kind of brute force method may totally remove the issue, the information gained from any removed regressors will be lost. The loss of information can be an unacceptable consequence to many researchers. (Montgomery, 2012)

The widely used method of dealing with multicollinearity is through the use of ridge regression, in which regression coefficients are calculated as such:

$$\hat{\beta}_R(k) = (X'X + kI)^{-1}X'y. \tag{1.1}$$

Here X is an $nx(p + 1)$ matrix, p being the number of independent variables used in the model.For values $k \geq 0$ the mean square error (MSE) is smaller than that of the least squares estimator:

$$\hat{\beta} = (X'X)^{-1}X'y. \tag{1.2}$$

The ridge regression method of dealing with multicollinearity was originally proposed by Hoerl and Kennard (1970). Many researchers have proposed estimates for k, including Lawless and Wang (1976), Kibria (2003), Alkhamisi and Shukur (2008), Kibria and Muñiz (2009), Kibria and Banik (2016), and Asar and Genç (2017), among others.

There has been limited research on ridge regression using non-Gaussian regression models, such as the Poisson regression model. Månsson and Shukur (2011) evaluated the performance of several k estimators for Poisson ridge regression using Monte Carlo simulations. The present thesis expands on that study by evaluating more k estimators and proposing some new k estimators for Poisson ridge regression models.

The Poisson regression model has a large amount of applications, especially in economics. A Poisson model is useful when the response variable includes countable, independent events, such as the total number of car accidents per week on a specific highway, the number of calls received per minute at a call center, or number of credit defaults per year (Greene, 2012). Further applications will be discussed in Section 4 of this thesis.

1.2 Objective of the Thesis

The problem of estimating k is the subject of many research papers and has not been solved yet. The different methods proposed in research each have advantages and disadvantages. It is necessary to compare different estimators under different error distributions under the same conditions. The purpose of this research is to compare popular ridge parameter estimators using the following criteria:

1. Smallest mean squared error (MSE)

2. Mean absolute percentage error (MAPE)

3. Performance of ridge regression estimator compared to the least squares estimator, in terms of the percentage of times in simulation that the ridge regression estimator produces a lower MSE than the least squares estimator.

The present research extends Poisson Ridge Regression (PRR) research by Månsson and Shukur (2011). Ridge regression estimators for Poisson regression models will be compared to the maximum likelihood estimation (MLE) method using Monte Carlo simulations.

The organization of the thesis is as follows. We define different types of ridge regression estimators of k in Chapter 2. A Monte Carlo simulation study has been conducted in Chapter 3. In Chapter 4, the empirical application of the Poisson ridge regression is presented. The summary and concluding remarks are given in Chapter 5.

CHAPTER 2

METHODOLOGY

2.1 Poisson Ridge Regression

In Poisson regression, it is standard to cross-section data in applications of n independent observations. The i^{th} observation is $(y_i, \mathbf{x}_i)$. Here y_i, the dependent variable, represents the number of occurrences of the event of interest and $\mathbf{x}_i' = \left\{x_{1i}, x_{2i}, ..., x_{pi}\right\}$ is a vector of p linearly independent regressors that are thought to determine y_i. y_i given x_i is Poisson distributed, with the density function:

$$f(y_i|x_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}, \; y_i = 0, 1, 2, ... \tag{2.1}$$

The mean parameter, which represents the average rate of occurrence of the event of interest, is calculated as:

$$\mu_i = e^{x_i'\beta}, \tag{2.2}$$

where β is a vector of parameters from the regression model.

The Maximum Likelihood Estimator (MLE) for β is found using the iterative weighted least square (IWLS) algorithm:

$$\hat{\beta}_{MLE} = (X'\hat{W}X)^{-1}X'\hat{W}\hat{z}, \tag{2.3}$$

$$\text{where } \hat{W} = diag(\hat{\mu}_i), \quad \hat{\mu}_i = e^{x_i\hat{\beta}}, \quad \text{and } \hat{z} = log(\hat{\mu}_i) + \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i}.$$

The Poisson ridge regression estimator is calculated as:

$$\hat{\beta}_R(k) = (X'\hat{W}X + kI)^{-1}X'\hat{W}\hat{z} = (X'\hat{W}X + kI)^{-1}X'\hat{W}X\hat{\beta}_{MLE} = U\hat{\beta}_{MLE}. \tag{2.4}$$

where $U = (X'\hat{W}X + kI)^{-1}X'\hat{W}X$

The MSE of $\hat{\beta}_R(k)$ can be calculated as follows:

$$MSE\left(\hat{\beta}_R(k)\right) = MSE\left(U\hat{\beta}_{MLE}\right) = E\left(U\hat{\beta}_{MLE} - \beta\right)'\left(U\hat{\beta}_{MLE} - \beta\right)$$

$$= E\left(\hat{\beta}'_{MLE}U'U\hat{\beta}_{MLE}\right) - E\left(\beta'U\hat{\beta}_{MLE}\right) - E\left(\hat{\beta}'_{MLE}U'\beta\right) + E\left(\beta'\beta\right)$$

$$\begin{aligned} &= E\left(\hat{\beta}'_{MLE}U'U\hat{\beta}_{MLE}\right) - E\left(\beta'U'U\hat{\beta}_{MLE}\right) - E\left(\hat{\beta}'_{MLE}U'U\beta\right) + E\left(\hat{\beta}'_{MLE}U'U\beta\right) \\ &\quad + E\left(\beta'U'U\beta\right) - E\left(\beta'U'U\beta\right) + E\left(\beta'U'U\hat{\beta}_{MLE}\right) \\ &\quad - E\left(\beta'U\hat{\beta}_{MLE}\right) - E\left(\hat{\beta}'_{MLE}U'\beta\right) + E\left(\beta'\beta\right) \end{aligned} \tag{2.5}$$

Lee and Silvapulle (1988) found that, for large values of n, the distribution of $\hat{\beta}_{MLE}$ is $\hat{\beta}_{MLE} \sim N\left(\beta, \left(X'\hat{W}X\right)^{-1}\right)$. Then:

$$\begin{aligned} MSE\left(\hat{\beta}_R(k)\right) &= E\left[\left(\hat{\beta}_{MLE} - \beta\right)' U'U\left(\hat{\beta}_{MLE} - \beta\right)\right] \\ &\quad - \beta'U'U\beta + \beta'U'U\beta + \beta'U'U\beta - \beta'U\beta - \beta'U'\beta + \beta'\beta \end{aligned}$$

$$= E\left[\left(\hat{\beta}_{MLE} - \beta\right)' U'U\left(\hat{\beta}_{MLE} - \beta\right)\right] + \beta'\left(U - I_p\right)'\left(U - I_p\right)\beta$$

$$= E\left[tr\left[\left(\hat{\beta}_{MLE} - \beta\right)' U'U\left(\hat{\beta}_{MLE} - \beta\right)\right]\right] + \beta'\left(U - I_p\right)'\left(U - I_p\right)\beta$$

$$= E\left[tr\left[U\left(\hat{\beta}_{MLE} - \beta\right)\left(\hat{\beta}_{MLE} - \beta\right)' U'\right]\right] + \beta'\left(U - I_p\right)'\left(U - I_p\right)\beta$$

$$= tr\left[UE\left[\left(\hat{\beta}_{MLE} - \beta\right)\left(\hat{\beta}_{MLE} - \beta\right)'\right]U'\right] + \beta'\left(U - I_p\right)'\left(U - I_p\right)\beta$$

$$= tr\left[U\left(X'\hat{W}X\right)^{-1}U'\right] + \beta'\left(U - I_p\right)'\left(U - I_p\right)\beta$$

$$\begin{aligned} &= tr\left[X'\hat{W}X + kI_p)^{-2}X'\hat{W}X\right] \\ &\quad + \beta'\left(-k\left(X'\hat{W}X + kI_p\right)^{-1}\right)'\left(-k\left(X'\hat{W}X + kI_p\right)^{-1}\right)\beta \end{aligned}$$

$$= tr\left(\left(X'\hat{W}X + kI_p\right)^{-2}X'\hat{W}X\right) + \beta'\left(k^2\left(X'\hat{W}X + kI_p\right)^{-2}\right)\beta$$

Let $C = X'\hat{W}X$ and $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_p)$ be the eigenvalues of C. Suppose there exists an orthogonal matrix D such that $D'CD = \Lambda$, then:

$$= tr\left(\left(X'\hat{W}X + kI_p\right)^{-2} X'\hat{W}X\right) + \beta'\left(k^2\left(X'\hat{W}X + kI_p\right)^{-2}\right)\beta$$

$$= tr\left(DD'\left(X'\hat{W}X + kI_p\right)^{-1} DD'\left(X'\hat{W}X + kI_p\right)^{-1} DD'X'\hat{W}X\right)$$
$$+k^2 (D\beta)'\left(D'X'\hat{W}XD + kDD'\right)^{-2} (D\beta)$$

$$= tr\left(D'\left(X'\hat{W}X + kI_p\right)^{-1} DD'\left(X'\hat{W}X + kI_p\right)^{-1} DD'X'\hat{W}XD\right) + k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2}$$

$$= tr\left(\left(D'X'\hat{W}XD + kI_p\right)^{-1}\left(D'X'\hat{W}XD + kI_p\right)^{-1} D'X'\hat{W}XD\right) + k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2}$$

$$= tr\left(\left(\Lambda + kI_p\right)^{-1}\left(\Lambda + kI_p\right)^{-1}\Lambda\right) + k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2}$$

$$= \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2} = Var\left(\hat{\beta}^R(k)\right) + \left(Bias\left(\hat{\beta}^R(k)\right)\right)^2 \tag{2.6}$$

where λ_i is the i^{th} eigenvalue of C and $\alpha = D\beta$ Månsson (2011)

2.2 Estimating the k Parameter

Let $\hat{\alpha}(k) = (X^{*'}X^* + kI)^{-1}X^{*'}y$, (2.7)

where $k = diag(k_1, k_1, ..., k_p)$, $k_i > 0$,

$$MSE(\hat{\alpha}(\hat{k})) = \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + \hat{k}_i)^2} + \sum_{i=1}^{p} \frac{\hat{k}_i^2\hat{\alpha}_i^2}{(\lambda_i + \hat{k}_i)^2} \tag{2.8}$$

The value of which k_i minimizes $MSE(\hat{\alpha}(\hat{k}))$ is:

$$\hat{k}_i = \frac{1}{\hat{\alpha}_i^2} \quad \text{Hoerl and Kennard (1970)} \tag{2.9}$$

2.3 k Estimators Used

This thesis will analyze the performance of 50 different k estimators. The k estimators used in this thesis are summarized below.

The first two k are from Hoerl and Kennard (1970):

$$HK = \hat{k}_{HK} = \frac{1}{\hat{\alpha}^2_{max}} \tag{2.10}$$

$$HK_2 = \hat{K}_{HK2} = \frac{1}{\sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{2.11}$$

A different estimator, produced by taking the harmonic mean of $\hat{k}_i$ was suggested by Hoerl et al. (1975):

$$HK_B = \hat{k}_{HKB} = \frac{p}{\sum\limits_{i=1}^{p} \hat{\alpha}_i^2} = \frac{p}{\hat{\alpha}'\hat{\alpha}} \tag{2.12}$$

The next k estimator was proposed by Lawless and Wang (1976):

$$\hat{k}_{LWi} = \frac{1}{\lambda_i \hat{\alpha}_i^2} \tag{2.13}$$

and suggest taking the harmonic mean of (2.13):

$$LW = \hat{k}_{LW} = \frac{p}{\sum\limits_{i=1}^{p} \lambda_i \hat{\alpha}_i^2} = \frac{p}{\hat{\alpha}'X'X\hat{\alpha}} \tag{2.14}$$

The next k estimator was proposed by Hocking et al. (1976):

$$HSL = \hat{k}_{HSL} = \frac{\sum_{i=1}^{p}(\lambda_i\hat{\alpha}_i)^2}{\left(\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2\right)^2} \tag{2.15}$$

The next three k estimators, proposed by Kibria (2003), take the arithmetic mean, geometric mean, and median of the ridge estimator $\hat{k}_i$:

$$AM = \hat{k}_{AM} = \frac{1}{p}\sum_{i=1}^{p}\frac{1}{\hat{\alpha}_i^2} \tag{2.16}$$

$$GM = \hat{k}_{GM} = \frac{1}{(\prod_{i=1}^{p}\hat{\alpha}_i^2)^{1/p}} \tag{2.17}$$

$$Med = \hat{k}_{Med} = Median\left(\frac{1}{\hat{\alpha}_i^2}\right) \tag{2.18}$$

Khalaf and Shukur (2005) suggested the following k estimator:

$$KS = \hat{K}_{KS} = \frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_{max}^2} \tag{2.19}$$

Alkhamisi et al. (2006) proposed the following:

$$KS_A = \hat{K}_{Arith}^{KS} = \frac{1}{p}\sum_{i=1}^{p}\left(\frac{\lambda_i}{(n-p)+\lambda_i\hat{\alpha}_i^2}\right) \tag{2.20}$$

$$KS_{Max} = \hat{K}_{Max}^{KS} = Max\left\{\frac{\lambda_i}{(n-p)+\lambda_i\hat{\alpha}_i^2}\right\} \tag{2.21}$$

$$KS_{Med} = \hat{K}_{Med}^{KS} = Median\left\{\frac{\lambda_i}{(n-p)+\lambda_i\hat{\alpha}_i^2}\right\} \tag{2.22}$$

Muñiz and Kibria (2009) proposed some new estimators, the first being the geometric mean of $\left(\dfrac{\lambda_i}{(n-p)+\lambda_i\hat{\alpha}_i^2}\right)$,

$$KM_1 = \hat{K}_{GM}^{KS} = \hat{K}_{KM1} = \prod_{i=1}^{p}\left(\frac{\lambda_i}{(n-p)+\lambda_i\hat{\alpha}_i^2}\right)^{1/p} \tag{2.23}$$

They also proposed the following six estimators, based on square root transformations, as suggested by Alkhamisi and Shukur (2008):

$$KM_2 = \hat{K}_{KM2} = Max\left\{\sqrt{\hat{\alpha}_i^2}\right\} \tag{2.24}$$

$$KM_3 = \hat{K}_{KM3} = Max\left\{\frac{1}{\sqrt{\hat{\alpha}_i^2}}\right\} \tag{2.25}$$

$$KM_4 = \hat{K}_{KM4} = \left(\prod_{i=1}^{p}\sqrt{\hat{\alpha}_i^2}\right)^{1/p} \tag{2.26}$$

$$KM_5 = \hat{K}_{KM5} = \left(\prod_{i=1}^{p}\sqrt{\frac{1}{\hat{\alpha}_i^2}}\right)^{1/p} \tag{2.27}$$

$$KM_6 = \hat{K}_{KM6} = Median\left\{\sqrt{\hat{\alpha}_i^2}\right\} \tag{2.28}$$

$$KM_7 = \hat{K}_{KM7} = Median\left\{\frac{1}{\sqrt{\hat{\alpha}_i^2}}\right\} \tag{2.29}$$

Muñiz et al. (2012) proposed the following estimators:

$$KM_8 = \hat{K}_{KM8} = Max\left\{\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_i^2}{\lambda_{max}}}\right\} \tag{2.30}$$

$$KM_9 = \hat{K}_{KM9} = Max\left\{\sqrt{\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_i^2}}\right\} \tag{2.31}$$

$$KM_{10} = \hat{K}_{KM10} = \prod_{i=1}^{p}\left(\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_i^2}{\lambda_{max}}}\right)^{1/p} \tag{2.32}$$

$$KM_{11} = \hat{K}_{KM11} = \prod_{i=1}^{p}\left(\sqrt{\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_i^2}}\right)^{1/p} \tag{2.33}$$

$$KM_{12} = \hat{K}_{KM12} = Median\left\{\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_i^2}{\lambda_{max}}}\right\} \tag{2.34}$$

The next two estimators were proposed by Kibria et al. (2011):

$$KM_{13} = \hat{K}_{KM13} = \prod_{i=1}^{p}\left(\frac{(n-p)+\lambda_{max}\hat{\alpha}_i^2}{\lambda_{max}}\right)^{1/p} \tag{2.35}$$

$$KM_{14} = \hat{K}_{KM14} = \prod_{i=1}^{p}\left(\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_i^2}\right)^{1/p} \tag{2.36}$$

Khalaf (2012) proposed the following modification of $\hat{k}_{HK}$:

$$GK = \hat{K}_{GK} = \hat{K}_{HK} + \frac{2}{(\lambda_{max}+\lambda_{min})'} \tag{2.37}$$

Nomura (1988) suggested the following:

$$HMO = \hat{K}_{HMO} = \frac{p}{\sum_{i=1}^{p}\left(\hat{\alpha}_i^2\Big/\left(1+\left(1+\lambda_i\sqrt{\hat{\alpha}_i^2}\right)\right)\right)} \tag{2.38}$$

The following four estimators were proposed by Dorugade (2013):

$$AD_{HM} = \hat{K}_{HM}^{AD} = \frac{2p}{\sum_{i=1}^{p}\lambda_{max}\hat{\alpha}_i^2} \tag{2.39}$$

$$AD_{Med} = \hat{K}_{Med}^{AD} = Median\left\{\frac{2}{\lambda_{max}\hat{\alpha}_i^2}\right\} \tag{2.40}$$

$$AD_{GM} = \hat{K}_{GM}^{AD} = \frac{2}{\lambda_{max}\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{1/p}} \tag{2.41}$$

$$AD_{AM} = \hat{K}_{AM}^{AD} = \frac{2}{\lambda_{max} p} \sum_{i=1}^{p} \frac{1}{\hat{\alpha}_i^2} \tag{2.42}$$

Asar and Aşır (2017) recently proposed the following nine estimators:

$$Y_1 = \hat{K}_{Y1} = \frac{1}{p} \sum_{i=1}^{p} \sqrt{\frac{1}{\lambda_i \hat{\alpha}_i^2}} \tag{2.43}$$

$$Y_2 = \hat{K}_{Y2} = \left(\prod_{i=1}^{p} \sqrt{\frac{1}{\lambda_i \hat{\alpha}_i^2}} \right)^{1/p} \tag{2.44}$$

$$Y_3 = \hat{K}_{Y3} = Median \left\{ \sqrt{\frac{1}{\lambda_i \hat{\alpha}_i^2}} \right\} \tag{2.45}$$

$$Y_4 = \hat{K}_{Y4} = Max \left\{ \sqrt{\frac{1}{\lambda_i \hat{\alpha}_i^2}} \right\} \tag{2.46}$$

$$Y_5 = \hat{K}_{Y5} = Median \left\{ \sqrt{\lambda_i \hat{\alpha}_i^2} \right\} \tag{2.47}$$

$$Y_6 = \hat{K}_{Y6} = Max \left\{ \sqrt{\lambda_i \hat{\alpha}_i^2} \right\} \tag{2.48}$$

$$Y_7 = \hat{K}_{Y7} = \frac{1}{p} \sum_{i=1}^{p} \sqrt{\lambda_i \hat{\alpha}_i^2} \tag{2.49}$$

$$Y_8 = \hat{K}_{Y8} = \frac{p}{\sum_{i=1}^{p} \sqrt{\lambda_i \hat{\alpha}_i^2}} \tag{2.50}$$

$$Y_9 = \hat{K}_{Y9} = \frac{p}{\sum_{i=1}^{p} \sqrt{\frac{1}{\lambda_i \hat{\alpha}_i^2}}} \tag{2.51}$$

Al-Hassan (2010) proposed the following estimator:

$$AH = \hat{K}_{AH} = \frac{\lambda_{max} \sum_{i=1}^{p} (\lambda_i \hat{\alpha}_i^2) + \left(\sum_{i=1}^{p} \lambda_i \hat{\alpha}_i^2 \right)^2}{\lambda_{max} \sum_{i=1}^{p} \lambda_i \hat{\alpha}_i^2} \tag{2.52}$$

Batah and Gore (2009) suggested the following k estimator:

$$FG = \hat{K}_{FG} = \frac{p}{\sum_{i=1}^{p}\left(\hat{\alpha}_i^2 \Big/ \left(\left(\frac{\hat{\alpha}_i^4 \lambda_i^2}{4} + 6\hat{\alpha}_i^4 \lambda_i\right)^{1/2} - 6\hat{\alpha}_i^2 \lambda_i\right)\right)} \tag{2.53}$$

Dorugade (2014) proposed the following:

$$AS = \hat{K}_{AS} = \frac{1}{\hat{\alpha}_{max}^2} + \frac{1}{\lambda_{max}} \tag{2.54}$$

$$AS_{Max} = \hat{K}_{Max}^{AS} = Max\left\{\frac{1}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right\} \tag{2.55}$$

$$AS_{Min} = \hat{K}_{Min}^{AS} = \frac{1}{Min\left\{\frac{1}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right\}} \tag{2.56}$$

Adnan et al. (2014) proposed the following estimators:

$$N_1 = \hat{K}_{HM}^{N1} = \frac{\sqrt{5}p}{\lambda_{max} \sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{2.57}$$

$$N_2 = \hat{K}_{HM}^{N2} = \frac{p}{\sqrt{\lambda_{max}} \sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{2.58}$$

$$N_3 = \hat{K}_{HM}^{N3} = \frac{2p}{\sum_{i=1}^{p}(\lambda_i^{1/4}) \sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{2.59}$$

$$N_4 = \hat{K}_{HM}^{N4} = \frac{2p}{\sqrt{\sum_{i=1}^{p} \lambda_i} \sum_{i=1}^{p} \hat{\alpha}_i^2} \tag{2.60}$$

The best performing of the 50 k estimators will be used as the basis for new k estimators. The performance of those new estimators will then be assessed using the same criteria, outlined below.

2.4 Criteria for Good Estimators

The following criteria will be used to judge the performance of the 50 k estimators:

1. Average MSE
2. Percentage of times PRR outperforms MLE (performance)
3. Average mean absolute percentage error (MAPE).

Average MSE is calculated as follows:

$$MSE_{Av} = \frac{\sum_{i=1}^{r} SE_i}{r} = \frac{\sum_{i=1}^{r} (\hat{\beta}-\beta)_i'(\hat{\beta}-\beta)_i}{r}, \quad \text{where } r = \text{number of replicates.} \tag{2.61}$$

The performance of PRR versus MLE is defined as the percentage of times, among the r replicates, that PRR has a smaller MSE than that of MLE.

MAPE is calculated as follows:

$$MAPE_{Av} = \frac{1}{r}\sum_{i=1}^{r} MAPE_i(\hat{\beta}), \tag{2.62}$$

$$\text{where} \quad MAPE_i(\hat{\beta}) = \frac{1}{p}\sum_{i=1}^{p} \frac{|(\beta-\hat{\beta})_i|}{|\beta_i|}. \tag{2.63}$$

2.5 Inducing Outliers

Since the thesis uses simulated data, it is important to test how the models perform with outliers, since real-world data sometimes contains outliers. Ten percent of the points in the response variable, y, were randomly selected and set to $y_i = 4\hat{\mu}_i$.

Outliers typically increase MSE. The increase in MSE, compared to the model with no outliers was analyzed with the following ratio:

$$MSE_{Ratio} = \frac{MSE_{AV}}{MSE_{AV}^{out}} = \frac{\sum_{i=1}^{r} SE_i}{\sum_{i=1}^{r} SE_i^{out}} . \tag{2.64}$$

In this context, the superscript ”out” refers to the model with outliers. The closer MSE_{Ratio} is to one, the more robust the model is to outliers. MSE_{Ratio} values close to zero reflect poor performance in the presence of outliers.

CHAPTER 3

THE MONTE CARLO SIMULATION

3.1 Simulation Technique

The main focus of this thesis is to compare the performance of different estimators in the presence of moderately to highly correlated independent variables. That means, the independent variables are a result of ρ, the coefficient of correlation. The thesis will use ρ values of 0.85, 0.90, 0.95, and 0.99. The independent variables are generated as follows:

$$x_{ij} = \sqrt{1-\rho^2} z_{ij} + \rho z_{ip}; \;\; i = 1, 2, ..., n; \;\; j = 1, 2, ..., p \tag{3.1}$$

where z_{ij} are pseudo-random numbers from the standard normal distribution.

The dependent variable (y) is generated in R using pseudo-random numbers from the Poisson distribution, with mean $\mu_i = e^{\beta_0+\beta_1 x_{i1}+...+\beta_p x_{ip}}$; i=1,2,...,n (3.2)

The values of β_j are chosen so that $\sum_{j=1}^{p} \beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$. These are common restrictions in many simulation studies (Månsson and Shukur, 2011).

The degrees of freedom are defined as $df = n - p - 1$. The value of β_0 will vary in this simulation, using the values $\beta_0 = -1, 0, 1$. Decreasing the value of β_0 results in a smaller value of μ_i, which leads to generating more values equal to zero. If a sample consists of only zeroes, this leads to non-convergence. For each intercept (β_0) value, IWLS converges with the following values for df.

$\beta_0 = 1 : df = 10, 15, 20, 30, 50$

$\beta_0 = 0 : df = 15, 20, 30, 50, 75$

$\beta_0 = -1 : df = 30, 50, 75, 100, 150$

3.2 Simulation Results

Figure 3.1 shows the MSE of each k estimator at each intercept, for $\rho = .85$. Tables for other correlation values can be found in the Appendix. The tables 3.1 through 3.3 show some of the simulation results. Full simulation results are available in the appendix.

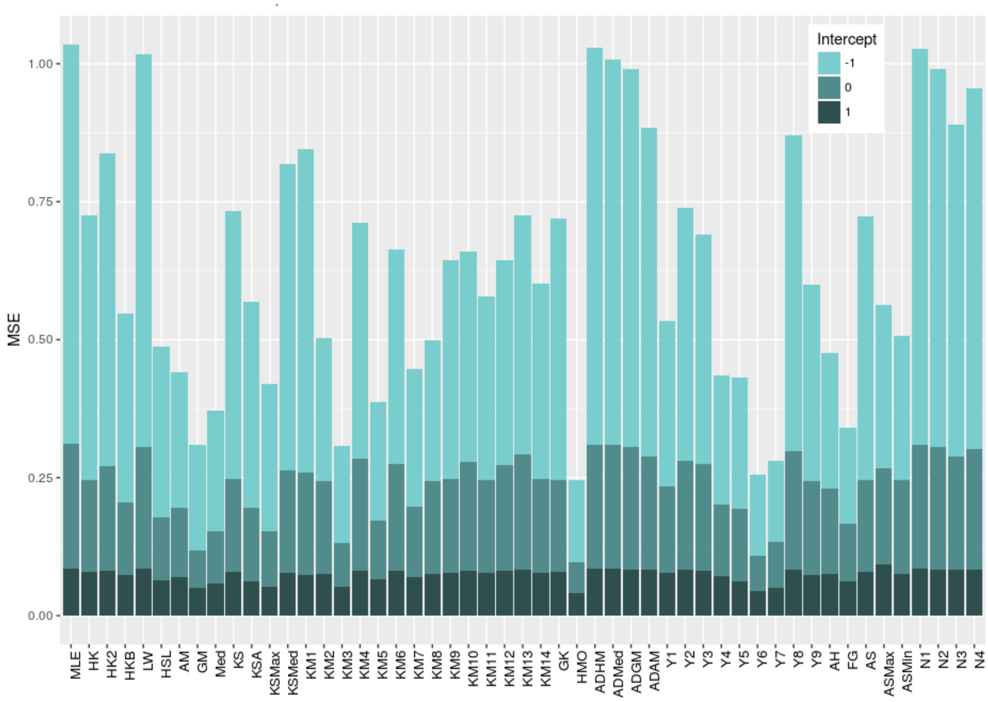

Figure 3.1: MSE for each k estimator, $n = 35$, $\rho = .85$

Table 3.1: Poisson Regression Simulation Results, P = 4, n = 35, Correlation(ρ) = .85

1. Performance = Percentage of times PRR outperforms MLE

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.$_1$	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.724	0	61.922	0.226	0	35.586	0.085	0	21.787
HK	0.480	93	51.108	0.167	92.36	30.862	0.079	80.44	21.108
HK_2	0.567	93.68	55.216	0.190	92.8	32.858	0.081	80.56	21.394
HKB	0.343	92	43.856	0.132	91.64	27.628	0.073	79.96	20.406
LW	0.713	94.4	61.635	0.220	93.36	35.238	0.085	81	21.785
HSL	0.309	90.96	40.885	0.115	91.12	25.740	0.063	77.48	19.189
AM	0.245	73.76	41.045	0.126	73.76	27.100	0.070	63.44	19.486
GM	0.190	84.2	34.500	0.069	85.24	20.684	0.050	74.48	17.292
Med	0.220	88.76	36.036	0.093	89.84	23.406	0.059	77.16	18.633
KS	0.486	93.08	51.422	0.168	92.4	30.990	0.079	80.44	21.115
KS_A	0.373	92.48	45.521	0.134	91.96	27.849	0.062	75.8	18.879
KS_{Max}	0.267	90.12	38.862	0.099	90.04	24.114	0.053	71.32	17.625
KS_{Med}	0.554	93.72	55.388	0.187	92.96	32.781	0.077	80.2	20.833
KM_1	0.586	93.76	56.312	0.185	92.88	32.433	0.074	79.72	20.467
KM_2	0.259	93.36	40.208	0.167	93	31.695	0.076	80.28	20.851
KM_3	0.176	84.08	34.124	0.080	86.68	22.225	0.052	74.4	17.572
KM_4	0.428	93.92	50.690	0.202	93.28	34.208	0.082	80.68	21.508
KM_5	0.214	90.32	35.985	0.107	91.6	25.337	0.065	79.04	19.392
KM_6	0.390	93.64	48.491	0.193	93.24	33.572	0.081	80.6	21.385
KM_7	0.248	91.4	38.607	0.129	92.16	27.668	0.069	79.52	20.023
KM_8	0.256	93.28	40.034	0.167	93	31.643	0.076	80.28	20.847
KM_9	0.397	93.12	47.955	0.169	92.72	31.319	0.078	80.44	21.045
KM_{10}	0.382	93.68	48.269	0.196	93.24	33.818	0.082	80.68	21.460
KM_{11}	0.334	93.2	44.925	0.168	92.76	31.490	0.077	80.44	20.980
KM_{12}	0.372	93.6	47.538	0.191	93.24	33.426	0.081	80.6	21.371
KM_{13}	0.433	94.08	51.165	0.210	93.36	34.768	0.083	80.92	21.654
KM_{14}	0.354	93.16	45.955	0.169	92.76	31.440	0.078	80.44	21.003
GK	0.474	93	50.959	0.167	92.36	30.855	0.079	80.44	21.107
HMO	0.148	81.76	31.857	0.056	80.6	19.618	0.041	59.88	15.994
AD_{HM}	0.718	94.4	61.721	0.225	93.36	35.561	0.085	81	21.785
AD_{Med}	0.699	94.28	61.030	0.224	93.36	35.483	0.085	81	21.779
AD_{GM}	0.685	94.28	60.405	0.222	93.36	35.334	0.084	81	21.771
AD_{AM}	0.594	92.16	56.878	0.206	92.6	34.048	0.083	80.52	21.548
Y_1	0.299	91.72	42.048	0.157	92.32	30.191	0.077	80.2	20.975
Y_2	0.458	93.48	51.180	0.198	93.12	33.712	0.083	80.8	21.597
Y_3	0.416	93.24	49.074	0.193	93	33.352	0.082	80.8	21.546
Y_4	0.235	89.52	38.039	0.130	91.4	27.502	0.071	79.44	20.269
Y_5	0.238	92.44	38.120	0.131	92.36	28.106	0.062	77.92	18.982
Y_6	0.147	83.96	31.808	0.064	87.56	19.812	0.045	70.68	16.565
Y_7	0.147	88.64	30.253	0.083	90.44	22.279	0.051	75.44	17.425
Y_8	0.574	94	56.748	0.213	93.32	34.760	0.084	80.92	21.680
Y_9	0.356	93.28	45.921	0.171	93	31.707	0.073	79.72	20.347
AH	0.245	92.72	38.736	0.154	92.76	30.378	0.076	80.4	20.785
FG	0.174	90.72	33.024	0.105	91.68	25.449	0.062	78.56	19.074
AS	0.477	93	51.031	0.167	92.36	30.859	0.079	80.44	21.108
AS_{Max}	0.295	67.32	46.471	0.174	65	32.004	0.093	56.24	21.985
AS_{Min}	0.262	93.44	40.257	0.170	93.12	31.983	0.075	80.2	20.756
N_1	0.717	94.36	61.698	0.225	93.36	35.558	0.085	81	21.785
N_2	0.685	94.24	60.485	0.221	93.36	35.254	0.084	80.96	21.758
N_3	0.602	93.92	56.995	0.205	93.2	34.033	0.083	80.84	21.615
N_4	0.654	94.08	59.318	0.217	93.32	34.968	0.084	80.96	21.732

Table 3.2: Poisson Regression Simulation Results, P = 4, n = 600, ρ = .85

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.01150986	0	8.512063	0.004234872	0	5.121368	0.001543436	0	3.098929
HK	0.01138037	63.24	8.46579	0.004207128	54.72	5.105159	0.001541863	53.36	3.097465
HK_2	0.01144787	63.68	8.490171	0.004220655	55.08	5.113087	0.001542557	53.44	3.098101
HKB	0.01127771	62.44	8.430031	0.004181141	54.4	5.089842	0.001540023	53.24	3.095712
LW	0.01150984	64.08	8.512056	0.004234812	55.16	5.121334	0.001543436	53.48	3.098929
HSL	0.01123679	61.68	8.416628	0.004169296	53.92	5.081385	0.001534335	52.48	3.090826
AM	0.03475409	39.56	12.721099	0.01681423	40.2	7.742635	0.008378082	40.44	4.529795
GM	0.01142872	55.2	8.48663	0.004286356	49.72	5.153869	0.001570687	50	3.108378
Med	0.01115915	60.48	8.392031	0.004134738	52.92	5.064132	0.001531667	52.32	3.08872
KS	0.01138331	63.24	8.466812	0.004207558	54.72	5.105391	0.001541867	53.36	3.097469
KS_A	0.01133133	62.68	8.451013	0.004198166	54.04	5.102479	0.001542575	51.28	3.097779
KS_{Max}	0.01132355	60.52	8.456149	0.00423504	52.6	5.133492	0.001709692	46.76	3.218501
KS_{Med}	0.0114438	63.68	8.488287	0.00421745	54.88	5.111084	0.001539763	53.16	3.095327
KM_1	0.01144208	63.56	8.487929	0.004212198	54.72	5.107901	0.001537675	53	3.093564
KM_2	0.01137574	63.2	8.464738	0.004223419	55.08	5.114695	0.001541466	53.28	3.097074
KM_3	0.01294655	53.92	8.877511	0.004527218	50.72	5.275558	0.001812893	49.48	3.197153
KM_4	0.0114616	63.84	8.495074	0.004231371	55.16	5.119295	0.001542867	53.44	3.098404
KM_5	0.01114499	61.56	8.382635	0.00416452	54.28	5.080265	0.00153611	52.92	3.09183
KM_6	0.01143557	63.6	8.48553	0.004229437	55.16	5.118169	0.001542666	53.44	3.098202
KM_7	0.01126475	62.4	8.427818	0.004188605	54.56	5.094419	0.001538607	53.04	3.094519
KM_8	0.01137431	63.2	8.464252	0.004223336	55.08	5.114648	0.001541463	53.28	3.097072
KM_9	0.01138219	63.24	8.466605	0.004214608	54.8	5.109547	0.001541768	53.32	3.097373
KM_{10}	0.01145188	63.72	8.491651	0.004230782	55.16	5.118969	0.001542822	53.44	3.09836
KM_{11}	0.01138017	63.2	8.466048	0.004218564	55.04	5.111862	0.001541672	53.32	3.097279
KM_{12}	0.01143267	63.6	8.484558	0.004229247	55.16	5.118063	0.001542658	53.44	3.098195
KM_{13}	0.01148367	63.92	8.5029	0.004233733	55.16	5.120697	0.001543209	53.48	3.098721
KM_{14}	0.01138096	63.24	8.466272	0.004217352	54.88	5.111153	0.001541706	53.32	3.097313
GK	0.01138036	63.24	8.465787	0.004207127	54.72	5.105159	0.001541863	53.36	3.097465
HMO	0.06170781	1.44	21.577229	0.029389854	0.12	16.474444	0.025302398	0	10.214126
AD_{HM}	0.01150984	64.08	8.512056	0.004234871	55.16	5.121367	0.001543436	53.48	3.098929
AD_{Med}	0.01150981	64.08	8.512045	0.004234867	55.16	5.121365	0.001543436	53.48	3.098929
AD_{GM}	0.0115097	64.08	8.512004	0.004234855	55.16	5.121358	0.001543435	53.48	3.098928
AD_{AM}	0.01160264	63.84	8.533477	0.004246769	55.16	5.125136	0.001580016	53.4	3.110466
Y_1	0.01147738	63.48	8.498829	0.004229116	55.08	5.117988	0.001545904	53.4	3.099343
Y_2	0.0115023	64.04	8.509383	0.004234152	55.16	5.12096	0.001543396	53.48	3.098891
Y_3	0.01149518	64	8.506978	0.004233192	55.16	5.120417	0.001543364	53.48	3.098861
Y_4	0.01150995	62.88	8.512104	0.004224116	54.96	5.114994	0.001558972	53.36	3.101794
Y_5	0.01068924	56.8	8.222426	0.004097358	53.56	5.037657	0.001522528	48.92	3.077115
Y_6	0.02257348	18.8	12.373545	0.005245744	33.88	5.844202	0.002009344	34.04	3.486024
Y_7	0.01181426	42.04	8.721614	0.003995929	47.48	4.9773	0.001565748	44.16	3.121525
Y_8	0.01150566	64.04	8.510631	0.004234499	55.16	5.121156	0.001543401	53.48	3.098895
Y_9	0.01096208	60.56	8.319336	0.004159612	54.04	5.075922	0.001524806	51.64	3.082006
AH	0.0113114	62.8	8.441103	0.004214862	54.92	5.109691	0.001541351	53.28	3.096957
FG	0.01098807	60.88	8.326171	0.004167353	54.32	5.081613	0.001535437	52.96	3.091333
AS	0.01138036	63.24	8.465789	0.004207128	54.72	5.105159	0.001541863	53.36	3.097465
AS_{Max}	0.05602494	29.88	16.310353	0.029175583	32.08	10.057508	0.014185851	33.12	5.722722
AS_{Min}	0.01137061	63.32	8.463101	0.00422612	55.16	5.116259	0.001541251	53.28	3.096858
N_1	0.01150984	64.08	8.512055	0.00423487	55.16	5.121367	0.001543436	53.48	3.098929
N_2	0.01150832	64.08	8.511526	0.004234663	55.16	5.121249	0.001543428	53.48	3.098921
N_3	0.01149267	63.96	8.506032	0.004231828	55.16	5.119603	0.001543289	53.48	3.09879
N_4	0.01150685	64.08	8.511014	0.004234464	55.16	5.121135	0.001543421	53.48	3.098914

Table 3.3: MSE Ratios (10% Outliers), P = 4, n = 35

	ρ = .85			ρ = .90			ρ = .95			ρ = .99		
k	-1	0	1	-1	0	1	-1	0	1	-1	0	1
MLE	0.548	0.236	0.094	0.524	0.228	0.091	0.505	0.223	0.086	0.518	0.218	0.086
HK	0.469	0.192	0.089	0.427	0.176	0.084	0.384	0.155	0.074	0.338	0.122	0.053
HK_2	0.497	0.21	0.091	0.46	0.196	0.087	0.421	0.179	0.079	0.392	0.151	0.064
HKB	0.407	0.166	0.085	0.358	0.146	0.078	0.306	0.12	0.064	0.258	0.086	0.038
LW	0.547	0.232	0.094	0.521	0.221	0.091	0.5	0.209	0.086	0.49	0.179	0.086
HSL	0.424	0.17	0.079	0.359	0.142	0.069	0.316	0.113	0.047	0.258	0.075	0.022
AM	0.628	0.296	0.138	0.52	0.207	0.097	0.353	0.121	0.053	0.144	0.039	0.016
GM	0.38	0.13	0.071	0.303	0.098	0.056	0.219	0.061	0.037	0.194	0.044	0.017
Med	0.366	0.142	0.077	0.304	0.12	0.065	0.24	0.088	0.048	0.2	0.057	0.025
KS	0.471	0.193	0.089	0.43	0.177	0.084	0.386	0.156	0.074	0.339	0.122	0.053
KS_A	0.465	0.193	0.082	0.418	0.176	0.072	0.383	0.148	0.054	0.307	0.104	0.035
KS_{Max}	0.441	0.182	0.082	0.385	0.154	0.066	0.336	0.121	0.044	0.234	0.074	0.022
KS_{Med}	0.509	0.216	0.09	0.481	0.208	0.085	0.462	0.197	0.078	0.461	0.182	0.069
KM_1	0.522	0.216	0.087	0.495	0.207	0.083	0.476	0.198	0.074	0.479	0.184	0.066
KM_2	0.398	0.217	0.093	0.365	0.215	0.088	0.335	0.21	0.085	0.294	0.201	0.098
KM_3	0.449	0.167	0.082	0.375	0.139	0.068	0.277	0.097	0.048	0.136	0.044	0.022
KM_4	0.465	0.229	0.094	0.434	0.229	0.091	0.413	0.226	0.089	0.381	0.242	0.111
KM_5	0.356	0.157	0.081	0.298	0.139	0.072	0.238	0.106	0.056	0.152	0.057	0.028
KM_6	0.447	0.225	0.094	0.418	0.223	0.09	0.395	0.218	0.087	0.346	0.218	0.101
KM_7	0.37	0.173	0.085	0.316	0.157	0.076	0.258	0.126	0.062	0.169	0.069	0.035
KM_8	0.397	0.216	0.093	0.363	0.215	0.088	0.334	0.209	0.085	0.293	0.2	0.097
KM_9	0.436	0.198	0.09	0.389	0.184	0.085	0.337	0.159	0.074	0.236	0.101	0.049
KM_{10}	0.438	0.225	0.094	0.406	0.224	0.09	0.383	0.219	0.088	0.341	0.228	0.108
KM_{11}	0.412	0.203	0.09	0.367	0.193	0.085	0.317	0.169	0.076	0.212	0.109	0.054
KM_{12}	0.438	0.224	0.094	0.406	0.221	0.09	0.383	0.215	0.087	0.333	0.214	0.1
KM_{13}	0.462	0.234	0.095	0.44	0.237	0.092	0.441	0.246	0.094	0.494	0.356	0.164
KM_{14}	0.419	0.201	0.09	0.373	0.189	0.085	0.321	0.165	0.075	0.214	0.103	0.051
GK	0.467	0.192	0.089	0.424	0.176	0.084	0.381	0.155	0.074	0.322	0.121	0.053
HMO	0.583	0.293	0.158	0.475	0.191	0.105	0.295	0.091	0.05	0.17	0.042	0.016
AD_{HM}	0.546	0.235	0.094	0.522	0.227	0.091	0.501	0.223	0.086	0.512	0.217	0.086
AD_{Med}	0.541	0.235	0.094	0.513	0.226	0.091	0.493	0.221	0.086	0.501	0.215	0.085
AD_{GM}	0.534	0.233	0.094	0.505	0.225	0.091	0.488	0.219	0.086	0.497	0.213	0.085
AD_{AM}	0.495	0.221	0.093	0.466	0.215	0.089	0.446	0.208	0.083	0.449	0.196	0.081
Y_1	0.374	0.189	0.09	0.315	0.173	0.084	0.245	0.138	0.072	0.117	0.063	0.041
Y_2	0.44	0.214	0.093	0.382	0.201	0.089	0.316	0.177	0.081	0.184	0.106	0.06
Y_3	0.425	0.212	0.092	0.371	0.198	0.088	0.296	0.172	0.08	0.153	0.087	0.055
Y_4	0.365	0.174	0.087	0.308	0.155	0.078	0.229	0.116	0.064	0.106	0.048	0.031
Y_5	0.441	0.225	0.096	0.412	0.223	0.091	0.421	0.227	0.089	0.427	0.259	0.114
Y_6	0.722	0.283	0.133	0.718	0.279	0.135	0.748	0.276	0.143	0.993	0.365	0.218
Y_7	0.513	0.232	0.109	0.495	0.234	0.108	0.494	0.241	0.115	0.629	0.299	0.181
Y_8	0.495	0.226	0.093	0.453	0.216	0.09	0.411	0.201	0.083	0.328	0.163	0.073
Y_9	0.477	0.232	0.094	0.465	0.233	0.092	0.459	0.237	0.092	0.439	0.242	0.111
AH	0.399	0.206	0.093	0.365	0.204	0.088	0.331	0.194	0.085	0.292	0.182	0.1
FG	0.38	0.179	0.085	0.337	0.169	0.078	0.299	0.148	0.066	0.23	0.109	0.051
AS	0.468	0.192	0.089	0.426	0.176	0.084	0.382	0.155	0.074	0.329	0.122	0.053
AS_{Max}	0.82	0.495	0.224	0.757	0.356	0.155	0.562	0.226	0.08	0.414	0.07	0.022
AS_{Min}	0.429	0.233	0.097	0.414	0.238	0.093	0.415	0.255	0.097	0.471	0.335	0.162
N_1	0.546	0.235	0.094	0.521	0.227	0.091	0.501	0.223	0.086	0.512	0.217	0.086
N_2	0.535	0.232	0.094	0.508	0.223	0.091	0.483	0.216	0.086	0.482	0.206	0.084
N_3	0.507	0.22	0.093	0.47	0.208	0.089	0.431	0.193	0.082	0.397	0.166	0.072
N_4	0.525	0.229	0.094	0.495	0.219	0.09	0.467	0.211	0.085	0.456	0.198	0.082

3.3 Performance as a Function of β_0

Simulations were varied by intercept value. The intercept values included were $\beta_0 = -1, 0, 1$. Performance, defined as the percentage of times that the estimator had a smaller MSE than the maximum likelihood model, decreased with an increase in intercept value. Both MAPE and MSE decreased with an increase in intercept value, which implies higher accuracy. The intercept value of $\beta_0 = 1$ produced the lowest MSE values.

3.4 Performance as a Function of ρ

Simulations were varied by correlation. The coefficient of correlation values used were $\rho = .85, .90, .95, .99$. Larger correlation values increased performance percentage, but also increased MSE and MAPE values, with the largest values at $\rho = .99$.

3.5 Performance as a Function of n

Simulations were varied by sample size (n). Simulations were run using the sample size $n = 35$ and $n = 600$. Generally, the larger sample size produced much smaller MSE values. It is also notable that the larger sample size affected the performance of individual k estimators. With $n = 35$, the k estimators with the lowest MSE values were Y_6, Y_7, HMO, FG, KM_2, KM_3, AM, AH, KM_2, KM_8, AS_{Min}, and AS_{Max}. With $n = 600$, the k estimators with the lowest MSE values were FG, Y_9, GM, HSL, Med, and Y_5.

3.6 Best Performing Estimators

Estimators were judged by their MSE value at both sample sizes, and performance with outliers. The k estimators that produced the lowest MSE values were selected. Performance with outliers was considered secondarily. The k estimators which produced the smallest MSE values are as follows: FG, Y_5, Y_9, AH, KM_2, and KM_8.

3.7 Proposed New Estimators

The following new estimators are proposed based on the seven best performing k estimators:

1. $CZ_1 = HarmonicMean\{FG, Y_5, Y_9, AH, KM_2, KM_8\}$

2. $CZ_2 = GeometricMean\{FG, Y_5, Y_9, AH, KM_2, KM_8\}$

3. $CZ_3 = ArithmeticMean\{FG, Y_5, Y_9, AH, KM_2, KM_8\}$

4. $CZ_4 = Median\{FG, Y_5, Y_9, AH, KM_2, KM_8\}$

5. $CZ_5 = Maximum\{FG, Y_5, Y_9, AH, KM_2, KM_8\}$

3.8 Performance of New Estimators

Simulations were then run again using these five new estimators. All five estimators performed fairly well. The best performing estimators were CZ_1, CZ_5, and CZ_3. CZ_1 frequently had a much smaller MSE than all other estimators. Figure 3.2 shows the MSE for the 15 best performing estimators (n=35, $\rho = .85, \beta_0 = -1$). See Tables 3.4 and 3.5 and tables A7 through A12 for simulation results with the new estimators.

Table 3.4: Poisson Regression Simulation Results (New Estimators), P = 4, n = 35, ρ = .85

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.7088039	0	62.29505	0.23496786	0	36.45803	0.08247555	0	21.63941
HK	0.480297	93.88	51.40929	0.17094316	93.28	31.42677	0.07678822	80.36	20.98787
HK_2	0.5602051	94.32	55.57943	0.19655611	93.8	33.56656	0.07922176	80.6	21.26998
HKB	0.3428405	92.6	43.95846	0.13468219	92.36	28.14434	0.0713326	79.56	20.32497
LW	0.7010262	95.28	62.03473	0.22952308	94	36.10883	0.08246094	80.84	21.63805
HSL	0.3031443	91.64	40.74378	0.12122675	91.32	26.40869	0.06166002	76.96	18.99735
AM	0.2329765	76.44	39.89231	0.11587944	75.36	26.28239	0.06998363	64.72	19.41486
GM	0.181186	85.52	33.60488	0.0704999	86.44	21.06886	0.04945014	73.96	17.26303
Med	0.2167196	89.24	35.78722	0.09771431	90.04	24.12621	0.0589686	77.36	18.64888
KS	0.4859627	93.96	51.73366	0.17250713	93.44	31.55979	0.076843	80.36	20.99371
KS_A	0.3694238	93.12	45.71068	0.13954408	92.4	28.43206	0.05979188	76.84	18.6792
KS_{Max}	0.2609286	91.16	38.70245	0.10474635	90.04	24.79633	0.05140393	71.96	17.39264
KS_{Med}	0.552789	94.44	55.83772	0.19488102	93.8	33.52551	0.07478304	80	20.72871
KM_1	0.5799505	94.56	56.6908	0.19231507	93.72	33.1996	0.07199818	79.36	20.35179
KM_2	0.2580438	93.64	40.19466	0.17640125	93.8	32.43258	0.07475317	80.12	20.77511
KM_3	0.1705738	85.52	33.35542	0.08165053	88.08	22.45	0.05130315	74.12	17.52435
KM_4	0.433351	94.8	51.20969	0.21112566	93.96	34.99528	0.08006497	80.68	21.3766
KM_5	0.2110569	91.08	35.63777	0.11213641	91.88	25.9181	0.06375041	78.96	19.34943
KM_6	0.3910307	94.32	48.77392	0.20137321	93.92	34.31872	0.07893522	80.6	21.25507
KM_7	0.2481702	92.36	38.53884	0.13514549	92.68	28.3034	0.06853487	79.44	19.98591
KM_8	0.2557311	93.6	40.01769	0.17572959	93.8	32.3795	0.07472062	80.12	20.77165
KM_9	0.4000639	94.08	48.30235	0.17439656	93.64	31.941	0.07629392	80.32	20.93768
KM_{10}	0.3857255	94.48	48.67987	0.20540942	93.96	34.61514	0.07969724	80.68	21.33783
KM_{11}	0.3363765	93.92	45.19374	0.17522211	93.8	32.15498	0.07578451	80.24	20.88461
KM_{12}	0.374274	94.32	47.84399	0.19939294	93.92	34.17779	0.07883548	80.6	21.24478
KM_{13}	0.4374917	94.92	51.68184	0.22003256	94	35.62622	0.08131545	80.84	21.51728
KM_{14}	0.3566891	93.96	46.2593	0.17499776	93.72	32.09033	0.07596436	80.24	20.90345
GK	0.4766073	93.84	51.28546	0.17084909	93.28	31.42034	0.0767832	80.36	20.98736
HMO	0.1471305	82.8	31.61548	0.05709397	80.92	19.82464	0.04045745	60.76	15.80407
AD_{HM}	0.70388	95.28	62.11222	0.23455228	94	36.43172	0.08246418	80.84	21.63829
AD_{Med}	0.6878477	95.24	61.46921	0.23377039	94	36.37625	0.08242619	80.84	21.63442
AD_{GM}	0.6741254	95.12	60.88254	0.23193067	94	36.25032	0.08235703	80.84	21.62629
AD_{AM}	0.5834015	93.24	56.93001	0.21807692	93.36	35.22643	0.08089365	80.52	21.44064
Y_1	0.2935828	92.48	41.82392	0.16556772	93.12	31.04446	0.07627836	80.48	20.93689
Y_2	0.4554355	94.32	51.55713	0.20672677	93.84	34.52761	0.08089034	80.84	21.47094
Y_3	0.4127159	94.2	49.18808	0.20015709	93.76	34.06478	0.08048913	80.8	21.42603
Y_4	0.2280989	90.24	37.39813	0.13570623	91.92	28.09225	0.07073657	79.72	20.26871
Y_5	0.2386109	92.92	38.27941	0.13788402	93.2	28.78409	0.06085483	77.84	18.91436
Y_6	0.1493494	83.52	32.00474	0.06747108	88.36	20.31241	0.0444064	70.2	16.50523
Y_7	0.1494385	88.84	30.3728	0.08726362	91.2	22.76762	0.05023304	74.92	17.36867
Y_8	0.5746807	94.92	57.15903	0.22168357	93.96	35.59712	0.08158783	80.84	21.5436
Y_9	0.3643156	93.92	46.33327	0.17723258	93.72	32.34682	0.07107182	79.68	20.22165
AH	0.2459471	93.32	38.73002	0.16063297	93.68	30.97715	0.0741905	80.16	20.71211
FG	0.1730899	90.92	32.80834	0.11048376	92.32	25.99092	0.06127913	78.64	19.03669
AS	0.4784081	93.84	51.34638	0.17089566	93.28	31.42352	0.07678569	80.36	20.98761
AS_{Max}	0.2856356	69.16	45.54903	0.16075411	65.92	31.17024	0.09155866	58.04	21.6205
AS_{Min}	0.2620687	93.88	40.27011	0.17957039	93.8	32.74567	0.0740133	80.08	20.69343
N_1	0.7033129	95.28	62.09097	0.23450362	94	36.42864	0.08246284	80.84	21.63816
N_2	0.6741733	95.16	60.89254	0.22975064	94	36.10502	0.08223508	80.88	21.61395
N_3	0.5955888	94.64	57.39412	0.21194221	93.88	34.80084	0.08102742	80.84	21.47984
N_4	0.6466877	94.96	59.73238	0.22533975	94	35.79882	0.08202104	80.84	21.59104
CZ_1	0.1407474	87.68	30.11826	0.07844236	91.32	22.04534	0.04887127	75.4	17.19815
CZ_2	0.2586082	93.32	40.06886	0.16161437	93.64	31.11928	0.07158889	79.88	20.37312
CZ_3	0.2306893	93.04	37.87982	0.14956652	93.48	30.00945	0.06836374	79.4	19.95697
CZ_4	0.2458594	93.44	39.16432	0.16362247	93.76	31.30681	0.07180167	79.92	20.37833
CZ_5	0.1698079	90.84	32.49987	0.10988402	92.28	25.91898	0.05880575	77.8	18.65761

Table 3.5: Poisson Regression Simulation Results (New Estimators), P = 4, n = 600, ρ = .85

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.01167838	0	8.502608	0.004365237	0	5.207782	0.001547143	0	3.124962
HK	0.01153794	65.6	8.453743	0.004341133	53.68	5.193034	0.00154551	52.76	3.123501
HK_2	0.01161029	65.96	8.478984	0.004353155	53.76	5.200267	0.001546264	52.76	3.12419
HKB	0.01142269	65.08	8.413799	0.004319928	53.28	5.179568	0.001543728	52.6	3.121959
LW	0.01167836	66.4	8.502601	0.004365187	53.8	5.207751	0.001547142	52.88	3.124962
HSL	0.01133801	63.96	8.390337	0.004313053	52.4	5.17182	0.001536315	51.76	3.115244
AM	0.03684882	40.8	12.899259	0.016292862	37.84	7.788854	0.008506284	40.08	4.567289
GM	0.01155362	56.72	8.448071	0.004425106	47.64	5.244466	0.001565284	49.44	3.141106
Med	0.01128418	62.8	8.368686	0.004294307	51.48	5.163363	0.001540061	51.52	3.118865
KS	0.01154119	65.6	8.454838	0.004341533	53.68	5.193264	0.001545515	52.76	3.123505
KS_A	0.01147227	65.2	8.427005	0.004330729	53.12	5.187119	0.001552364	50.48	3.13403
KS_{Max}	0.01144016	62.84	8.423687	0.004396916	51.08	5.232922	0.001720823	46.88	3.258432
KS_{Med}	0.01160742	65.96	8.478054	0.004350421	53.72	5.198647	0.001543709	52.52	3.122044
KM_1	0.01160453	66.04	8.47635	0.004345627	53.64	5.195448	0.001542026	52.44	3.120535
KM_2	0.01153004	65.6	8.451081	0.004355489	53.8	5.201719	0.001545196	52.72	3.123253
KM_3	0.01339461	55.12	8.904883	0.004580054	49.6	5.322459	0.001730742	48.96	3.211411
KM_4	0.01162717	66.16	8.484925	0.004362245	53.8	5.205928	0.001546538	52.84	3.124422
KM_5	0.01126371	63.88	8.358653	0.004305227	53	5.170349	0.001540311	52.36	3.118959
KM_6	0.01159795	65.96	8.474695	0.004360609	53.8	5.204937	0.001546317	52.76	3.12422
KM_7	0.01141122	65	8.409056	0.004325866	53.44	5.183524	0.001542712	52.52	3.121121
KM_8	0.01152844	65.6	8.450537	0.004355413	53.8	5.201674	0.001545193	52.72	3.123251
KM_9	0.01153903	65.6	8.4541	0.004347791	53.72	5.197036	0.001545445	52.72	3.123455
KM_{10}	0.01161566	66.08	8.480973	0.004361691	53.8	5.205598	0.001546498	52.76	3.124389
KM_{11}	0.01153611	65.6	8.453118	0.004351266	53.76	5.199145	0.001545369	52.72	3.123395
KM_{12}	0.01159474	65.92	8.473598	0.004360438	53.8	5.204832	0.001546309	52.76	3.124214
KM_{13}	0.01165043	66.32	8.493008	0.004364233	53.8	5.207169	0.001546896	52.88	3.12474
KM_{14}	0.01153721	65.6	8.453488	0.004350205	53.76	5.198501	0.001545397	52.72	3.123417
GK	0.01153793	65.6	8.453739	0.004341133	53.68	5.193034	0.00154551	52.76	3.123501
HMO	0.06203947	1.28	21.643385	0.029620923	0.52	16.546456	0.025305244	0	10.203905
AD_{HM}	0.01167836	66.4	8.502601	0.004365236	53.8	5.207781	0.001547142	52.88	3.124962
AD_{Med}	0.01167833	66.4	8.50259	0.004365233	53.8	5.20778	0.001547142	52.88	3.124962
AD_{GM}	0.01167821	66.4	8.50255	0.004365225	53.8	5.207774	0.001547142	52.88	3.124961
AD_{AM}	0.01207023	66.16	8.553265	0.004367057	53.72	5.208822	0.001551946	52.8	3.126643
Y_1	0.01164386	65.92	8.491623	0.004359091	53.8	5.203649	0.001546375	52.84	3.124096
Y_2	0.01166981	66.36	8.499625	0.004364614	53.8	5.207393	0.001547105	52.88	3.12493
Y_3	0.0116619	66.36	8.496907	0.004363814	53.8	5.206887	0.001547068	52.88	3.1249
Y_4	0.01167107	65.16	8.489066	0.004353103	53.56	5.199508	0.001545955	52.76	3.123742
Y_5	0.0107892	59.64	8.187567	0.004250423	52	5.13548	0.001525166	48.04	3.102835
Y_6	0.02221462	19.96	12.222615	0.005468852	33.24	5.967537	0.00195348	34.32	3.458929
Y_7	0.01173648	43.92	8.620383	0.004206045	46.28	5.107267	0.001557028	42.8	3.134523
Y_8	0.01167372	66.4	8.500968	0.004364905	53.8	5.207575	0.00154711	52.88	3.124935
Y_9	0.01110554	63.24	8.30004	0.004302298	52.64	5.167192	0.00152762	50.72	3.107004
AH	0.01146101	65.4	8.42728	0.004348057	53.72	5.197224	0.001544997	52.72	3.123055
FG	0.01110137	63.08	8.301136	0.004307974	53.16	5.172177	0.001539115	52.08	3.117793
AS	0.01153793	65.6	8.453741	0.004341133	53.68	5.193034	0.00154551	52.76	3.123501
AS_{Max}	0.05917371	31.12	16.556264	0.027935167	30.4	10.093997	0.014116382	34.2	5.726085
AS_{Min}	0.01152332	65.52	8.448792	0.004357812	53.8	5.203143	0.001545007	52.68	3.123095
N_1	0.01167835	66.4	8.5026	0.004365236	53.8	5.207781	0.001547142	52.88	3.124962
N_2	0.01167667	66.4	8.502024	0.004365055	53.8	5.20767	0.001547135	52.88	3.124955
N_3	0.01165942	66.36	8.496059	0.004362625	53.8	5.206162	0.001546995	52.88	3.124834
N_4	0.01167504	66.4	8.501467	0.004364881	53.8	5.207564	0.001547127	52.88	3.124949
CZ_1	0.01060478	57.92	8.121172	0.004256448	51.96	5.13928	0.001528654	50.72	3.108154
CZ_2	0.01137233	65.04	8.395642	0.004339028	53.72	5.191508	0.001541958	52.52	3.120337
CZ_3	0.01119154	63.76	8.33146	0.004315986	53.28	5.177007	0.001533876	51.6	3.112606
CZ_4	0.01132634	64.72	8.379769	0.004333294	53.6	5.187842	0.001542122	52.56	3.120499
CZ_5	0.01078788	59.64	8.186934	0.00424976	51.96	5.135044	0.001525163	48.04	3.102847

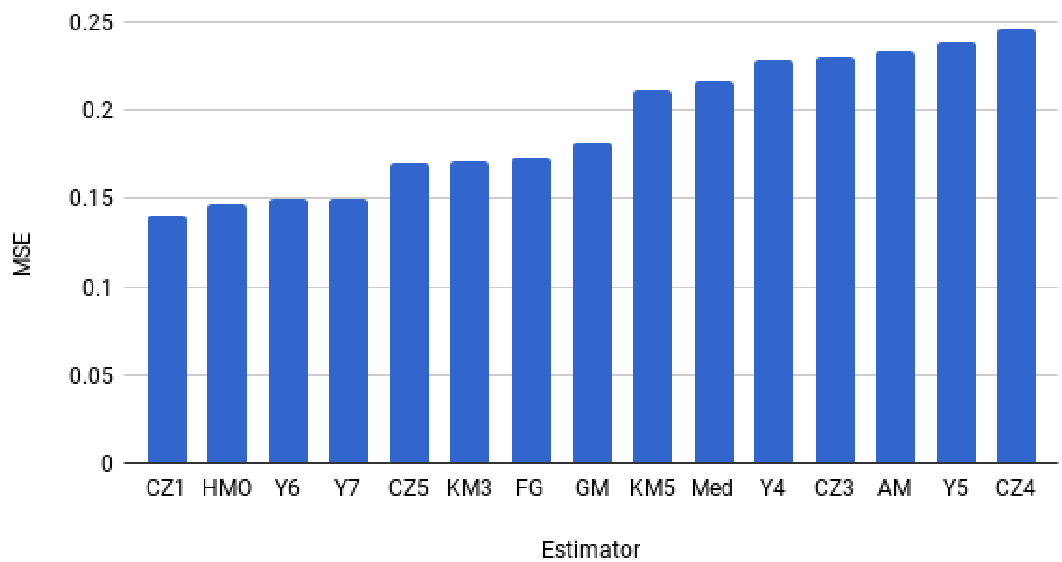

Figure 3.2: Estimators with Smallest MSE (n=35, ρ = .85, $\beta_0 = -1$)

CHAPTER 4

APPLICATION

The ridge estimators are applied here using data from a study of 659 individuals by Sellar et al. (1985), which estimated the value of recreational boating in East Texas. Four lakes were studied: Lakes Conroe, Livingston, Somerville, and Houston. The study tracked the number of visits an individual has made to the lakes (NTrip), as well as their income level, quality rating scores for the lakes, whether the individual has engaged in water-skiing at Lake Somerville (SKI), whether the individual has paid an annual fee at Lake Somerville (FEE), and the travel costs to Lakes Conroe, Somerville, and Houston (LC, LS, and LH). The three cost variables (LC, LS, and LH) are highly correlated. The level of multicollinearity is measured here using the Variance Inflation Factor (VIF). VIF is the ratio of variance in the full model, with all terms, divided by the variance of a reduced model, with only one term. With high VIF values (VIF greater than 3), the variance of the estimated coefficient is increased. Table 4.1 shows the VIF for each regressor. It is clear from this table that LC, LS, and LH are highly correlated. In the case of severe multicollinearity, a ridge regression model is recommended. Table 4.2 shows the MSE values of the MLE model and the 55 ridge regression models. All ridge estimators had a lower MSE than the MLE model. The MLE model had an MSE of 39.61, while all of the ridge models produced an MSE smaller than 35. Most k estimators performed fairly well, with an MSE of about 31, including the five newly introduced estimators. Table 4.3 shows the significance of the regression coefficients of MLE and two of the best performing new estimators, CZ_1 and CZ_3. Table 4.4 shows the estimated regression coefficients for each model. All coefficients had a positive slope, with the exception of LS, which consistently had a negative coefficient value. This means that the number of trips (NTrip) decreases as

LS, the travel cost to Lake Somerville, increases. The number of trips increases with an increase in the other variables (FEE, LC, and LH).

Table 4.1: VIF for each regressor

SKI	FEE	LC	LS	LH
1.088497	1.024602	11.838841	3.188796	10.908486

Table 4.2: MSE for Each k Estimator - Recreation Data

k	MSE	k	MSE	k	MSE	k	MSE
MLE	39.60541	KM_2	31.24646	HMO	33.11322	AH	31.24154
HK	31.23623	KM_3	31.88795	AD_{HM}	31.23278	FG	31.34948
HK_2	31.23444	KM_4	31.23298	AD_{Med}	31.23278	AS	31.23623
HKB	31.26526	KM_5	31.437	AD_{GM}	31.23278	AS_{Max}	34.99645
LW	31.23278	KM_6	31.2334	AD_{AM}	31.23278	AS_{Min}	31.25382
HSL	34.21166	KM_7	31.54585	Y_1	31.23322	N_1	31.23278
AM	33.78956	KM_8	31.24646	Y_2	31.2328	N_2	31.23278
GM	32.33407	KM_9	31.23828	Y_3	31.23288	N_3	31.2328
Med	33.32627	KM_{10}	31.23298	Y_4	31.23651	N_4	31.23278
KS	31.23623	KM_{11}	31.24073	Y_5	31.56081	CZ_1	31.66421
KS_A	32.66997	KM_{12}	31.23341	Y_6	33.47754	CZ_2	31.27203
KS_{Max}	34.24815	KM_{13}	31.23278	Y_7	32.29833	CZ_3	31.32226
KS_{Med}	31.63274	KM_{14}	31.24003	Y_8	31.23278	CZ_4	31.29118
KM_1	31.30317	GK	31.23623	Y_9	31.35176	CZ_5	31.56081

Table 4.3: Significance of Coefficients from MLE and Selected New Estimators

	MLE		CZ_1		CZ_3	
Coefficient	Z-Value	P-Value	Z-Value	P-Value	Z-Value	P-Value
SKI	9.737	0	16.20397	0	15.13735	0
FEE	14.380	0	32.1527	0	31.83864	0
LC	2.660	0.00781	15.3222	0	14.96995	0
LS	-39.974	0	-58.57733	0	-56.72167	0
LH	16.499	0	23.76201	0	22.95511	0

Table 4.4: Estimated Regression Coefficients for Each Model

	MLE	HK	$HK2$	HKB	LW	HSL	AM	GM	Med	KS	KS_A	KS_{Max}	KS_{Med}	KM_1
Intercept	0.91031873	3.19149285	3.18995509	3.19953738	3.18617459	1.55791431	1.79343382	2.83740657	2.0876324	3.19149267	2.57963846	1.53875576	3.16359274	3.2022094
SKI	0.52901662	1.20186633	1.2011092	1.206626	1.19934646	0.71884307	0.81273613	1.1507137	0.9222995	1.20186624	1.08201852	0.710987	1.21298568	1.20949062
FEE	1.131180122	8.65606841	8.78703856	7.77108972	9.0845962	0.4026696	0.51651339	1.87957988	0.7061991	8.65608374	1.26794246	0.39448975	4.51671343	7.15262984
LC	0.007795252	0.09211493	0.09193725	0.09326174	0.09152701	0.06406152	0.06923024	0.09140163	0.0756613	0.09211491	0.08620161	0.06364088	0.09597896	0.09399561
LS	-0.060464674	-0.22039964	-0.22008618	-0.2224643	-0.21936751	-0.1987449	-0.20436332	-0.22658836	-0.2111709	-0.2203996	-0.22180437	-0.19828154	-0.22855807	-0.22383996
LH	0.043712553	0.11681521	0.11665859	0.11789848	0.11630583	0.14383841	0.1415357	0.12945338	0.1384905	0.11681519	0.13288592	0.14402008	0.12257643	0.11868691

	KM_2	KM_3	KM_4	KM_5	KM_6	KM_7	KM_8	KM_9	KM_{10}	KM_{11}	KM_{12}	KM_{13}	KM_{14}	GK
Intercept	3.19589388	3.08581122	3.18753578	3.19575751	3.18854213	3.18073117	3.195894	3.19274871	3.18754508	3.1939092	3.18854485	3.18637595	3.19360618	3.19149285
SKI	1.20423228	1.20253324	1.19996724	1.21369767	1.20043575	1.21408061	1.2042324	1.20250705	1.19997153	1.20312075	1.20043703	1.19943742	1.20295826	1.20186633
FEE	8.23162786	3.30135553	8.98089436	5.80330161	8.90186727	5.0328802	8.2316134	8.5435417	8.98017362	8.43416842	8.90165024	9.06947037	8.46328555	8.65606828
LC	0.09267738	0.09552274	0.09167098	0.09531553	0.09177999	0.09580962	0.0926774	0.09226608	0.09167198	0.09241163	0.09178029	0.09154807	0.09237302	0.09211493
LS	-0.22140222	-0.22934942	-0.21961897	-0.22656073	-0.21980989	-0.22785419	-0.2214022	-0.22066746	-0.21962071	-0.22092642	-0.21981041	-0.21940425	-0.22085761	-0.22039964
LH	0.11732897	0.12499882	0.11642831	0.12053738	0.11652197	0.12171382	0.117329	0.11695047	0.11642916	0.11708258	0.11652223	0.11632366	0.11704734	0.11681521

	HMO	AD_{HM}	AD_{Med}	AD_{GM}	AD_{AM}	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
Intercept	2.23804748	3.18617459	3.18617488	3.18617467	3.18617501	3.1881686	3.18664548	3.18713862	3.19168875	3.17810105	1.98686418	2.8623963	3.18638515	3.20186293
SKI	0.97454355	1.19934646	1.19934659	1.1993465	1.19934665	1.20026084	1.19955964	1.19978465	1.20196484	1.21397732	0.88581792	1.1566807	1.19944158	1.21170148
FEE	0.83418849	9.08459633	9.08457424	9.08458978	9.08456466	8.93145005	9.04911065	9.01151371	8.63887168	4.93889436	0.6336561	1.9657133	9.06877815	6.57541435
LC	0.07892492	0.09152701	0.09152704	0.09152702	0.09152705	0.09173926	0.09157639	0.09162858	0.09213812	0.09585167	0.07346391	0.0918774	0.09154904	0.09461549
LS	-0.21454439	-0.21936751	-0.21936756	-0.21936753	-0.21936759	-0.21973849	-0.21945367	-0.21954483	-0.22044066	-0.22799366	-0.2088677	-0.2269908	-0.21940593	-0.22505906
LH	0.13685721	0.11630583	0.11630586	0.11630584	0.11630587	0.11648687	0.11634769	0.11639209	0.11683584	0.12186573	0.1395547	0.1290825	0.11632448	0.11945293

	AH	FG	AS	AS_{Max}	AS_{Min}	N_1	N_2	N_3	N_4	CZ_1	CZ_2	CZ_3	CZ_4	CZ_5
Intercept	3.19424116	3.20194123	3.19149285	1.1791987	3.1976683	3.18617459	3.18618088	3.18661168	3.18618713	3.15626682	3.20032668	3.2024367	3.20175267	3.17810105
SKI	1.20330072	1.21162004	1.20186633	0.558084	1.2053196	1.19934646	1.1993493	1.19954428	1.19935212	1.21229473	1.20726499	1.21049236	1.20874091	1.21397732
FEE	8.40177813	6.59945829	8.65606835	0.2633489	8.027086	9.08459629	9.08412477	9.05167163	9.08365617	4.34538136	7.64093891	6.90729812	7.32435597	4.93889436
LC	0.09245446	0.09459113	0.09211493	0.0557599	0.0929405	0.09152701	0.09152766	0.09157283	0.09152832	0.09599775	0.09342138	0.09426765	0.09379837	0.09585167
LS	-0.22100284	-0.22500974	-0.22039964	-0.1893894	-0.2218775	-0.21936751	-0.21936866	-0.21944746	-0.21936979	-0.22875395	-0.22275897	-0.22436667	-0.22346449	-0.22799366
LH	0.11712182	0.11942035	0.11681521	0.1472323	0.1175802	0.11630583	0.11630639	0.11634466	0.11630694	0.12287993	0.11806199	0.11900852	0.11846496	0.12186573

CHAPTER 5

SUMMARY AND CONCLUDING REMARKS

In this thesis, we investigated some ridge regression (RR) estimators for estimating the ridge regression parameter k for the Poisson regression model, when the explanatory variables are moderate to highly correlated. Since a theoretical comparison among the estimators is not possible, a Monte Carlo simulation study has been conducted to compare the performance of the proposed ridge regression estimators. In simulation, we evaluated proposed estimators of k for different experimental conditions: the degree of correlation, sample size, and intercept value. For each combination we performed 2500 replications. The evaluation of the performance of these estimators has been done by using the MSE, MAPE, and the proportion of times the RR estimators outperformed the ML estimator. The performance of the estimators in presence of outliers was also analyzed.

The simulation results show that increasing the correlation among the explanatory variables has a negative effect on the performance of the estimators (i.e. the MSE and MAPE increases). In most of the cases, ridge regression estimators outperform the ML estimator, even when the correlation between the explanatory variables is large. When the sample size increases, the MSE and MAPE values decreases for all estimators, including the ML estimator. From this analysis, the six best performing estimators were chosen. Based on these six best estimators, five new estimators were proposed, CZ_1, CZ_2, CZ_3, CZ_4, and CZ_5. Simulations were then run again with these five new estimators added in. These five new estimators performed very well, producing small MSE values. The best performing of these new estimators were CZ_1, CZ_3, and CZ_5, with CZ_1 often producing the lowest MSE. These three estimators are therefore recommended for PRR models.

Data from a study on recreational habits was used to illustrate the findings of the thesis. This study tracked the number of boating trips 659 individuals took to four lakes in East Texas. Among the variables studied, the cost of travel to three lakes: Conroe, Somerville, and Houston, were highly correlated, which produced severe multicollinearity with the MLE model. MSE was decreased with all 55 PRR models, so the accuracy of prediction was increased.

We have considered the Poisson regression model. As a future research, this thesis can further be extended for the following models:

i Two parameter Poisson regression model. That means, a model with both ridge regression estimator and Liu (1993) estimator.

ii Ridge Regression zero inflated Poisson regression model.

iii Ridge Regression Based on Some Robust Estimators (see Samkar and Alpu, 2010).

iv Ridge regression estimators for the restricted linear model.

v Restricted ridge regression estimators for a semiparametric regression model.

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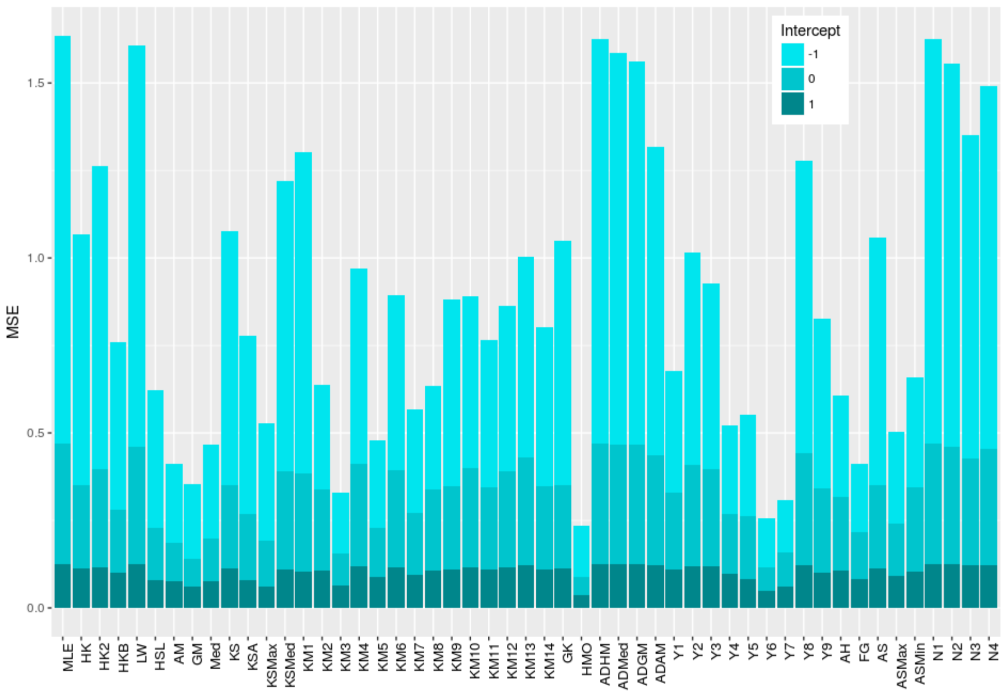

Figure A1: MSE for each k estimator, ρ = .90

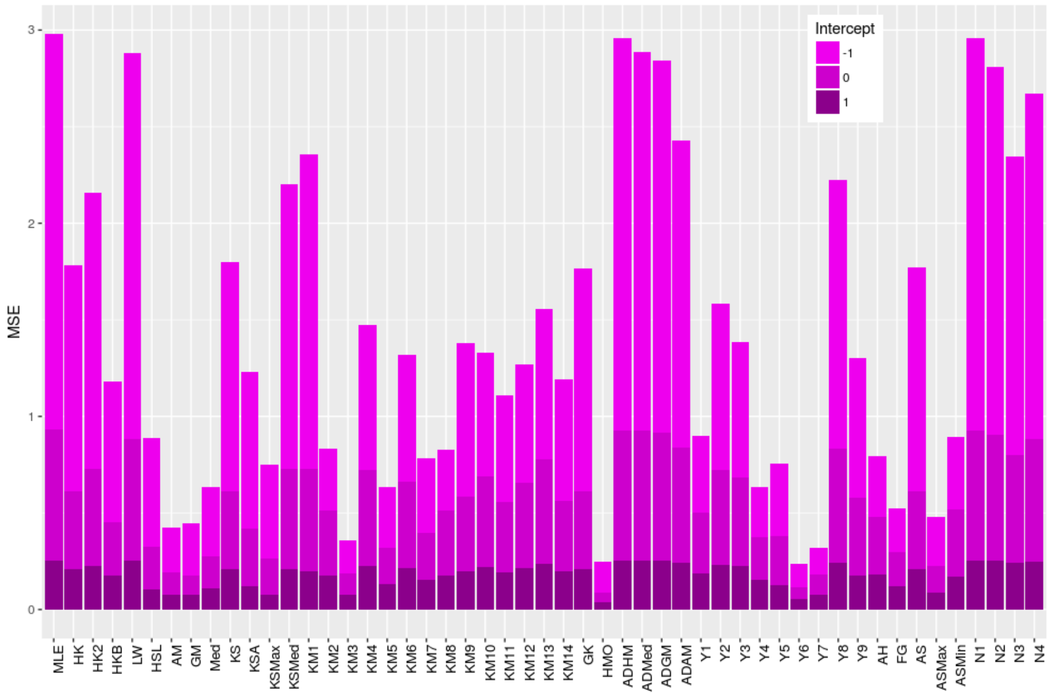

Figure A2: MSE for each k estimator, ρ = .95

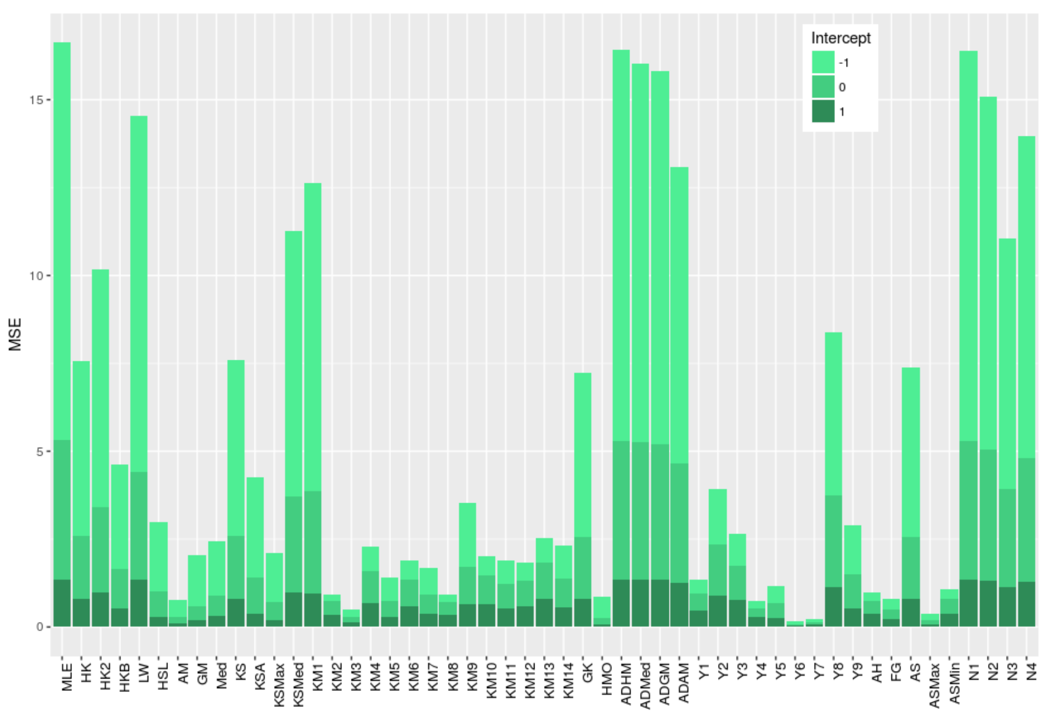

Figure A3: MSE for each k estimator, ρ = .99

Table A1: Poisson Regression Simulation Results, P = 4, n = 35, ρ = .90

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Performance	MAPE	MSE	Performance	MAPE	MSE	Performance	MAPE
MLE	1.164	0	77.745	0.347	0	44.192	0.124	0	26.685
HK	0.716	96.72	61.556	0.238	96.2	36.754	0.112	87.76	25.485
HK_2	0.865	96.96	67.510	0.280	96.64	39.886	0.117	87.84	26.008
HKB	0.478	95.96	50.944	0.180	95.88	32.190	0.101	87.32	24.329
LW	1.147	97.48	77.295	0.335	96.72	43.543	0.124	88.04	26.682
HSL	0.392	95.08	45.112	0.149	95.44	28.978	0.080	85.56	21.601
AM	0.226	85.96	39.106	0.112	84.04	25.580	0.075	77.04	20.003
GM	0.213	92.68	35.018	0.079	92.64	21.684	0.061	84.12	19.126
Med	0.270	94.36	38.699	0.121	94.52	26.397	0.077	85.16	21.391
KS	0.724	96.8	61.944	0.240	96.32	36.928	0.112	87.76	25.495
KS_A	0.510	96.36	52.994	0.187	95.88	32.655	0.080	85.44	21.584
KS_{Max}	0.335	94.96	42.734	0.131	95.08	27.229	0.062	82.56	19.065
KS_{Med}	0.830	97.16	68.239	0.279	96.6	40.129	0.110	87.6	25.242
KM_1	0.917	97.2	70.077	0.279	96.6	39.942	0.105	87.24	24.703
KM_2	0.298	96.8	43.659	0.232	96.68	37.713	0.107	87.64	25.053
KM_3	0.172	92.4	33.410	0.092	92.96	23.469	0.064	84	19.492
KM_4	0.558	97.16	58.970	0.294	96.68	41.656	0.118	87.92	26.158
KM_5	0.251	95.12	38.746	0.142	95.44	28.975	0.087	86.68	22.706
KM_6	0.499	96.96	55.749	0.278	96.72	40.603	0.116	87.88	25.941
KM_7	0.297	95.92	42.379	0.175	95.84	32.244	0.095	87.08	23.694
KM_8	0.295	96.8	43.452	0.231	96.68	37.640	0.107	87.64	25.047
KM_9	0.532	96.76	56.011	0.238	96.48	37.317	0.111	87.76	25.379
KM_{10}	0.489	97	55.555	0.284	96.72	41.078	0.117	87.88	26.085
KM_{11}	0.419	96.8	51.055	0.236	96.64	37.504	0.109	87.68	25.272
KM_{12}	0.474	96.96	54.496	0.274	96.72	40.394	0.116	87.84	25.922
KM_{13}	0.571	97.32	59.806	0.310	96.76	42.636	0.121	88.04	26.421
KM_{14}	0.454	96.8	52.698	0.237	96.56	37.451	0.110	87.68	25.310
GK	0.699	96.72	61.298	0.238	96.2	36.744	0.112	87.76	25.484
HMO	0.145	91.56	30.765	0.051	90.84	18.422	0.038	77.28	15.488
AD_{HM}	1.155	97.48	77.481	0.346	96.76	44.151	0.124	88.04	26.683
AD_{Med}	1.118	97.44	76.469	0.344	96.76	44.061	0.124	88.04	26.675
AD_{GM}	1.095	97.36	75.729	0.342	96.76	43.889	0.124	88.04	26.663
AD_{AM}	0.880	96.48	69.282	0.316	96.04	42.222	0.121	87.72	26.322
Y_1	0.350	95.72	45.575	0.218	96.32	35.489	0.110	87.68	25.245
Y_2	0.607	96.88	59.546	0.290	96.68	40.919	0.120	87.96	26.334
Y_3	0.531	96.92	56.096	0.277	96.64	40.150	0.119	88	26.244
Y_4	0.255	94.64	39.383	0.170	95.36	31.275	0.098	87.2	23.934
Y_5	0.291	96.44	42.523	0.179	96.36	33.092	0.082	86.2	22.113
Y_6	0.138	92.2	30.894	0.067	93.8	20.048	0.050	81.12	17.591
Y_7	0.149	94.88	30.351	0.097	95.4	24.106	0.062	84.44	19.357
Y_8	0.837	97.24	69.071	0.320	96.72	42.757	0.122	88.04	26.492
Y_9	0.483	97	53.847	0.242	96.64	38.099	0.101	87.2	24.351
AH	0.289	96.48	42.299	0.211	96.56	35.769	0.106	87.64	24.954
FG	0.195	95.68	35.018	0.136	95.92	29.033	0.081	86.72	22.165
AS	0.707	96.72	61.422	0.238	96.2	36.749	0.112	87.76	25.485
AS_{Max}	0.264	80.64	43.663	0.149	76.44	29.595	0.091	70.64	21.493
AS_{Min}	0.314	96.84	44.482	0.239	96.72	38.248	0.105	87.64	24.912
N_1	1.154	97.48	77.450	0.346	96.76	44.146	0.124	88.04	26.683
N_2	1.094	97.36	75.636	0.338	96.76	43.663	0.124	88.04	26.638
N_3	0.924	97.08	70.023	0.306	96.72	41.646	0.121	88	26.382
N_4	1.037	97.28	73.880	0.330	96.72	43.204	0.123	88.04	26.596

Table A2: Poisson Regression Simulation Results, P = 4, n = 35, ρ = .95

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Performance	MAPE	MSE	Performance	MAPE	MSE	Performance	MAPE
MLE	2.049	0	105.552	0.677	0	61.743	0.254	0	37.326
HK	1.172	99.32	78.321	0.402	99.08	47.253	0.209	95.84	34.239
HK_2	1.431	99.36	87.297	0.502	99.16	52.974	0.226	95.88	35.476
HKB	0.729	98.96	61.697	0.281	98.96	39.534	0.173	95.88	31.409
LW	1.996	99.52	104.443	0.630	99.2	59.746	0.254	95.92	37.315
HSL	0.567	98.56	52.090	0.217	99	33.202	0.105	94.68	24.079
AM	0.231	92.68	38.964	0.116	92.64	25.054	0.077	89.2	20.026
GM	0.270	97.08	37.412	0.098	98.08	22.809	0.078	94.12	21.027
Med	0.357	98	42.788	0.165	98.8	29.924	0.112	95.24	25.046
KS	1.183	99.36	78.762	0.405	99.08	47.480	0.209	95.84	34.259
KS_A	0.810	99.08	65.196	0.300	99	40.312	0.120	94.92	25.742
KS_{Max}	0.484	98.64	49.335	0.187	99	30.987	0.078	93.84	20.613
KS_{Med}	1.478	99.36	90.789	0.516	99.2	54.408	0.209	95.92	34.273
KM_1	1.629	99.36	94.486	0.527	99.2	54.715	0.199	95.92	33.325
KM_2	0.319	99.28	45.152	0.336	99.16	45.955	0.178	95.88	32.457
KM_3	0.176	96.64	33.264	0.106	98.08	24.352	0.078	94.12	21.283
KM_4	0.752	99.36	68.660	0.497	99.2	54.706	0.223	95.84	35.658
KM_5	0.315	98.6	42.054	0.191	98.92	33.198	0.130	95.56	27.601
KM_6	0.652	99.4	63.897	0.451	99.2	52.336	0.213	95.8	35.013
KM_7	0.383	98.92	46.876	0.248	98.96	38.149	0.151	95.76	29.624
KM_8	0.316	99.24	44.946	0.334	99.16	45.847	0.178	95.88	32.445
KM_9	0.797	99.32	67.610	0.382	99.04	47.172	0.200	95.88	33.780
KM_{10}	0.637	99.36	63.542	0.471	99.2	53.461	0.219	95.84	35.446
KM_{11}	0.553	99.32	58.243	0.363	99.16	46.763	0.192	95.88	33.339
KM_{12}	0.615	99.36	62.207	0.444	99.2	51.968	0.212	95.84	34.965
KM_{13}	0.778	99.32	69.923	0.542	99.2	57.014	0.234	95.84	36.408
KM_{14}	0.626	99.32	61.347	0.369	99.12	46.915	0.195	95.88	33.494
GK	1.154	99.32	77.946	0.401	99.08	47.233	0.209	95.84	34.236
HMO	0.159	97.24	30.803	0.049	98.12	17.311	0.037	92.04	15.200
AD_{HM}	2.029	99.52	105.074	0.675	99.2	61.668	0.254	95.92	37.320
AD_{Med}	1.960	99.52	103.352	0.671	99.2	61.475	0.254	95.92	37.300
AD_{GM}	1.924	99.52	102.170	0.663	99.2	61.112	0.253	95.92	37.267
AD_{AM}	1.591	98.88	91.583	0.598	98.88	57.623	0.242	95.88	36.416
Y_1	0.396	98.48	47.050	0.312	99.04	41.964	0.189	95.72	32.763
Y_2	0.860	99.16	69.620	0.491	99.16	53.178	0.232	95.84	36.003
Y_3	0.702	99.28	63.244	0.454	99.16	51.354	0.227	95.84	35.708
Y_4	0.260	97.72	38.791	0.219	98.68	34.616	0.153	95.52	29.451
Y_5	0.375	99.32	48.206	0.258	99.04	40.010	0.124	95.52	27.342
Y_6	0.123	96.96	29.398	0.062	98.6	18.746	0.053	94	17.934
Y_7	0.135	98.32	28.505	0.106	98.8	24.816	0.077	94.56	21.577
Y_8	1.388	99.4	88.611	0.592	99.16	58.209	0.243	95.84	36.661
Y_9	0.727	99.32	65.830	0.399	99.12	49.108	0.178	95.84	32.192
AH	0.316	99.16	43.973	0.298	99.16	42.855	0.180	95.84	32.489
FG	0.227	98.92	37.657	0.177	99	33.281	0.118	95.56	26.747
AS	1.163	99.32	78.130	0.402	99.08	47.243	0.209	95.84	34.238
AS_{Max}	0.252	88.96	42.437	0.140	87.84	27.662	0.088	85.24	20.479
AS_{Min}	0.376	99.28	48.041	0.346	99.16	46.736	0.172	95.92	32.038
N_1	2.027	99.52	105.019	0.675	99.2	61.660	0.254	95.92	37.319
N_2	1.903	99.52	101.729	0.653	99.2	60.669	0.252	95.92	37.194
N_3	1.541	99.36	91.113	0.562	99.2	56.209	0.240	95.84	36.429
N_4	1.788	99.48	98.597	0.632	99.16	59.722	0.250	95.92	37.073

Table A3: Poisson Regression Simulation Results, P = 4, n = 35, ρ = .99

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Performance	MAPE	MSE	Performance	MAPE	MSE	Performance	MAPE
MLE	11.291	0	243.819	3.972	0	149.278	1.354	0	86.213
HK	4.999	99.96	151.743	1.785	100	94.378	0.794	99.92	65.840
HK_2	6.761	99.96	179.106	2.438	100	112.498	0.984	99.96	73.217
HKB	2.978	99.96	112.984	1.108	100	72.709	0.542	99.88	53.999
LW	10.132	99.96	233.055	3.072	100	130.412	1.349	99.96	86.100
HSL	1.986	99.96	85.865	0.731	100	53.366	0.271	99.8	33.627
AM	0.491	99.2	45.323	0.174	98.84	27.074	0.101	98.16	20.857
GM	1.458	99.8	66.848	0.404	99.96	38.287	0.190	99.76	28.925
Med	1.532	99.8	74.990	0.589	100	49.650	0.310	99.8	38.097
KS	5.018	99.96	152.225	1.792	100	94.664	0.796	99.92	65.902
KS_A	2.827	99.96	111.730	1.034	100	69.362	0.385	99.84	42.161
KS_{Max}	1.372	99.96	74.557	0.521	100	46.202	0.198	99.76	28.773
KS_{Med}	7.552	99.96	202.297	2.732	100	125.047	0.983	99.96	74.327
KM_1	8.748	99.96	214.289	2.907	100	127.578	0.967	99.96	72.794
KM_2	0.187	99.96	33.587	0.366	100	49.325	0.359	99.92	47.803
KM_3	0.225	99.8	34.097	0.149	99.96	26.772	0.122	99.8	24.647
KM_4	0.689	99.96	64.736	0.917	100	77.520	0.681	99.96	64.905
KM_5	0.670	99.96	56.245	0.429	100	47.178	0.296	99.84	40.242
KM_6	0.549	99.96	57.661	0.752	100	70.107	0.591	99.96	60.787
KM_7	0.748	99.96	61.662	0.545	100	54.482	0.377	99.88	45.686
KM_8	0.186	99.96	33.483	0.365	100	49.200	0.358	99.92	47.768
KM_9	1.796	99.96	98.868	1.085	100	78.102	0.635	99.92	60.393
KM_{10}	0.539	99.96	57.701	0.820	100	73.589	0.653	99.96	63.681
KM_{11}	0.694	99.96	65.112	0.697	100	65.715	0.516	99.92	55.720
KM_{12}	0.516	99.96	56.060	0.736	100	69.347	0.587	99.96	60.584
KM_{13}	0.678	99.96	62.863	1.057	100	83.136	0.790	99.96	69.838
KM_{14}	0.952	99.96	75.185	0.808	100	69.697	0.555	99.92	57.310
GK	4.678	99.96	148.633	1.773	100	94.155	0.794	99.92	65.818
HMO	0.596	99.88	48.750	0.186	100	28.124	0.082	99.8	20.566
AD_{HM}	11.119	99.96	241.818	3.951	100	148.860	1.352	99.96	86.167
AD_{Med}	10.772	99.96	236.858	3.908	100	147.970	1.348	99.96	86.030
AD_{GM}	10.610	99.96	233.916	3.866	100	146.918	1.344	99.96	85.870
AD_{AM}	8.420	99.92	200.200	3.387	99.96	135.163	1.263	99.92	82.544
Y_1	0.388	99.92	43.789	0.491	100	50.151	0.466	99.88	50.109
Y_2	1.581	99.96	88.678	1.446	100	88.867	0.902	99.96	70.847
Y_3	0.911	99.96	67.369	0.975	100	73.775	0.764	99.96	66.127
Y_4	0.208	99.76	33.814	0.251	100	34.663	0.281	99.8	37.784
Y_5	0.474	99.96	53.107	0.426	100	52.373	0.261	99.88	40.030
Y_6	0.102	99.84	28.608	0.031	100	12.833	0.033	99.8	13.683
Y_7	0.083	99.96	23.477	0.066	100	18.568	0.070	99.8	20.074
Y_8	4.627	99.96	158.168	2.613	100	121.342	1.143	99.96	79.579
Y_9	1.413	99.96	87.511	0.963	100	76.483	0.526	99.96	56.370
AH	0.226	99.96	36.227	0.361	100	47.093	0.381	99.92	48.771
FG	0.313	99.96	43.910	0.280	100	42.022	0.216	99.84	36.616
AS	4.820	99.96	150.075	1.779	100	94.266	0.794	99.92	65.829
AS_{Max}	0.190	98.92	36.273	0.104	97.88	22.550	0.075	97	18.056
AS_{Min}	0.294	99.96	40.382	0.427	100	52.359	0.366	99.92	48.214
N_1	11.100	99.96	241.594	3.949	100	148.811	1.352	99.96	86.162
N_2	10.057	99.96	228.038	3.713	100	143.696	1.320	99.96	85.159
N_3	7.125	99.96	186.178	2.794	100	122.003	1.128	99.96	78.665
N_4	9.172	99.96	216.316	3.507	100	139.134	1.289	99.96	84.198

Table A4: Poisson Regression Simulation Results, P = 4, n = 600, ρ = .90

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.01644419	0	10.113587	0.006143805	0	6.180165	0.002307547	0	3.777529
HK	0.01618726	69.04	10.033184	0.006079381	58.16	6.149478	0.002303666	53.56	3.774601
HK_2	0.01631803	69.32	10.074814	0.006110535	58.28	6.164242	0.002305364	53.72	3.77588
HKB	0.01596544	68.64	9.965104	0.006016526	57.88	6.118989	0.002298984	53.36	3.771046
LW	0.01644415	69.72	10.113576	0.006143631	58.48	6.180082	0.002307547	53.8	3.777529
HSL	0.01564847	67.28	9.875409	0.005911837	56.48	6.067074	0.002277866	52.52	3.755612
AM	0.03625134	46.28	13.568661	0.015410892	44.52	8.005739	0.007942872	41.64	4.929629
GM	0.0153435	62.08	9.782934	0.005773678	53.04	6.012892	0.002284786	50.56	3.7576
Med	0.01551666	66.6	9.834667	0.005816435	56.48	6.016716	0.002279947	52.2	3.757871
KS	0.01619243	69.04	10.034793	0.006080327	58.2	6.149923	0.002303676	53.56	3.774608
KS_A	0.01602815	68.64	9.990048	0.005986324	57.28	6.104045	0.002290224	51.24	3.767123
KS_{Max}	0.01575215	66	9.908468	0.00590323	55.32	6.0639	0.002514652	47.88	3.909455
KS_{Med}	0.01633566	69.36	10.079913	0.006109549	58.32	6.163711	0.002299872	53.4	3.771559
KM_1	0.01632989	69.32	10.078199	0.006096174	58.2	6.157235	0.002295128	53.12	3.76808
KM_2	0.01617109	69.08	10.030396	0.006117343	58.36	6.16753	0.00230267	53.52	3.773845
KM_3	0.0165984	60.84	10.111341	0.006060672	54.16	6.153115	0.002408753	50.44	3.809029
KM_4	0.01634996	69.52	10.084645	0.00613544	58.36	6.176182	0.002306164	53.72	3.776482
KM_5	0.01566312	67.48	9.870435	0.005983392	57.68	6.103168	0.002290709	53.04	3.764524
KM_6	0.01630306	69.44	10.07023	0.006131674	58.36	6.17446	0.002305694	53.68	3.776108
KM_7	0.01591148	68.48	9.94989	0.006034205	57.96	6.127232	0.00229551	53.16	3.768585
KM_8	0.01616851	69.08	10.029618	0.006117164	58.36	6.167446	0.002302664	53.52	3.773841
KM_9	0.01618767	69	10.034053	0.006096904	58.28	6.157807	0.002303434	53.56	3.774426
KM_{10}	0.01633094	69.44	10.078972	0.006134324	58.36	6.175652	0.002306066	53.72	3.776409
KM_{11}	0.01618207	69.04	10.032884	0.006106117	58.28	6.162183	0.002303194	53.48	3.774243
KM_{12}	0.01629756	69.44	10.068597	0.006131255	58.36	6.174262	0.002305677	53.68	3.776097
KM_{13}	0.01639459	69.6	10.098491	0.006141157	58.44	6.178908	0.002307012	53.8	3.777122
KM_{14}	0.01618414	69	10.033338	0.006103302	58.28	6.160844	0.00230328	53.48	3.774309
GK	0.01618724	69.04	10.033179	0.00607938	58.16	6.149477	0.002303666	53.56	3.774601
HMO	0.04773388	7.04	18.785257	0.021432423	4.08	14.016495	0.020969096	0.28	9.292035
AD_{HM}	0.01644415	69.72	10.113576	0.006143801	58.48	6.180164	0.002307547	53.8	3.777529
AD_{Med}	0.01644408	69.72	10.113555	0.006143794	58.48	6.18016	0.002307546	53.8	3.777529
AD_{GM}	0.01644392	69.72	10.113506	0.006143771	58.48	6.180149	0.002307546	53.8	3.777529
AD_{AM}	0.01655466	69.48	10.129037	0.006138298	58.48	6.177157	0.002314696	53.72	3.779692
Y_1	0.01633752	69.08	10.080901	0.006131903	58.44	6.174458	0.002306634	53.76	3.776724
Y_2	0.01642679	69.68	10.108343	0.006142011	58.44	6.179318	0.00230745	53.8	3.777456
Y_3	0.01641039	69.64	10.103314	0.006139602	58.44	6.178179	0.002307351	53.8	3.777383
Y_4	0.01623017	68.48	10.050893	0.006117644	58.36	6.166706	0.002304977	53.68	3.77513
Y_5	0.01474017	64.88	9.583827	0.005833047	56.24	6.029812	0.002230718	50.12	3.718553
Y_6	0.02307261	30.04	12.479604	0.006432734	40.92	6.51465	0.002766181	36.4	4.093206
Y_7	0.01442551	51.6	9.56799	0.005417694	50.96	5.830787	0.0022376	46.16	3.724191
Y_8	0.01643531	69.68	10.110936	0.006142767	58.48	6.179677	0.00230746	53.8	3.777465
Y_9	0.01540934	67.52	9.789493	0.005962503	57.16	6.092728	0.002261595	52.16	3.741648
AH	0.01604748	68.96	9.991106	0.006098964	58.24	6.158739	0.00230243	53.48	3.77364
FG	0.01535843	67.24	9.774048	0.005986047	57.6	6.104388	0.002287307	53.04	3.762136
AS	0.01618725	69.04	10.033182	0.006079381	58.16	6.149477	0.002303666	53.56	3.774601
AS_{Max}	0.05668879	36.56	16.886818	0.02442504	37.36	9.793788	0.013369946	35.76	5.967883
AS_{Min}	0.01616012	69.12	10.027718	0.006123566	58.36	6.170499	0.002302119	53.52	3.773422
N_1	0.01644415	69.72	10.113575	0.006143801	58.48	6.180163	0.002307547	53.8	3.777529
N_2	0.01644124	69.72	10.112698	0.006143336	58.48	6.179943	0.002307528	53.8	3.777516
N_3	0.01640858	69.6	10.102734	0.006136516	58.4	6.17669	0.002307178	53.8	3.777252
N_4	0.01643839	69.72	10.111837	0.006142884	58.48	6.179728	0.002307511	53.8	3.777503

Table A5: Poisson Regression Simulation Results, P = 4, n = 600, ρ = .95

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.03458358	0	14.55558	0.012413174	0	8.726677	0.004580848	0	5.308665
HK	0.03344277	84.4	14.31904	0.012127558	71.4	8.633023	0.004564648	62.12	5.299951
HK_2	0.03403635	84.56	14.44451	0.012279257	71.48	8.682858	0.004571297	62.12	5.303527
HKB	0.03251126	84.08	14.12995	0.011902401	71.12	8.558453	0.004543209	61.96	5.288303
LW	0.03458342	84.68	14.55555	0.012412145	71.56	8.726348	0.004580848	62.16	5.308664
HSL	0.03158292	83.48	13.92679	0.011619709	70.68	8.459533	0.004397704	60.88	5.208903
AM	0.03841106	66.04	14.89378	0.015934679	62.6	9.023931	0.007218134	51.2	5.681794
GM	0.02670301	80.12	12.87794	0.01051479	68.24	8.094192	0.004362637	60.08	5.181228
Med	0.03063955	83.44	13.74432	0.011341153	70.52	8.367328	0.004433912	60.88	5.228096
KS	0.03346558	84.4	14.32379	0.012132006	71.4	8.634448	0.004564686	62.12	5.29997
KS_A	0.03330681	84.32	14.28787	0.012033281	71.2	8.602167	0.004457497	60.96	5.237475
KS_{Max}	0.03150779	83.32	13.91903	0.01155199	70.48	8.440359	0.00445313	58.52	5.222302
KS_{Med}	0.03426793	84.64	14.49105	0.01231925	71.52	8.695793	0.004556832	62.08	5.295453
KM_1	0.03429089	84.64	14.49534	0.012301508	71.48	8.689718	0.004543692	61.88	5.288418
KM_2	0.03342205	84.32	14.32359	0.012312141	71.52	8.693989	0.004559416	62.12	5.29713
KM_3	0.02776817	79.48	13.1347	0.011165324	69.12	8.343087	0.004435591	59.8	5.205943
KM_4	0.03417513	84.56	14.47428	0.012377034	71.52	8.71493	0.004574996	62.12	5.305502
KM_5	0.03116501	83.64	13.83916	0.011829545	71	8.535677	0.004509034	61.8	5.269708
KM_6	0.03393166	84.52	14.42362	0.012360586	71.52	8.709661	0.004573457	62.12	5.304689
KM_7	0.03245914	84.04	14.12245	0.012018559	71.32	8.597725	0.004525135	61.8	5.278422
KM_8	0.03341104	84.32	14.32138	0.012311407	71.52	8.693757	0.004559392	62.12	5.297118
KM_9	0.03346212	84.4	14.32597	0.012215379	71.48	8.662097	0.004563289	62.12	5.299218
KM_{10}	0.03409362	84.56	14.45799	0.012373017	71.52	8.71368	0.004574624	62.12	5.30531
KM_{11}	0.03345004	84.36	14.32566	0.012260031	71.48	8.676849	0.00456202	62.12	5.298534
KM_{12}	0.03390997	84.52	14.4194	0.012359105	71.52	8.709195	0.00457338	62.12	5.304649
KM_{13}	0.03436751	84.64	14.51297	0.01240143	71.52	8.722883	0.004578722	62.16	5.307512
KM_{14}	0.03345498	84.36	14.32595	0.012246537	71.48	8.672392	0.004562466	62.12	5.298775
GK	0.03344269	84.4	14.31902	0.012127554	71.4	8.633022	0.004564648	62.12	5.29995
HMO	0.03102247	43.64	14.81805	0.012802409	36	10.545495	0.013776803	6.8	7.87619
AD_{HM}	0.03458343	84.68	14.55555	0.012413161	71.56	8.726673	0.004580848	62.16	5.308664
AD_{Med}	0.03458323	84.68	14.55552	0.012413142	71.56	8.726667	0.004580847	62.16	5.308663
AD_{GM}	0.0345824	84.68	14.55535	0.012413088	71.56	8.726648	0.004580845	62.16	5.308663
AD_{AM}	0.03453621	84.6	14.54779	0.01239424	71.56	8.719292	0.004579676	62.16	5.308159
Y_1	0.03388606	84.48	14.41153	0.012346257	71.52	8.705437	0.004575746	62.16	5.306206
Y_2	0.03448593	84.68	14.536	0.012404888	71.52	8.72402	0.004580321	62.16	5.308387
Y_3	0.03438291	84.68	14.51618	0.01239283	71.52	8.720008	0.004579832	62.16	5.308121
Y_4	0.03307935	83.96	14.25003	0.012265708	71.44	8.681021	0.004566766	62.12	5.301912
Y_5	0.02856843	82.6	13.26743	0.011394106	70.56	8.38826	0.004250934	59.44	5.122194
Y_6	0.02423757	56.36	12.79749	0.008663162	57.28	7.568889	0.004352929	45.48	5.159493
Y_7	0.02020237	73.16	11.27589	0.009142933	66.12	7.556878	0.004002908	54.72	4.974802
Y_8	0.03454707	84.68	14.54839	0.012409395	71.52	8.725471	0.004580446	62.16	5.308451
Y_9	0.03115142	83.72	13.84299	0.011831967	71	8.531566	0.00442437	61.16	5.220933
AH	0.03287238	84.24	14.20922	0.012235144	71.48	8.668876	0.00455907	62.12	5.296945
FG	0.03000344	83.4	13.60061	0.011788187	71.08	8.521492	0.004492202	61.68	5.26027
AS	0.03344273	84.4	14.31903	0.012127556	71.4	8.633022	0.004564648	62.12	5.29995
AS_{Max}	0.05193563	56.68	16.93155	0.021052485	57.36	9.800755	0.010651341	44.24	6.242862
AS_{Min}	0.033397	84.32	14.32126	0.012339646	71.52	8.702994	0.004556814	62.08	5.295721
N_1	0.03458341	84.68	14.55555	0.012413159	71.56	8.726672	0.004580848	62.16	5.308664
N_2	0.03457087	84.68	14.55306	0.012411315	71.52	8.72608	0.004580769	62.16	5.308622
N_3	0.03441921	84.68	14.52254	0.012382018	71.52	8.716568	0.00457913	62.16	5.307747
N_4	0.03455844	84.68	14.5506	0.012409496	71.52	8.725495	0.004580691	62.16	5.308581

Table A6: Poisson Regression Simulation Results, P = 4, n = 600, ρ = .99

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.18582777	0	33.38409	0.06934168	0	20.485105	0.02582587	0	12.462976
HK	0.16103696	99.28	31.08224	0.06101044	98.92	19.262953	0.025309389	95.28	12.347252
HK_2	0.17218731	99.28	32.18449	0.06540785	98.92	19.92372	0.025516004	95.28	12.393898
HKB	0.14076035	99.28	29.19586	0.05584294	98.92	18.472268	0.024630175	95.24	12.193512
LW	0.18582269	99.32	33.38368	0.06927383	98.88	20.476116	0.025825857	95.4	12.462973
HSL	0.1251882	99.24	27.65938	0.05173606	98.96	17.883959	0.019922343	95.16	11.029444
AM	0.06746711	94.04	19.74635	0.02898679	97.44	12.692454	0.015968824	92.28	9.25563
GM	0.07352575	98.84	20.7999	0.03242938	98.6	14.099061	0.018417736	94.88	10.654796
Med	0.10341449	99.24	25.46218	0.04851616	98.92	17.209502	0.021116842	95.12	11.30816
KS	0.16141992	99.28	31.12135	0.06112187	98.92	19.280408	0.025310645	95.28	12.347531
KS_A	0.15987992	99.28	31.20037	0.06214022	98.92	19.503208	0.023067501	95.2	11.845224
KS_{Max}	0.12634891	99.24	27.80045	0.05192521	98.96	17.91604	0.019076399	95.12	10.819446
KS_{Med}	0.18376568	99.32	33.21055	0.06858305	98.88	20.380982	0.025599719	95.32	12.412219
KM_1	0.18330701	99.32	33.17393	0.06825879	98.88	20.336434	0.025402886	95.28	12.368199
KM_2	0.1521237	99.28	30.54248	0.06598494	98.92	20.032949	0.025118562	95.28	12.30623
KM_3	0.0809866	98.56	21.81411	0.03957523	98.56	15.522133	0.018384524	94.76	10.610565
KM_4	0.17339477	99.28	32.34645	0.06812812	98.88	20.319118	0.025650753	95.32	12.423714
KM_5	0.11779492	99.24	26.83777	0.05349711	98.92	18.157887	0.023443623	95.16	11.930177
KM_6	0.16909797	99.28	31.91552	0.0673468	98.88	20.218415	0.025584012	95.32	12.410592
KM_7	0.13264506	99.28	28.67243	0.0591206	98.92	19.006117	0.024065474	95.2	12.058362
KM_8	0.15189849	99.28	30.52089	0.06596174	98.92	20.029693	0.02511776	95.28	12.306054
KM_9	0.15896305	99.28	30.97693	0.06324888	98.92	19.619188	0.025258807	95.28	12.336437
KM_{10}	0.17132732	99.28	32.17257	0.06796037	98.88	20.297641	0.025632265	95.32	12.420112
KM_{11}	0.15666941	99.28	30.83439	0.06447017	98.92	19.807299	0.025212343	95.28	12.326459
KM_{12}	0.16842528	99.28	31.86174	0.06730336	98.88	20.212435	0.025581273	95.32	12.409989
KM_{13}	0.17881039	99.32	32.81283	0.06892091	98.88	20.42842	0.025764736	95.36	12.449451
KM_{14}	0.15747976	99.28	30.88543	0.06409434	98.92	19.749945	0.025228616	95.28	12.329958
GK	0.16103502	99.28	31.08207	0.06101034	98.92	19.262939	0.025309383	95.28	12.347251
HMO	0.02143056	97.52	11.48191	0.0067354	98.04	6.678344	0.005439857	89	5.770576
AD_{HM}	0.18582359	99.32	33.38374	0.06934125	98.88	20.485046	0.025825857	95.4	12.462973
AD_{Med}	0.18581178	99.32	33.38301	0.06934083	98.88	20.484989	0.025825805	95.4	12.462961
AD_{GM}	0.18578998	99.32	33.38103	0.06933866	98.88	20.484713	0.025825738	95.4	12.462949
AD_{AM}	0.1842233	99.2	33.21633	0.06911412	98.88	20.45229	0.025768784	95.4	12.4514
Y_1	0.15845314	99.2	30.85455	0.06531888	98.88	19.948544	0.025254713	95.28	12.34591
Y_2	0.18067006	99.32	32.95868	0.06882375	98.88	20.415953	0.025790487	95.4	12.455433
Y_3	0.17424336	99.28	32.41774	0.06820323	98.88	20.325546	0.025764585	95.36	12.449608
Y_4	0.13826624	98.96	28.72046	0.06055693	98.92	19.261674	0.024410606	95.28	12.156346
Y_5	0.11324397	99.24	26.40875	0.05236886	98.92	17.998914	0.019945996	95.04	11.046408
Y_6	0.01704792	96.04	10.57663	0.00770451	98.2	6.954882	0.007688465	92.04	6.870258
Y_7	0.02553499	98.16	12.25815	0.01845972	98.6	10.589707	0.012484885	94.16	8.742412
Y_8	0.18464944	99.32	33.29048	0.06921206	98.88	20.467955	0.025810482	95.4	12.459637
Y_9	0.1408882	99.28	29.27931	0.06100411	98.92	19.257476	0.023811955	95.2	11.985711
AH	0.14285014	99.28	29.57246	0.06367471	98.92	19.700715	0.025124756	95.28	12.307368
FG	0.10324389	99.2	25.31756	0.05284833	98.92	18.057997	0.023136292	95.16	11.848432
AS	0.16103599	99.28	31.08216	0.06101039	98.92	19.262946	0.025309386	95.28	12.347252
AS_{Max}	0.06374912	90.04	18.87231	0.02607487	95.68	11.387236	0.017109906	89.88	8.560494
AS_{Min}	0.15006923	99.28	30.43423	0.06685016	98.88	20.156896	0.0250312	95.28	12.287288
N_1	0.18582309	99.32	33.38369	0.0693412	98.88	20.485039	0.025825856	95.4	12.462973
N_2	0.18548661	99.32	33.35509	0.06928358	98.88	20.477026	0.025823159	95.4	12.462382
N_3	0.18097761	99.32	32.96569	0.06826402	98.88	20.333745	0.025761355	95.36	12.448718
N_4	0.18515079	99.32	33.32651	0.06922627	98.88	20.469052	0.025820482	95.4	12.461796

Table A7: Poisson Regression Simulation Results (New Estimators), P = 4, n = 35, ρ = .90

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	1.0028166	0	72.95136	0.32629316	0	43.01376	0.11910255	0	26.0815
HK	0.6259404	95.96	58.09197	0.22148176	96.04	35.65563	0.106847	88.52	24.89263
HK_2	0.7530041	96.4	63.53335	0.26191816	96.28	38.70548	0.111842	88.56	25.38898
HKB	0.4258919	95.16	48.38645	0.16717313	95.72	31.16153	0.09566814	88.08	23.69835
LW	0.9786389	96.84	72.46727	0.31386388	96.52	42.31657	0.1190661	88.64	26.0788
HSL	0.3656912	94.32	43.11879	0.13951993	95.28	28.17122	0.07439321	85.8	20.93152
AM	0.2267087	82.44	39.36818	0.11746684	83.12	25.96672	0.0713103	75.92	19.52213
GM	0.1969764	90.32	34.20196	0.07444337	93.16	21.0231	0.05718861	84.12	18.60961
Med	0.2503527	93.16	37.37616	0.11162958	94.44	25.38812	0.07157075	86.04	20.63378
KS	0.6336287	96	58.46267	0.2236132	96.12	35.83325	0.10694869	88.52	24.90179
KS_A	0.4697694	95.64	50.34719	0.17568273	95.64	31.73933	0.0745835	85.72	20.95392
KS_{Max}	0.3162343	94.28	41.01017	0.12127531	94.68	26.2924	0.05811833	82.64	18.54279
KS_{Med}	0.7505575	96.6	64.51203	0.26387883	96.16	39.04913	0.10454921	88.4	24.61761
KM_1	0.8078521	96.6	65.96458	0.26249854	96.2	38.82387	0.09959921	87.92	24.04782
KM_2	0.2822983	96.08	42.31207	0.22208673	96.28	36.70338	0.10135592	88.52	24.40289
KM_3	0.1669752	90.4	33.07998	0.08720549	93.44	22.80391	0.06000056	83.32	19.00394
KM_4	0.5266961	96.6	56.58369	0.2815697	96.44	40.64315	0.11309634	88.64	25.5555
KM_5	0.2323945	94.16	37.23336	0.13173783	95.6	27.89768	0.08164897	87.32	22.05008
KM_6	0.4689782	96.52	53.57232	0.26590633	96.4	39.6355	0.11088386	88.64	25.34751
KM_7	0.2807417	94.92	40.75974	0.16272259	96	31.06651	0.08912002	87.56	22.96327
KM_8	0.2796845	96.08	42.11955	0.22112238	96.28	36.63017	0.10129182	88.52	24.39693
KM_9	0.4911797	96	53.43322	0.22389406	96.16	36.2408	0.10542708	88.56	24.77087
KM_{10}	0.4592144	96.52	53.35941	0.27171044	96.44	40.05113	0.11229289	88.64	25.48406
KM_{11}	0.393491	96.08	49.06906	0.22329482	96.2	36.45214	0.10405479	88.56	24.64972
KM_{12}	0.4458503	96.52	52.40537	0.26248275	96.4	39.4203	0.11066108	88.64	25.328
KM_{13}	0.5343598	96.8	57.25454	0.29700206	96.52	41.61702	0.11598298	88.64	25.82724
KM_{14}	0.4240855	96.08	50.54584	0.22356519	96.2	36.39067	0.1045354	88.56	24.69246
GK	0.6132865	95.96	57.81581	0.22133406	96.04	35.64657	0.10683554	88.52	24.89166
HMO	0.1412853	89.36	30.50401	0.04986811	91.48	18.189	0.0372033	75.2	15.20702
AD_{HM}	0.9889481	96.84	72.65942	0.32560875	96.52	42.97512	0.1190767	88.64	26.07942
AD_{Med}	0.9577302	96.76	71.73185	0.32366889	96.52	42.86068	0.11891765	88.64	26.06726
AD_{GM}	0.9389417	96.72	70.97555	0.32060306	96.52	42.66949	0.1188024	88.64	26.05595
AD_{AM}	0.7856605	95.76	65.23363	0.29631595	96.12	40.84617	0.11658461	88.2	25.80934
Y_1	0.3178844	95.12	43.52736	0.20269421	95.96	34.16569	0.10461263	88.32	24.6274
Y_2	0.5550701	96.32	56.66075	0.27222518	96.44	39.67159	0.11480534	88.64	25.70346
Y_3	0.4936268	96.16	53.56132	0.2623788	96.32	39.00832	0.11387384	88.64	25.61925
Y_4	0.2357272	93.4	38.07677	0.15630175	95.32	29.83102	0.09381947	87.6	23.41911
Y_5	0.2792435	95.4	41.42623	0.17039413	96.08	32.25024	0.07743168	86.96	21.491
Y_6	0.139985	89.84	31.0727	0.06492581	94.12	19.82318	0.0482807	80.88	17.20025
Y_7	0.147844	93.36	30.15303	0.0945474	95.2	23.7027	0.05924895	84.52	18.8997
Y_8	0.7496732	96.68	65.16319	0.30064948	96.48	41.56626	0.11668233	88.64	25.87398
Y_9	0.4587053	96.28	51.86445	0.23190608	96.2	37.20217	0.09614185	88.04	23.73097
AH	0.2730753	95.8	41.03468	0.20125881	96.2	34.84613	0.10110996	88.56	24.35904
FG	0.1876072	94.36	34.24967	0.12898325	95.6	28.21911	0.07719196	87.08	21.55358
AS	0.6181626	95.96	57.93472	0.22140739	96.04	35.65107	0.10684123	88.52	24.89214
AS_{Max}	0.2742711	77.12	44.57399	0.15916801	74.92	30.58714	0.08849338	70	21.0621
AS_{Min}	0.2933738	96.36	42.81369	0.22775428	96.36	37.2011	0.09969512	88.44	24.23928
N_1	0.9874724	96.8	72.62615	0.32552863	96.52	42.97059	0.11907365	88.64	26.07918
N_2	0.9351905	96.76	70.92703	0.31745497	96.52	42.48446	0.11855795	88.64	26.03429
N_3	0.8015774	96.44	65.83389	0.28621763	96.36	40.45018	0.11571181	88.64	25.77111
N_4	0.8863345	96.68	69.28141	0.30991734	96.52	42.02044	0.11806585	88.64	25.9911
CZ_1	0.1422857	92.24	29.8559	0.08607488	95.32	23.07984	0.05671529	85	18.60739
CZ_2	0.293801	95.8	42.94517	0.2026431	96.12	35.10511	0.09581987	88.32	23.77897
CZ_3	0.2563547	95.64	40.21304	0.18491624	96.16	33.61944	0.08998263	87.92	23.10407
CZ_4	0.273171	95.96	41.57966	0.20495087	96.16	35.326	0.09649753	88.32	23.84428
CZ_5	0.1802911	94.36	33.62271	0.12794375	95.6	28.10898	0.07310121	86.72	20.98845

Table A8: Poisson Regression Simulation Results (New Estimators), P = 4, n = 35, ρ = .95

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	2.0490858	0	106.03229	0.68272769	0	61.95466	0.24061898	0	37.15363
HK	1.1725077	98.76	78.8465	0.40728531	99.08	47.55207	0.19736449	96.24	33.98442
HK_2	1.4319195	99.08	87.84817	0.50489904	99.16	53.14615	0.21482845	96.32	35.29142
HKB	0.7290244	98.6	62.17478	0.28168156	99.04	39.58486	0.16484727	96.16	31.20953
LW	1.9929189	99.12	104.85498	0.63329823	99.12	59.90733	0.24047855	96.36	37.14581
HSL	0.5412752	98.52	51.72647	0.21692201	98.8	33.33152	0.10813219	95.08	24.61559
AM	0.2301093	93.48	38.58909	0.10384248	93.12	23.92963	0.07412403	90.32	19.68704
GM	0.2721952	97.36	37.763	0.09707626	97.88	22.59061	0.07657127	94.88	21.01672
Med	0.3600331	98.24	43.08517	0.16587689	98.52	29.86711	0.11001182	95.44	25.10373
KS	1.1835254	98.8	79.3131	0.41064721	99.08	47.78009	0.19761989	96.24	34.00381
KS_A	0.7907261	98.76	64.74362	0.302241	99.04	40.4889	0.11844286	95.72	25.81678
KS_{Max}	0.481012	98.56	49.20137	0.19085873	98.76	31.36301	0.08047272	94.48	21.00754
KS_{Med}	1.4827148	99.08	91.49611	0.52151369	99.12	54.68046	0.19950655	96.32	34.10793
KM_1	1.6230686	99.08	94.86765	0.5307369	99.12	54.88984	0.18918462	96.28	33.15111
KM_2	0.323035	98.88	45.88539	0.33266813	99.16	45.86309	0.17497648	96.28	32.47562
KM_3	0.1767835	97.4	33.18967	0.10101773	97.88	23.85419	0.07702668	95.28	21.19404
KM_4	0.7600862	99.08	69.48382	0.49227297	99.12	54.68187	0.21504289	96.36	35.54016
KM_5	0.3195479	98.44	42.70172	0.19165164	98.84	33.26325	0.12752203	95.92	27.57298
KM_6	0.6586645	99.08	64.67232	0.44909575	99.12	52.3946	0.20616443	96.36	34.9292
KM_7	0.3875409	98.68	47.48327	0.24732505	99.12	38.17018	0.14749409	96.12	29.57832
KM_8	0.3201347	98.88	45.66968	0.33111535	99.16	45.75659	0.17481966	96.28	32.46306
KM_9	0.8005974	98.92	68.25772	0.38276738	99.12	47.33338	0.19126238	96.28	33.59283
KM_{10}	0.6427024	99.08	64.3688	0.46865158	99.12	53.50207	0.21216136	96.36	35.34502
KM_{11}	0.557195	98.92	58.97653	0.36143427	99.16	46.82	0.18563741	96.24	33.21985
KM_{12}	0.6209108	99.08	62.96582	0.4422939	99.12	52.03191	0.20548961	96.36	34.88151
KM_{13}	0.7859176	99.12	70.79209	0.54060519	99.12	57.08268	0.22607365	96.36	36.31531
KM_{14}	0.6297965	98.92	62.0605	0.36837659	99.16	47.00536	0.18759267	96.28	33.35084
GK	1.1551138	98.76	78.45102	0.40662959	99.08	47.52865	0.19732762	96.24	33.982
HMO	0.1637733	97.28	31.20332	0.04958168	98	17.23325	0.03630062	93.2	15.08393
AD_{HM}	2.0266048	99.12	105.50334	0.68077217	99.12	61.8761	0.24053273	96.36	37.14834
AD_{Med}	1.9582841	99.12	103.77518	0.67664656	99.12	61.70503	0.24019791	96.36	37.12626
AD_{GM}	1.9099815	99.08	102.38225	0.67056636	99.12	61.39907	0.23970765	96.36	37.09392
AD_{AM}	1.587795	98.88	92.14567	0.61877895	98.68	58.59256	0.22988062	96.32	36.19579
Y_1	0.4064848	98.6	48.07854	0.31259027	99.08	42.08791	0.18520016	96.2	32.7972
Y_2	0.8609	99	70.29519	0.49307262	99.12	53.40402	0.22211599	96.36	35.88703
Y_3	0.7065151	99.08	63.85167	0.44857292	99.16	51.35753	0.21711296	96.36	35.57629
Y_4	0.2643382	97.84	39.18557	0.21647461	98.48	34.34622	0.15229879	95.68	29.62515
Y_5	0.3836497	98.76	49.04038	0.25980268	99.12	40.14446	0.12242455	95.88	27.29015
Y_6	0.1250526	96.96	29.67694	0.05782843	98.48	18.27247	0.0508195	94.48	17.63896
Y_7	0.1384759	98.16	28.90967	0.1019143	98.72	24.44709	0.07375584	95.24	21.19372
Y_8	1.3635925	99.08	88.64646	0.5915253	99.12	58.2824	0.23113484	96.36	36.52314
Y_9	0.7262624	98.92	66.02968	0.39106758	99.16	49.01431	0.17058496	96.28	31.97344
AH	0.3250603	98.88	44.73257	0.29538071	99.16	42.81667	0.17369723	96.24	32.35836
FG	0.2309144	98.6	38.22989	0.17617754	99.04	33.22163	0.11526454	96	26.65131
AS	1.1634984	98.76	78.6432	0.40695493	99.08	47.5403	0.19734599	96.24	33.9832
AS_{Max}	0.247976	90.24	41.87416	0.12915501	87.84	26.97225	0.08177367	86.96	19.89988
AS_{Min}	0.3749072	98.92	48.46086	0.34470998	99.12	46.72963	0.17014767	96.24	32.103
N_1	2.0241626	99.12	105.44458	0.68054455	99.12	61.86692	0.24052256	96.36	37.14771
N_2	1.8990718	99.12	102.13499	0.65769605	99.12	60.85668	0.23871935	96.36	37.02852
N_3	1.5378549	99.04	91.58143	0.56447416	99.12	56.36571	0.22785017	96.36	36.26661
N_4	1.7836742	99.12	98.98846	0.63639693	99.12	59.89399	0.23697393	96.36	36.91192
CZ_1	0.1353147	98.16	28.38846	0.09294083	98.8	24.15561	0.07055345	95.2	20.9999
CZ_2	0.3703684	98.88	49.02101	0.30626418	99.16	43.87562	0.16195592	96.2	31.3205
CZ_3	0.3104124	98.8	44.86097	0.27307893	99.16	41.49025	0.14797286	96.16	30.02208
CZ_4	0.3238396	98.84	45.96995	0.30784873	99.16	44.1004	0.16411785	96.24	31.55122
CZ_5	0.202858	98.6	35.84632	0.17257306	99.04	32.93484	0.1088958	95.92	25.91458

Table A9: Poisson Regression Simulation Results (New Estimators), P = 4, n = 35, ρ = .99

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	11.59862064	0	248.89533	3.92752004	0	146.02998	1.32806587	0	86.05758
HK	5.16901858	99.96	155.40998	1.77483503	99.96	92.17015	0.78228575	99.96	65.89084
HK_2	6.90309291	99.96	183.30872	2.4230536	99.96	109.98407	0.97449011	99.96	73.40056
HKB	2.99875138	99.96	115.39624	1.11382591	99.96	71.03546	0.54211062	99.96	54.38781
LW	10.43098252	100	237.88167	3.07875993	99.96	128.44568	1.323704	99.96	85.94975
HSL	2.05559927	99.92	87.5429	0.76317238	99.96	52.92151	0.2874116	99.88	34.85554
AM	0.4015014	99.48	43.19435	0.1900052	98.96	27.26891	0.12001248	98.28	22.46368
GM	1.34270517	99.72	67.17091	0.43354699	99.92	37.67977	0.20286711	99.84	29.57603
Med	1.48597362	99.92	76.31322	0.59851432	99.96	48.71072	0.31843104	99.92	38.75027
KS	5.18756899	99.96	155.90178	1.78197103	99.96	92.46829	0.78369033	99.96	65.95807
KS_A	2.89638962	99.96	114.10815	1.05639641	99.96	68.88654	0.3871428	99.96	42.47837
KS_{Max}	1.43009884	99.92	75.91346	0.5241464	99.96	45.45461	0.20268776	99.88	29.22191
KS_{Med}	7.62486669	100	205.82244	2.69268656	99.96	122.47297	0.97311624	99.96	74.53462
KM_1	8.96633329	100	218.62747	2.90633372	99.96	125.25256	0.95625724	99.96	72.91907
KM_2	0.18411166	99.96	33.54524	0.36497934	99.96	48.92098	0.37004544	99.96	48.67378
KM_3	0.20857339	99.84	33.38231	0.15239147	99.92	26.50987	0.13609176	99.8	25.70931
KM_4	0.67791077	100	64.42767	0.89003637	99.96	76.14187	0.68946887	99.96	65.62485
KM_5	0.6591489	99.96	57.16986	0.42800704	100	46.20267	0.30376206	99.96	40.77465
KM_6	0.54418465	99.96	57.51251	0.72844981	99.96	68.80285	0.60114942	99.96	61.51946
KM_7	0.75211324	99.96	62.8852	0.53659044	99.96	53.46954	0.38235568	99.96	46.28176
KM_8	0.18302347	99.96	33.44503	0.36323335	99.96	48.78915	0.36946942	99.96	48.63503
KM_9	1.81379776	99.96	100.63576	1.05920063	99.96	76.42052	0.63261382	99.96	60.7316
KM_{10}	0.53907833	99.96	57.75614	0.795186	99.96	72.2469	0.6576507	99.96	64.2807
KM_{11}	0.70688798	99.96	66.07507	0.67655884	99.96	64.44622	0.52076794	99.96	56.27929
KM_{12}	0.51352531	99.96	55.95328	0.7129509	99.96	68.06294	0.59611315	99.96	61.28097
KM_{13}	0.66209415	100	61.81075	1.02232746	99.96	81.51618	0.79199266	99.96	70.35548
KM_{14}	0.96815134	99.96	76.37489	0.78382182	99.96	68.27711	0.5568637	99.96	57.79554
GK	4.69020908	99.96	151.83489	1.76182077	99.96	91.93604	0.7816406	99.96	65.86984
HMO	0.58235547	99.8	49.29534	0.18748374	100	27.51802	0.0805633	99.84	20.60103
AD_{HM}	11.41344599	100	246.80516	3.90535099	99.96	145.59537	1.32655119	99.96	86.01167
AD_{Med}	10.94043716	100	241.48483	3.86624192	99.96	144.79084	1.32093833	99.96	85.82183
AD_{GM}	10.87683784	100	238.8368	3.82532963	99.96	143.753	1.31421659	99.96	85.58268
AD_{AM}	8.62323277	99.96	206.55567	3.37458248	99.92	132.0764	1.22429409	99.92	81.71107
Y_1	0.3889054	99.92	44.11143	0.47750714	99.96	49.11002	0.47835829	99.92	50.8935
Y_2	1.63122164	99.96	90.95591	1.42371912	99.96	87.0571	0.89584854	99.96	70.95743
Y_3	0.86121277	99.96	67.23187	0.93968523	99.96	72.31002	0.7626844	99.96	66.36362
Y_4	0.20732689	99.8	33.5111	0.24735886	99.96	34.11375	0.29407683	99.88	38.63228
Y_5	0.4747413	99.96	53.63538	0.42385213	99.96	51.69867	0.27387732	99.92	41.10216
Y_6	0.10175007	99.76	28.61451	0.03075036	100	12.75715	0.03543134	99.72	13.98985
Y_7	0.08112181	99.96	23.19876	0.06652247	99.96	18.56974	0.07436891	99.92	20.53821
Y_8	4.87645724	100	162.47759	2.60723129	99.96	119.33392	1.1254286	99.96	79.51832
Y_9	1.31426707	100	86.50562	0.92196148	99.96	75.08891	0.5303737	99.96	56.89025
AH	0.22133229	99.96	35.7475	0.350242	99.96	46.30618	0.3853064	99.96	49.29951
FG	0.3175933	99.96	44.54639	0.27209444	99.96	41.26734	0.22002628	99.96	37.10178
AS	4.82189324	99.96	153.30412	1.76820306	99.96	92.05165	0.78196254	99.96	65.88032
AS_{Max}	0.17837797	98.88	35.13783	0.10881857	97.52	22.88834	0.09079489	96.72	19.82571
AS_{Min}	0.28871191	99.96	39.98735	0.42808638	99.96	52.11682	0.37345673	99.96	48.8162
N_1	11.39291187	100	246.57221	3.90280706	99.96	145.54516	1.32637339	99.96	86.00629
N_2	10.32548969	100	232.97517	3.66809181	99.96	140.48712	1.29595228	99.96	85.02855
N_3	7.29557829	99.96	190.65332	2.76709845	99.96	119.23483	1.11450628	99.96	78.74319
N_4	9.41463899	100	221.119	3.46491363	99.96	135.99109	1.26739478	99.96	84.09359
CZ_1	0.10197737	99.96	23.74936	0.08447563	99.96	20.46401	0.08003285	99.84	21.65286
CZ_2	0.3227991	99.96	45.22677	0.42804446	99.96	52.87905	0.3676123	99.96	48.35791
CZ_3	0.22957846	99.96	37.82974	0.35245554	99.96	47.9687	0.31684246	99.96	44.99306
CZ_4	0.24479372	99.96	39.17488	0.37064755	99.96	49.35962	0.34924029	99.96	47.46317
CZ_5	0.13577832	99.96	28.88463	0.20803069	99.96	36.1392	0.20001428	99.92	35.46409

Table A10: Poisson Regression Simulation Results (New Estimators), P = 4, n = 600, ρ = .90

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.01620663	0	10.073359	0.006043302	0	6.132135	0.002303969	0	3.795928
HK	0.01596561	69.64	9.999736	0.005982533	58.08	6.103113	0.002300022	54.24	3.793071
HK_2	0.01608755	69.88	10.037473	0.006012503	58.24	6.117298	0.002301814	54.28	3.794349
HKB	0.01575442	68.96	9.937435	0.005925712	57.52	6.075488	0.002295519	54.24	3.789747
LW	0.0162066	70.24	10.073349	0.006043144	58.4	6.132059	0.002303969	54.36	3.795928
HSL	0.01545002	67.88	9.854192	0.00585651	56.32	6.039524	0.002277344	53.32	3.776254
AM	0.03763598	47.2	13.610251	0.014407748	42.88	7.854689	0.008309309	41.84	5.006056
GM	0.01510674	62.2	9.762749	0.005698167	52.6	5.974297	0.002299301	51.4	3.783184
Med	0.01530615	66.72	9.808704	0.005743778	55.96	5.985457	0.002290523	52.76	3.785477
KS	0.01597037	69.64	10.001204	0.005983433	58.08	6.103533	0.002300033	54.24	3.793079
KS_A	0.01579574	69.08	9.950039	0.005916562	57.2	6.066896	0.002273679	52.84	3.769709
KS_{Max}	0.01545169	66.72	9.867583	0.00591033	54.4	6.078436	0.002433825	48.6	3.870748
KS_{Med}	0.01610357	69.92	10.041939	0.006011022	58.24	6.116513	0.002296337	54.24	3.790177
KM_1	0.0160963	69.88	10.039843	0.006000161	58.16	6.111025	0.002291947	54.12	3.786961
KM_2	0.01595115	69.64	9.997134	0.006018864	58.24	6.120349	0.002299218	54.24	3.792453
KM_3	0.01637027	61.36	10.116814	0.005980907	54.04	6.110477	0.002431234	51.08	3.834475
KM_4	0.01611974	70	10.047074	0.006035602	58.32	6.128491	0.002302555	54.28	3.794914
KM_5	0.01545909	67.92	9.848996	0.00589478	57.2	6.058393	0.002287243	54	3.783136
KM_6	0.01607603	69.92	10.034012	0.006032015	58.28	6.12675	0.00230208	54.28	3.794561
KM_7	0.01570461	68.8	9.921889	0.005940836	57.64	6.082591	0.002292742	54.04	3.787475
KM_8	0.01594879	69.64	9.996431	0.006018698	58.24	6.120271	0.002299212	54.24	3.792449
KM_9	0.01596636	69.72	10.000528	0.005999479	58.16	6.111119	0.002299844	54.24	3.792927
KM_{10}	0.01610224	69.96	10.041915	0.006034522	58.32	6.127953	0.002302457	54.28	3.794839
KM_{11}	0.01596134	69.72	9.99944	0.006008281	58.2	6.11531	0.00229965	54.24	3.792777
KM_{12}	0.01607092	69.92	10.032501	0.00603163	58.28	6.126566	0.002302063	54.28	3.794548
KM_{13}	0.01616162	70.08	10.05979	0.006040849	58.4	6.130979	0.002303409	54.32	3.795531
KM_{14}	0.01596321	69.72	9.999855	0.006005601	58.2	6.114031	0.00229972	54.24	3.79283
GK	0.01596559	69.64	9.999731	0.005982532	58.08	6.103113	0.002300022	54.24	3.793071
HMO	0.04727399	6.44	18.677745	0.021399008	3.4	14.008172	0.021042368	0.28	9.260016
AD_{HM}	0.0162066	70.24	10.073349	0.006043299	58.4	6.132133	0.002303969	54.36	3.795928
AD_{Med}	0.01620653	70.24	10.073328	0.006043292	58.4	6.13213	0.002303968	54.36	3.795928
AD_{GM}	0.01620638	70.24	10.073288	0.006043278	58.4	6.132122	0.002303967	54.36	3.795927
AD_{AM}	0.01622416	70.08	10.084206	0.006048489	58.28	6.13577	0.002303241	54.32	3.795775
Y_1	0.01609838	69.88	10.044909	0.006031048	58.28	6.125522	0.002302848	54.36	3.794846
Y_2	0.01619025	70.2	10.068549	0.006041661	58.4	6.131331	0.002303868	54.36	3.795852
Y_3	0.01617591	70.08	10.064164	0.006039315	58.36	6.130194	0.002303773	54.36	3.795783
Y_4	0.01602469	69.16	10.023441	0.006020957	58.08	6.120243	0.002301386	54.16	3.793901
Y_5	0.01456897	65.32	9.57149	0.005760909	56.56	5.995512	0.002231423	50.2	3.744394
Y_6	0.0227597	30.44	12.428761	0.006372493	37.64	6.476322	0.00282635	36.16	4.153734
Y_7	0.01429512	52.8	9.57644	0.005355392	49.68	5.798834	0.002245392	45.68	3.753227
Y_8	0.01619815	70.2	10.070842	0.006042374	58.4	6.131685	0.002303882	54.36	3.795864
Y_9	0.0152334	68	9.773365	0.005876743	57.16	6.052871	0.002256839	52.76	3.761816
AH	0.01584167	69.4	9.963026	0.006000852	58.2	6.111753	0.002298833	54.24	3.792202
FG	0.01518805	67.6	9.764218	0.005897268	57.4	6.061783	0.002283981	53.68	3.781352
AS	0.0159656	69.64	9.999734	0.005982532	58.08	6.103113	0.002300022	54.24	3.793071
AS_{Max}	0.05731455	34.68	16.800904	0.022757226	34.8	9.644118	0.014090732	35.52	6.058172
AS_{Min}	0.0159408	69.64	9.994646	0.006024652	58.28	6.123116	0.002298759	54.24	3.792112
N_1	0.0162066	70.24	10.073348	0.006043299	58.4	6.132133	0.002303969	54.36	3.795928
N_2	0.01620388	70.24	10.07254	0.006042869	58.4	6.131927	0.00230395	54.36	3.795914
N_3	0.01617317	70.08	10.063328	0.006036557	58.28	6.128891	0.0023036	54.32	3.795658
N_4	0.01620122	70.2	10.071748	0.006042451	58.4	6.131727	0.002303932	54.36	3.795901
CZ_1	0.014144	63.84	9.441078	0.005754112	56.24	5.991919	0.0022547	52.44	3.760182
CZ_2	0.01568753	68.96	9.917033	0.005977624	58.12	6.100708	0.002291577	54.08	3.786899
CZ_3	0.01538705	68.16	9.82522	0.005922708	57.64	6.074603	0.002272918	53.32	3.773588
CZ_4	0.01560638	68.84	9.891297	0.005959494	57.84	6.092162	0.002291772	54.08	3.787008
CZ_5	0.01456474	65.32	9.570222	0.005757852	56.56	5.993975	0.002231415	50.2	3.744385

Table A11: Poisson Regression Simulation Results (New Estimators), P = 4, n = 600, ρ = .95

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.03283609	0	14.30179	0.012641723	0	8.784325	0.004458053	0	5.219407
HK	0.03178934	83.04	14.0862	0.012375353	72.08	8.695585	0.004443436	59.32	5.211708
HK_2	0.0323332	83.28	14.20044	0.012513937	72.2	8.741851	0.004449535	59.36	5.21495
HKB	0.0309322	82.96	13.9125	0.012153615	71.52	8.621232	0.004424512	59.24	5.201768
LW	0.03283595	83.36	14.30177	0.012640724	72.48	8.784002	0.004458053	59.4	5.219407
HSL	0.03008376	82.36	13.74228	0.011878693	70.8	8.520262	0.004311223	58.24	5.138742
AM	0.04255273	65.52	15.30739	0.015959753	61.88	9.036874	0.008156198	48.92	5.747974
GM	0.02577061	78.96	12.77741	0.010731835	67.8	8.149531	0.00426156	56.88	5.113322
Med	0.0291073	82.16	13.54314	0.011620123	70.36	8.438405	0.004327712	58.32	5.148027
KS	0.03180976	83.04	14.09046	0.012379305	72.08	8.696897	0.004443471	59.32	5.211726
KS_A	0.03164172	83	14.06504	0.012258344	71.76	8.650547	0.00433573	57.84	5.15112
KS_{Max}	0.02997134	82.08	13.71779	0.011808755	70.52	8.498511	0.004369716	54.84	5.15665
KS_{Med}	0.03254171	83.32	14.24221	0.012549976	72.24	8.753561	0.004436427	59.24	5.207772
KM_1	0.03255968	83.36	14.24624	0.012532299	72.16	8.747502	0.004424548	59.2	5.201423
KM_2	0.03177804	83.04	14.09055	0.012544726	72.28	8.75237	0.00443888	59.28	5.209419
KM_3	0.02730549	77.92	13.1245	0.011461416	69	8.388831	0.004332923	56.76	5.140523
KM_4	0.03245824	83.36	14.22646	0.012608234	72.44	8.773222	0.004452896	59.4	5.216709
KM_5	0.02972028	82.4	13.65365	0.012071814	71.36	8.59626	0.004390743	59.04	5.184039
KM_6	0.03225032	83.24	14.18237	0.012592082	72.44	8.768044	0.004451287	59.4	5.215872
KM_7	0.03085605	82.88	13.9054	0.012264699	71.68	8.658274	0.004408135	59.12	5.192882
KM_8	0.03176835	83.04	14.08858	0.012544041	72.24	8.752148	0.004438859	59.28	5.209408
KM_9	0.03180853	83.04	14.09239	0.012455156	72.16	8.722434	0.004442279	59.28	5.21113
KM_{10}	0.03238796	83.36	14.21229	0.01260411	72.44	8.771925	0.004452535	59.4	5.216525
KM_{11}	0.03179953	83.04	14.09217	0.012496219	72.2	8.736195	0.004441177	59.28	5.210577
KM_{12}	0.032231	83.24	14.17859	0.012590679	72.44	8.767589	0.004451218	59.4	5.215837
KM_{13}	0.03263812	83.36	14.26255	0.012630881	72.44	8.780751	0.004456176	59.4	5.218426
KM_{14}	0.03180332	83.04	14.0924	0.012483768	72.16	8.732027	0.004441566	59.28	5.210772
GK	0.03178928	83.04	14.08619	0.01237535	72.08	8.695584	0.004443436	59.32	5.211708
HMO	0.03099073	43.04	14.77732	0.012848872	36.12	10.591991	0.013841309	6.04	7.868303
AD_{HM}	0.03283595	83.36	14.30177	0.012641711	72.48	8.784321	0.004458053	59.4	5.219407
AD_{Med}	0.03283577	83.36	14.30173	0.012641693	72.48	8.784315	0.004458052	59.4	5.219406
AD_{GM}	0.03283501	83.36	14.30158	0.01264165	72.48	8.784305	0.00445805	59.4	5.219405
AD_{AM}	0.03294206	83.16	14.31444	0.012732522	72.4	8.798715	0.004467534	59.32	5.22056
Y_1	0.03221598	83.12	14.17024	0.012585463	72.32	8.767816	0.004454799	59.4	5.217855
Y_2	0.03274855	83.36	14.28433	0.012633751	72.44	8.78176	0.004457554	59.4	5.219154
Y_3	0.0326553	83.36	14.26597	0.012621675	72.44	8.777717	0.004457105	59.4	5.218912
Y_4	0.03156408	82.68	14.03055	0.012507701	72.12	8.745355	0.004450864	59.36	5.215587
Y_5	0.02730253	81.32	13.12718	0.011685675	70.68	8.462146	0.00417271	56.96	5.063284
Y_6	0.02395025	56.72	12.74712	0.008875967	56.52	7.672693	0.004410006	43.12	5.190265
Y_7	0.01973611	73	11.23881	0.009490266	65.36	7.676083	0.003967121	52.44	4.933192
Y_8	0.03280328	83.36	14.29533	0.012638151	72.44	8.783157	0.004457683	59.4	5.219218
Y_9	0.02958007	82.52	13.62526	0.012096074	71.44	8.598974	0.004322631	58.28	5.146124
AH	0.03129342	83	13.98778	0.01247223	72.2	8.728233	0.00443834	59.28	5.209042
FG	0.02864674	82.2	13.42772	0.012045024	71.24	8.585597	0.004378797	58.8	5.177546
AS	0.03178931	83.04	14.08619	0.012375352	72.08	8.695585	0.004443436	59.32	5.211708
AS_{Max}	0.05783985	55.36	17.53089	0.020777327	55.44	9.879471	0.012011483	41.64	6.421425
AS_{Min}	0.03175868	83.08	14.08869	0.012570698	72.44	8.760995	0.004436557	59.28	5.208242
N_1	0.03283594	83.36	14.30176	0.012641709	72.48	8.78432	0.004458053	59.4	5.219407
N_2	0.03282464	83.36	14.29951	0.012639954	72.48	8.783747	0.004457982	59.4	5.21937
N_3	0.0326862	83.36	14.27176	0.012612044	72.44	8.774537	0.004456521	59.4	5.218614
N_4	0.03281344	83.36	14.29728	0.012638221	72.44	8.783181	0.004457913	59.4	5.219335
CZ_1	0.02437126	80.16	12.4596	0.011500744	70.32	8.400618	0.004256092	57.92	5.111398
CZ_2	0.03083215	82.96	13.89413	0.012391401	71.96	8.701143	0.004412646	59.12	5.195458
CZ_3	0.02989816	82.68	13.69615	0.012214943	71.68	8.641694	0.004355329	58.68	5.164699
CZ_4	0.03051094	82.84	13.82631	0.012316673	71.8	8.675677	0.004410763	59.08	5.194413
CZ_5	0.02720618	81.28	13.10821	0.011667485	70.68	8.456167	0.004172455	56.96	5.063144

Table A12: Poisson Regression Simulation Results (New Estimators), P = 4, n = 600, ρ = .99

	Intercept = -1			Intercept = 0			Intercept = 1		
k	MSE	Perf.	MAPE	MSE	Perf.	MAPE	MSE	Perf.	MAPE
MLE	0.18575322	0	33.39513	0.068427073	0	20.262242	0.025228958	0	12.365021
HK	0.16108059	99.8	31.13023	0.060303723	99.16	19.066425	0.024718464	95.72	12.249292
HK_2	0.17201607	99.84	32.20104	0.064587992	99.16	19.715722	0.02492239	95.72	12.295444
HKB	0.14037237	99.8	29.23153	0.055243693	99.12	18.301745	0.02404708	95.6	12.093563
LW	0.18574814	99.8	33.39472	0.068361936	99.2	20.253626	0.025228946	95.72	12.365018
HSL	0.12407945	99.8	27.63612	0.051362889	99.08	17.754366	0.019241924	95.32	10.891945
AM	0.06935484	93.28	19.99176	0.028467213	97.2	12.651804	0.016383923	92.68	9.250544
GM	0.07187112	99.32	20.63493	0.031973179	99	13.987419	0.017995052	95.16	10.552087
Med	0.10268139	99.76	25.46108	0.047576487	99.04	17.046886	0.020555026	95.28	11.203223
KS	0.16145497	99.8	31.16791	0.060414998	99.16	19.083768	0.024719661	95.72	12.24956
KS_A	0.15925927	99.84	31.17988	0.061532577	99.16	19.323255	0.02243161	95.52	11.738342
KS_{Max}	0.12512679	99.8	27.75739	0.051507124	99.08	17.780414	0.018517296	95.28	10.70953
KS_{Med}	0.18368755	99.8	33.22232	0.067698052	99.2	20.161269	0.024998575	95.72	12.312788
KM_1	0.18319005	99.8	33.18357	0.067380442	99.2	20.118133	0.024805505	95.72	12.26894
KM_2	0.15184199	99.84	30.5562	0.065180835	99.16	19.825442	0.02453063	95.72	12.206899
KM_3	0.07944184	98.88	21.64551	0.038786032	99.08	15.342605	0.01808759	95	10.533774
KM_4	0.17340776	99.84	32.37779	0.067274365	99.2	20.102365	0.025055713	95.72	12.325651
KM_5	0.11654098	99.8	26.77256	0.052779269	99.08	17.981383	0.022866208	95.52	11.826447
KM_6	0.16896816	99.84	31.93598	0.066537125	99.2	20.005499	0.024992092	95.72	12.312727
KM_7	0.13203515	99.8	28.6857	0.058330934	99.12	18.818653	0.023473838	95.56	11.956262
KM_8	0.15161818	99.84	30.53511	0.065157882	99.16	19.82225	0.024529862	95.72	12.206727
KM_9	0.15889524	99.8	31.01352	0.062508873	99.16	19.419016	0.024668368	95.72	12.238029
KM_{10}	0.17127775	99.84	32.19889	0.067096922	99.2	20.08048	0.025038672	95.72	12.32219
KM_{11}	0.15652386	99.84	30.86283	0.063705297	99.16	19.604115	0.024622627	95.72	12.227719
KM_{12}	0.16828726	99.84	31.88236	0.066493808	99.2	19.999699	0.024989404	95.72	12.31213
KM_{13}	0.17874017	99.84	32.83335	0.068024776	99.2	20.207311	0.02516918	95.72	12.351627
KM_{14}	0.15736019	99.84	30.91668	0.063337674	99.16	19.54773	0.024638629	95.72	12.231329
GK	0.16107865	99.8	31.13006	0.060303627	99.16	19.066412	0.024718459	95.72	12.249291
HMO	0.02159962	97.8	11.5884	0.006763336	98.04	6.679439	0.005439398	90.12	5.815336
AD_{HM}	0.18574903	99.8	33.39478	0.068426647	99.2	20.262184	0.025228947	95.72	12.365018
AD_{Med}	0.18573705	99.8	33.39404	0.06842622	99.2	20.26213	0.02522889	95.72	12.365006
AD_{GM}	0.18571268	99.8	33.39196	0.068423978	99.2	20.261847	0.02522881	95.72	12.364992
AD_{AM}	0.18392625	99.76	33.19535	0.068127507	99.2	20.217109	0.025178383	95.72	12.352744
Y_1	0.15817449	99.8	30.81975	0.06416872	99.2	19.699911	0.024699205	95.72	12.251141
Y_2	0.18047048	99.8	32.96338	0.067909342	99.2	20.194044	0.025193617	95.72	12.357398
Y_3	0.17408472	99.84	32.43736	0.067288404	99.2	20.104173	0.025168847	95.72	12.351664
Y_4	0.13811889	99.6	28.66598	0.05941747	99.16	19.012799	0.023895898	95.68	12.068691
Y_5	0.11275178	99.72	26.4088	0.052358816	99.12	17.898684	0.019449818	95.24	10.942003
Y_6	0.01699761	96.6	10.6205	0.007717411	98.2	6.967176	0.007698615	92.68	6.882011
Y_7	0.02560858	98.56	12.37504	0.018646749	98.68	10.623028	0.012263217	94.4	8.684406
Y_8	0.18454713	99.8	33.3009	0.068301712	99.2	20.245696	0.025213185	95.72	12.361618
Y_9	0.14077657	99.72	29.34628	0.060500098	99.16	19.073304	0.023222481	95.52	11.884412
AH	0.14312499	99.8	29.63748	0.062928888	99.16	19.500631	0.02453753	95.72	12.208707
FG	0.10288743	99.68	25.37899	0.052414126	99.12	17.907158	0.022577247	95.48	11.746482
AS	0.16107962	99.8	31.13015	0.060303675	99.16	19.066418	0.024718462	95.72	12.249291
AS_{Max}	0.06818008	89.48	19.48342	0.026784764	95.04	11.55682	0.016550373	90.24	8.482102
AS_{Min}	0.14989257	99.84	30.45295	0.066017445	99.16	19.946276	0.024445403	95.72	12.187449
N_1	0.18574853	99.8	33.39474	0.068426597	99.2	20.262177	0.025228945	95.72	12.365018
N_2	0.18541081	99.8	33.36644	0.068369919	99.2	20.254362	0.025226317	95.72	12.364431
N_3	0.18087447	99.84	32.97953	0.067372375	99.2	20.11484	0.025165434	95.72	12.350717
N_4	0.18507373	99.8	33.33816	0.068313547	99.2	20.246585	0.025223707	95.72	12.363848
CZ_1	0.05962356	99.64	19.50729	0.043111906	99.12	16.312522	0.019479372	95.24	10.940886
CZ_2	0.14005222	99.8	29.37986	0.061901624	99.16	19.346659	0.023937727	95.56	12.06735
CZ_3	0.13009424	99.76	28.37981	0.059149452	99.16	18.941761	0.022943457	95.52	11.831759
CZ_4	0.13895658	99.76	29.25385	0.061514119	99.16	19.265371	0.023927464	95.56	12.06258
CZ_5	0.09949787	99.68	24.99212	0.050677502	99.12	17.621805	0.019444271	95.24	10.941036