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Arvind Kumar Busam
Florida International University

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FLORIDA INTERNATIONAL UNIVERSITY

Miami, Florida

OPTIMIZATION OF WAITING TIME AT TOLL PLAZAS

A thesis submitted in partial fulfillment of the

requirements for the degree of

MASTER OF SCIENCE

in

INDUSTRIAL AND SYSTEMS ENGINEERING

by

Arvind Kumar Busam

2005

To: Dean Vish Prasad
College of Engineering and Computing

This thesis, written by Arvind Kumar Busam, and entitled Optimization of Waiting Time at Toll Plazas, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this thesis and recommend that it be approved.

Martha A. Centeno

Albert Gan

Seongmoon Kim, Major Professor

Date of Defense: July 20, 2005

The thesis of Arvind Kumar Busam is approved.

Dean Vish Prasad
College of Engineering and Computing

Dean Douglas Wartzok
University Graduate School

Florida International University, 2005

DEDICATION

I dedicate this thesis to my parents, Mr. B.A.K. Mukkanteshwara Rao and Mrs. B. Sujatha and my loving sister Aruna Kumari Busam. Thanks for supporting me. I love you.

ACKNOWLEDGMENTS

I would like to thank my committee members, Dr. Seongmoon Kim, Dr. Martha A. Centeno and Dr. Albert Gan. Thank you for serving on my committee. Most importantly, I would like to express my sincere gratitude to Dr. Seongmoon Kim, my major professor. Thank you very much for your help, time and patience.

ABSTRACT OF THE THESIS

OPTIMIZATION OF WAITING TIME AT TOLL PLAZAS

by

Arvind Kumar Busam

Florida International University, 2005

Miami, Florida

Professor Seongmoon Kim, Major Professor

Toll plazas have several toll payment types such as manual, automatic coin machines, electronic and mixed lanes. In places with high traffic flow, the presence of toll plaza causes a lot of traffic congestion; this creates a bottleneck for the traffic flow, unless the correct mix of payment types is in operation. The objective of this research is to determine the optimal lane configuration for the mix of the methods of payment so that the waiting time in the queue at the toll plaza is minimized. A queuing model representing the toll plaza system and a nonlinear integer program have been developed to determine the optimal mix. The numerical results show that the waiting time can be decreased at the toll plaza by changing the lane configuration. For the case study developed an improvement in the waiting time as high as 96.37 percent was noticed during the morning peak hour.

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CHAPTER 1

INTRODUCTION

Traffic congestion has emerged to be a major concern both to the public and to the transportation engineers (Zarrillo, et.al, 1997). Traffic congestion is being experienced daily by many travelers both on the inner city roads as well as on the highways and toll plazas are one of the reasons for the congestion on the highways. Even though toll plazas are a reason for traffic congestion, they cannot be removed for the various reasons. Traffic signals help in the smooth flow of the vehicles at intersections, and the revenue generated from the toll plazas is one of the ways of maintaining the highways. Traffic congestion causes delays and wastes fuel, which costs over $40 billions a year. In addition, the effects of traffic congestion increase air pollution ("Curbing Gridlock", a special report by TRB)

A toll road is a road on which a toll authority collects fee for using the road. A toll plaza is the location where tolls are collected, and these are called toll plaza structures. In the United States tolls roads have been in use for over two centuries with the construction of the Lancaster turnpike in 1790's. Initially, even though users disliked the idea of paying toll to use the road, not only for the reason that they have to pay toll but also for the long delay caused at the toll booths, toll roads had to be implemented because they were the main source of revenue (California PATH Research Report).

According to Klodzinski and Al-Deek (2002), toll plaza lanes can be one of the following five types. 1) Manual toll lanes (transactions are handled by a toll collector), 2) Automatic coin machine toll lanes (coin machines, hereafter referred to as ACM), 3)

dedicated electronic toll lanes (hereafter referred to as ETC), 4) mixed ETC and manual lanes and 5) mixed ETC and ACM lanes. Manual and ACM lanes are called conventional lanes.

Manual toll collection method requires a toll collector, who collects the toll based on the type of the vehicle. In ACM lanes the tolls are paid manually, but instead of a toll collector there is a coin machine which collects the toll. Both these toll lanes (manual and ACM) require the motorists to stop at the plaza and pay the toll. The average speed through the conventional lane types is the least because of the stop and pay system and, hence, queues grows significantly during peak hours (Pietrzyk, 1994).

ETC technology uses transponders and external sensors to collect toll. The external sensors determines if the transponders attached to the vehicles in the toll lane are valid or not, and it identifies the vehicle's classification and deduct the necessary toll from the transponder's account. Transponders are nothing but electronic tags mounted on the vehicles that assess the toll based on the vehicle classification. Mixed ETC lanes combine the ETC technology with the conventional toll payment type. According to the Transportation Research Board, the average capacity for a manual lane is 350 vehicles per hour, for an automatic lane is 500 vehicles per hour and for a dedicated ETC lane is 1200 vehicles per hour.

The remainder of this chapter discusses the problem statement in detail, outlines the goals and objectives of the research, and includes research methodology and results. Chapter 2 is a review of the pertinent literature. Chapter 3 outlines the formulation that was used in the research. Chapter 4 explains how the data have been collected, how the

analysis was done and the results obtained. Finally, Chapter 5 summarizes the conclusion and the future research that can be done.

1.1 Problem statement

Toll plaza may create a bottleneck to the traffic flow on the highways. The main reason for this is the conventional toll collection types which require the vehicles to stop and pay the toll. The process of stopping to pay the toll reduces the traffic flow, increases the waiting time, queue length and thus reduces the efficiency of the toll plazas. According to Ceballos, when each vehicle must come to a stop in order to pay the toll, the throughput of the lane drops from freeway rates of 2000 vehicles per hour to toll plazas rates in the range of 350 to 500 vehicles per hour.

Before the implementation of ETC, the only accepted way to improve traffic operations was to construct additional lanes or increase capacity by decreasing the time of each transaction through the use of tokens or coupons, but there was not much change in the throughput and the costs for the expansions were high. At some plazas, it was impossible to expand because of no right of way. To meet the increasing demand in traffic, increasing the number of lanes at a toll plaza is no longer a feasible solution. Therefore other solutions were needed for reducing the congestion at the toll plazas. Improvements had been sought to decrease the time of each toll transaction so that users experience either reduced delays or no delay at all. ETC was developed and implemented.

Even though electronic toll collection has been implemented in most toll plazas, we still see traffic congestion at these plazas. One way of reducing the delay at the toll

plazas is to change all the lanes to electronic toll collection type. However, this cannot be implemented until all the toll facility users have electronic transponders. So until all the users of the toll facility become ETC patrons, this is not a feasible solution. Hence, we should have conventional toll collection types along with the electronic toll collection and the facilities have to manage the lane configurations in such a way which maximizes the throughput, reduces waiting time and queue length at the toll plazas. Thus, the problem is the inefficient operation of toll plazas even with such advanced technology.

Several approaches have been identified to reduce the waiting time at toll plazas, including simulation and analytical models. However, none of the mathematical models that have been developed are entirely realistic in nature and those models that have been built lack realism with respect to the arrival rates and the service distributions. The problem of waiting time at the toll plazas requires to be addressed by taking into account the stochastic nature of the arrival rates and service distributions.

In the real world most of the toll lanes are operational all through the day even though many toll lanes can be closed during the non rush hour. This results in an increase in the operational costs (Junga, 1990). Therefore, for reducing the operational costs of the toll plaza it is important to open a specified number of toll lanes at a particular time of the day. Hence these toll plaza lanes have to be changed dynamically. This dynamic change of lanes can be done by electronic messaging instead of the regular boards that are present.

1.2 Goal and Specific Objectives

The main goal of this research is to reduce the waiting time at the toll plazas by developing a mathematical model, which determines the optimal lane configuration of the toll plaza, resulting in minimum waiting time and maximum throughput. The traffic operations are assessed by evaluating the capacity of the toll plaza and the waiting time over different toll plaza configurations. The specific objectives to achieve this goal included:

- Detailed literature review on toll plaza system, discovering improvements that have been made to reduce the delay at the toll plazas
- Designed a case study for toll plaza operations, and collected data for the arrival rates, service times of the toll transactions and analyze the data for the required statistical properties.
- Developed a mathematical model and evaluated the waiting time at toll plazas and identified the plaza configuration with the minimum waiting time.

1.3 Research Methodology

The methodology that was used in this research to reduce the waiting time at the toll plazas is as follows:

1. Queuing theory was used to determine the model that represents the toll plaza operations. Vehicles arriving to the toll plaza and paying the toll in the respective toll lanes were analyzed to determine the type of queuing system the toll plaza operations belong to. The operations at the toll plaza when each lane is considered

separately follow M/G/1 queuing model. The arrival process follows the Poisson process and the service process follows a general process.

2. Nonlinear integer programming was used to develop the mathematical model. The model was developed to minimize the waiting time at the toll plazas.
3. Designed a case study to implement the model and found the potential improvement or the potential decrease in the waiting times that can be achieved.

1.4 Results

The analysis was done to determine the potential improvement in the waiting times for the 8 lane and the 15 lane toll plaza model. The analysis was carried for the whole day. The results show that the maximum improvement in the waiting time could be obtained during the rush hour and it is more sensible to change the lane configuration during the rush hour. It is advisable to change the lane configuration even during the non rush hour for decreasing the operational costs but it is not required as there was no improvement in waiting time. The analysis showed that the maximum improvement that can be obtained by implementing the optimal lane configuration was over 96 percent.

CHAPTER 2

LITERATURE REVIEW

This chapter is a review of the research in the relevant areas. This review helps in understanding the various strategies that have been developed to reduce the traffic congestion at the toll plazas. The issue of traffic congestion at the toll plazas is a long standing problem. The literature dates back to 1954 which addresses the problem of waiting time at the toll booths. Even though this has been a long aged problem and many improvements and advances have been made in the way these toll plazas are operated, the problem of traffic congestion at toll plazas is still in existence, the main reason for which being the increase in the traffic volumes and its nature

Capacity planning for the toll plazas involves the study of queues and how these queues can be controlled efficiently. Initial studies on optimal control policies for queues focus on optimal hysteric policies by Gebhard (1967) and on threshold policies for M/G/1 and G/G/1 queuing systems by Heyman (1968) and Sobel (1969). Multi-channel queuing systems are investigated by Singh (1970), Seth (1977). Controlling the queue length by varying the service rates is considered in Crabill (1972, 1974), Huang et al. (1977) and others.

These studies even though are related to queue lengths and their control, they however do not apply to toll plaza operation. This is because of the assumption in the studies that the decisions can be made at random points. In the case of toll plazas decisions cannot be made at random points but are decided prior to the actual implementation.

The literature on the toll plaza operations can be categorized by the simulation models and the mathematical models developed to improve the toll plaza operations. The following is the literature review of the simulation models developed and used to improve the traffic operations at the toll plaza, followed by the review of the mathematical models developed and used to analyze and improve the traffic operations at the toll plaza.

2.1 Simulation Models

Pietryzk (1994) suggested that ETC can provide a feasible solution to the problem of toll plaza expansion with limited right-of-way. In this study, efficiency of ETC was evaluated with a comprehensive cost effective analysis relative to the implementation of ETC over a period of 20 years (1995-2015). The benefits of implementing ETC compared to the conventions plans for lane expansion on Florida Turnpike systems was 61 fewer toll lanes at mainline plazas, millions of savings in capital investments for the operation and maintenance

Lin and Su (1994) developed a methodology for the level-of-service analysis of main-line toll plazas. They developed a methodology which relies on a computer simulation model (TPSIM). This model was used to examine the operating characteristics of toll plazas through field observations and computer simulation. According to the model a stable performance can generally be expected if the average queue length of a toll lane does not exceed three vehicles and an average queue length of more than 10 vehicles represents an unstable and undesirable operation. In this model, average queue length and average waiting time spent in toll collection system are used as measures of

effectiveness to classify quality of service in to six different levels namely A through F. The simulation model used generates random arrivals according to a shifted negative exponential function and the service time is determined according to a specified probability distribution.

Al-Deek et al, (1996) evaluated the improvements in the traffic operations at a real-life toll plaza with electronic toll collection. The improvements in the traffic operations with electronic toll collection were studied at the electronic toll plazas of the Orlando-Orange County Expressway Authority (OOECA). The significant improvements of installing one dedicated electronic toll collection lane includes a potential lane capacity increase of 194 percent, average savings of 5 seconds in service time per vehicle, a reduction of 1 minute in average queuing delay, a reduction of 2.5 minutes in maximum queuing delay per vehicle and an average savings of 8.5 vehicle-hours in total queuing delay per peak hour. The variability in the inter-vehicle time has been reduced significantly in the electronic toll collection lane. These results focus on the Orlando's Holland East Plaza. Electronic toll collection lane(s) did not have any affect on the capacity, inter-arrival times and service times of the mixed lanes, but the arrivals have shifted to the dedicated electronic toll collection lane thus reducing the delays at the mixed lanes and improving the traffic operations of the toll plaza.

Al-Deek et al, (1997) investigated the operational benefits of electronic toll collection and the improvements in traffic operations at the plazas due to the electronic toll collection. This investigation indicates that for the dedicated electronic toll collection lanes, the measured capacity has tripled; the service time has decreased by five seconds per vehicle, the average queuing delay has decreased by one minute per vehicle, the

maximum queuing delay has decreased by 2.5-3 minutes per vehicle and the total queuing delay has decreased by 8.5-9.5 vehicle-hours per morning peak hour for that lane. Variability in the head way has also been reduced significantly in the dedicated electronic toll collection lanes. Capacity, service times and headway of the mixed lanes did not change significantly, but the arrivals have shifted to the dedicated electronic toll collection lanes and hence reducing delays at the mixed lanes and improving traffic operations for the entire toll plaza.

In this study, implementation of electronic toll collection went through four different stages. In the first stage, there were no electronic toll collection lanes. In the second stage, mixed electronic toll collection lanes were implemented. In stage three, only one dedicated electronic toll collection lane per direction were implemented with all the other lanes being mixed electronic lanes. Finally in stage four, two dedicated electronic toll lanes were implemented with all the other lanes being mixed electronic toll lanes. The improvements in the traffic operations were evaluated by the studies conducted before and after installation of electronic toll collection lanes. The introduction of electronic toll collection system has eliminated the variability in service times due to operators. In mixed lanes, there is no statistical significance at the 5 percent level in the inter-vehicle time except for the dedicated electronic toll collection lanes. This is because of the lower electronic percentage usage in the mixed lanes. Electronic toll users are more inclined to use the dedicated electronic toll lanes rather than the mixed electronic lanes because of the faster service. There are no statistically significant changes in the inter-vehicle times for the manual lanes and hence there is no significant change in the

capacity. The study shows that the average queuing delay, maximum delay and total delay per lane are reduced for all mixed lanes because of the electronic toll collection.

Zarrillo et al, (1998) developed a Toll Plaza model (TPModel), which is a macroscopic analytical queuing model, for the purpose of estimating the rush hour delay at toll plazas. It illustrates the impact of ETC on traffic operations at toll collection facilities; it also considers that queues consist of a mixture of vehicles requiring different types of services that require different service times. It also addresses the variation in the processing rates. A delay sensitivity analysis was also performed on each of the model's input variables (throughput, percentage of trucks, and percentage of ETC users). This model estimates increasing delay with increasing approach volumes, increasing percentage of trucks utilizing the manual toll collection and estimates a decreasing delay with the increase in the percentage of ETC usage and increase in the service rate for the various services. Uniform arrival rates were used to develop the model.

Al-Deek (2001) developed and validated a toll plaza evaluation toll called TPSIM to find the optimal lane configuration and also investigated the sensitivity of the peak hour plaza delay to the use of ETC in real world. This is a discrete-event stochastic object-oriented microscopic simulation model, to evaluate the operational performance of the toll plazas. Toll lane selection algorithm, lane-changing algorithm and modified versions of car following algorithms are integrated in to this model. The model output is the measures of effectiveness that are used to evaluate the performance of the toll plaza. An interesting result of this study is that for the toll plaza configurations simulated with manual payment lanes operating over capacity, total plaza queuing delay can be reduced by half, average queuing delay per vehicle can be reduced by more than 90s and the plaza

throughput can increase by more than 20 percent, if only 10 percent of the users switch to the electronic lanes. It was also shown that adding more dedicated ETC lanes prematurely can cause an increase in plaza queuing delay and a decrease in the throughput.

Klodzinski and Al-Deek (2002) analyzed various measures of effectiveness (MOE) that can be used to evaluate a toll facility for its level of service, including density, volume-to-capacity ratio and delay by developing a macroscopic methodology for measuring the level of service (LOS). He concluded that 85 percentile of the cumulative individual vehicular delay is the most credible measure of effectiveness for evaluating the level of service of a toll plaza based on the field research and the data analysis. Service time was examined to determine the level a driver begins to feel discomfort and a level of service hierarchy was established.

Ceballos and Curtis have analyzed multi-server queues to estimate the average waiting times and queue lengths at toll plazas for given arrival rates, number of servers and service rates. They have developed a mathematical model to analyze the toll plaza operation. The assumptions made in this model are Poisson arrival rates and Poisson service times. These models only approximate the performance and they do not fully reflect the realistic operation of a toll plaza.

2.2 Mathematical Models

Mierzcjewski et al (1991) analyzed the ETC technology and its potential application on Florida's Turnpike. They reviewed ETC technology, surveyed the attitudes and characteristics of Turnpike patrons and evaluated the specific application of ETC technology to Florida's Turnpike. They suggested that to handle year 2015 demand levels

with conventional plaza configuration requires 26 toll lanes. With the use of 10 percent, 30 percent and 50 percent ETC the number of toll lanes required at the same time would reduce to 22, 18 and 14 respectively.

Boronico and Siegel (1998) investigated capacity planning analysis for toll plaza operations. They developed a cost minimizing capacity planning model that minimized expected costs which include operating as well as user costs. Also optimal capacity levels were determined for both peak and off peak hours. This model suggested significant savings with the implementation of a decision support system that may be developed from the mathematical models developed through the efficient allocation of capacity utilizing manpower planning analysis. In this model both direct costs as well as user costs are included in the objective function, which is minimized. In this study, models for both the capacity planning and manpower planning are developed. Empirical results for the capacity planning model are presented. Optimal shift scheduling is obtained from the manpower planning model. Even though empirical results provide the current standards, what we need is the optimal standards. The solution obtained by the use of these two models provides an approximation but the combination of the two models into a large scale model would necessitate the use of heuristics and consequently provide an approximation to the optimal solution. Finally this investigation does not take into account general service rates which lack the practicality of real world. Even though the results in this study suggests potential benefits, the inclusion of the above mentioned considerations would increase the benefits through the implementation of a decision support models based on the analytical models.

Polus and Reshetnik (1997) developed a methodology for the development of a toll plaza design manual for estimation of the configuration of an urban toll plaza. This manual is meant for plaza where conventional methods of toll collection are combined with electronic methods. In this study a new concept called throughput equivalent volume was established and this concept was used to convert different volume combinations of various toll collection methods and different vehicle classes at a specific plaza into a common equivalent volume. This method made it possible to determine the number of lanes required for each toll collection type, which was done by dividing the sum of throughput volumes by the throughput capacity of the equivalent lane.

Zarrillo et al, (1997) showed that one possible solution to the optimal use of the existing toll plaza facilities is to efficiently manage the plaza's configuration pattern. He developed a mathematical model that closely describes the traffic operations at the toll plaza. This model describes the plaza performance and provides a methodology to calculate the queue length and delay at the toll plaza for a particular plaza configuration. He quoted that the analysis of the queue length and the delay for various plaza configurations can be used to maximize the convenience to the users. The model built by Zarrillo has some important considerations like the type of services available, the driver-types, the approach volume rates and the maximum processing service rates. This model also takes into account mixed lanes that provide more than one type of service and mixed lanes that provide service to more than one type of vehicle. The assumptions made to build this model are, the driver chooses the lane with the shortest queue length and once committed the drivers do not change the lanes. For building this model, the input to the model, the traffic approach volume rate has been assumed to be a continuous function of

time that was approximated by a series of uniform step functions. As a future research, Zarrillo recommends to use random arrival and processing rates, so that the approach volume and the processing rates are generated by frequency distributions rather than constant percentages of a uniform series of step functions. He also recommends the use of real data to build and validate the model.

Levinson and Chang (2003) examined the deployment of ETC and developed a model to maximize social welfare associated with a toll plaza. The payment choice model developed estimates the percentage of traffic using ETC as a function of delay, price and a fixed cost of acquiring the in-vehicle transponder. According to them delay is in turn dependent on the number of ETC and conventional manual toll collection lanes. This model recommended the pace at which ETC should be deployed at California's Carquinez Bridge in terms of the number of dedicated ETC lanes and the appropriate ETC discount.

Lin and Lin (2001) modeled traffic delays to improve the traffic operations at Northern New York border crossings. This study identified the processing time characteristics and the delay characteristics at the above site and developed a delay model for planning applications. For this study because of unavailability of the data and because of the difficulty in data collection, a simulation model, TPSIM was used to examine the nature of delays and also to generate data to develop the delay model.

Boxma and Takine (2003) derived the joint queue length distribution in the M/G/1 queue with several classes of customers and FIFO service discipline. The model considered is a single server queue with K customer classes, served without priorities and

according to a FIFO discipline. The customers of each class arrive according to a Poisson process.

Klodzinski and Al-Deek (2004) also analyzed the implementation of open road tolling. In this analysis, two ETC lanes were added at a mainline toll plaza in each of the direction and this improved operating condition during the rush hour. The analysis showed a reduction in the average delay per vehicle of 7 seconds for the ACM lanes and 8 seconds for the manual cash lanes. The average delay per lane was reduced by over 4560 seconds for the ACM lanes and 3360 seconds for the manual lanes. The implementation of open road tolling increased the capacity by 43.8 percent.

Even though much work has been done in terms of reducing the waiting time at the toll plazas, by using simulation and developing mathematical models, the fact that these models either use deterministic arrival and service rates is not quite appropriate. The arrival rates and the service times at the toll plazas are stochastic in nature and not deterministic. Thus the research conducted uses Poisson arrival process and general service process and determines the optimal lane configuration with a help of a mathematical model that was developed to find the same.

CHAPTER 3

FORMULATION

This chapter describes the formulation of the mathematical model developed to evaluate the optimal plaza configuration. The notation used is explained and followed by the mathematical model developed.

3.1 Notation

The nomenclature below summarizes the various symbols used in this chapter.

$t =$ Time index

$\lambda_t =$ Average vehicle arrival rate to the toll plaza at time t

$\lambda_t^k =$ Average vehicle arrival rate to the lane-type k at time t, for $k = U, V, W, X, Y, Z$

$p^i =$ Percentage of driver-type i, for $i = M, E, A, T$

$n_t =$ Total number of available lanes at a toll plaza at time t

$n_t^k =$ Number of available lanes of lane-type k at time t

$\mu^k =$ Average service rate for one lane of lane-type k, for $k = U, V, W, X, Y, Z$

$\sigma^K =$ Standard deviation of service time of lane-type k, for $k = U, V, W, X, Y, Z$

Driver types:

1. M : A car driver who requires change or a receipt.
2. E : A car driver who has an ETC pass.
3. A : A car driver with the exact change at hand
4. T : A truck driver.

Lane types:

1. U: A dedicated lane type which provides manual service by a toll collector; service is provided to both cars and trucks.
2. V: A dedicated lane type which provides ACM service; service is provided only to cars.
3. W: A dedicated lane type which provides ETC service; service is provided to both cars and trucks.
4. X: A dedicated lane type for trucks which provides manual service by a toll collector.
5. Y: A mixed lane type which provides both manual toll collection service and ETC service; service is provided only to cars.
6. Z: A mixed lane type which provides both ACM service and ETC service; service is provided only to cars.

δ^{MU} = The proportion of the manual drivers using the U lane

δ^{MY} = The proportion of the manual drivers using the Y lane

δ^{AV} = The proportion of the automatic drivers using the V lane

δ^{AZ} = The proportion of the automatic drivers using the Z lane

δ^{EW} = The proportion of the electronic drivers using the W lane

δ^{EY} = The proportion of the electronic drivers using the Y lane

δ^{EZ} = The proportion of the electronic drivers using the Z lane

δ^{TU} = The proportion of the truck drivers using the U lane

δ^{TW} = The proportion of the truck drivers using the W lane

δ^{TX} = The proportion of the truck drivers using the X lane

l_t^k = Average queue length at time t, for the lane-type k, for $k = U,V,W,X,Y,Z$

l_t = Average total queue length of the toll plaza at time t

w_t^k = Average waiting time at time t, for the lane-type k, for $k = U,V,W,X,Y,Z$

w_t = Average total waiting time of the toll plaza at time t

3.2 Mathematical Model

The toll plaza system can be viewed as a queuing model with random arrival and general service distribution. Also, the arrival rate is a function of time and the arrival rate at a particular minute in any of the lanes is independent of the arrival rate in the previous minute in the respective lanes. The service times follow a general distribution.

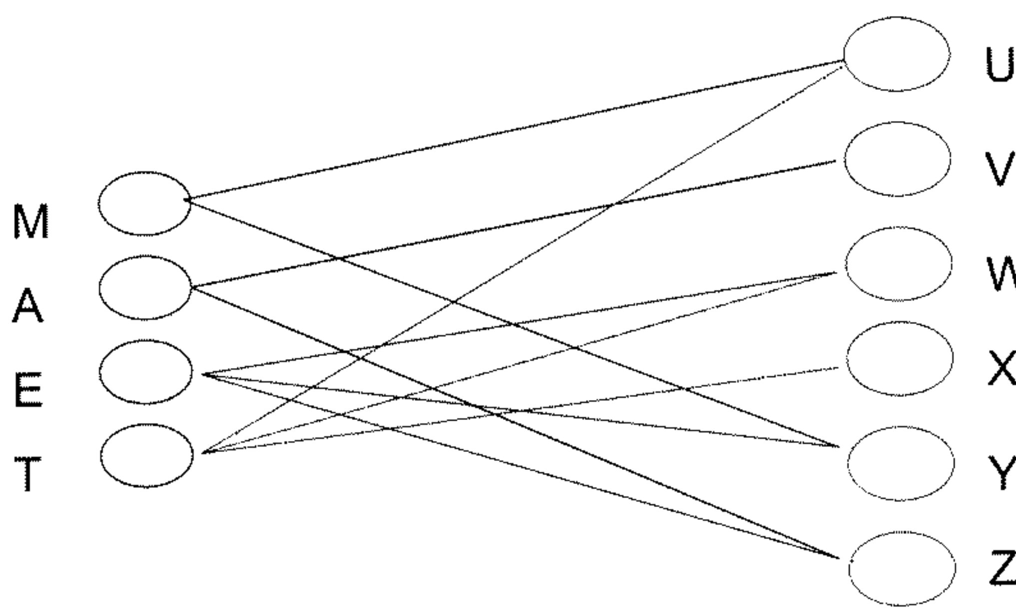


Figure 1: Driver-types and Lane-types

Input Process

During any time of a day, the arrival process and the traffic volume rate are random. The arrival process is Poisson process because of the following reasons: 1) the number of arrivals at time 0 is 0, 2) the arrival rates are independent, and 3) the arrival process has a mean arrival rate of λ_t in the time period $(t, t+1)$. p^i is the percentage of the i^{th} driver type. n_t^k is the number of toll booths of lane type k at time t. The mean arrival rate of the vehicles to each lane-type k is λ_t^k at time t

$$\text{Total arrival rate to the toll plaza, } \lambda_t = \sum_k \lambda_t^k n_t^k$$

$$\text{Arrival rate to the U lane type, } \lambda_t^U n_t^U = \lambda_t p^M \delta^{MU} + \lambda_t p^T \delta^{TU}$$

$$\text{Arrival rate to the V lane type, } \lambda_t^V n_t^V = \lambda_t p^A \delta^{AV}$$

$$\text{Arrival rate to the W lane type, } \lambda_t^W n_t^W = \lambda_t p^E \delta^{EW} + \lambda_t p^T \delta^{TW}$$

$$\text{Arrival rate to the X lane type, } \lambda_t^X n_t^X = \lambda_t p^T \delta^{TX}$$

$$\text{Arrival rate to the Y lane type, } \lambda_t^Y n_t^Y = \lambda_t p^M \delta^{MY} + \lambda_t p^E \delta^{EY}$$

$$\text{Arrival rate to the Z lane type, } \lambda_t^Z n_t^Z = \lambda_t p^A \delta^{AZ} + \lambda_t p^E \delta^{EZ}$$

The arrival rates to each of the different lane-types can be calculated using the above formulae. The arrival rate to one lane of a particular lane-type k can be controlled by changing the number of lanes of that lane-type, n_t^k.

Service Process

Service time is the length of the payment process at a tollbooth. Service time does not include the waiting time in the queue. Service process cannot be a Poisson process

because if it was to be a Poisson process then the amount of time the driver would be waiting in the queue would be immaterial of the amount of time he already waited in the queue, i.e. the amount of waiting time should have been memory less, this is not the case because if the driver has been waiting for sometime to get served, it obviously decreased his waiting time in the queue from when he joined the queue. Also the service process cannot be deterministic because, consider the case of a manual lane, the service times not only depend on the toll attendant's efficiency but also depends on the driver, depending on whether he is paying the exact amount or if he needs change.

The actual service time may be influenced by a number of factors, the main being the lane-type, the experience of a toll collector in case manual service is provided in that particular lane, the number of coins to be processed by the automatic coin machine, the type of vehicle and the efficiency of the ETC technology. The mean service rate of the vehicles during the time t at a lane-type k is μ_t^k. The standard deviation of the service time during time t at a lane-type k is σ_t^k. The mean service rate and the standard deviation of the service times are calculated from the time study that was done to find the service times. The time study included noting down the arrival and departure times of the vehicles in various lane-types at the toll plaza. This study was done at Okeechobee, SR 112 and SW 8th Street toll plazas.

Queue Length and Waiting Time

The system being considered is a Non-Homogeneous Poisson queuing model. After λ_t^k for $k = U, V, W, X, Y, Z$ are obtained, the queue length, l_t^k in one lane of lane-

type k during time t is calculated. Each lane is considered as a $M/G/1$ queuing system with the following assumptions and the *Pollaczek-Khintchine* formula is applied.

Assumptions

1. The arrival process to a toll plaza is a Poisson process with mean arrival rate λ_t during time t. This subsequently implies that the arrival process to each lane of lane-type k is also a Poisson process with the mean arrival rate with mean λ_t^k during the time t.

2. Service time distribution at each tollbooth of lane-type k is stationary over time with the mean $1/\mu_k$ and the standard deviation σ_k.

$$l_t^k = \frac{\left(\lambda_t^k \sigma^k\right)^2 + \left(\dfrac{\lambda_t^k}{\mu_k}\right)^2}{2\left(1 - \dfrac{\lambda_t^k}{\mu^k}\right)}$$

$$= \frac{\left(\lambda_t^k\right)^2 \left(\sigma^k\right)^2 + \dfrac{\left(\lambda_t^k\right)^2}{\left(\mu^k\right)^2}}{2\left(1 - \dfrac{\lambda_t^k}{\mu^k}\right)}$$

$$= \frac{\left(\lambda_t^k\right)^2 \left(\left(\sigma^k\right)^2 + \dfrac{1}{\left(\mu^k\right)^2}\right)}{2\left(1 - \dfrac{\lambda_t^k}{\mu^k}\right)}$$

$$= \frac{\left(\frac{\lambda_t^k}{\mu^k}\right)^2 \left((\sigma^k)^2 (\mu^k)^2 + 1\right)}{\frac{2}{\mu^k}\left(\mu^k - \lambda_t^k\right)}$$

Using the above formula, the queue length in each of the toll lanes is calculated. The total queue length is then calculated by summing up all the individual queue lengths.

$$l_t = \sum l_t^k n_t^k$$

Since service rate is different for each lane-type, the number of vehicles waiting in the queue may not be a good measure of criteria for performance comparison. Instead, we are interested in w_t^k, the waiting time of the vehicles in the queue in one lane of lane-type k during time t. By the Little's law

$$w_t^k = \frac{l_t^k}{\lambda_t^k} = \frac{\lambda_t^k((\mu^k \sigma^k)^2 + 1)}{2\mu^k(\mu^k - \lambda_t^k)}$$

Now the total waiting time in all the lanes of all the lane-types during time t at the toll plaza is calculated using

$$w_t = \sum_k w_t^k n_t^k = \sum_k \frac{\lambda_t^k n_t^k((\mu^k \sigma^k)^2 + 1)}{2\mu^k(\mu^k - \lambda_t^k)}$$

The waiting time or the delay of the toll plaza is a better measure of effectiveness because delay has the most effect on the driver's level of inconvenience and also it is a direct result of the traffic conditions. Even when the service rate is high we might have a very long queue depending on the arrival rates, and hence delay is a much better measure of effectiveness than queue length of the toll plaza. The configuration that yields the minimum total delay is the optimal plaza configuration.

Nonlinear Integer Programming Model

The objective of this research is to find an optimal configuration of the toll plaza configuration over time t so that the total waiting time of the toll plaza is minimized i.e., Optimal configuration of the lanes such that w_t is minimized. The optimal configuration, the number of lanes to be open for each lane-type, in order to minimize the total waiting time at the toll plaza during time t can be determined by using the following Non-linear integer programming problem

Minimize $$w_t = \sum_k \frac{\lambda_t^k n_t^k ((\mu^k \sigma^k)^2 + 1)}{2\mu^k (\mu^k - \lambda_t^k)}$$

Subject to: $$\lambda_t = \sum_k \lambda_t^k n_t^k$$

$$\lambda_t^U n_t^U = \lambda_t p^M \delta^{MU} + \lambda_t p^T \delta^{TU}$$

$$\lambda_t^V n_t^V = \lambda_t p^A \delta^{AV}$$

$$\lambda_t^W n_t^W = \lambda_t p^E \delta^{EW} + \lambda_t p^T \delta^{TW}$$

$$\lambda_t^X n_t^X = \lambda_t p^T \delta^{TX}$$

$$\lambda_t^Y n_t^Y = \lambda_t p^M \delta^{MY} + \lambda_t p^E \delta^{EY}$$

$$\lambda_t^Z n_t^Z = \lambda_t p^A \delta^{AZ} + \lambda_t p^E \delta^{EZ}$$

$$n_t = \sum_k n_t^k$$

$$\mu^k > \lambda_t^k$$

$$n_t^k \geq 0$$

The variable in this problem is the number of lanes of each lane-type at the time t, n_t^k. From the objective function, it is clear that this problem is a non-linear programming problem. Also the variable belongs to a set of integers. Therefore the problem under consideration is a Non-Linear Integer programming problem. Even though this is a Non-Linear Integer programming problem, because the range of n_t is comparatively small and also because the total number of lanes per direction of a plaza is a small number (no more than 20), this problem can be easily solved.

CHAPTER 4

ANALYSIS AND RESULTS

4.1 Data Collection

The arrival rates were provided by the FDOT for the Okeechobee and Cypress Creek Toll plazas for five different days. The data provided were for three weekdays and weekend. The mean of the weekday traffic arrival rate is being taken into account for the analysis. A time study was done at three different plazas in order to obtain the service times for the available types of toll collection except for the truck only lanes. Arrival rates and the service times for the truck only lanes have been assumed. The time study was done at the Okeechobee Toll Plaza on the Florida Turnpike, Miami Airport Expressway (SR 112) and SW 8th Street Toll Plaza. The times the vehicles leave the toll plaza after paying toll have been collected at three different plazas. The study was done at three different plazas because not all lane types were available at a single plaza. The data was collected during the morning peak hour, in between 7:00 AM and 9:00 AM at all the three plazas on three different days. One hundred observations were made for each lane type at each of the plaza. To do the analysis mean service rate and the standard deviation of the service time were calculated from the time study.

The times the vehicles leave the toll plaza are noted by using a stop watch. The instant the vehicle stops to pay the toll the stop watch was started and the instant the vehicles leaves a reference line after paying the toll, the stop watch was stopped and the reading was noted. For the next vehicle, the instant the vehicle stops at the toll booth, without resetting the stop watch; it was started and once the vehicle under consideration

crosses the reference mark, it was stopped and the time of the vehicle leaving the toll plaza was noted.

At the Okeechobee Toll Plaza departure times for the manual lane, dedicated ETC lane and the mixed ETC with manual lane are collected. At the SR 112 Plaza, the departure times for the manual lane, ACM lane, ETC lane and mixed ETC manual lane are collected. Finally at the SW 8th Street Toll Plaza, the departure times for the mixed ETC ACM lane are collected.

The driver type percentages were taken from Klodzinski, (2002). The proportions δ^{ik} had to be assumed because of the unavailability of data for the truck only lanes.

4.2 Analysis

The analysis to obtain the optimal lane configuration included the following steps:

1. All the possible lane-type configurations were determined for the various lane types and the total number of lanes considered.
2. The lane configurations that had no dedicated lanes of at least one of the manual, electronic and automatic lane-types were removed from the feasible set.
3. The configurations that cannot serve at least one of the various driver types were removed from the feasible set. For example, the configuration that includes only the lane-types V, W, X, Z cannot serve the cars which pay manually; such configurations have been removed from the feasible set.
4. Service rates for each of the lane-types were calculated.
5. Arrival rates of one lane for all the lane-types were calculated over time.

6. The configurations that result in the arrival rate of a particular lane type greater than the service rate for that lane-type, have been removed from the feasible set because when the arrival rate is more than the service rate, the queue will explode.
7. Queue length and waiting time at the various lane-types were calculated for one lane configuration
8. Total queue length and waiting time of the toll plaza were calculated for that configuration.
9. Above steps were repeated for all the feasible combinations to obtain the optimal configuration.

The mathematical model developed could not be validated because of the unavailability of the waiting time by the hour. The waiting time by the hour was not available nor was it easy to collect the waiting time of each vehicle over the entire hour and for the entire day. To estimate the waiting time of a vehicle, it is necessary to know when it exactly entered the queue and started waiting in the queue to get served and when exactly it was served and this time study to estimate the waiting times would be rather difficult to do. The analysis was implemented with the help of a program written in JAVA. The compiled and executed program lists out the various feasible combinations of lane-types for a given total number of lanes of a toll plaza, calculates the arrival rates of the lane-types, calculates the arrival rates of one lane of each lane-type k, computes the individual queue length and waiting time of each lane-type and also the total queue length and total waiting time of the toll plaza.

After all the possible configurations were found, and the respective waiting times were calculated, the frequency distribution graph was generated to determine the how

many configurations result in how much waiting time. From this graph the minimum, the maximum and the average waiting times were determined. Then the minimum and the average waiting times were compared to determine the potential improvement in the waiting time for a particular time period. Similarly the maximum waiting time and the minimum waiting time were compared to find the improvement in the worst case scenario. This comparison was done for all the hours of the day.

4.3 Results

The program written to do the analysis was executed to find the optimal plaza configurations for 8 lane and 15 lane plaza. The program was run for each hour of the day for both 8 lanes and 15 lanes to find optimal configuration. With this program the various feasible combinations were generated and then the corresponding queue length and waiting time were calculated. The waiting times were then analyzed to find the optimal configuration. The following is the waiting time frequency distribution chart that shows the number of configurations that result in the various waiting times.

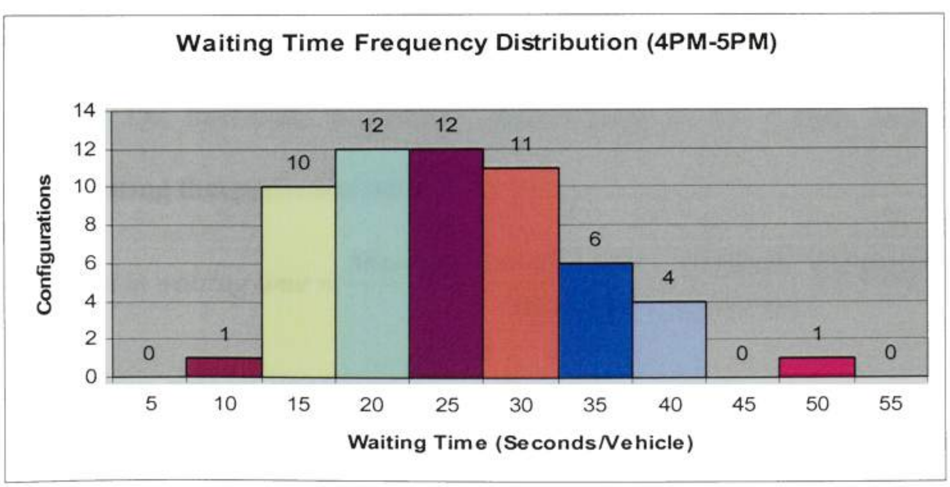


Figure 2: Waiting Time Frequency Distribution for 8 lane plaza

This is the waiting time frequency distribution for the 8 lane toll plaza during the peak hour 4PM-5PM. This chart explains how many different lane configurations result in the various waiting times. For example the number of configurations that result in a waiting time in the range of 20 seconds to 25 seconds is 12. It can be clearly seen that there is a large difference between the minimum waiting time and the maximum waiting time. The minimum waiting time is 8.43 seconds and the average waiting time is 22.84 seconds. The percentage improvement in the waiting time when the lane configuration is changed from the configuration that results in the average waiting time to the configuration that results in the minimum waiting time is calculated using the below formula. The percentage improvement in this case is 63.11 percent.

$$\%\,improvement\ in\ waiting\ time = \frac{Average\ waiting\ time - Minimum\ waiting\ time}{Average\ waiting\ time} * 100$$

The maximum waiting time for the 8 lane configuration during the same hour is 46.64 seconds. The percentage improvement in the waiting time when the lane configuration is changed from the configuration that results in the maximum waiting time to the configuration that results in the minimum waiting time is calculated using the below formula. The maximum percentage improvement in the waiting time could be 81.93 percent during that particular hour.

$$\%\,improvement\ in\ waiting\ time = \frac{Maximum\ waiting\ time - Minimum\ waiting\ time}{Maximum\ waiting\ time} * 100$$

The various percent improvements in the waiting time by the hour for the 8 lane plaza are as shown in the following table. From the table it can be seen that the maximum percent improvement and the average percent improvement were the highest during the

morning rush hour 6:00 AM-7:00 AM, the maximum percent improvement being 96.05 percent and the average percent improvement being 91.37 percent and these improvements are significant. The decrease in waiting time is indirectly a decrease in cost and hence improves the economy. The following table shows the minimum, average, maximum waiting times and the average and maximum percent improvements in the waiting times for all the 24 hours in a day for the 8 lane plaza.

Table 1: Percentage Decrease in the Waiting Times

	Waiting Time			% Potential Improvement	
Time of the day	Minimum	Average	Maximum	Average	Maximum
MidNight-1AM	0.73	1.63	2.98	55.15	75.45
1AM-2AM	0.45	0.98	1.77	53.98	74.45
2AM-3AM	0.36	0.97	1.39	62.99	74.14
3AM-4AM	0.36	0.77	1.39	53.38	74.14
4AM-5AM	0.45	0.97	1.77	53.51	74.45
5AM-6AM	1.12	2.61	4.82	57.20	76.84
6AM-7AM	5.09	59.00	128.70	91.37	96.05
7AM-8AM	13.13	76.91	176.30	82.93	92.55
8AM-9AM	12.15	56.03	125.70	78.32	90.33
9AM-10AM	8.07	21.12	42.74	61.80	81.12
10AM-11AM	4.54	28.48	60.05	84.04	92.43
11AM-Noon	3.65	14.45	29.16	74.76	87.49
Noon-1PM	3.77	15.65	31.75	75.90	88.12
1PM-2PM	3.65	14.45	29.16	74.76	87.49
2PM-3PM	3.77	15.65	31.75	75.90	88.12
3PM-4PM	5.81	12.62	24.19	53.99	75.99
4PM-5PM	8.43	22.84	46.64	63.11	81.93
5PM-6PM	9.37	28.08	58.71	66.65	84.05
6PM-7PM	6.41	14.58	28.34	56.02	77.37
7PM-8PM	3.40	12.44	24.84	72.64	86.30
8PM-9PM	2.15	5.86	11.18	63.36	80.80
9PM-10PM	1.62	4.06	7.62	60.10	78.73
10PM-11PM	1.42	3.44	6.42	58.84	77.95
11PM-MidNight	1.93	5.09	9.64	62.02	79.95

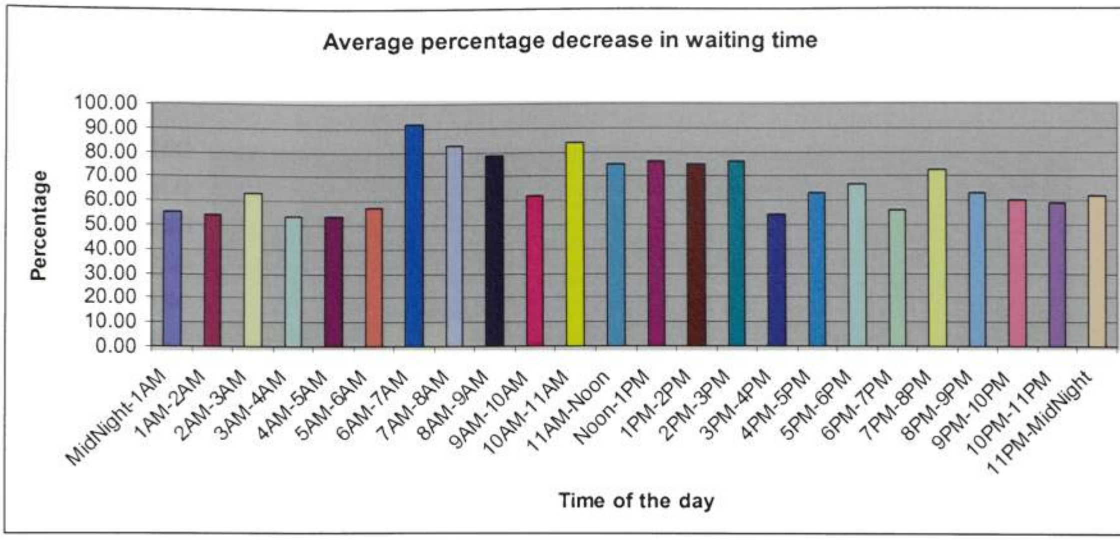


Figure 3: Average percentage decrease in waiting times

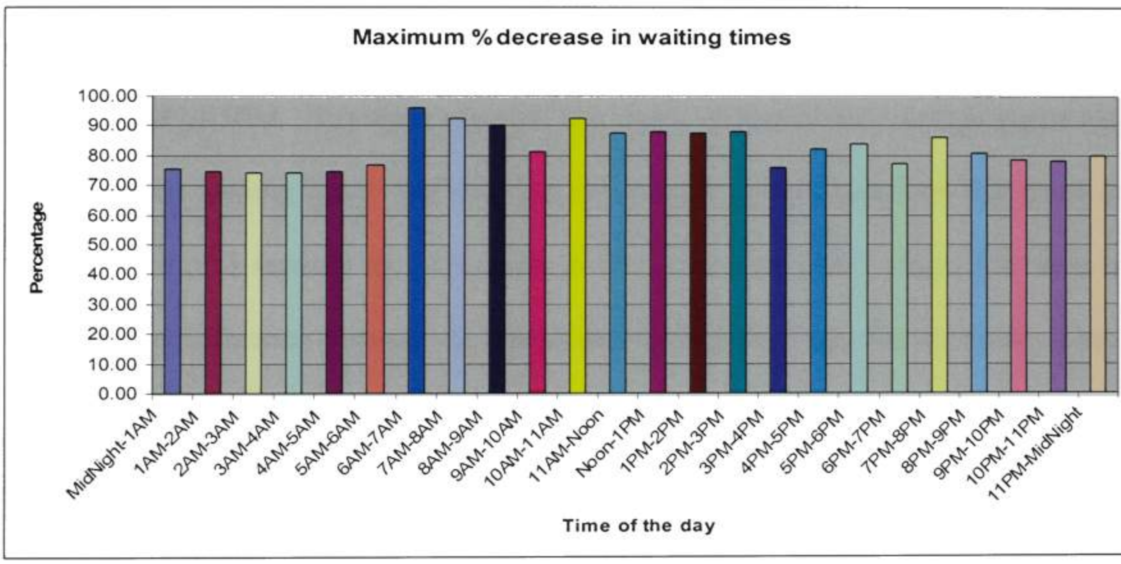


Figure 4: Maximum percentage decrease in waiting times

The above two charts show the percent improvement in the average waiting times and the maximum waiting times over the entire day by the hour.

For the 15 lane plaza configuration, the following is the waiting time frequency distribution chart which tells how many different configurations result in the various waiting times.

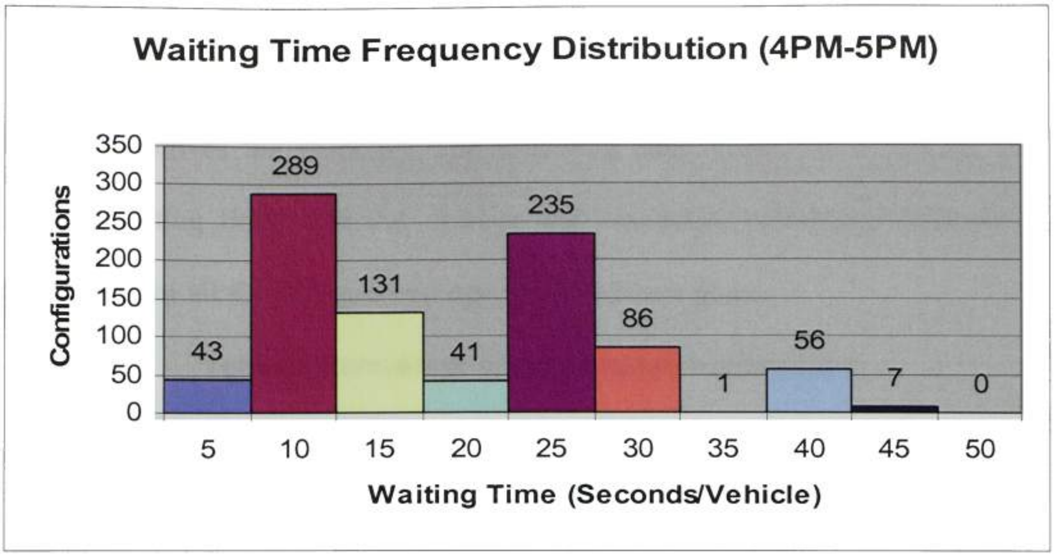


Figure 5: Waiting time frequency distribution for 15 lane plaza

This is the waiting time frequency distribution for the 15 lane toll plaza during the peak hour 4PM-5PM. This chart explains how many different lane configurations result in the various waiting times. For example the number of configurations that result in a waiting time of 5 seconds to 10 seconds is 289. It can be clearly seen that there is a considerable difference between the minimum waiting time and the maximum waiting time. The minimum waiting time is 3.78 seconds; the average waiting time is 16.67 seconds and the maximum waiting time is 41.42 seconds. This means that the average improvement in the waiting time could be 77.32 percent and the maximum improvement in the waiting time could be 90.87 percent during that particular hour. The peak hour 4PM-5PM was taken as an example to illustrate the percentage decrease in the waiting times. The various decreases in waiting time percentages by the hour for the 15 lane plaza are as shown in the following table. From the table it can be seen that the maximum and the average improvements in the waiting times were the largest during the morning rush hour 7:00 AM-8:00 AM, the maximum improvement in waiting time being 96.37 percent

and the average improvement in waiting time being 89.10 percent and these improvements are significant. The decrease in waiting time is indirectly a decrease in cost and hence improves the economy. The following table shows the minimum, average, maximum waiting times and the average and maximum percentage decrease in the waiting times for all the 24 hours in a day for the 15 lane plaza.

Table 2: Percentage decrease in the waiting times

	Waiting Time			% Potential Improvement	
Time of the day	Minimum	Average	Maximum	Average	Maximum
MidNight-1AM	0.39	0.99	1.95	61.01	80.15
1AM-2AM	0.24	0.61	1.95	60.66	87.66
2AM-3AM	0.19	0.48	0.94	60.21	79.57
3AM-4AM	0.19	0.48	0.94	60.21	79.57
4AM-5AM	0.24	0.61	1.18	60.66	79.66
5AM-6AM	0.59	1.54	3.04	62.01	80.74
6AM-7AM	2.45	8.18	18.19	70.07	86.54
7AM-8AM	5.38	49.31	148.10	89.10	96.37
8AM-9AM	5.07	37.18	107.00	86.36	95.26
9AM-10AM	3.65	15.53	38.10	76.50	90.42
10AM-11AM	2.21	7.12	15.56	68.95	85.79
11AM-Noon	1.81	5.5	11.69	67.09	84.52
Noon-1PM	1.86	5.71	12.19	67.36	84.71
1PM-2PM	1.81	5.49	11.69	67.07	84.53
2PM-3PM	1.86	5.71	12.19	67.36	84.71
3PM-4PM	2.75	9.69	22.03	71.61	87.51
4PM-5PM	3.78	16.67	41.42	77.32	90.87
5PM-6PM	4.13	20.08	51.64	79.45	92.01
6PM-7PM	3.00	11.07	25.67	72.90	88.31
7PM-8PM	1.70	5.08	10.72	66.63	84.19
8PM-9PM	1.10	3.05	6.22	63.97	82.33
9PM-10PM	0.84	2.26	4.54	62.92	81.55
10PM-11PM	0.74	1.97	3.92	62.64	81.22
11PM-MidNight	0.99	2.73	5.53	63.59	82.01

The following two charts show the percentage decrease in the average and the maximum percentage decrease waiting times over the entire day by the hour.

Figure 6: Average percentage decrease in waiting times

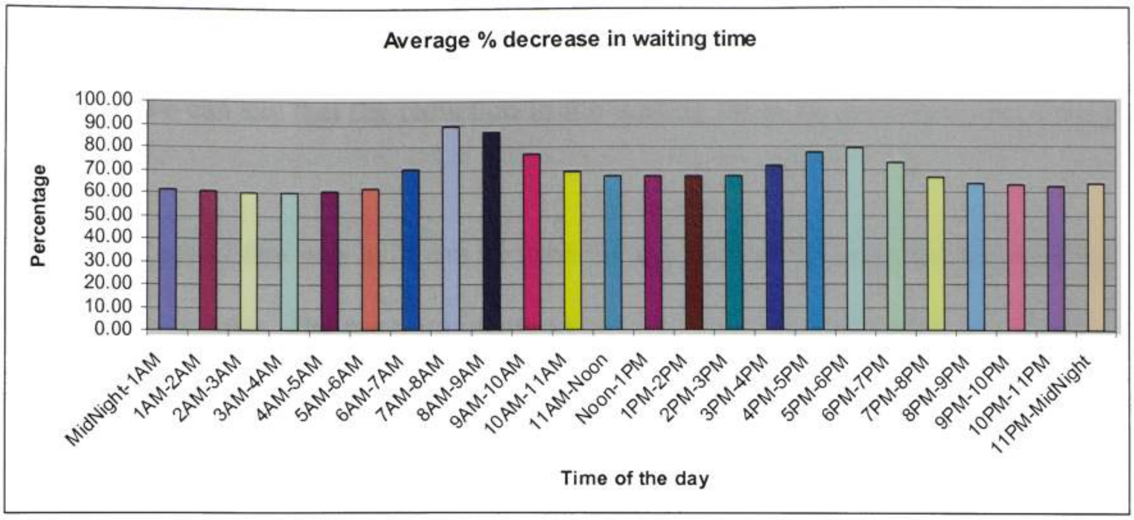
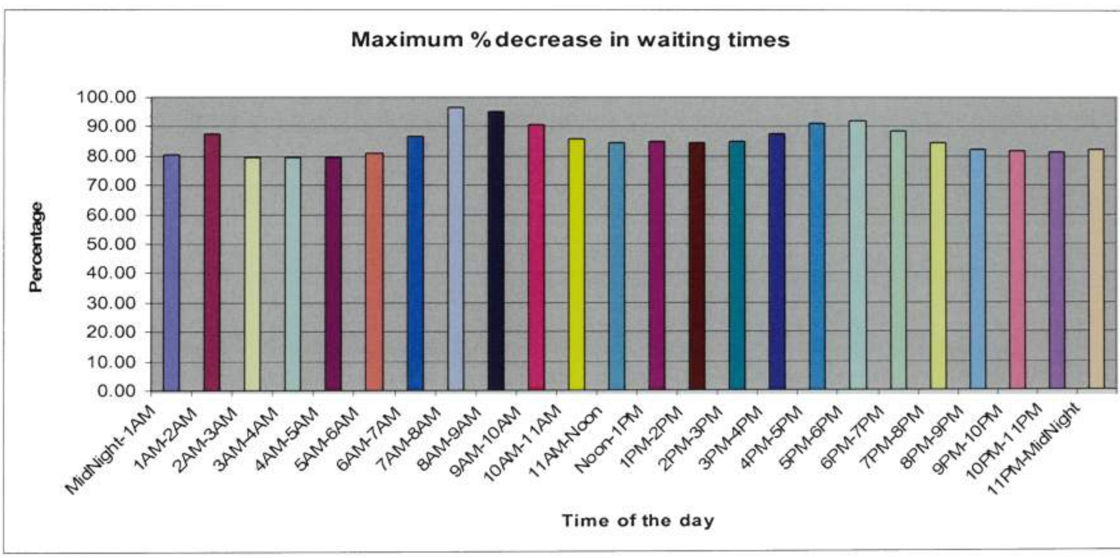


Figure 7: Maximum percentage decrease in waiting times



Thus by changing the lane configuration to a desirable configuration the total waiting time of the toll plaza can be considerably decreased, which saves a lot of money, reduces pollution and along with the other uses.

It can be seen that the average waiting times for the 15 lane toll plaza is much lesser than the average waiting times for the 8 lane toll plaza. This is because of the increase in the number of lanes. In the future traffic increases and as a result the waiting

times will also increase. Thus even though the average waiting times are less for this study, in reality the average waiting times can be as high as the maximum waiting times and hence we can say that the reduction in the waiting times for a 15 lane toll plaza can be greater.

The model that was built can be used for any number of lanes and any number of lane and driver types. The program incorporating the mathematical model developed was written in such a way that once it is provided with the necessary inputs it will give the optimal lane configuration for any number of lanes and any number of lane types.

CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

Traffic congestion has emerged to be the number one concern not just to the public but also to the transportation engineers (Zarrillo et. al, 1997). Traffic congestion is being experienced daily by many travelers both on the inner city roads as well as on the highways. The delay and wasted fuel from being stuck in traffic costs over $40 billions a year. Also one of the effects of traffic congestion is the increase in air pollution ("Curbing Gridlock", Special report by TRB)

Tolls are one of the most effective means of financing and maintaining highways. Hence, the toll plazas cannot be eliminated. Even though electronic toll collection has been implemented in most toll plazas, we still see traffic congestion at these plazas. Also as time passes there will be an increase in the traffic. Due to this increase in traffic, congestion and the delay caused due to congestion also increase. The current methods of toll collection at the toll plazas are also one of the reasons for the congestion and the delay. Even though these are efficient methods of toll collection, still we see constant traffic congestions and delays at the toll plazas. This is because of the stochastic nature of the traffic arrival rates, which depends on the time of the day and also the day of the week. The other obvious reason for the congestion at the toll plazas considering the fact that the arrivals are stochastic in nature is the static lane configuration all through out the day.

The best way to reduce the congestion problem would be by changing all the lanes to electronic lanes, which is not feasible at this point in time. The alternative

approach to reduce the problem of congestion is to dynamically change the plaza configuration. The optimal plaza lane configuration should have the minimum queue length, minimum waiting time for each customer, minimum total delay of the system and maximum throughput for the plaza.

The optimal plaza lane configuration for toll plazas have been evaluated previously (Zarrillo et. al, 1997) but for constant arrival and service rates. In real world, neither the arrival rate nor the service rate is a constant. Hence, we need to take into account the stochastic nature of the arrival and service rates to find the optimal lane configuration. This optimum lane configuration helps the management of the toll plazas to run an efficient and a user-satisfied toll collection system until all users of the facility become ETC patrons. If implemented, this system can do the following:

- Reduce the waiting time associated with each vehicle in the system and hence the total waiting time of the system (which is the measure of performance of a toll plaza).
- Traffic flow can be balanced and congestion decreases.
- The queue length can be decreased and hence the waiting time.
- Because of the above reasons the economy can be improved and also the other side effects of congestion and long queues, mobile emissions and hence pollution can be reduced.

As discussed above changing all lanes to ETC, is not a feasible solution at this point in time until all the patrons are ETC enabled. Hence, we should have conventional toll collection types along with the electronic toll collection and the facilities have to manage the lane configurations in such a way which maximizes the throughput, reduces waiting

time and queue lengths at the toll plazas. Thus, optimal lane configurations have to be determined to maximize the level of service at toll plazas.

The specific objectives to achieve this goal included:

1. Designing a case study for toll plaza operations and collecting data for the arrival rates, service times of the toll transactions and analyze the data.
2. Developing a mathematical model to evaluate the delay at toll plazas and identify the plaza configuration with the minimum delay.

The methodology that was used in the research is

1. Queuing theory was used to determine the model which represents the toll plaza operations.
2. Nonlinear integer programming was used to develop the mathematical model.

The analysis was carried out to determine the optimal configuration, for this arrival rates were obtained from the FDOT, the service rates were calculated from the time study done to obtain the service times. A program in JAVA was written to obtain the feasible set of lane configurations and then queue length and waiting times for these configurations were calculated The results showed that the maximum decrease in the waiting time could be obtained during the rush hour and the waiting time can be decreased up to 96.05 percent in the case of the 8 lane toll plaza configuration and 96.37 percent in the case of the 15 lane toll plaza configuration.

In this research, the waiting time at the toll plazas was reduced by considering the toll plaza operations as M/G/1 model with stochastic arrival and service processes, which represented the realistic arrival and service processes.

Thus, this work has added to the body of the research about reducing the waiting times at toll plazas by developing a non-linear integer programming model. Future research may involve the following:

1. Evaluating the optimal lane configuration taking in to account all the various possible lane-types.
2. Calculating the cost savings with the implementation of the optimal toll plaza configuration.
3. Evaluating the reduction in the air pollution.
4. Implementing any new lane-types that have been proposed and evaluating the optimal configuration.
5. Calculating the increased capacity of the toll plaza with the optimal lane configuration.

In conclusion, even though, the study addressed the reduction in waiting time at the toll plazas with stochastic arrival and service processes and an improvement of over 96 percent was noticed, yet the above future research can be done

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APPENDIX

Table 3: Mean Arrival Rates by the Hour

Hour	M	A	E	M/E	A/E	Total
1	140	161	167	6	0	474
2	70	139	98	3	0	310
3	43	126	62	3	0	234
4	37	139	73	1	0	250
5	51	141	118	6	0	316
6	160	161	416	8	0	745
7	598	200	1917	32	22	2769
8	1051	358	3393	137	397	5336
9	1052	334	3211	113	374	5084
10	920	306	2573	69	44	3912
11	715	197	1574	18	19	2523
12	609	182	1255	23	13	2082
13	650	162	1315	20	7	2154
14	674	160	1253	19	13	2119
15	694	185	1267	21	12	2179
16	920	213	1866	26	25	3050
17	1129	303	2510	34	38	4014
18	1089	310	2818	36	63	4316
19	950	219	2087	31	28	3315
20	630	164	1142	17	8	1961
21	446	129	724	12	4	1315
22	354	99	547	11	3	1014
23	304	138	430	7	0	879
24	214	139	285	9	0	647
Total	13500	4665	31101	662	1070	

Table 4: Service Times

Observation Number	M	A	E	M/E	A/E
1	3.79	9.41	0.21	4.92	2.9
2	1.11	4.91	1.91	4.1	4.6
3	0.86	2.37	1.17	2.85	4.1
4	2.26	3.05	0.21	1.97	3.9
5	2.12	2.16	0.27	4.79	3.3
6	2.73	3.29	0.22	2.53	2.2
7	21.96	1.84	0.43	11.1	2.7
8	3.04	2.73	0.67	6.92	0.8
9	1.42	1.54	0.42	1.22	1.9
10	2.37	0.62	0.89	4.02	1.7
11	2.41	3.29	0.35	0.54	15
12	4.68	4.62	0.24	1.41	4.8
13	2.25	2.98	0.15	2.04	4.6
14	2.62	6.22	0.21	2.34	2.8
15	2.1	3.48	0.22	2.1	3.5
16	13.72	3.7	0.33	1.78	2.6
17	4.5	18.3	0.58	17.94	4.6
18	1.4	5.55	0.47	1.54	4.5
19	4.67	5.34	0.28	20.59	3
20	5.29	3.99	0.17	1.75	6.4
21	15.78	5.17	0.22	4.91	4.9
22	1.85	8.85	1.53	24.59	3.3
23	1.28	6.05	1.1	11.67	4.3
24	4.5	4.35	0.15	1.19	3.4
25	1.48	4.11	0.3	2.99	3.1
26	17.1	6.41	0.25	0.64	7.4
27	3.12	4.74	0.28	1.04	4.4
28	2.25	5.53	0.14	0.56	2.6
29	1.09	2.22	0.41	0.72	6.8
30	2.47	1.68	0.17	1.85	2.7
31	2.68	5.07	0.19	4.59	2.8
32	12.42	36.97	0.47	0.68	3.7
33	2.59	4.22	0.28	23.1	5.2
34	2.6	22.41	0.48	1.23	2.3
35	1.88	2.84	0.29	33.85	2.2
36	13.34	3.93	0.55	20.23	5
37	1.22	6.57	0.24	24.25	12.4
38	0.94	3.24	0.79	44.05	3.5
39	5.57	4.03	0.36	3.43	2.2
40	6.98	15.23	0.54	2.28	2.7
41	4.86	3.73	0.41	1.73	9.4

	42	2.41	4.68	0.41	10.34	4.4
	43	1.24	5.29	0.29	0.92	5
	44	5.87	5.07	0.47	0.43	2.1
	45	2.47	4.68	0.28	8.56	3.1
	46	2.24	3.91	0.37	29.07	3.5
	47	2.61	4.85	0.55	3.41	4.7
	48	3.29	5.67	0.98	2.34	5
	49	3.78	6.4	0.63	3.1	5.1
	50	2.18	5.47	0.33	2.49	4.9
	51	2.05	4.18	0.62	7.52	5
	52	7.93	6.74	0.44	3.65	4
	53	1.47	3.92	0.78	50.35	2.4
	54	3.72	2.97	0.33	4.84	2.6
	55	1.79	1.97	0.48	0.85	1.9
	56	5.09	2.24	0.18	10.55	4.2
	57	1.36	20.53	0.29	1.03	3.8
	58	4.86	2.19	0.22	2.35	3.7
	59	4.47	3.62	1.21	0.4	3.3
	60	1.23	5.09	0.28	3.66	5.9
	61	2.84	3.29	0.35	12.91	4.7
	62	2.04	3.74	0.6	0.73	3
	63	3.79	2.13	0.44	4.47	9.6
	64	40.15	5.11	0.43	2.91	2.9
	65	3.8	4.92	0.97	0.35	12.9
	66	1.56	3.68	0.28	2.98	4
	67	16.42	4.81	0.31	1.93	2.5
	68	17.94	4.93	0.42	1.6	6.6
	69	3.72	5.54	0.25	15.68	2.1
	70	1.22	1.18	0.19	2.34	2.7
	71	7.91	2.48	0.19	9.1	1.8
	72	3.94	7.55	0.23	2.28	3.7
	73	6.62	5.66	0.26	7.36	15
	74	4.28	3.19	0.24	1.36	5.1
	75	2.86	1.25	0.29	0.21	4.1
	76	1.49	3.95	0.25	2.02	6.3
	77	4.76	2.59	0.24	1.02	2.1
	78	8.67	4.82	0.24	0.14	3.1
	79	1.31	2.55	0.3	2.22	3.4
	80	1.67	5.51	0.5	0.8	3.2
	81	1.48	13.78	1.44	1.12	3.6
	82	1.54	5.61	1.17	11.05	2.3
	83	66.05	7.49	0.43	2.04	4.3
	84	3.06	2.49	0.36	1.05	3.7
	85	0.6	4.34	0.37	1.98	3.1

86	0.61	4.81	0.28	5.26	4.2
87	2.22	4.47	0.33	6.43	5.8
88	8.44	3.56	0.46	4.23	5.9
89	0.8	2.8	0.42	3.07	8.9
90	1.86	2.98	0.6	2.95	3.7
91	2.17	2.13	0.74	4.23	3.6
92	5.39	3.28	0.48	3.8	2.4
93	5.53	2.17	0.35	3.36	3
94	2.59	4.24	0.49	2.93	2
95	2.66	2.49	0.5	4.73	4.3
96	1.53	4.56	0.36	19.54	3
97	1.41	2.44	0.23	1.84	4.1
98	11.03	5.04	0.86	8.11	5.1
99	9.35	4.3	0.56	2	4
100	10.09	3.54	0.66	1.04	4.2
Mean Service Time	5.27	5.12	0.46	6.15	4.29
Mean Service Rate	11.39	11.73	131.12	9.76	13.99
Standard Deviation	0.14	0.08	0.01	0.15	0.04