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Using a Repeated Measures ANOVA Design to Analyze the Effect Writing in Mathematics Has on the Mathematics Achievement of Third Grade English Language Learners and English Speakers

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USING A REPEATED MEASURES ANOVA DESIGN TO ANALYZE THE EFFECT WRITING IN MATHEMATICS HAS ON THE MATHEMATICS ACHIEVEMENT OF THIRD GRADE ENGLISH LANGUAGE LEARNERS AND ENGLISH SPEAKERS

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Zoe Ansorena Morales

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To: Dean Michael R. Heithaus
College of Arts, Sciences and Education

This dissertation, written by Zoe Ansorena Morales, and entitled Using A Repeated Measures ANOVA Design to Analyze the Effect Writing in Mathematics Has on the Mathematics Achievement of Third Grade English Language Learners and English Speakers, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

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Florida International University, 2016

DEDICATION

I dedicate this doctoral dissertation to my children Elizabeth Zoe Morales and Jose Francisco Morales, and my husband Jose Manuel Morales. Our family is my inspiration. To my mother Maria Elena Ansorena and my father Jorge Ansorena. Your example of faith, strength, and persistence has guided me through this extraordinary journey.

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ABSTRACT OF THE DISSERTATION

USING A REPEATED MEASURES ANOVA DESIGN TO ANALYZE THE EFFECT WRITING IN MATHEMATICS HAS ON THE MATHEMATICS ACHIEVEMENT OF THIRD GRADE ENGLISH LANGUAGE LEARNERS AND ENGLISH SPEAKERS

by

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Professor Maria L. Fernandez, Major Professor

The gap that exists between English language learners and English speaking students' achievement in mathematics continues to grow. Moreover, students are now required to show evidence of their mathematics knowledge through writing in standardized assessments and class assignments.

The purpose of this study was to analyze students' writing in mathematics and the metacognitive behaviors they portrayed through their writing as they solved mathematics problems. The instruments included a pretest, two biweekly tests, and a posttest. The writing instruction encompassed students learning to solve problems by using Polya's four phases of problem solving which was completed in 12 sessions over a period of 6 weeks. Garofalo and Lester's framework which renamed Polya's phases into orientation, organization, execution, and verification, was used to look at the metacognitive behaviors students used. The participants included 67 students enrolled in four third grade classes, who were English language learners and English speakers.

This research followed a quasi-experimental design, with a treatment group and a control group. A one-way repeated ANOVA was used to analyze the data. The findings showed no significant difference between the mathematics achievement scores of treatment and control. However, growth trends in achievement scores revealed that the treatment group scores were increasing faster than the control group scores across the four tests during the 6-week study. Moreover, significant differences were found between the treatment and the control groups when the problem solving with metacognitive behaviors scores were analyzed. Descriptive statistics showed the frequency of occurrence of each of the problem solving phases increased steadily across the four tests for the students in the treatment group. During the posttest, 100% of treatment group students wrote about metacognitive behaviors they used during the orientation and organization phases, 91.4% wrote about their metacognition for executing the solution, and 80% wrote about the verification process they followed.

These findings are useful to education professionals who are interested in creating programs for teaching mathematics at the elementary level that include effective problem solving practices. This evidence-based method may be adopted in school districts with large populations of ELLs in order to assist these students when solving problems in mathematics.

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CHAPTER I

INTRODUCTION

Mathematics education has undertaken numerous changes in the last century in response to student needs and to educational, social, and economic issues. The latest reform in mathematics education, the Common Core State Standards for Mathematics (CCSS, 2010), builds on the National Council of Teachers of Mathematics' Principles and Standards (NCTM, 2000) and Curriculum Focal Points to prepare students for college and career success (CCSS, 2010; Dacey & Polly, 2012). The CCSS include eight mathematics practices that engage students in higher order thinking as students (a) make sense and persevere in solving problems, (b) reason abstractly and quantitatively, (c) construct viable arguments and critique the reasoning of others, (d) model with mathematics, (e) use appropriate tools strategically, (f) attend to precision, (g) look for and make use of structure, and (h) look for and express regularity in repeated reasoning (CCSS, 2010). These mathematics practices call for students to problem solve, communicate, and reason.

With an emphasis on problem solving, expectations for student learning are high so it is important for students to have meaningful learning experiences in their mathematics classes. George Polya's (1957) problem solving phases state that when students solve problems in mathematics they should (a) understand the problem, (b) devise a plan to solve it, (c) carry out the plan, and (d) examine the solution. These phases are regularly found in mathematics textbooks and used as a framework when teaching students how to problem solve in mathematics classrooms. By using Polya's framework in the classroom, researchers have found that students are able to successfully solve mathematics problems (Garofalo & Lester, 1985; Pugalee, 2001, 2004).

Additionally, the CCSS for Mathematics (2010) expect students to use evidence to support their thinking. Through the use of writing in mathematics, students can construct knowledge, reflect on their work, and clarify their thoughts (NCTM, 2000). They can also use writing as a tool to explain and show the expected evidence by describing how they came to understand what the problem was asking, to decide on the most appropriate strategies, to work to find a sound answer, and to verify their work. The CCSS for Mathematics demand more rigorous thinking than previously (White & Dauksas, 2012). Students' thinking and understanding can be evidenced through drawings, and through using and explaining concrete models, place-value strategies, inverse relationships, and properties of operations (Dacey & Polly, 2012).

National Council of Teachers of Mathematics (2000) emphasizes instruction that supports *all* students by providing them with the experiences to learn mathematics in a comprehensible way. This includes supporting the mathematical development of English Language Learners (ELLs) to help them meet the high expectations of recent reforms (e.g., CCSS for Mathematics). English language learners are students who are being served in appropriate programs of language assistance (NCES, 2013). They are part of the largest growing minority group in U.S. classrooms, reaching 10% or an estimated 4.4 million K-12 students during the 2010-2011 school year (NCES, 2013). In the same year, 2010-11, states located in the west had the highest percentages of ELLs in their public schools. California was in the lead with 29% ELL enrollment; followed by Oregon, Hawaii, Alaska, Colorado, Texas, New Mexico, and Nevada with 10% ELL enrollment.

In addition, Florida, the District of Columbia, Oklahoma, Arkansas, Massachusetts, Nebraska, North Carolina, Virginia, Arizona, Utah, New York, Kansas, Illinois, and Washington had ELL school enrollment percentages between 6% and 9.9%. The K-12 ELL population is continually growing, especially in the District of Columbia and the aforementioned 21 states, thus looking at the experiences ELLs are having in mathematics and their performance in comparison to other students is important.

According to the NCES (2013), ELLs' performance in the mathematics section of the 2013 National Assessment of Educational Progress (NAEP) assessment was below that of non-ELLs. The NAEP assessment uses three achievement levels: basic, proficient, and advanced. When the scores are reported, the achievement-level cut scores result in four ranges: below basic, basic, proficient, and advanced. The NCES reported that while 55% of non-ELLs are below the proficient level, 86% of ELLs performed at below basic or basic levels on the fourth grade 2013 NAEP mathematics assessment. Additionally, only 1% of ELLs performed at the advanced or superior performance level on the same test (NCES, 2013). The advanced level indicates that students are able to apply procedural and conceptual knowledge to complex and real-world problem solving in the five NAEP mathematics content areas: (a) number properties and operations, (b) measurement, (c) geometry, (d) data analysis, statistics and probability, and (e) algebra (NCES, 2013).

Additionally, the new Common Core State Standards (CCSS, 2010) and the new assessments linked to these standards expect students to show evidence of their thinking and of their conceptual and procedural understanding of mathematics. The new standardized assessment used in the state of Florida is the Florida Standards Assessment

(FSA). Students are expected to write in the mathematics section of this standardized test to explain their answers (Dixon, Leiva, Larson & Adams, 2013; FSA, 2014). It has been suggested in the literature that writing in mathematics can help students develop into analytical thinkers as writing requires students to clarify their ideas and to reflect on what they are learning (Berkenkotter, 1982; Cooper, 2012; Nelson, 2012; Parker & Breyfogle, 2011). English language learners are also being required to show evidence of their mathematical understanding in the same way their English speaking classmates do, through writing. Research studies that look at the way ELLs perform in mathematics when writing is used as an instructional practice is lacking. However, the literature has suggested that writing can help students develop their mathematical thinking and problem solving. On the basis of these premises, research on instruction that includes writing in mathematics to assist students, especially ELLs, in becoming better problem solvers as they achieve higher in mathematics is important.

Theoretical Framework

Mathematics education has been filled with an overwhelming number of theories. One of the most frequently applied theories in mathematics education is constructivism. Constructivism follows the ideas of John Dewey and Jean Piaget, and it is founded on the belief that students construct their own understanding of mathematical concepts. John Dewey believed that students need to engage in real-world experiences and practical activities that provide social interactions and creativity that in turn lead to meaningful learning (Dworkin, 1959). Furthermore, Jean Piaget believed that children need to be actively involved in their learning, use their prior knowledge to build on new knowledge, and move through stages of cognitive development (Piaget & Inhelder, 1969).

Piaget believed that children's knowledge is first constructed by using concrete models that can be translated into symbolic representations and later into abstract ideas (Piaget & Inhelder, 1969). Piaget and Inhelder argued that concepts are learned from the actions or operations of our experiences. The concrete operations stage, as explained by Piaget and Inhelder (1969), provides a transition between schemes and general logical structures. Children at this stage relate operations (e.g., adding or union, subtracting or separating) directly to objects. With time, they are able to interpret a given concept more abstractly which together with their automaticity of procedures may lead to conceptual competence (Hiebert & Lefevre, 1986).

An example of how Piaget's ideas were used with young children learning mathematics can be seen in Carpenter and colleagues' work (1999). Carpenter, Fennema, Franke, Levi, and Empson (1999) used Piaget's ideas concerning how children solve problems in the concrete operations stage to study how children develop conceptual and procedural understanding of mathematical concepts. Cognitively Guided Instruction (CGI) was developed as the researchers showed how young children think and solve mathematics problems at different levels of cognition (Carpenter et al., 1999). Cognitively Guided Instruction is derived from the belief that in order to solve problems, children first model the actions in the problem, thus using concrete objects. These "concrete" or physical strategies become more effective counting strategies. Once they understand number relations children rely on more complex strategies (number facts) which in turn, become more abstract ways of solving mathematics problems (Carpenter et al., 1999). Carpenter and colleagues research shows how children develop conceptual and procedural understanding of concepts as they move from using concrete manipulatives to doing mental mathematics.

Furthermore, Hiebert and Wearne (1986) explained that "poor performance in school mathematics can be traced to a separation between students' conceptual and procedural knowledge of mathematics" (p.199). Conceptual learning leads to knowledge permanence, understanding, and the ability to apply what is learned to new problems. For instance, Thompson and Saldanha's (2007) study showed that in order to have conceptual understanding of fractions students need to analyze different schemes. These schemes were separated into division schemes, including sharing or partitioning of equal parts; multiplication schemes or systems for creating units of units; measurement schemes or segmented quantities; and fraction schemes which can be a collection with some shaded parts (Thompson & Saldanha, 2007). The authors found that providing opportunities for students to work with these different schemes, students were able to conceptually understand fractions before learning a set of rules or procedures to solve computation problems (Thompson & Saldanha, 2007). Furthermore, Kling (2011) explained that when students have conceptual understanding of addition facts, including the ability to decompose and recompose numbers, they became fluent in adding basic facts. It can be concluded that when children learn conceptually in mathematics they can make important connections between procedures and concepts (Stylianides & Stylianides, 2007).

Moreover, Hiebert and Wearne (1986) found that there were several points in the problem solving process where links between concepts and procedures are specifically critical. However, they explained that the way in which these links are established cannot yet be specified or assessed. On the basis of the fact that ordinary instruction programs rely mainly on procedural skills, the authors argued that students' mathematical behavior often shows students looking at surface features of problems and recalling and applying symbolic rules which produce mathematically unreasonable answers (Hiebert & Wearne, 1986). The authors also found that students need to make three important connections between concepts and procedures during the problem solving process. The first is at the beginning of the problem solving process when the problem is being interpreted or understood (correlated to Polya's [1957] first phase) and the students make connections between mathematical symbols (e.g., numerical or operational) and their conceptual referents. For instance, the division sign in $\frac{7}{8} \div \frac{1}{4}$ can be interpreted as connecting the symbol with the algorithm *invert and multiply*, or as connecting the symbol with the conceptual notion of *how many fourths are in seven eighths* (Hiebert & Wearne, 1986). The second connection is made when the students are solving the problem (correlated to Polya's second and third phases) in which procedures are selected and applied, sometimes without linking these rules to their conceptual rationales. The third point looks at the connections between the procedures used and the conceptual knowledge of the symbols to evaluate if the answer is reasonable (correlated to Polya's fourth phase).

The Principles and Standards for School Mathematics' process standards (NCTM, 2000) argue for the importance of making connections between conceptual and procedural understanding and calling for students to problem solve, communicate, represent, reason and make connections as they learn mathematics. Additionally, in the CCSS (2010), the first mathematical practice outlined is focused on problem solving and establishes that children can use concrete manipulatives to help in conceptualizing and solving problems, checking their answers to problems using different methods, and understanding others' methods in solving problems (CCSS, 2010). These closely relate to Piaget's belief of how children first use concrete operations to understand concepts. The mathematical practice also has its basis in Polya's (1957) phases of problem solving as it encourages children to use different strategies and to look back and verify their answers. As students construct mathematical knowledge, tools such as diagnostic interviews (Ashlock, 2006), collaborative conversations (Gibson & Hasbrouck, 2009), and writing activities (Pugalee, 2001, 2004) can be used to assess their thinking process and mathematical knowledge. In particular, writing activities have been used with secondary school students to assess the levels students reach when they problem solve (Pugalee, 2001, 2004). Pugalee (2001, 2004), and Garofalo and Lester (1985) studied the metacognitive behaviors students demonstrate when they problem solve by using writing activities. Pugalee's (2001) research shows how writing in mathematics supports students in developing problem solving, communication, and reasoning skills as they use specific metacognitive behaviors that mirror Polya's phases to explain their thinking. Students in Pugalee's study (2001) wrote to describe how they solved mathematics problems during and after the process. Then, their descriptions were categorized into behaviors that showed how the students used the four phases to solve the mathematics problems. These studies used Polya's (1957) four phases as well as constructivist views as their theoretical lens.

Constructivists' views in education include students developing structures that are more complex, abstract, and powerful than the ones they already have (Clements, 1997). They also focus on analyzing students' thinking in order to identify their mathematical understanding (Clements, 1997). The NCTM process standards state that students need to be encouraged “to use the new mathematics they are learning to develop a broad range of problem-solving strategies, to formulate challenging problems and to learn to monitor and reflect on their own ideas in solving problems” (NCTM, 2000, p. 116). Pugalee (2001) studied students’ written reflections during the problem solving process. He found that students in a high school Algebra 1 class showed specific metacognitive behaviors. Pugalee’s study was based on Garofalo and Lester’s (1985) metacognitive framework that described four metacognitive behaviors: Orientation, Organization, Execution, and Verification. These behaviors follow George Polya’s (1957) four phases of problem solving, and can be related to those as follows: orientation as understanding the problem, organization as devising a plan, execution as carrying out the plan, and verification as examining the solution or looking back. Garofalo and Lester (1985) explained that students’ metacognitive behaviors while solving mathematics problems may include selecting strategies to help understand the problem, planning a course of action and the strategies to solve it, monitoring execution activities while implementing strategies, and revising if the plan used was effective. Pugalee (2001) found that the high school students’ written responses could be connected to Garofalo and Lester’s metacognitive framework.

Basis for the Current Study

In spite of the national, state, and school districts’ efforts in developing and updating standards to improve academic achievement in mathematics across grade levels for *all* students, ELLs still fall far behind their English speaking counterparts. In fact, the results from the National Center for Education Statistics (2013) reported that in 1996 ELLs’ average score on the National Assessment of Educational Progress (NAEP) in mathematics was 201 as opposed to 225 for English speakers, a difference of 24 points; while in 2013, ELLs' average score in the mathematics NAEP was 219 while non-ELLs' average was 244, showing a difference of 25 points (NCES, 2013). There is no statistical significance reported for the difference in average scores between the groups. Even though the NCES reports that the changes within the ELLs' scores from 1996 to 2013 are statistically significant at $p < .05$, the achievement gap revealed by this test shows it is increasing when comparing the ELLs to the English speaking students. In addition, the 2011 Trends in Mathematics and Science Study (TIMSS) found that U.S. fourth graders scored above the international average on knowing facts and procedures but below the international average on applying procedural and conceptual knowledge in problem solving and on reasoning in problem solving. The 2011 TIMSS measured students' cognitive domains defined as: (a) *knowing* or the knowledge of mathematics facts, concepts, tools, and procedures; (b) *applying* or the ability to apply knowledge and conceptual understanding in a problem situation; and (c) *reasoning* or going beyond the solution of routine problems that encompass unfamiliar situations, complex contexts, and multi-step problems (Mullis, Martin, Foy, and Arora, 2012). Results showed that the U.S. fourth grade students scored the highest in the domain of *knowing* with an average scale score of 556, 15 points higher than the international mathematics average of 541. This showed that in the domain of *knowing*, the score for the United States was significantly higher than the overall mathematics score (Mullis et al., 2012). However, the fourth graders scored at an average of 539 in the *applying* cognitive domain, two points lower than the international average. They scored the lowest in the *reasoning* domain with an average scale score of 525, 16 points below the international mathematics average score.

The scores for both the *applying* cognitive domain and the *reasoning* cognitive domain were significantly lower than the overall mathematics score (Mullis, et al., 2012). Problem solving is particularly connected to the second and third cognitive domains discussed by Mullis et al. (2012): applying and reasoning. During problem solving, students are expected to think about what the problem is asking, apply their knowledge, and reason about ways to solve it. As observed in the 2011 TIMSS report, the United States' students scored below the country's average score of 541 in both these cognitive domains. Furthermore, English language learners, being part of 10% of the student population in the U.S. classrooms, score much lower in assessments (e.g., NAEP) that require mastery of these types of domains (NCES, 2013); thus perpetuating the achievement gap between ELLs and English speakers. In addition, researchers have found that ELLs are not likely to choose science, technology, engineering, and mathematics (STEM) careers and so are underrepresented in these fields (Brown, Cady, & Taylor, 2009). Careers in STEM require professionals to have knowledge of mathematics facts, concepts, tools and procedures, but more importantly be able to apply these concepts and use reasoning to go beyond routine problem solving.

Another recent change in curriculum has made significant changes to the way mathematics should be taught and learned. The Common Core State Standards has refocused mathematics into a more rigorous and problem solving subject. Students are required to be persistent, create and use diverse strategies, and show evidence of their thinking (CCSS, 2010). Additionally, students are required to use writing to show evidence of their understanding in both classroom assignments and assessments. Most of the literature describing writing in mathematics has been conducted from middle school to college level (Barlow & Drake, 2008; Bicer, Capraro, & Capraro, 2013; Brown, Cady & Taylor, 2009; Pugalee, 2001, 2004; Taylor & McDonald, 2007). Writing in mathematics has been used as instructional and assessment practices. The current study investigated the effect writing in mathematics had on students' mathematics achievement and on students' problem solving assessments. Writing activities included students' (a) explanations to their answers to problems, and (b) explanations of the processes they followed in solving the problems. Given that writing is now a needed form of communication in the mathematics classroom for both construction of knowledge and for assessment starting at the elementary level, research on how writing in mathematics affects the mathematics achievement of elementary ELLs is important.

Statement of the Problem

The gap that exists between ELLs and English speaking students' achievement in mathematics continues to grow (NCES, 2013). Students are now required to show evidence of their mathematics knowledge through writing in standardized assessments (FSA, 2014). Elementary age students struggle when writing in mathematics as this is a practice that is not typically included in daily instruction. However, students at very early grades are being asked to support their answers to mathematics problems during classroom assignments, weekly assessments, and standardized tests. Thus far, writing in mathematics is not a common practice at the elementary level even though research studies at higher levels (e.g., middle school, high school, and college) have found it is an effective tool for students to develop and show their mathematical knowledge (Baxter, Woodward, & Olson, 2005; Bicer, Capraro, & Capraro, 2013; Cooper, 2012; Porter & Masingila, 2000; Pugalee 2001, 2004; Seto & Meel, 2006; Taylor & McDonald, 2007;

Williams, 2003). There is a lack of studies that show the connection between student mathematics achievement and using writing to promote conceptual understanding and problem solving at the elementary school level.

Purpose of the Study

The purpose of this study was to investigate third grade ELLs' and English speakers' writing during mathematics and the relationship between this writing and the students' achievement in mathematics. The students wrote to explain answers to mathematical problems during whole group instruction, and received instruction on the four problem solving phases developed by George Polya during a six week period. A one-way repeated ANOVA was used to analyze students' achievement scores in four different assessments.

Research Questions

The research questions that guided this study are as follows:

1. Are the mathematics achievement scores of English language learners and English speakers using writing in mathematics significantly higher than the achievement scores of students not using writing in mathematics?
    a. Are the mathematics achievement scores of the English language learners using writing in mathematics significantly higher than the achievement scores of the English language learners not using writing?
    b. Are the mathematics achievement scores of the English speakers using writing in mathematics significantly higher than the achievement scores of the English speakers not using writing?

2. Are the problem solving with metacognitive behaviors' scores of English language learners and English speakers using writing in mathematics significantly higher than the problem solving with metacognitive behaviors' scores of students not using writing in mathematics?
3. Which of the four metacognitive behaviors (orientation, organization, execution, verification) do third grade English language learners and English speakers most often demonstrate when they write during problem solving on the achievement tests?

Significance of the Study

This study included third grade students in a predominantly Hispanic school in the southwest area of Miami Dade County Public Schools (MDCPS). The current study focused on third grade students given that they are being required to use writing to explain their answers to mathematics problems in high stakes testing as well as in weekly assessments and classroom assignments under the new Mathematics Florida Standards (MAFS, 2015). The current study examined the effects of writing during problem solving on the mathematics achievement of third grade ELLs and English speakers to determine if the use of writing helps students to achieve higher in the subject. It also analyzed the metacognitive behaviors the students used on the biweekly tests and on the posttest.

The fourth grade 2013 NAEP mathematics assessment showed a large percentage of English language learners (86%) performing at below basic and basic levels in mathematics academic achievement. The 2013 NAEP study showed that the ELLs' average score in mathematics was higher in 2013 than in 1996. However, the 2013 NAEP results also show that there is a wider gap between the ELLs and the English speakers in the 2013 test than in the 1996 NAEP mathematics test, showing that although the two groups are scoring higher, the English speakers are advancing at a faster rate than the ELLs. Additionally, the 2011 TIMSS results showed that on the whole, the U.S. fourth grade students scored the highest in the domain of *knowing*, scored at an average level in the *applying* cognitive domain, and scored the lowest in reasoning when compared to international mathematics average scores (Mullis, et al., 2012).

Given these results, this study might assist Miami-Dade County Public Schools third grade teachers and other similar school districts third grade teachers in deciding if using writing in mathematics can increase the mathematics achievement of students by restructuring instructional practices to include more writing in their mathematics lessons. This study can assist in developing instructional practices and strategies that can be used to improve the delivery of the elementary mathematics curriculum.

Delimitations

The study's delimitation are as follows: only students enrolled in third grade classes at the school selected were invited to participate in this research study; two teachers participated in the study and one of the teachers was the researcher. Only students in the treatment group had access to writing instruction during mathematics. The students in the control group did not have access to writing instruction in mathematics during the length of the study. The ELLs could choose to write in their native language to explain their answers.

Operational Definitions

Adaptive reasoning. It is referred to the capacity for logical thought, reflection, explanation, and justification (NRC, 2001).

Conceptual understanding. It refers to comprehension of mathematical concepts, operations, and relations (NRC, 2001).

Mathematical proficiency. It is described as having five components or strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001).

Metacognition. Someone's ability for knowing their own thinking, sometimes used for monitoring their understanding of a given topic.

Procedural fluency. It is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately (NRC, 2001).

Productive disposition. It is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (NRC, 2001).

Strategic competence. It is the ability to formulate, represent, and solve mathematical problems (NRC, 2001).

Writing in math. It refers to the written explanations to mathematics word problems or open-ended questions.

CHAPTER II

REVIEW OF THE LITERATURE

Engaging students in meaningful learning in mathematics is important. The use of manipulatives, literature connections, “math talks,” and writing actively are ways of engaging students in learning new mathematical concepts. All these strategies can be used to promote problem solving which is described by the NCTM (2000) as “engaging in a task for which the solution method is not known in advance… [in which] students draw on their knowledge and through this process…develop new mathematical understandings” (p. 52). In an effort to improve student learning in mathematics, the NCTM (2000) and CCSS (2010) require students to build new knowledge through solving problems, applying their knowledge to new situations, and monitoring and reflecting about their solutions. During the problem solving process, students should acquire conceptual understanding and develop procedural fluency of mathematical concepts. Connecting problem solving with writing in mathematics engages students in learning and results in higher academic achievement in the mathematics classroom (Berkenkotter, 1982; Burton & Mims, 2012; Parker & Breyfogle, 2011). However, studies that look at these connections have only been conducted with students at the secondary or college level. Literature about the effect of writing in mathematics at the elementary or primary grades is non-existent even when it is becoming a requirement in both instructional practices and high stakes tests (FSA, 2014).

Chapter II is divided into four major sections. The first section describes the connection between mathematical problem solving, and conceptual learning and procedural fluency. The second section discusses instructional practices and assessment.

Section three addressed how students develop strategies to solve problems in math. Lastly, section four includes a review of the research on the problem solving phases and metacognitive behaviors related to each phase.

Conceptual Understanding and Procedural Fluency during Problem Solving

Problem solving should be part of daily tasks in the mathematics classroom, as it has been recommended by NCTM (2000) and by the CCSS (2010). The first CCSS mathematical practice states that students will “make sense and persevere in solving problems” (CCSS, 2010). The CCSS include grade-level specific standards, but do not define specific interventions or instructional approaches for students who are ELLs. It does, however, assume that “all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives” (CCSS, 2010, p. 4). Keeping this in mind, teachers need to use appropriate instructional approaches to engage all their students, including ELLs, in mathematical problem solving for them to be college and career ready. NCTM (2000) stated that students should be given opportunities to investigate problems, evaluate results, organize information, and communicate their findings. They should also be able to recognize, apply, and interpret what to do in each problem; and create a system of effective methods to solve mathematics problems (NCTM, 2000). These requirements can be fulfilled by using George Polya’s problem solving phases which include understanding the problem, devising a plan, carrying out the plan, and examining or verifying that the answer makes sense (Polya, 1957).

A variety of instructional practices help students develop valuable strategies in mathematics. Commonly, textbooks, manipulatives, hands-on experiences, writing, and literature connections are used to teach the mathematics standards and to help students acquire mathematical proficiency. NRC (2001) argued that for students to have mathematical proficiency they need to have conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition. For the purpose of this study, adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification; conceptual understanding is comprehension of mathematical concepts, operations, and relations; procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Productive disposition refers to the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy, and strategic competence is the ability to formulate, represent, and solve mathematical problems (NRC, 2001). The NRC (2001) argues that when students have a deep understanding of mathematics topics they connect pieces of knowledge and making these connections is what helps them solve problems effectively. Additionally, when students develop the ability for knowing their own thinking and for monitoring their mathematical understanding and their problem solving strategies, they are able to achieve strategic competence and adaptive reasoning.

Instructional and Assessment Practices

The NCTM (2000) and the CCSS (2010) have stressed the importance of having students develop conceptual learning as well as procedural fluency in mathematics. The Principles and Standards for School Mathematics (NCTM, 2000) state that all student learning needs to be supported by instructional practices that make mathematics comprehensible; thus, it is important to use appropriate strategies to teach ELLs mathematics including visual cues, graphic organizers, realia, concrete and visual manipulatives, teaching of vocabulary with multiple meanings, and working in supportive groups (Kersaint, Thompson & Petkova, 2009; Robertson, 2009). When researching about the use of writing in mathematics, it can be found that most studies have been done with students at higher levels: middle school, high school, and college (Pugalee, 2001, 2004; Taylor & McDonald, 2007; Williams, 2003). These students bring content knowledge from instruction received in their native language (Brown, Cady, & Taylor, 2009) which may help in understanding and building on the new concepts they are learning. Contrarily, elementary school students, especially in the early grades, do not have the background knowledge to build on from, and learn both English language skills and mathematical content simultaneously. Thus, it is essential for teachers to use the tools that support ELLs in learning language and mathematics concurrently at this level (Brown, Cady, & Taylor, 2009; Robertson, 2009). Borgioli's (2008) anecdotal piece about mathematics strategies used in her classroom shows that choosing the appropriate tasks, tools, and classroom norms can lead ELLs to be successful in the mathematics classroom. Some of these effective strategies include using literature and writing in mathematics (Columba, 2013; Gerretson & Cruz, 2011; Kersaint, Thompson, & Petkova, 2009; Nelson, 2012), using manipulatives and hands-on activities (Brown, Cady, & Taylor 2009; Reimer & Moyer, 2005; Robertson, 2009); extending on children-invented strategies (Empson, 2001), and using problems that engage learners with their connection and relevance to students' lives (Meyer, 2012; Ramirez & Celedon-Pattichis, 2012).

Current Instructional Practices

At the elementary level, students use a variety of materials and strategies to understand and apply mathematics concepts in order to construct conceptualizations that will serve as the foundation for more advanced mathematics. Studies have found the mathematics textbook is the most commonly used instructional tool in mathematics classes (Charles, 2009; Hodges, Landry & Cady, 2009). However, when analyzing textbooks, Charles (2009) argued that most of them communicate only mathematical skills knowledge or procedural fluency rather than conceptual understanding. Additionally, Hodges, Landry, and Cady (2009) claimed that conventional mathematics textbooks usually provide an overabundance of resources for teachers but are deficient on pedagogical approaches to promote students' conceptual learning.

The states that have made or will make the transition to CCSS need to address these issues as the new standards call for conceptual understanding and procedural fluency and have adopted new textbooks to reflect these changes. The use of conventional textbooks as the primary curriculum and instructional resource in mathematics classrooms tends to focus instruction on the mastery of skills rather than in conceptual understanding. In the case of Florida, the mathematics textbooks used at the elementary level in some districts, GO Math, attempt to address the CCSS (2010) by starting each lesson with an investigation-type question in which students are given real-life scenarios with questions that introduce the topic for the day. However, districts that focus on the gradual release model of instruction may undermine attempts to address the CCSS within the textbooks. Within the gradual release of responsibility model (Fisher & Frey, 2008); first, the teacher models how to solve the question and some subsequent questions by giving the students a variety of strategies they can use to solve similar problems. Next, the students are expected to work in pairs or small groups; and lastly, they work independently to solve some questions that mainly assess procedural knowledge. At the end of each lesson, there are one to four questions that require students to show evidence of their understanding of the day's concept through writing. These last questions might ask students to explain what strategy is most appropriate to solve a problem; to write a question for a given data set, picture, or graph; or to write to explain if a given situation makes sense or not. The gradual release model used in the textbook setup is meant to help students acquire conceptual understanding and procedural fluency as they develop strategic competence. However, Meyer (2012) states that textbook questions should be modified to "induce in the student a perplexed, curious state, a question in her/his head that mathematics can help answer," in order to have students develop their own strategies to solving problems or be able to build upon the strategies previously learned to facilitate retention and conceptual understanding. This process help students in developing in-depth conceptual understanding as they become efficient in choosing procedures and strategies to solve the problems (Meyer, 2012). However, the way in which the mathematics textbooks at the elementary level are being used, do not provide students with enough experiences to develop different strategies to problem solve. Rather the strategies are directly taught throughout each lesson.

In addition to textbooks, teachers also use student journal entries, admit-exit slips (Altieri, 2009), small group responses, observations, and activity sheets to teach and evaluate students. Celedon-Pattichis and Turner (2012) explained that ELLs from different cultural and linguistic backgrounds can solve challenging mathematics tasks similar to those presented to English speakers by Carpenter et al. in the CGI model given that certain instructional practices are used. Some of these practices include making the content relevant to the ELLs' lives, and giving the students opportunities to speak, write, read and listen in the mathematics classroom. These results are from a study of 45 Latino and Latina low-income students in Kindergarten (Turner & Celedon-Pattichis, 2011). Furthermore, Kinzer and Rincon (2012) argued in their narratives that when teachers of ELLs use meaningful problems with relevant content, students can achieve as high as their monolingual English speaking counterparts in solving mathematics problems. These instructional practices enhance the concepts that students need to understand in order to be college and career ready.

Similarly, Hodges, Landry, and Cadi (2009), and McLeman and Cavell (2009) agreed that vocabulary that is developed through the collaboration among students and teachers help students in using the terms over time giving students ownership of the mathematical terminology. Approaches such as having a word wall for mathematics vocabulary, playing word games, making connections to literature, and keeping a mathematics journal are practices that develop language and content knowledge. Mathematics word walls that include multiple meaning words are essential to minimize misconceptions. For example, words such as *foot*, *reflection, square, and second*, may be used incorrectly in mathematics because of the multiple meanings these words have in everyday English (Ashlock, 2006). Thus, word walls also present ELLs with visual aids that can be used to write about mathematics.

Another effective practice that promotes higher achievement in mathematics is the use of literature. Authors of children books such as Marilyn Burns, Greg Tang, Mitsumasa Anno, Jon Scieszka, and Shel Silverstein have captured the essence of mathematics concepts in their literature. When mathematics concepts are introduced or reviewed with a story most students are receptive and involved in learning (Bintz & Moore, 2012; Christy, Lambe, Payson, & Carnevale, 2011; Columba, 2013; Gerretson & Cruz, 2011; Nelson, 2012). Altieri (2009) explained that by integrating literacy and writing into mathematics instruction her students improve both their language and mathematics skills. Christy and colleagues (2011) also reported that when children are presented with innovative ideas based on a story (in this case, The Wizard of Oz) they are engaged and excited while learning mathematics topics. The use of literature connections in mathematics also provides the opportunity for students to use writing to reflect on their thinking and for teachers to reflect on what the students know and understand thoroughly. By analyzing students' written entries, teachers' instruction can be improved to provide both conceptual and procedural learning experiences to students through engaging problem solving tasks (Bintz & More, 2012; Burton & Mims, 2012). Ramirez & Celedon-Pattichis (2012) added that using stories that students can relate to as part of their mathematics instruction benefits ELLs as the vocabulary is related to their experiences and they can solve challenging mathematics problems as well as English speakers.

Altieri (2009) described another instructional practice, word association in which ELLs can use written (word, definition, and sentence) and visual (picture) clues to help them remember the meaning of mathematical terms. She also uses multi-meaning word cards in which students write words that have multiple definitions and draw pictures to remind them of the different meanings. For example, the students may write the word volume and draw a picture of a container and write "amount" and on the side draw someone shouting and write "how loud someone is" (Altieri, 2009). These are instructional practices that help ELLs express their conceptual and procedural understanding and develop writing skills to write effectively in mathematics.

Taylor and McDonald (2007) found that when first-year college students wrote to explain the process they followed to solve problems in mathematics their conceptual understanding increased as they were able to better understand the different strategies they used. The results of this study indicated that formal writing activities in mathematics assisted students in reflecting in ways that correlated to Polya's phases and was conducive to solving problems effectively. This research study aimed to find if similar results happen when writing in mathematics is used with elementary students.

Writing in Mathematics

Pugalee (2004) and Williams (2003) found that high school students who wrote about their problem solving processes score significantly higher in problem solving assignments than students who did not use writing. Pugalee's (2004) study compared students' written and oral (think-aloud) explanations when they solved mathematics problems. A test for differences between proportions of how students answered the problems (using either written or oral descriptions) showed than when students used written descriptions they had significantly lower errors than when they used oral descriptions (Pugalee, 2004). Pugalee's (2004) study also analyzed the students' responses based on the four metacognitive behaviors described by Garofalo and Lester (1985) and found the number of orientation and execution behaviors were significantly higher for students providing written explanations. These two metacognitive behaviors were positively correlated to success with problem solving tasks. In turn, Williams (2003) used a pretest–posttest design with a control and treatment group. The participants in his study included 42 beginning algebra high school students. Williams (2003) studied the students' written responses to how they solved five weekly non-routine mathematics word problems. Williams (2003) found that there was a positive significant difference in the posttest scores of the students who used writing during problem solving, and concluded that writing about the executive process resulted in gains in the problem solving performances of beginning algebra high school students.

In addition to the research that supports using writing to help middle school, high school, and college students be successful problem solvers, new textbooks as well as new standardized assessments are requiring students to show evidence of their understanding through written explanations across grade levels (FSA, 2014). Furthermore, new curricula focus on increasing students' conceptual understanding of mathematical topics. A portion of each lesson of the mandated elementary textbook in some districts in Florida requires students to write their thinking process to solve problems (Dixon et al., 2013) and new standardized assessments (FSA, 2014) also require students to explain their answers through written responses in order to show their conceptual understanding and procedural fluency. Writing is a practice that has shown improvement of student achievement in content areas such as mathematics at the middle school, high school, and college levels. Taking into account the need to help ELLs progress to higher standards of mathematical knowledge and application, it is important to study the effect that writing can have in promoting conceptual learning and procedural fluency while ELLs problem solve in mathematics at the elementary school level.

Writing, which is usually implemented during Reading and Language Arts, is a valuable tool to also use in mathematics to encourage ELLs to write about what they are learning. Bresser, Melanese, and Sphar (2008) recommend the use of sentence frames, prompting questions that model the structure of a well-crafted answer, and open-ended questions to help ELLs to feel confident when using writing in mathematics. Writing in mathematics can assist ELLs in understanding mathematical topics that are new to them or that are used in different ways in the U.S. mathematics curriculum (Access Center, 2008; Berkenkotter, 1982; Williams & Casa, 2012). Most ELLs have difficulty with problem solving due to culturally-linked content and to vocabulary found in mathematics word problems (Brown, Cady, & Taylor, 2009; Robertson, 2009). There is a general belief that ELLs can learn mathematics without much difficulty because "mathematics is universal" (Robertson, 2009). However, ELLs struggle daily with mathematics problems that address situations that are unfamiliar to them or that have vocabulary that is new or ambiguous (Ramirez & Celedon-Pattichis, 2012). Explicit teaching of mathematical language and vocabulary is critical for ELLs to understanding concepts and thus for understanding problems in word form. Ashlock (2006) suggested that when introducing a concept to young or elementary-age children the appropriate terms should be used even if it is informally.

Assessment

The need to align high-stakes assessments to the mathematics curriculum has also had an impact on mathematics textbooks, instruction, and in how teachers help students develop conceptual understanding. The NCTM's (2012) position statement on assessment discusses the importance of formative and summative assessments that evaluate students' mathematical knowledge versus the use of high-stakes testing as the sole source of information about students' mathematical knowledge. Traditional classrooms commonly use summative assessments which are given at the end of a lesson, chapter, or unit in order to assess students' understanding of mathematics concepts and mainly look at students' procedural knowledge. However, assessment should be a tool of final evaluation of student knowledge as well as an ongoing measurement (formative assessment) of students' progress. By using open-ended questions as a formative assessment tool, teachers can assess students' conceptual development and procedural fluency (Borgioli, 2008).

Furthermore, tasks given to ELLs need to be meaningful to facilitate the connections students need to make that can lead to conceptual understanding (Borgioli, 2008; Celedon-Pattichis & Turner, 2012; Kinzer & Rincon, 2012). For instance, Borgioli explains how a project called Children's Math Worlds funded by the National Science Foundation uses stories from the children's lives to create mathematics problems that the students have to solve. In this way students can learn mathematical concepts by using texts, word problems or examples involving scenarios with which they are familiar. Celedon-Pattichis and Turner (2012) also explain the impact of using information from students' lives in developing mathematics problems and the positive effect this has on conceptual understanding and in problem solving.

Developing Strategies to Solve Problems in Mathematics

Cognitively Guided Instruction (CGI) developed by Carpenter, Fennema, Franke, Levi and Empson (1999) showed how young children think and solve mathematics problems at different levels of cognition. It is based on the belief that first children model the actions in a mathematical problem and during this process they develop strategies to solve it. The research conducted by Carpenter et al. (1999) demonstrated that children used strategies depending on the type of mathematics problem they were solving, and the strategies became more effective as they had more experience solving problems. For example, to solve an addition problem students started by using direct modeling strategies, which included using fingers or objects to represent each addend; eventually, they used counting strategies which included counting on from one of the addends (Carpenter, et al., 1999). The counting strategies were more efficient and abstract than the direct modeling strategies as the students started noticing the relationship between numbers and used the physical object as a way to keep track of the counting sequences. Additionally, the children used number facts which included recalling doubles facts (e.g., 7+7, 2+2) or using derived facts (e.g., 4+5 is one more than 4+4) (Carpenter et al., 1999). To solve multiplication and division problems students also started out by using direct modeling by making sets with the specified number of objects in each, and then slowly moved to using counting strategies which included skip counting; lastly, they used derived facts (Carpenter et al., 1999).

Similar to the findings by Carpenter and colleagues with White non-Hispanic students, Ramirez and Celedon-Pattichis (2012) discussed that Kindergarten ELLs can respond to challenging mathematics problems with questions and comments. NCTM (2013) also stated on a position statement regarding Teaching Mathematics to ELLs that "expanded learning opportunities and instructional accommodations should be available to English language learners (ELLs) who need them to develop mathematical understanding and proficiency" (p.1).

Specific topics in mathematics might be challenging for some ELLs. An example is fractions, as students may come from places where decimals are emphasized over fractions, or whose cultures do not have conceptual representations of fractions (Kersaint, Thompson, & Petkova, 2009). Another cultural-mathematical difference is numerals as there are countries where notations such as commas and periods are used in opposite ways than in the United States. For instance, South American and European countries use commas in numbers where U.S. uses periods, and use periods where commas are used in the U.S. (e.g., *5.231* will be read as five thousand two hundred thirty one, rather than as a decimal notation). Students may also have difficulty with money as the U.S. coin values cannot be derived from the size of the coin (e.g., a dime is smaller in size than the nickel but larger in value) and also because the coin value is not written on the coins (e.g., the dime has "one dime" written on it, instead of the numerical representation of "10₵"). Another difficult topic for ELLs is measurement as students may only understand the metric system (Kersaint, Thompson, & Petkova, 2009) rather than the U.S. customary system of measurement. These culturally different conceptions in mathematics create many challenges for the English language learners.

Consequently, writing in mathematics can inform the teacher about misconceptions, error patterns, or the topics that need to be reviewed or re-taught (Parker & Breyfogle, 2011; Williams & Casa, 2011). It can help students describe their mathematical thinking, show the process they go through to solve problems, and show competence in using problem solving strategies (Fernandez, Hadaway, & Wilson, 1994; Pugalee, 2004). Writing can also provide students with a medium to switch between verbal and visual modes of thought as students can express their thinking in the form of diagrams, flow charts, and drawings (Berkenkotter, 1982). Thus, writing is especially beneficial to ELLs as they develop language skills and become mathematics problem solvers.

Students become better problem solvers by writing about problem solving (Berkenkotter, 1982; Burton & Mims, 2012; Parker & Breyfogle, 2011, Pugalee, 2004). Teachers can promote the use of the eight mathematical practices by challenging students to look for clues to understand the problem or as George Polya wrote, asking questions that help students develop a set of strategies to solve problems independently (Polya, 1957). Students who are learning to problem solve need to listen to others' ideas and compare these with their own, justify their thinking, write about the process they go through to solve problems, and struggle with such in order to find a variety of solutions (NCTM, 2000). By using writing in mathematics students are able to share the strategies they use to solve mathematics problems with others in the class and by writing about the process these strategies become familiar to them. Students start creating a repertoire of strategies they can use during problem solving inside and outside the mathematics classroom (Burton & Mims, 2012; Columba, 2013; Gerretson & Cruz, 2011; NCTM, 2000; Nelson, 2012; Parker & Breyfogle, 2011;Williams & Casa, 2012). Writing in mathematics can also be used to help ELLs share their experiences, develop their writing skills in English using mathematics terminology, verbalize their learning, and explain their solutions (Kersaint, Thompson, & Petkova, 2009).

Students can write to explain their thinking process (Berkenkotter, 1982; Bicer, Capraro, & Capraro, 2013; Fernandez, Hadaway, & Wilson, 1994; Pugalee, 2001, 2004; Taylor & McDonald, 2007; Williams, 2003), write about their feelings towards mathematics (Baxter, Woodward, & Olson, 2005) or write to show their understanding about a given concept (Altieri, 2009; Barlow & Drake, 2008; Bintz & Moore, 2012; Brown, Cady, & Taylor, 2009; Cooper, 2012; Fernandez, Hadaway, & Wilson, 1994; Rosli, Goldsby, & Capraro, 2013; Seto & Meel, 2006; Wiest, 2008; Williams & Casa, 2012). For the purpose of this study "writing in mathematics" included writing activities with entries in which the students explained their thinking process when solving mathematics problems.

Problem Solving Phases and Metacognitive Behaviors

The methodologies, strategies, and activities used in a classroom to introduce a concept lay the foundation for students' understanding and application of the concept. George Polya (1957) was a visionary when it came to teaching his students to solve problems. He devised a model in which students followed four phases in order to solve problems: understanding the problem, making a plan by seeing how the various items are connected in the problem, carrying out the plan in which students use different strategies, and looking back to review and discuss the solution (Polya, 1957). Polya's phases of problem solving can be utilized with students at all grade levels, and ELLs can benefit greatly from his model as it presents them with a clear guide to solve problems. For instance, during the first phase: *understanding the problem*, ELLs can work on key vocabulary and cultural elements to facilitate understanding. In phase two, *devising a plan*, the ELLs can make a plan by using strategies to find the right pathway to solve the problem. During phases three and four, *carrying out the plan* and *looking back*, ELLs can solve the problem and can go back to the strategy they used to solve the problem by writing to explain about the process they followed. Using the problem solving phases helps students acquire mathematical specific terminology and problem solving strategies that can be used in real-life scenarios. Additionally, they will be able to “make sense and persevere in solving problems” as cited in the first CCSS mathematical practice (CCSS, 2010) and be closer to meet the high standards required to succeed in their future college and career paths.

According to researchers such as Garofalo and Lester (1985), Pugalee (2001), and Yimer and Ellerton (2006) it is important to analyze what students are thinking while they solve problems in mathematics to inform the instructor what the student knows about a certain topic. The analysis can lead to a deep understanding of what students do to solve the problem from their initial engagement to a final verification of strategies used and steps taken. Garofalo and Lester (1985) showed that there are four main categories of metacognitive behaviors students show when solving problems in mathematics: (a) Orientation, (b) Organization, (c) Execution, and (d) Verification. This framework is based on George Polya’s problem solving model. In the orientation phase, the students show strategic behaviors to assess and understand a problem such as using comprehension strategies, analysis of information and conditions, and assessment of familiarity with the task (Garofalo & Lester, 1985). During the organization stage, the students plan what to do and choose their actions; so they identify the main goal or what the problem is asking to solve as well as the sub-goals or smaller goals they need to reach before finding the main answer to the problem. This is similar to “devising a plan” under George Polya’s phases. The execution stage is where the students carry out their plan, use the strategies they had planned on using, monitor their progress, and/or discard their plan if it does not work. The last stage, verification, is where students reflect and look back at what they have done: what has worked and what has not (Garofalo & Lester, 1985). The verification stage can be done throughout the problem solving process, as students might verify their work as they advance in solving the problem, not necessarily only at the end. This process, however, is not a linear model, but rather a process in which students follow a more cyclical model and go back and forth from understanding the problem to verifying their answer (Garofalo & Lester, 1985; Polya, 1957; Wilson, Fernandez, & Hadaway, 1993). The process they follow to solve problems in mathematics includes thinking about what they do and why they do it. This metacognition leads them to be successful problem solvers (Fernandez, Hadaway, & Wilson, 1994). By analyzing what students had written as they solve mathematical problems, researchers can observe if some behaviors are related to students' mathematics achievement. Given this, it is important to analyze students' writing when solving problems and their metacognitive behaviors to see if this leads to higher achievement in mathematics.

CHAPTER III

METHODOLOGY

Chapter III describes the methodology used to test the hypotheses presented below. It includes the research questions, hypotheses, setting and participants, instrumentation, research design and procedures, data analysis procedures, and limitations.

Research Questions

The research questions that guided this study were as follows:

1. Are the mathematics achievement scores of English language learners and English speakers using writing in mathematics significantly higher than the achievement scores of students not using writing in mathematics?
    a. Are the mathematics achievement scores of the English language learners using writing in mathematics significantly higher than the achievement scores of the English language learners not using writing?
    b. Are the mathematics achievement scores of the English speakers using writing in mathematics significantly higher than the achievement scores of the English speakers not using writing?
2. Are the problem solving with metacognitive behaviors' scores of English language learners and English speakers using writing in mathematics significantly higher than the problem solving with metacognitive behaviors' scores of students not using writing in mathematics?
3. Which of the four metacognitive behaviors (orientation, organization, execution, verification) do third grade English language learners and English speakers most

often demonstrate when they write during problem solving on the achievement tests?

Hypotheses

The hypotheses that guided this study were as follows:

1. The students who use writing to solve problems in mathematics will score significantly higher on the mathematics posttest than those students who do not use writing during problem solving.
    a. The achievement scores of the ELLs using writing in mathematics will be significantly higher than the achievement scores of the ELLs not using writing.
    b. The achievement scores of the English speakers using writing in mathematics will be significantly higher than the achievement scores of the English speakers not using writing.
2. The students who use writing to solve problems in mathematics will score significantly higher on the problem solving with metacognitive behaviors questions than those students who do not use writing during problem solving over time.
3. Elementary English language learners and English speakers demonstrate orientation, organization, and execution metacognitive behaviors more often than verification metacognitive behaviors when writing during problem solving on the achievement tests.

Setting and Participants

The study took place during the spring of 2016 at an elementary public school located in a suburban area in South Florida. The participants included 67 students enrolled in four third-grade classes. The classes were taught by two different teachers, one being the researcher. The researcher trained the third grade teacher whose students also participated in the study so both could follow the same model to teach the students in the treatment group. The participants in this study were students at three different English language proficiency levels. The first group included English language learners (ELLs) ranging from English for Students of Other Languages (ESOL) level 1 to level 4. The second group consisted of students who exited the ESOL program in the past one to two years (ESOL level 5). The third group (English speakers) consisted of students who were fluent English speakers. The ESOL 5 students were part of the ELLs group given that they still were being monitored by the ESOL teacher to assure continued progress in the academic areas. Also based on the CALLA Handbook (Chamot & O'Malley, 1994) an ELL student develops academic language skills after five to seven years of learning the language and the participants in the group who are ESOL 5 students have been learning English in an academic setting for less than five years. All the ELLs participating in this study were of Hispanic background. The students' ages ranged from 8-10 year olds. Results were reported in aggregated ways so that the names of the participants are non-identifiable.

The researcher obtained approval from the university's Institutional Review Board (IRB) department and from the school district research committee prior to starting the data collection. Since the participants were under 18 years of age, the parents signed and returned a parental consent form for their child to participate in this research study. The students were invited to participate and they also signed a student assent form before the beginning of the study.

Instrumentation

The current research study analyzed students' mathematics achievement and writing in mathematics as they solved mathematics problems and the metacognitive behaviors that they portrayed through their writing. The instruments included a pretest, two biweekly tests, and a posttest.

The pretest, the biweekly tests, and the posttest had the same number of questions and the same format. Additionally, the pretest, biweekly tests, and posttest represented equivalent forms of the same test. Each test consisted of 10 questions divided into four multiple choice questions, five short response questions, and one problem solving question. The multiple choice questions were scored as correct or incorrect and had a value of one point. The short response questions had a value of two points and were scored as follows: two points if the student showed the correct answer and appropriate explanation, one point if the answer was partially correct and/or a partial explanation was given, or zero points if the answer was incorrect. Each question had its own criteria depending on the content and on the possible written responses (Appendix C shows the scoring criteria for the posttest). The problem solving question from each test was scored using a rubric from zero to four depending on the content and complexity level of the responses. If the student scored a one to four, the rubric from Table 1 was used to further score the written responses of the participants and to analyze the metacognitive behaviors they showed in their writing. The total number of points for each test was 18 points. An additional maximum of 16 points was given to the written responses of the problem solving question and these were the scores used to test hypothesis #2. However, the overall score of 18 points was the one taken into account to analyze the effect of writing in achievement over time, and to test hypotheses #1, #1a, and #1b. Hypothesis #3 will encompass the behaviors the students write about in the problem solving question in each of the achievement tests. All students in the treatment group received instruction on the four phases of problem solving, strategies, and behaviors to be used in each phase. The writing instruction was only received by the participants in the treatment group.

Table 1
Rubric for Assessing Students' Levels of Mathematics Problem Solving

Description of Metacognitive Behaviors in each Problem Solving Category	1 Beginning **(Well below grade level)**	2 Developing **(Working towards grade level standards)**	3 Accomplished **(On grade level)**	4 Exemplary **(Above grade level)**
Orientation *(Assessing problem, Understanding problem situation):* • Read/reread problem • Make initial representation • Analyze information conditions	Student defines the problem incorrectly, does not show any understanding of the problem.	Student shows some understanding of the problem and misses some key information.	Student clearly defines the problem and outlines problem conditions in an effective manner.	Student clearly defines the problem and outlines problem conditions in an effective manner. Understanding includes extending the problem in some way.
Organization *(Making a plan of action)*: • Identify goals • Make a plan on how to solve problem	Student cannot identify goals or make a plan to solve the problem.	Student shows evidence of some goals but does not have a plan of action.	Student shows evidence of goals and makes a plan to solve the problem.	Student shows evidence of goals, makes a plan to solve the problem, and goes beyond expectations by planning to solve it in more than one way.

Execution *(Performance of goals, Performing calculations):* • Carry out details of the plan • Solve equations • Manipulate numeric information	Student does not show any procedures or strategies to solve the problem, and has no solution.	Student shows minimal use of procedures and strategies but not enough to solve the problem, uses some mathematics language and symbols, and has an incorrect solution.	Student shows clear procedures, good use of mathematics language and symbols, uses a variety of strategies, may have minimal mistakes in solution but uses correct arithmetic.	Student shows organized, clear procedures, excellent use of mathematics language and symbols, uses a variety of strategies, has a correct solution with use of correct arithmetic. Correct solution to extension to the problem is also given.
Verification *(Evaluation of plan of action and results/Can occur throughout the process not only at the end)* • Check reasonableness of work • Checks for accuracy of work	Given there is no solution verification does not apply.	Student does not check for reasonableness or accuracy of work.	Student evaluates if chosen strategies have worked, makes adjustments to procedures or strategies if necessary.	Student evaluates if chosen strategies have worked, makes adjustments to procedures or strategies if necessary, checks for accuracy of work throughout the process and at the end.

Note. Adapted from Lester and Garofalo's metacognitive behaviors framework (1985), which is based on Polya's four phases of problem solving (1957).

Additionally, all tests were tested for validity and reliability to confirm that they would serve the purpose for which they were designed, and that they had consistency of what was being measured. In order to determine if the pretest, biweekly tests, and posttest questions had content validity, the researcher aligned each test item to the Florida Mathematics Standards (MAFS) to find if each test question reflected the expectations of the standard being measured (Carbaugh, 2014). Furthermore, another third grade mathematics teacher reviewed the questions and followed the same process to align the items to the content standards in the MAFS. Each teacher used a list with all the third grade mathematics standards under MAFS, and matched each question to the standard it measured. Each teacher's alignment was revised to make sure each question was valid and measured the same standard. The Mathematics Florida Standards (MAFS) that were used in the multiple choice questions included questions that assessed Operations and Algebraic Thinking (3.OA) and Measurement and Data (3.MD) and included standards MAFS.3.OA.1.1, MAFS.3.OA.1.2, MAFS.3.OA.1.3, and MAFS.3.MD.2.3. More specifically, the standards assessed the students' knowledge in interpreting products of whole numbers (MAFS.3.OA.1.1), interpreting whole-number quotients of whole numbers (MAFS.3.OA.1.2), using multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (MAFS.3.OA.1.3). The questions also evaluated if the students were able to solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs (MAFS.3.MD.2.3).

The standards used in the short response questions included questions that assessed the following three domains: Operations and Algebraic Thinking (OA), Number

Base Ten (NBT), and Measurement and Data (MD). The specific standards included in the short response questions assessed the students' ability to identify arithmetic patterns including patterns in the addition table or multiplication table, and to explain these patterns using properties of operations (MAFS.3.OA.4.9), use place value understanding to round whole numbers to the nearest 10 or 100 (MAFS.3.NBT.1.1), and fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (MAFS.3.NBT.1.2).

The standard used in the problem solving question for all four tests was MAFS.3.OA.4.8: students are able to solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity, and assess the reasonableness of answers using mental computation and estimation strategies including rounding. Each of the teachers assessing the instruments for validity purposes checked each question and linked it to the standard. This was the strategy used in order to achieve strong content validity in each of the tests used.

In addition, a mathematics education professor, an ESOL (English for Students of Other Languages) professor, and two third grade mathematics teachers gave feedback on the test questions. The feedback included suggestions about changing vocabulary and sentence length to make the problems equally accessible to the ELLs and the English speakers. The questions were revised and edited to show the changes that were suggested. The problem solving questions were also analyzed for complexity and frequency of language or how many times certain words appeared in the test by using LexTutor (Cobb, 2015; Heatley, Nation, & Coxhead, 2002) in order to simplify the language and make it more accessible for all students. The vocabulary that was new or was subject-related was explicitly taught to all students. All students in both treatment and control group took the same tests (pretest, biweekly test 1, biweekly test 2, and posttest).

The original problem solving questions for the weekly assignment and for the four tests were chosen from a variety of sources including Florida Mathematics Standards bank of problems, the NCTM Navigation Series in Grade 3-5 (Anderson, Gavin, Dailey, Stone, & Vuolo, 2005; Chapin, Koziol, MacPherson, & Rezba, 2002; Cuevas & Yeatts, 2001; Duncan, Geer, Huinker, Leutzinger, Rathmell, & Thompson, 2007; Gavin, Belkin, Spinelli, & Marie, 2001), questions found in Teaching Children Mathematics (an NCTM journal) and other NCTM publications, questions found in Carpenter, Franke, and Levi's (1999) *Thinking Mathematically*, questions found in Carpenter, Fennema, Franke, Levi, and Empson's (2003) *Children's Mathematics*, or questions found in the National Research Council's (2001) *Adding it up: Helping Children Learn Mathematics*. After the content validity was ascertained, the final questions were tested for reliability with a demographically similar sample of fourth graders at the beginning of the school year (Fall of 2015) in order to assess the questions' consistency with what was being measured. Ambiguous questions were either deleted or changed. The reason fourth graders were used at the beginning of the school year is due to the similarity in content knowledge the third graders who participated in the study would have at the end of the academic year.

After the content analysis, the final problem solving questions used for the pretest, biweekly tests, posttest and weekly assignments were a combination of problems from the Florida Mathematics Standards bank of problems, the NCTM Navigation Through Algebra Series in Grade 3-5 (Cuevas & Yeatts, 2001), questions found in Teaching Children Mathematics (an NCTM journal) and other NCTM publications, questions found in Carpenter, Franke, and Levi's (1999) *Thinking Mathematically*, questions found in Carpenter, Fennema, Franke, Levi, and Empson's (2003) *Children's Mathematics*, or questions found in the National Research Council's (2001) *Adding it up: Helping Children Learn Mathematics* . Some of the problems were adapted for wording as appropriate for accessibility by all students.

The feedback received from the third grade teacher and the college professors was used to change the wording in the problems. Additionally, by using LexTutor, the words in each of the problem solving questions were categorized into four categories: (a) the most frequent 1000 word families, (b) the second 1000 more frequently used words, (c) the Academic Word List, which included words that students learn in academic settings across subjects, and (d) words that do not appear on any of the previous three categories or those words more specific to mathematics topics (Cobb, 2015; Heatley, Nation, & Coxhead, 2002). Modifications included changing some of the vocabulary found in the questions and reducing the length of some sentences. The language in the problems were simplified, however the mathematics content and complexity was not. The academic specific vocabulary was taught to the students prior to them answering the problem solving questions.

A rubric was used to analyze the students' written responses to the problem solving questions and to determine students' levels of mathematics problem solving. Table 1 shows the rubric that helped delineate the students' levels of mathematics problem solving and the metacognitive behaviors that they demonstrated when they wrote about the problem solving process they used. For the purposes of this study, the rows show the four categories of problem solving phases described by Lester and Garofalo (1985): orientation, organization, execution, and verification. Under the title for each phase, there are metacognitive behaviors that can be used during problem solving. The columns show the levels of mathematics problem solving of the students in relation to each of the problem solving categories. Each of the students' written problem solving solutions on the pretest, biweekly tests, and posttest were scored using this rubric.

The answers to all test items and the written responses to all the problem solving questions were checked by the study's researcher and a veteran third grade teacher in order to establish inter-rater agreement on scores after students completed each one of the tests. Both teachers were fluent in Spanish and they were able to understand the writing of students who wrote their explanations in Spanish on the free response and problem solving questions.

The teacher who scored the third graders' test answers was trained on the scoring guidelines using the Scoring Criteria sheet (sample found in Appendix C) for the achievement scores and the Rubric found in Table 1 for the problem solving with metacognitive behaviors questions for each achievement test. The training consisted of a total of four meetings in which the researcher instructed the teacher about test questions, possible student answers, and scoring procedures. The teacher and the researcher used the same item criteria for scoring the multiple choice and the short response questions. The test answers from the sample of fourth graders drawn on to trial test items were used during the training in order to practice scoring the different types of test items. At this time, scoring discrepancies were addressed in order to have both scorers utilizing the scoring criteria sheet and the rubric in the same manner. These training sessions took place before the beginning of the treatment.

In order to calculate the inter-rater agreement on the actual test scores the Intraclass Correlation Coefficient (ICC) was calculated. The ICC is used to tell how much variance is accounted for by agreement, in which higher levels indicate more agreement between the raters (Griffin & Gonzalez, 1995; Landers, 2015; Shrout & Fleiss, 1979). The ICC was the most appropriate reliability test for this study given that the two raters' responses were measured on a scale level of measurement, the categories were mutually exclusive, and each response had the same number of categories. Additionally, the two raters were independent and they were the same raters for all subjects. The ICC analysis showed the level of agreement or inter-rater reliability between the teacher and the researcher for each of the third graders' achievement test answers and for the problem solving student answers.

The rubric in Table 1 was used by the teacher and the researcher scoring the tests to assess the students' problem solving written answers. It was revised and edited after receiving feedback from a third grade mathematics teacher, an English Language Arts college professor, and a Mathematics Education professor. During the weekly instructional sessions, the students used a child-friendly rubric (Appendix B) to learn the four phases and the behaviors that can be associated to each. As the students learned to use the four phases of problems solving and to write about them, they worked in pairs or small groups and used their rubric to critique their own answers. This child-friendly rubric was developed based on the same information from the rubric on Table 1. The vocabulary and format differs from the original rubric in Table 1 given that it needed to have information that was understandable and easy to use independently by the children.

Research Design and Procedures

The research followed a quasi-experimental design. One of the goals in this multipurpose study was to analyze the effect of using writing in mathematics in the achievement scores of ELLs and English speakers in a convenience sample of students, which indicates that participants were not randomly assigned. Under this circumstances, the researcher was not in the capacity to manipulate the independent variable (the use of writing in mathematics) neither the dependent variable (the achievement in mathematics measured in each test). The study also assessed the influence of the treatment in the achievement of students who were exposed to it as well as the metacognitive behaviors that all the students showed through their written problem solving solutions. Consequently, the phenomenon under investigation, the effect of writing in mathematics achievement, was studied as it manifested. According to Gall, Gall, and Borg (2007) lack of manipulation of variables and randomization of samples are common characteristics of quasi-experimental research designs.

In the present study each teacher had a control group and a treatment group for whom writing in mathematics was used as a whole unit of instructional strategy. This step served as a way to decrease teacher effect on the final statistical analysis. Additionally, it would have been impossible to isolate the control group of students when the writing instruction was given during treatment sessions if both treatment and control students were in the same classroom. Given that the students were already assigned to the four classes by the school administration, the student demographics were used to choose which classes will receive the treatment and which will serve as control. The four classes were similar on most demographics; however, the two classes from each teacher who received the treatment had the majority of ELLs who were ESOL levels 1, 2, or 3.

Both treatment and control groups received the same time for mathematics instruction, an hour daily in each class. All groups used the same instructional resources that included the mathematics textbook, literature connections (e.g., children's stories which included the study of mathematics topics), manipulatives, hands-on activities, and technology during mathematics instruction. In addition, during the 6 weeks of the study, all participants followed their regular class schedules and practices including assignments, home learning, and assessments.

All participants were given the pretest at the beginning of the study. The students in the treatment group were taught to use writing to explain their thinking processes when solving mathematics problems in whole group instruction. The writing instruction focused on having students learn to solve problems by using Polya's four phases: understanding the problem, devising a plan, carrying out the plan, and looking back while writing about their process. The writing instruction was done twice a week for 6 weeks during the mathematics instructional time. Each session lasted 30 to 45 minutes.

During the first week, the teacher instructed the students about the problem solving phases, and the strategies that could be used to effectively solve word problems. Each week had two sessions that were organized as follows. During the first session the students worked together with the teacher to solve a word problem so that vocabulary was explicitly taught, students could receive feedback on strategies and skills learned, and ESOL strategies such as sentence frames, sentence structure, mathematical vocabulary, and new content vocabulary was explained to students. During the second session of each week, the students worked in pairs or groups of three to discuss possible solutions and strategies to solve the problem with each other. Researchers have found that it is more beneficial for students to work in groups or pairs to solve math problems than to do so independently (Fernandez, Hadaway, & Wilson, 1994). By working in pairs or small groups, they helped each other to write about the process he or she followed in solving the problems. The ELLs were able to write in their first language if they were not able to explain their answers in English. Additionally, they were allowed to verbally say their answers in Spanish and the teacher or another group member helped them translate those answers. There was a lesson plan for each of the 12 sessions (two sessions a week for 6 weeks) that demonstrated how students were guided to use the target writing strategies and the problem solving question to be used for each lesson. During this 6 weeks of treatment, the students in the control group solved different mathematics problems from their textbook. These problems included computation and application questions in the format of multiple choice or open-ended questions.

In addition, every two weeks the students in both treatment and control groups completed a biweekly test independently. At the end of the 6 weeks, the treatment and control groups also completed a posttest. All tests were completed during the students' mathematics time but not during the writing in mathematics instructional sessions.

Data Analysis Procedures

A repeated measures ANOVA was the statistical method used to analyze the data from this study given that in this type of analysis a variable is measured several times to determine the effect of a treatment or intervention. That was precisely the main goal of this study: to examine the effect of writing in mathematics on the academic progression of third graders in mathematics over time. Therefore, a repeated-measures ANOVA was the appropriate statistical method to be used in this study.

Variables

The independent variable used in research question one was writing in mathematics, and the dependent variable was the achievement in mathematics measured by each test. The achievement in mathematics scores was a result of taking the sum of the points received in all of the questions (1 point for multiple choice questions, 2 points for short response and 4 points for the problem solving question). The independent variable used in research question two was writing in mathematics, and the dependent variable was the problem solving with metacognitive behaviors' scores measured within each test. The problem solving scores were measured using the scores of only the problem solving question of each test. Each of these questions was scored using the rubric found in table 1. Each student had a score for each phase of problem solving based on their writing. The final score for each problem solving question showed the sum of the points received on each of the four phases. The scores from the problem solving question was also used to determine the frequency of the metacognitive behaviors and to conduct the analysis for research question three.

Statistical Procedures

This research study adopted a quantitative analysis to respond to each research question. A repeated measures ANOVA design was used to detect differences of achievement across students' level of English language proficiency and treatment-control groups. The repeated measures ANOVA used the scores from the pretest, each of the biweekly tests, and the posttest to determine mathematics achievement over time. The data were analyzed using the Statistical Package for Social Sciences (SPSS) version 22.0. An alpha value (*α)* of .05 (level of significance) was used for each statistical analysis.

Limitations

A limitation of this study is the generalizability of the results. This study used a small sample of a population. This small population may be very different from populations in other parts of the country as well as other parts of the world. In addition, having two teachers also brings diversity to the lessons given that teachers have their own teaching style, and use different methods and instructional approaches. However, to address this limitation, the second teacher was trained on how to use writing in mathematics, and both teachers used the same problems for instruction and followed the same format for each of the lessons in order to decrease the teacher effect on the statistical analysis.

Summary

This chapter described the methodology that was used in this study. It included the research questions, hypotheses, setting and participants, instrumentation, research design and procedures, data analysis procedures, and limitations. The following chapter reports the results and the analysis of the data collected to determine if the use of writing in mathematics helped students improve their academic achievement in mathematics as well as the metacognitive behaviors the students demonstrated in their written mathematics problem solving work.

CHAPTER IV

RESEARCH FINDINGS

Chapter IV describes the findings of the study, including the demographic information about the participants and the results of the data analysis.

Demographic Information

The setting for this study was an elementary school in a suburban area in South Florida. There were a total of 81 students invited to participate in the study who were enrolled in third grade at the selected school. Only 68 students received parental consent to participate in the research study and one student transferred to a new school after two weeks of treatment; resulting in a total of 67 students participating during the six-week period. A total of 35 students were part of the treatment group and 32 students were part of the control group. There were 16 English speakers and 19 ELLs in the treatment group and 15 English speakers and 17 ELLs in the control group. Students' ages ranged from 8 to 10 years old. The majority of the participants, 65 students (97.1%) were of Hispanic heritage, one student was African American and one student was White. Table 2 shows the race/ethnicity of the students for each of the treatment and the control groups. Additionally, English Language Learners (ELLs) or students who are being served in appropriate programs of language assistance made up 52.2% of the students in the treatment group and 53.1% of the students in the control group. Table 2 also presents the number and percentages of students considered either ELL or English Speakers for the treatment and control groups.

Table 2
Frequencies and Percentages of Demographics of Students Participating in Study

	Frequency		Percentage	
	Treatment *N*=35	Control *N*=32	Treatment *N*=35	Control *N*=32
Race/Ethnicity				
Hispanic	35	30	100.0	93.8
Black	0	1	0.0	3.1
White	0	1	0.0	3.1
English Lang. Proficiency				
ELLs	19	17	54.3	53.1
English Speakers	16	15	45.7	46.9
SES				
High	5	7	14.3	21.9
Medium	4	2	11.4	6.3
Low	26	23	74.3	71.9
Gender				
Female	17	17	48.6	53.1
Male	18	15	51.4	46.9
Age				
8 years old	10	9	28.6	28.1
9 years old	24	23	68.6	71.9
10 years old	1	0	2.9	0.0
Siblings				
No siblings	6	7	17.1	21.9
Younger siblings	10	4	28.6	12.5
Older siblings	12	15	34.3	46.9
Younger and older	7	6	20.0	18.8
Family Structure				
Married parents	27	22	77.1	68.8
Divorced parents	2	7	5.7	21.9
Single parents	6	3	17.1	9.4
Parental Education				
Both graduated college	12	6	34.3	18.8
1 parent graduated college	13	16	37.1	50.0
None graduated college	10	10	28.6	31.2

Tests of Hypotheses

Descriptive Statistics

Table 3 presents the descriptive data for the achievement scores of students in the experimental and control groups. Each of the four achievement tests had the same format: four multiple choice questions worth one point each, five short response questions worth two points each, and one problem solving question worth four points. Therefore, the maximum number of points a student could earn was 18 points.

Table 3
Descriptive Data for the Achievement Scores of Students in the Experimental and Control Groups

	TREAT_CONTR	ELL_NON	Mean	Std. Deviation	*N*
PRETEST	Treatment	English Speaker	9.81	2.316	16
		ELL	10.11	3.478	19
		Total	9.97	2.965	35
	Control	English Speaker	12.33	1.234	15
		ELL	12.06	2.926	17
		Total	12.19	2.264	32
	Total	English Speaker	11.03	2.243	31
		ELL	11.03	3.334	36
		Total	11.03	2.860	67
BIWEEKLY_1	Treatment	English Speaker	12.06	3.660	16
		ELL	12.37	2.477	19
		Total	12.23	3.030	35
	Control	English Speaker	13.73	2.344	15
		ELL	14.06	2.817	17
		Total	13.91	2.570	32
	Total	English Speaker	12.87	3.160	31
		ELL	13.17	2.741	36
		Total	13.03	2.923	67

Table 3 (Continued)
Descriptive Data for the Achievement Scores of Students in the Experimental and Control Groups

	TREAT_CONTR	ELL_NON	Mean	Std. Deviation	*N*
BIWEEKLY_2	Treatment	English Speaker	13.00	2.944	16
		ELL	12.68	2.473	19
		Total	12.83	2.662	35
	Control	English Speaker	13.80	3.075	15
		ELL	14.53	2.831	17
		Total	14.19	2.923	32
	Total	English Speaker	13.39	2.985	31
		ELL	13.56	2.772	36
		Total	13.48	2.852	67
POSTTEST	Treatment	English Speaker	15.56	2.804	16
		ELL	16.21	1.512	19
		Total	15.91	2.188	35
	Control	English Speaker	13.40	1.549	15
		ELL	14.53	1.419	17
		Total	14.00	1.566	32
	Total	English Speaker	14.52	2.502	31
		ELL	15.42	1.680	36
		Total	15.00	2.132	67

Statistical Analysis for Hypothesis 1

Hypothesis #1: The students who use writing to solve problems in mathematics will score significantly higher on the mathematics posttest than those students who do not use writing during problem solving.

A One-Way Repeated Measures Analysis of Variance was used to analyze the difference writing in mathematics had on the achievement scores of the students in the treatment and control groups. The between-subject variable was treatment/control while the within-subject variables were the scores of the pretest, the two biweekly tests, and the posttest. The analysis showed that there was no significant difference between the mathematics achievement scores of the students in the treatment group and the control group ($F(1,65) = 3.411$, $p=.069$) across time. Table 4 shows the results of the repeated measures ANOVA conducted for question 1. Figure 1 shows a line graph with the growth trends for the achievement scores for the treatment and control groups, displaying the achievement scores means for each of the pretest, biweekly 1 test, biweekly 2 test, and posttest.

Table 4
Repeated Measures Analysis of Variance on Achievement Scores

Effect	Mean Square	df	F	Sig.	Partial Eta Squared
Intercept	46271.516	1	3388.561	<.001	.981
TREAT_CONTR	46.576	1	3.411	.069	.050
Error	13.655	65			

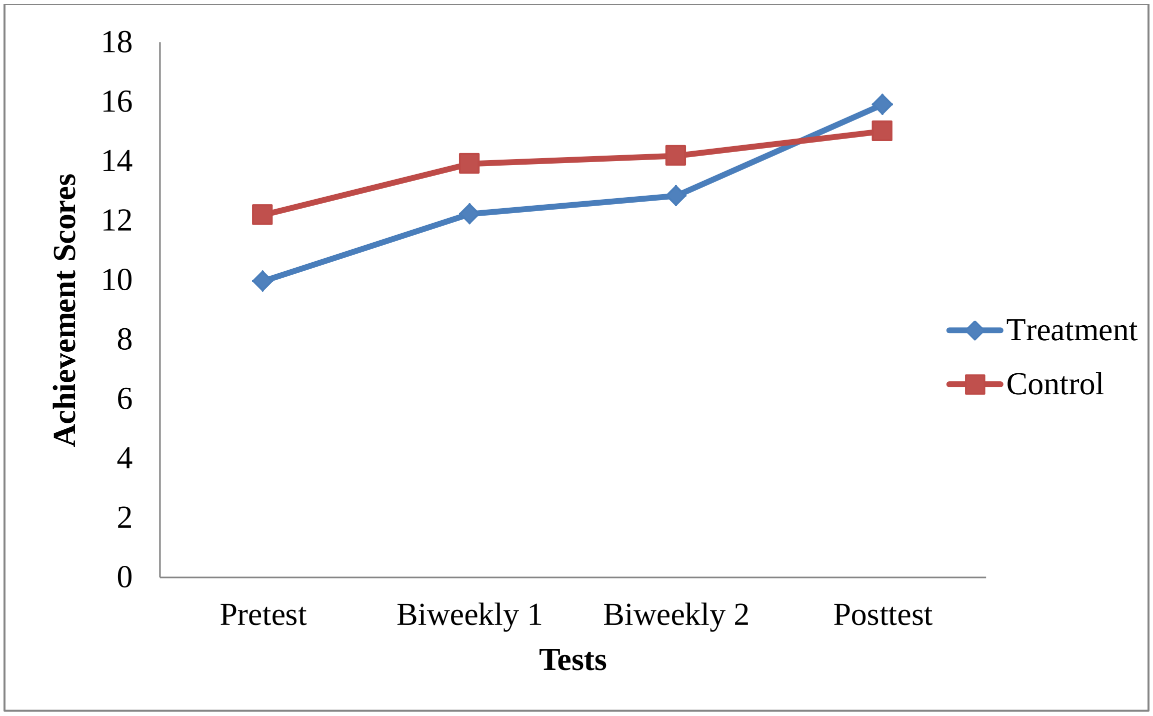


Figure 1. Line graph presenting the growth trends for the achievement scores for the treatment and control groups on each of the four tests: Pretest, Biweekly Test 1, Biweekly Test 2, and Posttest.

Statistical Analysis for Hypothesis 1a

Hypothesis #1a: The achievement scores of the ELLs using writing in mathematics will be significantly higher than the achievement scores of the ELLs not using writing.

A One-Way Repeated Measures ANOVA was also used to test the differences between the ELL students in the treatment group, hence using writing, to the ELLs in the control group, who were not using writing. When reviewing the distribution of the data in order to meet the assumption of normality, it was found that the value of skewness on the pretest score (-1.054) and the value of kurtosis on the posttest (-1.153) were a bit off the standard range of -1 to 1. However, by taking a close examination at the distribution of scores of the biweekly 1 test, the biweekly 2 test, and the posttest, it can be corroborated that these scores resemble the behavior of a normal distribution.

Using the same variable setting as for question 1, the repeated measures ANOVA determined that the achievement scores means of the ELLs were not statistically significantly different between time points ($F(1,34) = 2.632$, $p=.114$). Table 5 shows the results of the repeated measures ANOVA conducted for question 1a. Figure 2 shows the growth trends for the achievement scores of the ELLs in each the treatment and control groups, displaying the achievement scores means for each of the pretest, biweekly 1 test, biweekly 2 test, and posttest.

Table 5
Repeated Measures Analysis of Variance on Achievement Scores for ELLs in Treatment Group and ELLs in Control Group

Effect	Mean Square	df	F	Sig.	Partial Eta Squared
Intercept	25462.749	1	2060.177	<.001	.984
TREAT_CONTR (ELLs only)	32.527	1	2.632	.114	.072
Error	12.359	34			

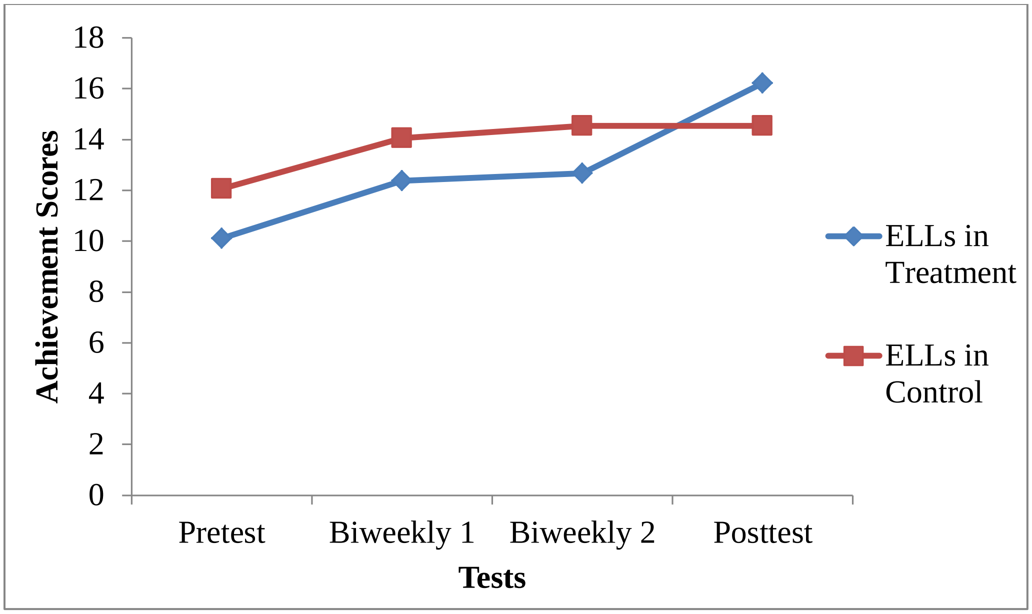


Figure 2. Line graph presenting the growth trends for the achievement scores for the ELL students in the treatment and control groups on each of the four tests: Pretest, Biweekly Test 1, Biweekly Test 2, and Posttest.

Statistical Analysis for Hypothesis 1b

Hypothesis #1b: The achievement scores of the English speakers using writing in mathematics will be significantly higher than the achievement scores of the English speakers not using writing.

A One-Way Repeated Measures ANOVA was also used to test the differences between the English speakers in the treatment group, who were using writing, to the English speakers in the control group, not using writing. Similar to the distribution of ELLs, the English speakers' distribution of the data also meets the assumption of normality. It was found that the value of skewness and kurtosis on all of the tests were within the standard range of -1 to 1. Using the same variable setting as for questions 1 and 1a, the repeated measures ANOVA determined that the achievement scores means of the English speakers were not statistically significantly different between time points ($F(1,29) = .980$, $p=.330$). Table 6 shows the results of the repeated measures ANOVA conducted for question 1a. Figure 3 shows the growth trends for the achievement scores of the English speakers in each of the treatment and control groups, by displaying the achievement scores means for each of the pretest, biweekly 1 test, biweekly 2 test, and posttest.

Table 6
Repeated Measures Analysis of Variance on Achievement Scores for English Speakers in Treatment Group and English Speakers in Control Group

Effect	Mean Square	df	F	Sig.	Partial Eta Squared
Intercept	20815.266	1	1.317E3	<.001	.978
TREAT_CONTR (English Speakers only)	15.492	1	.980	.330	.033
Error	15.801	29			

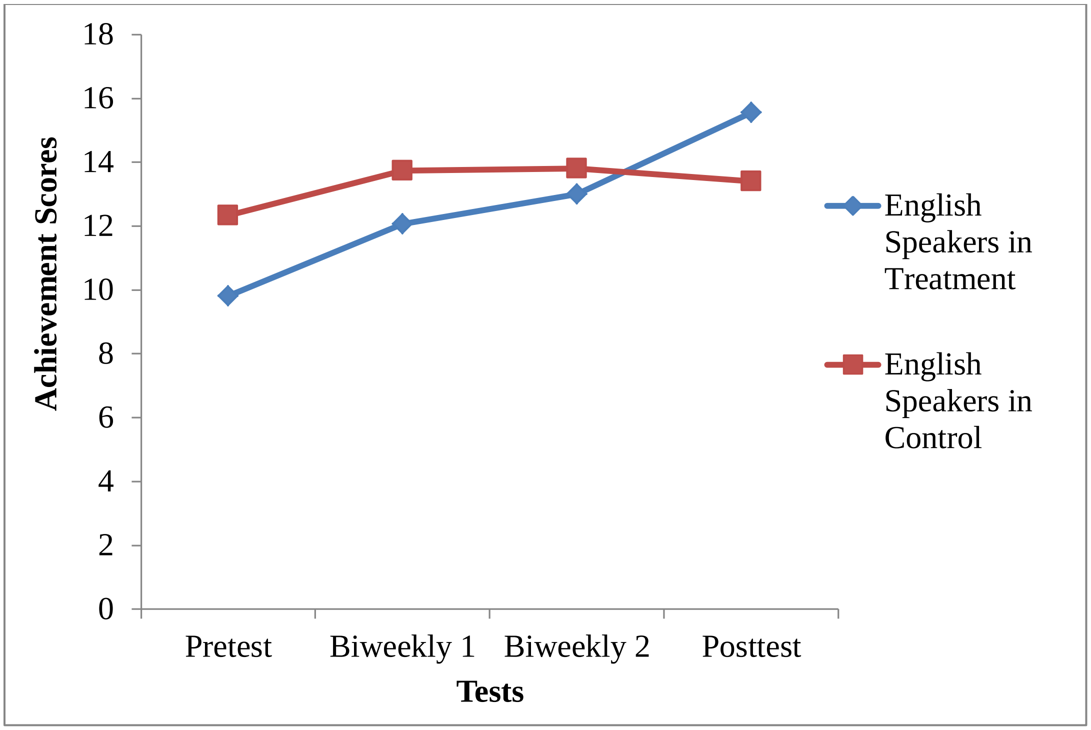


Figure 3. Line graph presenting the growth trends for the achievement scores for the English speakers in the treatment and control groups on each of the four tests: Pretest, Biweekly Test 1, Biweekly Test 2, and Posttest.

Statistical Analysis for Hypothesis 2

Hypothesis #2: The students who use writing to solve problems in mathematics will score significantly higher on the problem solving with metacognitive behaviors questions than those students who do not use writing during problem solving over time.

A One-Way Repeated Measures ANOVA was also used to test the differences between the scores on the problem solving with metacognitive behaviors questions of the students using writing in the treatment group to the students in the control group. The between-subject variable was treatment/control while the within-subject variables were the scores of the problem solving with metacognitive behaviors questions on the pretest, the two biweekly tests, and the posttest. Statistically significant differences were found between the students receiving the treatment and the students not receiving the treatment at the α = .05 level of significance ($F(1,65) = 75.971$, $p < .001$) as summarized in Table 7. Post Hoc tests using the Bonferroni correction were conducted to analyze the differences among each set of tests. Table 8 summarizes these results of the Post Hoc tests.

Table 7

Repeated Measures ANOVA on Problem Solving with Metacognitive Behaviors Scores

Effect	Mean Square	df	F	Sig.	Partial Eta Squared
Intercept	9638.331	1	732.920	<.001	.919
TREAT_CONTR	999.062	1	75.971	<.001	.539
Error	13.151	65			

Table 8

Post Hoc Tests with Bonferroni Adjustment on Problem Solving with Metacognitive Behaviors Scores

(I) time	(J) time	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
Pretest	Biweekly 1	-5.823*	.363	<.001	-6.811	-4.834
	Biweekly 2	-8.648*	.398	<.001	-9.731	-7.565
	Posttest	-8.674*	.393	<.001	-9.743	-7.605
Biweekly 1	Pretest	5.823*	.363	<.001	4.834	6.811
	Biweekly 2	-2.825*	.477	<.001	-4.123	-1.527
	Posttest	-2.851*	.416	<.001	-3.983	-1.719
Biweekly 2	Pretest	8.648*	.398	<.001	7.565	9.731
	Biweekly 1	2.825*	.477	<.001	1.527	4.123
	Posttest	-.026	.441	1.000	-1.225	1.173
Posttest	Pretest	8.674*	.393	<.001	7.605	9.743
	Biweekly 1	2.851*	.416	<.001	1.719	3.983
	Biweekly 2	.026	.441	1.000	-1.173	1.225

*. The mean difference is significant at the .05 level.

Additionally, Figure 4 shows the growth trends for the problem solving with metacognitive behaviors questions of the students in the treatment group and the students in the control group.

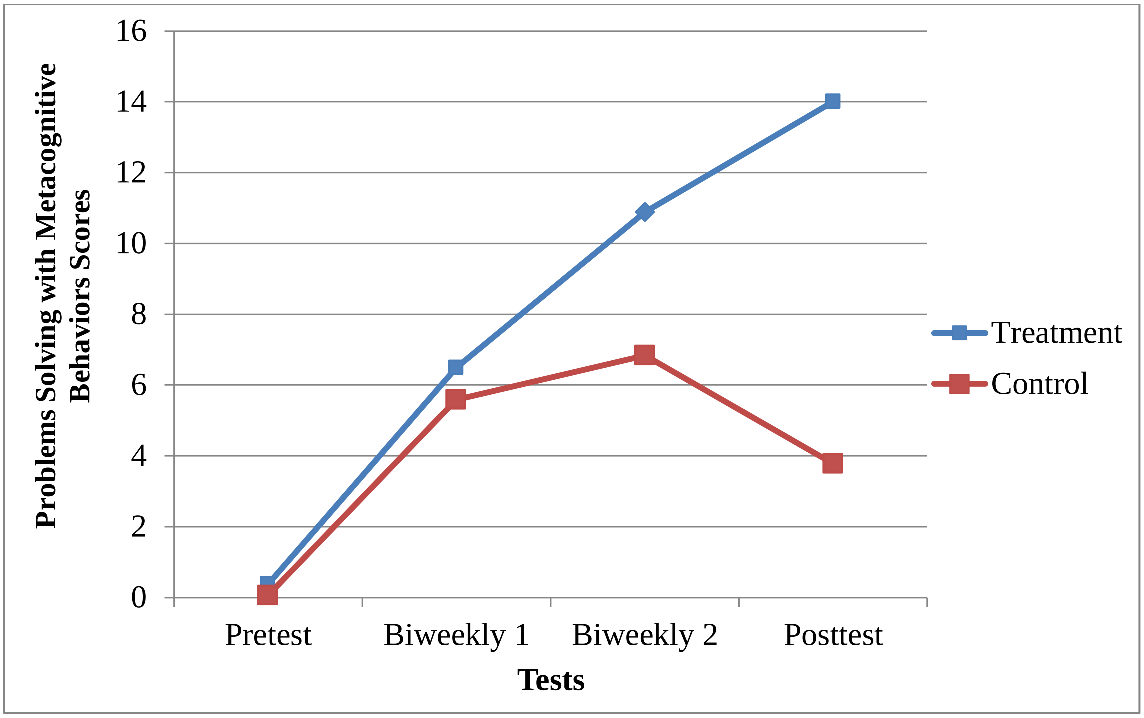


Figure 4. Line graph presenting the growth trends for the problem solving with metacognitive behaviors scores for the students in the treatment group and for students in the control group on each of the four tests: Pretest, Biweekly Test 1, Biweekly Test 2, and Posttest.

Descriptive Analysis for Hypothesis 3

Hypothesis #3: Elementary English language learners and English speakers demonstrate orientation, organization, and execution metacognitive behaviors more often than verification metacognitive behaviors when writing during problem solving on the achievement tests.

Descriptive statistics were used to find the frequency of occurrence of each of the problem solving phases (orientation, organization, execution, and verification) on each of the tests (pretest, biweekly 1, biweekly 2, and posttest). Table 9 shows the frequencies and percentages of each problem solving phases on each test.

Table 9
Frequency and Percentages of Problem Solving Phases Used in Each Test for Students in the Treatment Group

		Frequency		Percentage	
		Treatment *N*=35	Control *N*=32	Treatment *N*=35	Control *N*=32
Pretest	Orientation	2	2	5.7	6.2
	Organization	2	0	5.7	0
	Execution	1	0	2.9	0
	Verification	0	0	0	0
Biweekly 1	Orientation	5	5	14.3	15.6
	Organization	5	4	14.3	12.5
	Execution	3	3	8.6	9.4
	Verification	1	2	2.9	6.2
Biweekly 2	Orientation	29	10	82.9	31.2
	Organization	26	10	74.3	31.2
	Execution	19	7	54.3	21.9
	Verification	12	3	34.3	9.4
Posttest	Orientation	35	2	100	6.2
	Organization	35	2	100	6.2
	Execution	32	3	91.4	9.4
	Verification	28	1	80	3.1

Figure 5 shows a bar graph displaying the percentages of students who wrote about the metacognitive behaviors they used to problem solve in each of the four tests.

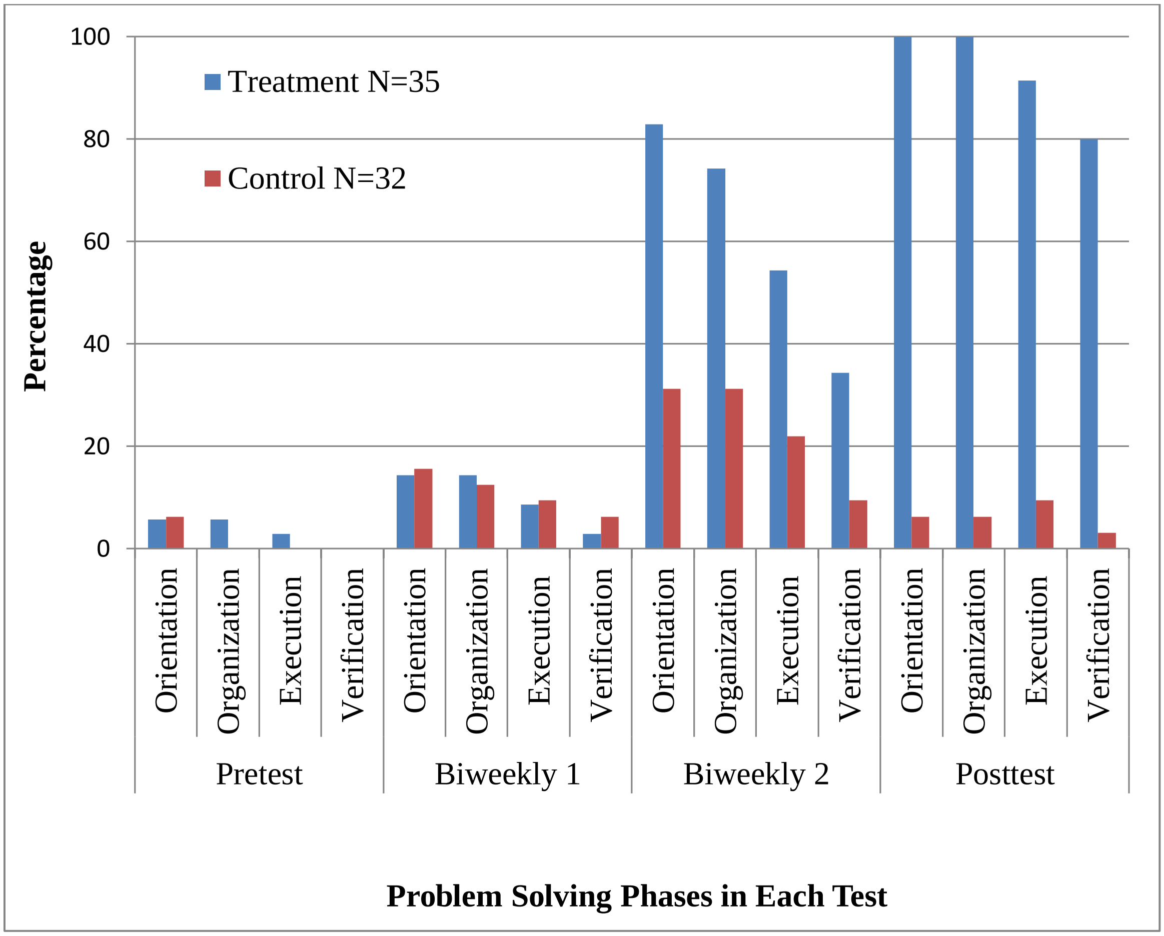


Figure 5. Bar graph presenting the percentages of problem solving phases used in each test.

Tests of Reliability

Reliability measures the consistency of outcomes of an assessment. This study used inter-rater reliability, which refers to the level of agreement between different examiners when assessing students' work in order to establish if such measurements are indeed reliable. According to Jones and Inglis (2015), the lower the inter-rater reliability, the more dependent the students' outcomes are on the idiosyncrasies of the examiner, and so the less fair the assessment. Inter-rater reliability is usually investigated by recruiting different examiners to mark the same students' work and comparing the outcomes, typically using the Pearson product-moment correlation coefficient. For the purposes of study, however, the Intraclass Correlation Coefficient (ICC) was calculated. The ICC is used to tell how much variance is accounted for by agreement, in which higher levels indicate more agreement between the raters or examiners (Griffin & Gonzalez, 1995; Landers, 2015; Shrout & Fleiss, 1979). The ICC was the most appropriate reliability test for this study given that the two raters' responses were measured on a scale level of measurement, the categories were mutually exclusive, and each response had the same number of categories. Additionally, the two raters were independent and they were the same raters for all subjects. The ICC analysis showed the level of agreement or inter-rater reliability between the teacher and the researcher for each of the third graders' test answers.

Table 10 shows the results of the ICC calculation in SPSS for each mathematics achievement test, and Table 11 shows the ICC calculation for each problem solving with metacognitive behaviors question. The ICC results show that all four achievement tests were reliable with an intraclass correlation of 1.00 for the pretest, .992 for the biweekly 1 test, .990 for the biweekly 2 test, and .979 for the posttest as shown in table 10. Similarly, table 11 shows the problem solving questions with metacognitive behaviors questions were also highly reliable with a coefficient of .998 for the pretest, .990 for the biweekly 1, .957 for the biweekly 2 test, and .999 for the posttest. The unusual high scores for the

ICC calculations for both achievement tests and problem solving with metacognitive behaviors questions can be attributed to the detailed explanations in the scoring criteria and the rubric used to score each test, as well as the scoring training sessions. The scoring criteria used to score the posttest can be found in Appendix C. Most of the time during the training sessions (to train the second teacher prior to starting the intervention) was spent practicing scoring the 4th grade sample pilot tests. Both scorers had discussed possible student answers based on the student answers from the pilot tests that helped in clarifying specific scoring for each test item in the study. Additionally, for the purpose of this study the average measures Intraclass Correlation was used and according to McGraw and Wong (1996), and Shrout an Fleiss (1979) this ICC is always higher that the Single measures ICC.

Table 10
Results of the ICC Calculation for Achievement Tests

		95% Confidence Interval		F Test with True Value 0			
	Intraclass Correlation	Lower Bound	Upper Bound	Value	df1	df2	Sig
Pretest							
Average Measures	1.000	1.000	1.000	.	66	.	.
Biweekly 1 Test							
Average Measures	.992	.988	.995	130.750	66	67	<.001
Biweekly 2 Test							
Average Measures	.990	.984	.994	104.274	66	67	<.001
Posttest							
Average Measures	.979	.966	.987	47.528	66	67	<.001

One-way random effects model

Table 11
Results of the ICC Calculation for the Problem Solving with Metacognitive Behaviors Question

		95% Confidence Interval		F Test with True Value 0			
	Intraclass Correlation	Lower Bound	Upper Bound	Value	df1	df2	Sig
Pretest							
Average Measures	.998	.997	.999	607.364	66	67	<.001
Biweekly 1 Test							
Average Measures	.990	.983	.994	96.242	66	67	<.001
Biweekly 2 Test							
Average Measures	.957	.930	.974	23.252	66	67	<.001
Posttest							
Average Measures	.999	.999	1.000	1.304E3	66	67	<.001

One-way random effects model

Summary

This chapter discussed the research questions, the research hypotheses, the demographics of the participants in this study, the data analysis, and the reliability tests used. It also included the repeated measures ANOVA analysis of the study to examine the effect writing in mathematics has on the students' achievement tests, as well as the repeated measures ANOVA analysis to examine the effect writing has on the problem solving with metacognitive behaviors questions. It also displayed the descriptive analysis of the metacognitive behaviors students portrayed in their writing for the problem solving question of each achievement test. The following chapter will discuss the results of the study in detail and offer suggestions for future studies.

CHAPTER V

DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

This chapter presents a discussion of the findings, conclusions, and delimitations of the study. It also presents implications and recommendations for future research studies.

Discussion of the Results

The main purpose of this study was to determine if the use of writing during mathematics had any effect on the results of students' achievement tests in mathematics.

Demographic Factors Analysis

At the beginning of the study, demographic data were collected about the students in order to test for homogeneity in the groups. Eight factors were evaluated by looking at the frequency and percentage in each group. The factors included students' race/ethnicity, English language proficiency, socioeconomic status, gender, age, student's siblings (e.g., student has older siblings, younger siblings, both younger and older, or no siblings), family structure (e.g., married parents, divorced parents, or single parents), and parental education background (e.g., both parents graduated from college, one parent graduated from college, or none of the parents graduated from college). These data are summarized in table 2 which shows the frequencies and percentages for these demographic factors for each treatment and control groups.

Hypothesis 1

The data analysis showed mixed findings. The study showed that there was no significant difference between the mathematics achievement scores of the students in the treatment group and the control group ($F(1,65) = 3.411$, $p=.069$) over time. This finding may be a result of a couple of factors. First, it is important to take into account the students' writing abilities. Students in third grade are still developing writing skills and on the way to mastering beginning writing strategies. Also the students in the study had never been exposed to writing extensively in the content areas. Another factor that may have influenced the results of the study was the participants' exposure to high stakes testing during the intervention period. The students took the Florida Standards Assessment also during the spring of 2016. Students have been found to have test anxiety, and feelings of uneasiness and apprehension before, during, and after high stakes testing which can result in lower academic achievement in core subjects (Segool, 2009). Lastly, as Figure 1 shows, the scores for the treatment group are increasing more quickly than those for the control group to the point that at the last measure (posttest) the treatment group surpassed the control group, although initially, the control group had higher scores. One of the reasons the control group scored higher in the pretest might be due to the fact that the majority of the lower ESOL level ELLs were in the treatment group. This is important when looking at the posttest average scores given that these same ELLs at ESOL levels 1, 2, and 3 together with other ELLs and English speakers in the treatment group surpassed the higher ESOL level ELLs and English speakers in the control group. These results suggest that if the treatment would have been done for a longer period of time it might have produced a significant difference between the groups. It may be beneficial to include additional writing in mathematics intervention sessions in which the children can further practice their writing during mathematics problem solving in future studies.

Similar results were found in a study conducted by English (1998) also with third grade students who received instruction on writing during problem solving during two months. The students in that study had to pose their own problems about addition and subtraction and solve them while explaining their creations and strategies through writing. Even though significant differences were found in the complexity of the written problems from pretest to posttest ($Z(n1 = 54, n2 = 52) = –3.14, p < .001$), no significant differences were found between the intervention and control groups in regards to posttest achievement scores.

English's study (1998) found that the children had difficulties in recognizing the formal symbolism as representations of problem situations and these were evident during the intervention program and in the posttest results. The author also stated that the activities of the current program being used were insufficient to broaden the children's interpretations of problems and that although the intervention improved their abilities to generate problems, it would be beneficial to include several more program sessions in which children could both solve and pose problems that extended beyond the basic approach they had used (English, 1998).

Hypothesis 1a

Additionally, when comparing the English Language Learners from the treatment group to the ELLs in the control group, the repeated measures ANOVA analysis showed that the achievement scores means of the ELLs were not statistically significantly different between time points ($F(1,34) = 2.632, p=.114$). This finding can be explained in part by the fact that the ELLs who participated in the study were still developing language and writing skills and on the way to mastering beginning writing strategies not only in English but also in their native language. Additionally, these ELLs were taking the Florida Standards Assessment also during the spring of 2016, and as stated previously, students tend to have feelings of anxiety, uneasiness, and apprehension before, during, and after high stakes testing that can lead to lower academic achievement in core subjects (Segool, 2009).

The length of time of the intervention is another factor that may have caused these non-significant results such as English (1998) suggested in his study. However, figure 2 shows the growth trends for the ELLs participating in the study. It can be observed that the ELLs in the treatment group had an achievement mean score higher than the control group in the posttest. This indicates that if the writing in mathematics intervention would have lasted longer than 6 weeks, a statistically significant difference may have been seen between the pretest and the posttest scores of the ELLs in each group. It is important to highlight that the use of ESOL strategies was essential for the ELLs to produce the writing they did in the biweekly tests and on the posttest. The lessons included teaching new and subject-related vocabulary explicitly, using sentence frames, and teaching grammar mini-lessons in order to assist the ELLs when writing to explain their thinking.

Hypothesis 1b

The same variable setting used for question 1 and for question 1a, was used for question 1b. The repeated measures ANOVA determined that the achievement scores means of the English speakers were not statistically significantly different between time points ($F(1,29) = .980$, $p=.330$). These results can also be attributed to the developing writing skills of this group of students, the test anxiety factor, and the length of the intervention. The line graph in Figure 3 shows the growth trends for the achievement scores of the English speakers in each of the treatment and control groups. It demonstrates that the English speakers in the treatment group had higher scores at the posttest than the English speakers in the control group, which indicates that if the intervention was extended additional time, the results may have shown a statistically significant difference between the groups.

Additionally, a one-way repeated measures ANOVA was used to analyze the differences between ELLs and English speakers in the treatment group ($F(1,33) =.105$, $p= .747$). Even though this analysis was not conducted in support of a research question, it was completed to see if there was a difference between the scores of the students based on the English proficiency level factor. The results showed no statistical difference between the ELLs and the English speakers who scored similarly on average over time. However, the growth trends data showed growth in achievement scores from the pretest to the posttest scores resulting in a positive relationship between their writing and their achievement scores.

Moreover, a one-way repeated measures ANOVA was used to look at the ELLs in the treatment to the English speakers in the control in order to find if there was a significant different between the achievement scores. The results also showed there was no significant difference ($F(1,32) =.654$, $p = .425$). This analysis was conducted in order to observe if the language proficiency factor together with the writing instruction had any effect on the achievement scores of the ELLs in treatment when compared to the English speakers in the control.

Hypothesis 2

The analysis of the One-Way Repeated Measures ANOVA for the second hypothesis showed that the effect of writing in mathematics on the students' demonstration of metacognitive behaviors while solving mathematics problems was statistically significant, $F(1,65) = 75.971$, $p < .001$.

It can be concluded that writing in mathematics had a positive effect on the scores for the problem solving questions of the students in the treatment versus the control group. Additionally, Post Hoc tests using the Bonferroni correction revealed that the treatment elicited an increase in students' problem solving scores from the pretest administration to the first biweekly test with a mean difference of 5.823 resulting in a statistically significant difference ($p<.001$). There was also a significant difference between the mean scores of pretest and biweekly 2 (mean difference = 8.648, $p<.001$), between the pretest and the posttest (mean difference = 8.674, $p<.001$), between the biweekly 1 and biweekly 2 (mean difference = 2.825, $p<.001$), and between biweekly 1 and posttest (mean difference of 2.851, $p<.001$). Table 8 summarizes the results of the Post Hoc Tests. It can be concluded that writing during problem solving in mathematics increases students' scores on problem solving with metacognitive behaviors questions over time.

Moreover, by looking at figure 4 and the growth trends for the students in treatment and control, it can be observed how the students in the treatment group showed continuous growth from pretest to posttest. It was also interesting to see a drop in the problem solving with metacognitive behaviors scores in the control group at the end in the posttest. The students completed the posttest after they had taken the Florida

Standards Assessment (FSA) in mathematics. They had stopped receiving test taking skills instruction which included leaning to explain their answers with a brief explanation. The children in the control group might have thought the need to explain through writing was no longer required or necessary given the FSA was over.

The results for the second research question are important because they show that the use of writing in mathematics helped to improve both ELLs and English speakers problem solving scores. Similar results were found in a study conducted by Rudnitsky, Etheredge, Freeman, and Gilbert (1995) that used two treatments: writing and solving to help students improve mathematics problem solving. The writing treatment consisted of lessons designed to engage students in creating mathematical problems, while the solving treatment included lessons about problem-solving procedures referred to as problem-solving steps, tips, rules, procedures, or guidelines. The treatment lasted 10 weeks and used a pretest-posttest design to assess student problem solving abilities. There was a statistically significant difference between the posttest results of the students in the treatment groups, which used either writing or solving, to the posttest results of the control group. Although both treatment groups outperformed the control group, the results showed that the writing treatment was superior to the solving treatment.

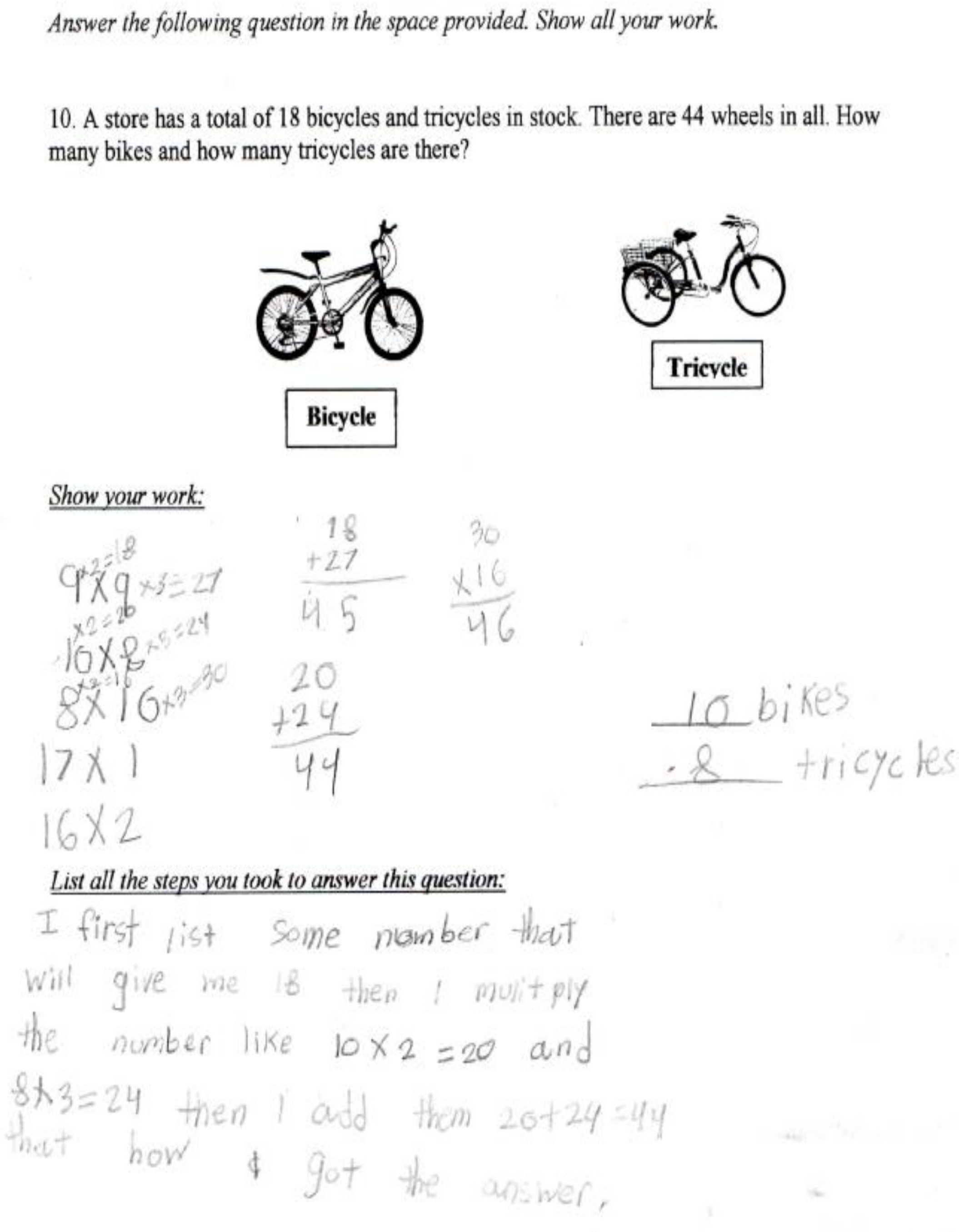
In the same way, the present study revealed significant differences in the problem solving with metacognitive behaviors questions scores of the treatment group versus the control group. These findings also support the theories of Garofalo and Lester (1985), and Fernandez, Hadaway, and Wilson (1994) about the significance of having students think about the processes they follow when they problem solve.

Hypothesis 3

Finally, the third hypothesis looked at the frequency of occurrence of each problem solving phase in each achievement test. By testing the students every two weeks, the frequency in which the students used the phases of problem solving was analyzed, and by using writing to describe their thinking processes the metacognitive behaviors demonstrated in each test was examined. This information can be used by teachers to scaffold the students' learning and help them in the problem solving process. Additionally, it could be observed that as time passed most treatment group students were using all the phases of problem solving, a step in becoming effective mathematics problem solvers.

Figure 5 shows the percentages of students who wrote about the metacognitive behaviors they used to problem solve in each of the four tests. The rubric in table 1 was used to score the students' responses for the problem solving questions which included the students writing about their metacognition. The students had to score 3 or 4 points in order to be coded as using the problem solving phase adequately. When analyzing the frequency of metacognitive behaviors during the pretest, it can be observed that the majority of students in both treatment and control did not write about metacognitive behaviors for any of the phases of problem solving. During the biweekly 1 test, which was given two weeks after the beginning of the treatment, both treatment and control show similar results in regards to writing about the metacognitive behaviors used during problem solving. It was interesting to see that the control group was writing also even when they were not being taught using the treatment. This may be due to the fact that students were being prepared for the FSA mathematics test which includes questions in which students have to explain their answers with an equation or a brief explanation (FSA, 2014). The data for the biweekly 2 test, given four weeks after the beginning of the intervention, start to show a much greater difference in the use of metacognitive behaviors by the students in the treatment group. The students in the treatment group use orientation (82.9%), organization (74.3%), and execution (54.3%) metacognitive behaviors much more frequently during the biweekly 2 test administration than during the previous two tests. This finding shows that after only four weeks of learning and using writing during problem solving there was a change in the way students were solving mathematics problems and thinking about the process. The students in the treatment group seem to be using metacognitive behaviors in all four phases of problem solving during the posttest, the last assessment conducted at the end of the 6 weeks of intervention. The data show the highest percentage of treatment students using the orientation and organization phases, with 100% of the treatment students describing the metacognitive behaviors they used to solve the problems in each of these two phases. Also during the posttest, 91.4% of treatment students wrote about their metacognition for executing the solution and 80% of them wrote about the verification process they followed.

The rubric in table 1 was used to score the students' responses for the problem solving questions which included the students' writing about their metacognition. The students had to score 3 or 4 points in order to be coded as using the problem solving phase (orientation, organization, execution, and verification) adequately. Figure 6 shows an example of an ELL student's answer with a score of 4 on the posttest's problem solving question, and Figure 7 shows an example of an ELL student's answer with a score of 3 on the posttest's problem solving question. In addition, Figure 8 presents an English speaker student work sample in which the student scored 4 points in the posttest problem solving question, and Figure 9 shows an English speaker student work sample in which the student scored 3 points in the posttest problem solving question.



Answer the following question in the space provided. Show all your work.

10. A store has a total of 18 bicycles and tricycles in stock. There are 44 wheels in all. How many bikes and how many tricycles are there?

Show your work:

9X9 x2=18 x3=27
10X8 x2=20 x3=24
8X16 x2=16 x3=30
17X1
16X2

18 +27 = 45

20 +24 = 44

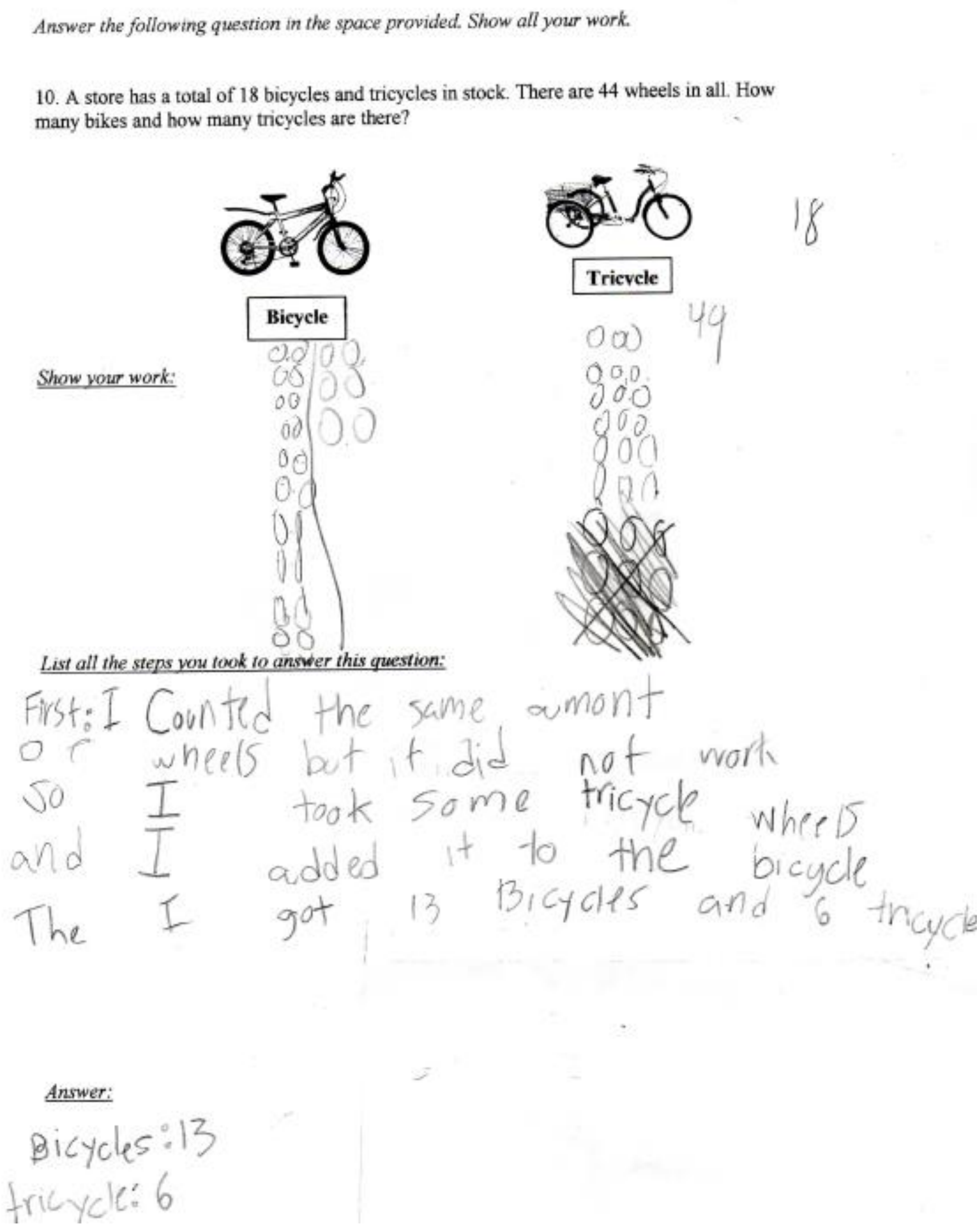
30 x16 = 46

10 bikes
8 tricycles

List all the steps you took to answer this question:

I first list some nomber that will give me 18 then I mulitply the number like 10X2=20 and 8X3=24 then I add them 20+24=44 that how I got the answer,

Figure 6. Sample work from an ELL student who scored 4 points on the posttest problem solving question. Transcription of student's written response: "I first list some number that will give me 18 then I multiply the number like 10x2=20 and 8x3=24 then I add them 20+24=44 that how I got the answer."


Answer the following question in the space provided. Show all your work.

10. A store has a total of 18 bicycles and tricycles in stock. There are 44 wheels in all. How many bikes and how many tricycles are there?

Bicycle **Tricycle** 18 44

Show your work:

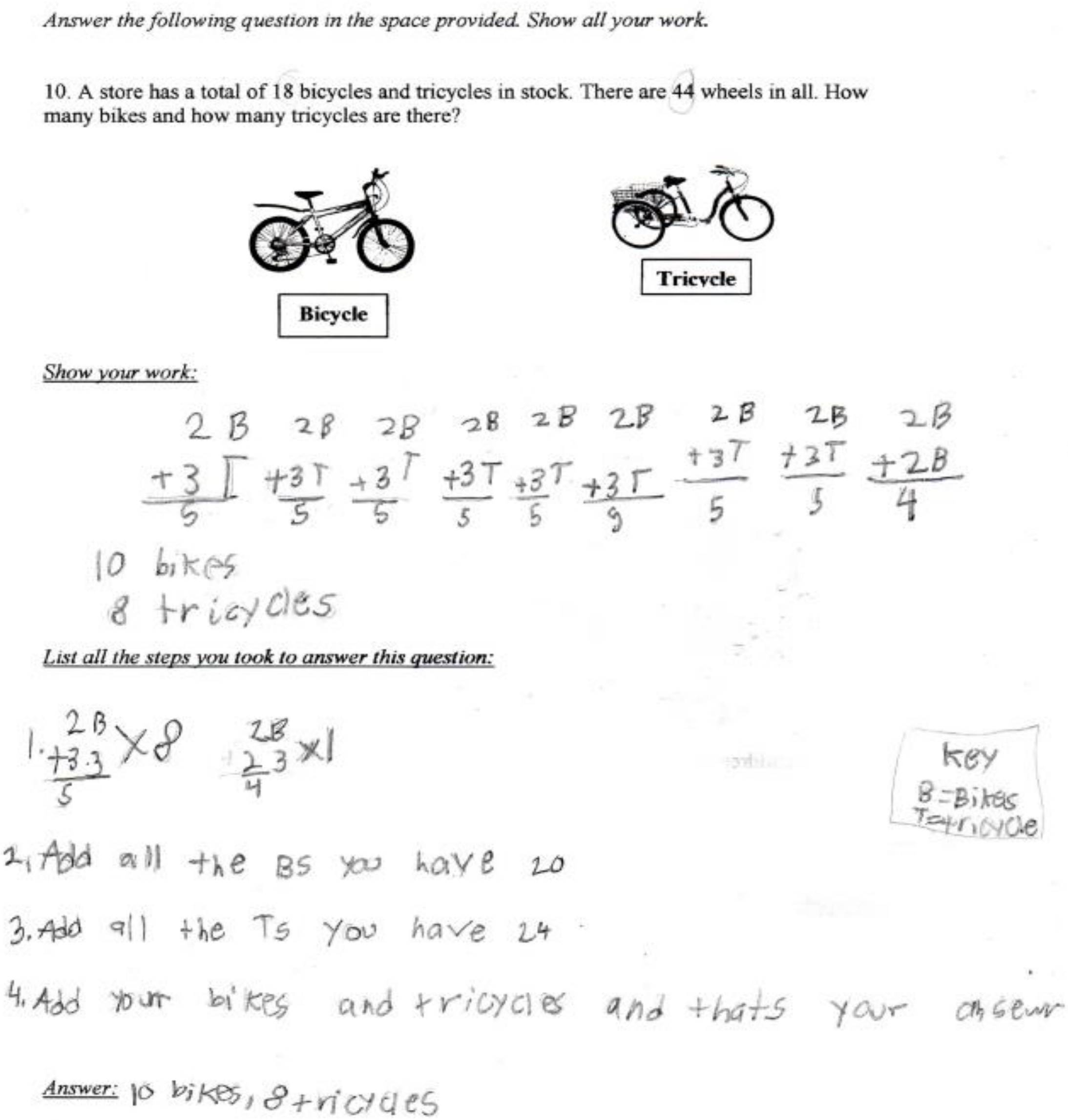
List all the steps you took to answer this question:

First: I Counted the same amont or wheels but it did not work so I took some tricycle wheels and I added it to the bicycle The I got 13 Bicycles and 6 tricycle

Answer:

Bicycles: 13
tricycle: 6

Figure 7. Sample work from an ELL student who scored 3 points on the posttest problem solving question. Transcription of student's written response: "First: I counted the same [amount] of wheels but it did not work so I took some tricycle wheels and I added it to the bicycle. [Then] I got 13 bicycles and 6 tricycle[s]." The student forgot to add the bicycles and tricycles in the final answer to satisfy the problem condition that there were a total of 18 bikes and tricycles all together.



Answer the following question in the space provided. Show all your work.

10. A store has a total of 18 bicycles and tricycles in stock. There are 44 wheels in all. How many bikes and how many tricycles are there?

Bicycle

Tricycle

Show your work:

2 B +3 T = 5; 2B +3T = 5; 2B +3T = 5; 2B +3T = 5; 2B +3T = 5; 2B +3T = 5; 2B +3T = 5; 2B +3T = 5; 2B +2B = 4

10 bikes
8 tricycles

List all the steps you took to answer this question:

1. 2B +3 3 = 5 X 8   2B 2 3 = 4 X 1

key
B = Bikes
T = tricycle

2. Add all the Bs you have 20
3. Add all the Ts you have 24
4. Add your bikes and tricycles and thats your ansewr

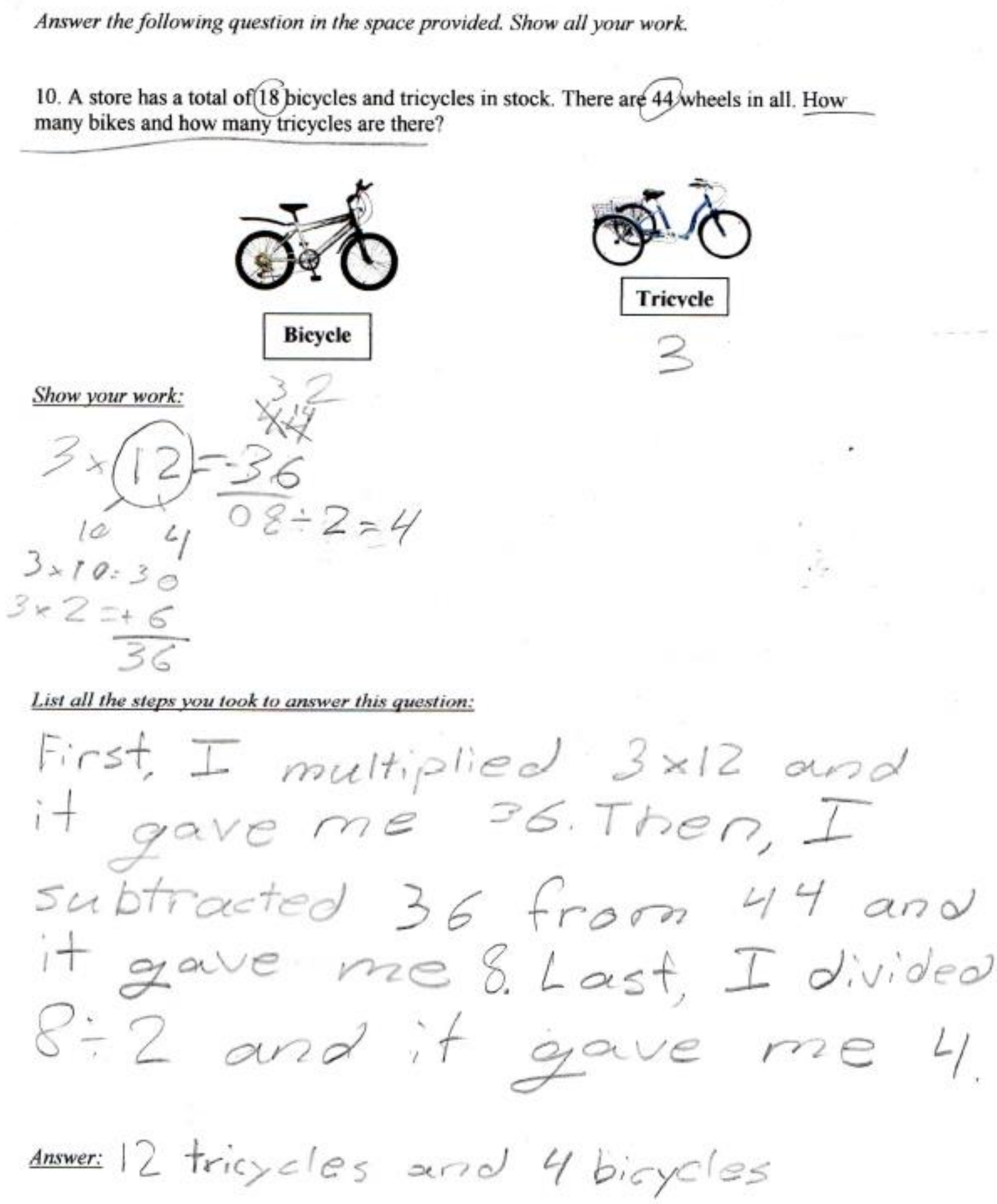
Answer: 10 bikes, 8 tricycles

Figure 8. Sample work from an English speaker student who scored 4 points on the posttest problem solving question. Transcription of student' written response:

"(1) 2 B +3 3 / 5 X 8   2 B 2 3 / 4 X 1

[The student represents the tricycles by writing a "3" rather than by writing a "T" like the student wrote in step 3.]

(2) Add all the Bs (bicycles) you have 20.
(3) Add all the Ts you have 24. 4. Add your bikes and tricycles and that's your ansewr [answer]
Answer: 10 bikes, 8 tricycles."


Answer the following question in the space provided. Show all your work.

10. A store has a total of 18 bicycles and tricycles in stock. There are 44 wheels in all. How many bikes and how many tricycles are there?

Bicycle

Tricycle

3

Show your work:

3 x 12 = 36

08 ÷ 2 = 4

3 x 10 = 30

3 x 2 = + 6

36

List all the steps you took to answer this question:

First, I multiplied 3x12 and it gave me 36. Then, I subtracted 36 from 44 and it gave me 8. Last, I divided 8÷2 and it gave me 4.

Answer: 12 tricycles and 4 bicycles

Figure 9. Sample work from an English speaker student who scored 3 points on the posttest problem solving question. Transcription of student' written response: "First, I multiplied 3x12 and it gave me 36. Then, I subtracted 36 from 44 and it gave me 8. Last, I divided 8÷2 and it gave me 4.
Answer: 12 tricycles and 4 bicycles."

Delimitations

There are some delimitations to this study that need to be discussed. First, the study was conducted with a small sample size (less than 100). This limits the generalizability of the results. This population may be very different from populations in other parts of the United States as well as other parts of the world. In addition, having two teachers also brings diversity to the lessons. Even though teachers have their own teaching style and use different instructional methods, in this study the second teacher was extensively trained by the researcher and first teacher on how to use writing in mathematics, and both used the same problems for instruction and followed the same format for each of the lessons.

Implications

The findings from this study are useful to teachers, mathematics curriculum specialists, professional development coordinators, principals, and district superintendents who are interested in creating programs for teaching mathematics at the elementary level that include engaging students in problem solving and increasing students' problem solving ability. Given that the writing in mathematics instruction improved the problem solving scores of those students who wrote about their metacognitive behaviors, other school organizations may also benefit from using this evidence-based method for teaching mathematics at the elementary school level. Particularly, in school districts such as Miami-Dade County in which the ELL population is high, it may be beneficial to include writing in mathematics in order to assist all students but especially the ELLs when solving problems in mathematics.

Recommendations for Future Research

There are a number of changes that can be made in future studies that might want to build on or expand on the findings of this study. Although mathematics achievement scores were not significantly different for treatment and control groups, trends in the mean scores of the repeated measures reveal that the scores for the treatment group are growing faster than those for the control group over the six-week period of the study. These trends suggest extending the extent of time for the study. The incorporation of problem solving and writing in the teaching of mathematics is supported by the CCSS (2010) and MAFS (2015). This practice is appropriate to be used in mathematics classes, and thus the intervention used in this study is something that can be incorporated in teaching mathematics throughout an entire semester or during an entire academic year. Given that problem solving is an on-going learning process, a longer period of time may also be beneficial in order to capture how and what metacognitive behaviors students develop as they become efficient problem solvers.

Additionally, future studies can concentrate on finding a larger group of students to participate in a similar context. A larger sample size may bring potentially more widely generalizable results in terms of the significant differences in using writing and the problem solving with metacognitive behaviors scores. Future studies should also investigate how students can be supported as they develop these metacognitive behaviors. A qualitative study including case studies of students in the treatment group who can be interviewed about the processes they used when solving problems, might be beneficial in finding out how students acquire metacognitive behaviors that may not be present in their writing.

Given that this study was conducted during the spring semester at the time in which students were also taking a high stakes assessment, it might be important to conduct a similar study during the fall semester given that test anxiety and/or test preparation would not interfere with the study results. Since the students would not be receiving test practice including writing during the fall semester, then writing in mathematics for test practice will not interfere with the results of the scores especially for those in the control group.

Summary

This chapter discussed the One-Way Repeated Measures Analysis of Variance results of this study, the delimitations, implications for the professionals in the field, and the recommendations for future research studies. Although the study did not produce significant mathematics achievement results over a 12 session, 6-week treatment that taught ELL and English Speaker third graders to use writing in mathematics, the intervention revealed faster growth trends overtime between the mean achievement scores for the treatment group over the control group. Additionally, the study produced statistically significant results between writing in mathematics and problem solving, an important finding for education professionals interested in implementing problem solving in mathematics programs at the elementary level.

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Appendix A

Pretest

Name: ___________________________________ Date:___________________

Teacher: __________________________________

Answer the following questions by choosing the best answer:

1. Tomas planted 7 rows of flowers. He planted 3 flowers in each row. How many flowers did he plant in all?

    a. 10

    b. 21

    c. 5

    d. 7

2. Jose has 30 photographs in a photo album. He placed 5 photographs on each page. How many pages did Jose fill?

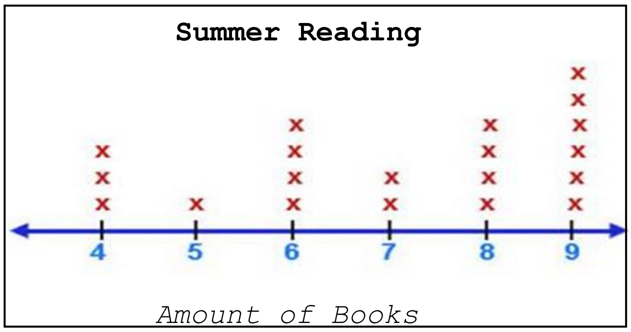
    a. 6

    b. 25

    c. 35

    d. 5

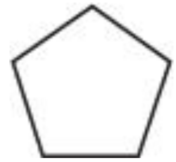
3. Ana asked her classmates about the amount of books they read during the summer and completed the following line plot.



How many children read 4 books?

a. 8 children
b. 3 children
c. 5 children
d. 6 children

4. Mary made a shape with 5 toothpicks. If she wants to make 8 shapes in all, how many toothpicks does she need?



a. 5
b. 8
c. 13
d. 40

Answer the following questions in the space provided. Show all your work.

5. Look at the following table. Describe the pattern and write the missing numbers

Flashlight	3	4	5	6	7	8
Batteries	12	16	20	24		

Show your work:

6. Sam sees 15 red balloons, 18 blue balloons, and 12 yellow balloons. How many balloons does he see in all?

Show your work:

Answer: ____________

7. Katy had 213 cards in her collection and Juan had 117 cards. How many more cards did Katy have than Juan?

Show your work:

Answer: ____________

8. Dan rounds number 234 to the nearest ten, and he writes 240. Does his answer make sense? Explain.

____

____

____

____

____

____

9. The picture graph shows the birthdays of some children in Alina's class.

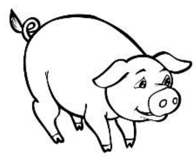
Birthdays	
September	☺ ☺ ☺ ☺ ◖
October	☺ ☺ ☺
November	☺ ☺ ☺ ☺ ☺ ☺
Key: Each ☺ = 2 children	

How many more birthdays are there in November than in September?

Answer: ____________

Answer the following question in the space provided. Show all your work.

10. A farmer has both pigs and chickens on his farm. There are 78 feet and 27 heads. How many pigs and how many chickens are there?



Pig

Chicken

Show your work:

List all the steps you took to answer this question:

Answer:

Appendix B

Child-Friendly Rubric

ORIENTATION

- ☐ Read
- ☐ Re-read
- ☐ Looked at clues or key words
- ☐ Looked at pictures
- ☐ Looked at numbers
- ☐ Made initial chart or picture
- ☐ Looked up words I don't understand

ORGANIZATION

- ☐ Made a plan
- ☐ Thought about using different strategies like:
    - ○ ____________
    - ○ ____________
    - ○ ____________

VERIFICATION

- ☐ Checked my answer by ____________________
- ☐ Does it make sense?
    - ○ Yes
    - ○ No

EXECUTION

- ☐ Solved the problem using the following:
    - ○ ______________
    - ○ ______________

Name:

Appendix C

Scoring Criteria for Posttest

Posttest Question	***Type of Question***	***Total Amount of Possible Points***	***Scoring complexity***
1. Daniela recorded her classmates' favorite ice cream flavors in the following picture graph. How many more children like strawberry ice cream than cookies & cream ice cream? a. 6 b. 9 c. 4 d. 5	multiple choice	1	**0 points** = wrong answer is chosen **1 point** = correct answer is chosen
2. Karla has 42 crayons. She wants to give 7 crayons to each of her friends. How many friends can she give crayons to? a. 6 b. 49 c. 8 d. 2	multiple choice	1	**0 points** = wrong answer is chosen **1 point** = correct answer is chosen
3. Elias buys 5 bags of cookies for his class. Each bag has 7 cookies. How many cookies does Elias have in all? a. 12 b. 2 c. 35 d. 7	multiple choice	1	**0 points** = wrong answer is chosen **1 point** = correct answer is chosen
4. Sammy reads 3 books each week. How many books does he read after 9 weeks? a. 6 b. 27 c. 3 d. 12	multiple choice	1	**0 points** = wrong answer is chosen **1 point** = correct answer is chosen

5. A bakery sells 24 cupcakes, 26 guava pastries, and 52 Cuban breads. How many things does the bakery sell in all? Show your work:	short response	2	**0 points** = does not add **1 point =** adds but makes computation mistake **2 points =** adds and gets correct answer (sum=102)
6. Look at the following table. Write the missing numbers and describe the pattern:	short response	2	**0 points** = wrong answers for both missing numbers **1 point** = one missing number is right and written description is right OR both missing numbers are right but there is no description or incomplete description given **2 points** = both missing numbers are right (6, 7) and description is complete and correct
7. The baseball team has 41 baseball balls, 76 gloves, and 33 bats. How many items do they have in all? A.	short response	2	**0 points** - does not add all 3 numbers **1 point** = adds all 3 numbers but makes computation mistake **2 points** = adds and gets correct answer (sum=150)
8. Carl rounds number 652 to the nearest ten, and he writes 650. Does his answer make sense?	short response	2	**0 points** = answers NO **1 point** = answers YES, and gives incomplete explanation or part of explanation is wrong **2 points** = answers YES and has a complete explanation

9. The bar graph shows the activities students do after school. How many more children are in chess than in basketball?	short response	2	**0 points** = does not subtract numbers **1 point** = subtracts but makes computation mistake **2 points** = subtracts and gets correct answer OR shows correct answer (even when subtraction work is not shown)
10. A store has a total of 18 bicycles and tricycles in stock. There are 44 wheels in all. How many bikes and how many tricycles are there?	problem solving question	4	**0 points** = does not show any work OR beginning work does not show understanding of the problem and there is no writing that shows understanding **1 point** = starts to show steps to solve the problem by showing no or limited understanding of the problem, AND writing does not show much understanding of the problem **2 points** = starts to solve for bicycles and tricycles by using the total number of wheels as a start to show initial understanding of problem, writes about some steps **3 points** = starts using one or more strategies to find amount for each the bikes and tricycles, has some computation mistakes but was on the right path to solve the problem AND writes some steps about process he/she is following **4 points** = solves correctly for the amount of bikes (10) and tricycles (8) AND writes about most steps used

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