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Weak electron scattering for the 3He-3H transition and the weak nuclear form factors

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WEAK ELECTRON SCATTERING FOR THE $^3He \leftrightarrow {}^3H$ TRANSITION

AND

THE WEAK NUCLEAR FORM FACTORS

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IN

PHYSICS

by

Michael A. Barnett

1995

To: Dean Arthur W. Herriott
College of Arts and Science

This thesis written by Michael A. Barnett, and entitled Weak Electron Scattering for the $^3He \to {}^3H$ Transition and the Weak Nuclear Form Factors, having been approved in respect to style and intellectual content, is refered to you for your judgement.

We have read this thesis and recommend that it be approved

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Richard Bone

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Ramon Lopez de la Vega

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Stephan L. Mintz, Major Professor

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The thesis of Michael A. Barnett is approved.

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Dean Arthur W. Herriott
College of Arts and Sciences

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Dr Richard L. Campbell
Dean of Graduate Studies

Florida International University, 1995

T0 THE MEMORY OF

MY FATHER AND MOTHER

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ABSTRACT OF THE THESIS

WEAK ELECTRON SCATTERING FOR THE $^3He \leftrightarrow ^3H$ TRANSITION AND THE WEAK NUCLEAR FORM FACTORS

by

Michael a Barnett

Florida International University, 1995

Professor Stephan L. Mintz, Major Professor

We calculate the differential cross section for weak electron scattering reaction, $e + {}^3He \rightarrow {}^3H + \nu_e$, for energies from 100 MeV to 6 GeV as a function of outgoing nucleus angle from 0 to $\pi/2$ radians. We find that the differential cross section at low $|q^2|$ increases with electron energy from 0.1GeV to 6.0 GeV, such that the peak value at 6.0 GeV is approximately 3.2×10^{-40} cm ^{2} / ster, a factor of 10 larger than the peak value at 0.1 GeV. We also find that the width of the peak falls very rapidly with increasing electron energy. At high $|q^2|$ we find that the differential cross section falls by approximately three orders of magnitude making experimental observation at this time unlikely. The contributions of the individual form factors are obtained for electron energies of 0.5GeV and 2.0 GeV. It is found that at low $|q^2|$,the form factors, $F_A(q^2)$ and $F_V(q^2)$, make contributions of similar size to the differential cross section and might be simultaneously determined , but for the case of $F_M(q^2)$ we find that the contribution is too small to determine. It is also found that at large $|q^2|$values, the contribution of $F_M(q^2)$ is substantially enhanced , but that the cross section is probably too small to enable a direct determination of $F_M(q^2)$.

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CHAPTER 1

INTRODUCTION

In this thesis we shall be looking at electron scattering processes of an essentially electroweak nature, specifically, scattering processes in which the interaction between the electron and the target nucleus are of the weak interaction type. This is one of the four ways in which elementary particles may interact with each other.A brief description of these four interactions is given below. [1]

(i) Gravitational Interactions

These interactions are attractive only, and have an associated force which has an inverse - square relationship with the distance between the particles. The mass serves as the "charge" for this interaction.

It has been postulated that the graviton (an elementary particle), is the mediator of the gravitational interaction. (That is the interaction takes place via exchange of this particle.) The graviton has spin J=2, and zero mass, (since the gravitational field has an infinite range).

The gravitational force is the weakest of all the basic forces and we shall not consider it further here. The strength of the gravitational interaction is about 10^{-36} that of the coulomb interaction.

(ii) Electromagnetic Interactions

These take place between particles, (which have either charges and / or magnetic moments), via the exchange of a photon. The associated forces, i.e. the electric and magnetic forces, are well known. The coulomb force, (a particular example of the electric force) has an inverse -square relationship with the distance between the particles, and is proportional to the product of the charge.[3] Of particular importance here is the dimensionless coupling constant

$$\alpha = \frac{e^2 2\pi}{hc} \approx 1/137.035$$

This constant essentially sets the strength of the electromagnetic interaction.

(iii)Strong interactions

These occur amongst hadrons, which are not actually elementary particles, but consist of quarks. Strong interactions loosely speaking take place through the exchange of particles built from quarks. An example would be nucleon interactions which can take place via pion exchange. The fundamental interaction however between hadrons actually takes place between their quark constituents via spin one, massless particles called gluons.

Crudely speaking the scale of the strong interactions is set by a coupling constant

$$\frac{g^2 2\pi}{hc} \approx 15.$$

The quark gluon coupling is provided by a running coupling constant, $\alpha_s(q^2)$ which falls for increasing $|q^2|$. Quantum Chromodynamics, (QCD)[4] is now believed to be the correct theory of strong interactions. It is essentially a non-Abelian gauge theory describing the interactions of quarks and gluons, and calculations are extremely difficult.

(iv)Weak Interactions-

The weak interaction is the focus of this thesis. This interaction occurs among and between leptons and hadrons. Leptons are a group of particles consisting of the electron and the electron-neutrino, the muon and the muon-neutrino, the tau and the tau-neutrino, along with their anti-particles. The weak force is much weaker than either the strong or electromagnetic forces. An estimate of the interaction strength is about 10^{-14} that of the strong interaction. The weak interaction is mediated by the exchange of intermediate vector-bosons which are extremely massive. The specific vector-bosons that are being refered to in this case are the W^-, W^+, and the Z^0 vector- bosons.of mass 80 to 90 Gev, and consequently the weak interaction is very short ranged.

The Standard Model

The ideas just presented have given rise to a widely accepted "standard model" of electro-weak interactions, created principally by Weinberg, Salam, and Glashow.[2,3]

At exceedingly high energy (high enough for W and Z particles to be created as readily as photons), the interactions mediated by these two different forces are essentially indistinguishable.Thus this theorectical unification is accomplished by assigning the photon and intermediate vector bosons to the same family of four particles.

The corresponding non-Abelian or non- commutative gauge theory for weak interactions is quantum flavour dynamics (QFD). It should be noted that the non-Abelian gauge theories (QCD) and (QFD), are clearly more complex than (QED), quantum electrodynamics, in that the symmetry group for (QED), a U(1) theory is Abelian and has therefore a much simpler structure than the symmetry group for QFD, or the W-S-G theory.

The subject of weak interactions was once limited to nuclear beta decay, but has grown immensely and now includes the decays of muons and tau leptons, the slow decays of mesons and baryons, muon capture by nuclei, and all neutrino interactions with matter to name just a few.

Since the 1960's when ideas based on local gauge invariance emerged,[2] a basis for the unification of weak and electro magnetic interactions has been provided, despite extraordinary differences in their observed chracteristics. This unified theory received experimental confirmation in the 80's with the discovery of the W and Z particles. Thus to summarize, both electromagnetic and weak interactions can be regarded as different manifestations of a single more fundamental electroweak interaction.

Before we can discuss the process which we are considering here we must first spend some time discussing the fundamental structure of matter. At an earlier time we would have included a wide range of strongly interacting particles such as the proton , neutron , pion,etc, as being elementary ,but now the particles which appear to be elementary consist of the leptons, quarks, and gluons,. i.e. essentially true elementary particles.

Types of Elementary Particles as well as the various classes of subatomic matter

Elementary particles consist of leptons , quarks, gluons,the photon, the weak intermediate vector- bosons, and the Higgs bosons.

We consider first the leptons. The lepton family consists of the electron (e^-), the muon (μ^-), and tau (τ^-) leptons, their associated neutrinos, as well as the anti-particles for all of these. Each charged lepton has spin (1/2), and for each there is a corresponding uncharged neutrino, also of spin 1/2. The leptons differ from the generally heavier hadrons primarily by being insensitive to the strong nuclear force, the dominant short -range force that binds together the particles of the atomic nucleus. The leptons share this immunity to the strong force with the photon, the massless carrier of the electromagnetic force, which forms a one member class of its own. Unlike the photon , however, both the leptons and the hadrons are capable of interacting by means of the weak force, the even shorter range force responsible for the radioactive beta decay of nuclei, as well as other processes which we shall discuss later.

One of the most striking things about the leptons is that there are so few of them. Here again they stand in contrast to the hadrons, of which there are now several hundred distinct particles arrayed in various sub-classes.There is no evidence that the leptons are anything but pointlike objects, and therefore, are considered elementary particles in the true sense.

Quarks

These particles[4] are believed to be the building blocks of Hadrons , and have fractional charge, and half integral spin They come in different varieties which are described by the terms color and flavour. So far there is a strong belief in the existence of six "flavours", although evidence for the top (t) quark is not quite as firm as that for the others.

	Charge	Mass
"up"u	2/3e	2-8 Mev
"down"d	-1/3e	5~15 Mev
"strange"s	-1/3e	100~300 Mev
"charmed"c	2/3e	1.3~1.7 Gev
"bottom"b	-1/3e	4.7~5.3 Gev
"top"t	2/3e	~170 Gev

We can arrange the quarks in distinct left handed doublets , or generations:

ie $(u,d)_L$,$(c,s)_L$, $(t,b)_L$ and right handed singlets : u_R , d_R , c_R , s_R , t_R , b_R.

For each quark there is a corresponding anti-quark, with opposite charge.

Quarks and anti-quarks experience strong, electromagnetic and weak interactions. Each quark has a number that is known as a baryon number, which has to be conserved.

Anti-quarks have baryon number = -1/3, whilst quarks have baryon number = 1/3.

Two quarks are required to build the meson class of hadrons, (a quark-antiquark pair underlie the structure of all mesons). The mesons include the pions, kaons, ρ-mesons etc.

The baryon class of hadrons however are more complicated, than this. They are

comprised of three quarks. Examples include the proton, neutron, Λ- particle, etc.
The term color was chosen because the rules for forming hadrons can be expressed succintly, by requiring all allowed combinations of quarks to be "white" or "colorless". The quarks are assigned the primary colors, green, red and blue., whilst the antiquarks have the corresponding anti-colors "cyan", "magneta", and "yellow".
Each one of the six flavours of quarks come in three different colors, thus we have effectively eighteen (18) distinct quarks or thirty-six, if we count the anti-quarks..
From the available colors for quarks there are two ways to create white: (i) by mixing all three primary colors (or the three anti-primary colors), or (ii) by mixing one primary color with its corresponding anti-color.
The baryons are made via the first scheme (i.e. three quarks, each one of a different primary color), whilst the mesons are made via the second scheme, (two quarks, one primary colour accompanied by its corressponding anti-color).
In the case of both of these classes the interaction which binds them is the exchange of gluons. It became clear very early that because some baryons would be made of the same quarks in the same state , thus violating the Pauli exclusion principle , quarks needed an additional degree of freedom to prevent this, which is called color.

Gluons

An important aspect of quarks , that should be considered in addition to their general chracteristics is the way in which they are able to interact with each other. They do so via exchange of gluons, which are essentially the mediators of all strong quark-quark interactions. There are eight of them , and they are all massless and have a spin angular momentum of one unit. In other words they are massless vector bosons, like the photon. The gluons are electrically neutral, but they are not color-neutral. Each gluon carries one color, and one anti-color.

There are nine possible combinations of a color and an anti-color, but one of them is equivalent to white, and is excluded, leaving eight distinct gluon fields.

The gluons preserve color symmetry in the following way. A quark is free to change its color, and it can do so independently of of all other quarks., but every color transformation must be accompanied by the emission of a gluon. The gluon, moving at the speed of light, is then absorbed by another quark, which will thus have its color shifted in exactly the same way, needed to compensate for the original change.

Suppose, for example, that a green quark changes its color to red, and in the process emits a gluon that bears the colors green and anti-red. The gluon is then absorbed by a red quark, whereby the red of the quark and the anti red of the gluon annihilate each other , thus leaving the second quark with an overall color of green.Hence in the final state, just as in the initial state we still have one red quark and one green quark . Thus we have no net colour change in the hadron despite the movement of gluons.

ELECTROWEAK PHENOMENA, AND THE SEARCH FOR INTERMEDIATE VECTOR BOSONS

One of the major developments in particle physics over the last 25 years has been that of the unified theory for weak and electromagnetic forces.Until this and other unified theories were introduced, the four observable forces of nature seemed to be quite independent of one another. To recap, the electromagnetic force governs the interactions of electrically charged particles , the weak nuclear force is responsible for such processes as the beta decay of a radioactive nucleus,the strong nuclear force holds the nucleus together, whilst gravity holds the universe together.

The prevailing view of the interactions between elementary particles is that a force is transmitted between two particles via the exchange of a third intermediary particle.

In the electro magnetic and weak interactions, the exchanged particle is a member of the family called the vector bosons. This term refers to a classification of particles according to one of their most basic properties, spin angular momentum. A boson is a particle whose spin when measured in fundamental units is an integer, such as 0,1,2.

A "vector- boson" has spin value equal to one. In the case of electromagnetism the exchanged vector boson is the photon,whilst the corresponding force carrier in weak interaction is the intermediate vector boson, (intermediate simply because of its mediating role between particles).

The idea that the nature of a force and the mediating particle are closely related was introduced by Yukawa[5] in 1935. Yukawa noted that the range of the force should be inversely proportional to the mass of the particle that transmits it. For example the range of the electromagnetic force is infinite in accordance with the masslessness of the photon. In the case of the nuclear force though, only a limited range applies, which suggests that they are carried by particles with mass. Specifically, Yukawa postulated the existence of a moderately heavy particle , later named the pion, the exchange of which gives rise to the strong attractive force between the proton and the neutron.

The weak nuclear force has a still shorter range than the strong force that acts between protons and neutrons. Thus the intermediate vector bosons of the weak force can be expected to have a mass larger than that of the pion, which is the intermediary particle for the strong interaction. Early attempts to detect the the intermediary particles associated with the weak force were unsucessful due to the larger masses of these bosons putting them out of reach of the then existing particle accelerators.

A good estimate for the masses of these weak force particles was not obtained until the advent of the unified electroweak theory in the late 1960's. This theory, which now forms the basis of the standard model of the electromagnetic and weak interactions , for the first time made specific and testable predictions about the properties of these intermediate vector bosons, including their masses. Furthermore the theory required that there be three such particles , with electric charges of +1, (W^+), -1(W^-), and zero(Z^o). The masses of these particles are now known to be 82Gev, for the the W bosons, and 92Gev for the Z^0 boson.

One approach to understanding the unified electroweak theory, begins with an imaginary primordial state in which the photon and the intermediate vector bosons were all equally massless. It was the breaking of a symmetry of nature that endowed the W^+, the W^- and the Z^0, with large masses, while leaving the photon massless. A mechanism for this symmetry breaking[6] was first discussed in 1964, by Peter Higgs of the University of Edinburgh. Interestingly enough the Higgs particle is able to supply masses for the W and Z bosons.

Many, electroweak interactions of matter entail an exchange of electric charge. For example a proton might give up its charge of (+1) to a neutrino (a massless particle of no charge). As a result the proton becomes a neutron, whist the neutrino is converted into a positron. All of these events can be accounted for by the the exchange of the charged (W^+ and W^-) vector bosons. But there are weak interactions in which the particles maintain the same charges they had before the event, as they do in the electromagnetic interactions. Here we have the exchange of the uncharged (Z^0) particle. Once the existence of neutral weak currents had been fully confirmed , it was only natural to try and find a way to detect the Z^0 as well as the W particles. The task of creating particles with such a large mass , however remained daunting. The largest particle accelerators at that time, consisted of machines, where in which a single beam of protons is raised to high energy and then directed onto a fixed target. In the ensuing collision of a beam particle with a target particle most of the energy released goes into moving the two particle system rather than demolishing it; only a small fraction of the beams energy is made available for the creation of new particles.

The best way then of observing an intermediate vector boson is to use a colliding beam machine, where the accelerated particles meet head on , transforming essentially all their energy into new particles.

The charged intermediate vector bosons, W^+ and W^-,were discovered in January 1983, whilst the neutral intermediate vector boson was discovered several months later. These discoveries were made by international teams of scientists using the antiproton-proton collider at CERN, in Geneva.

BETA DECAY

The first known weak interaction to be discovered by was that of beta particle emission. This reaction was first observed in nuclei, but it may be discussed in a simpler form as

$$n \rightarrow p + e^{-} + \bar{\nu}$$

I.e. a neutron decays into a proton plus an electron as well as an anti electron-neutrino. Thus the parent nucleus effectively undergoes an increase of atomic number by one, the daughter nucleus having an extra proton, although the number of nucleons remains the same. As this process proceeds an electron is emitted, in addition to an anti-neutrino.

We now come to a point closer to the work to be described here. On a fundamental level the weak interaction of a lepton with hadronic matter may be described[2] by an interaction of the type:

$$g\overline{\Psi_{\nu}}\gamma_{\mu}(1-\gamma_5)\Psi_e \frac{1}{q^2 - M_w{}^2} \overline{\Psi_d}\gamma^{\mu}(1-\gamma_5)\Psi_{\mu} \qquad (1)$$

Here we have used Standard Bjorken and Drell [7] notation .

This particular interaction term represents the first order weak exchange of a W boson by an electron and an up (u) quark, which converts the electron into an electron neutrino and the u quark to a d quark. We choose this particular example because it underlies the process of interest to us.

The mass in the denominator M_w is the mass of the W intermediate vector boson which is of the order of 80 Gev, whilst. the largest q^2 values that we consider are of the order of 30 $(Gev)^2/c^2$, which is negligible when compared to $6400(Gev)^2/c^2$ resulting from the $(M_W)^2$ term. Thus the q^2 dependence of the denominator will not be detectable and the interaction may be written as

$$\frac{G}{\sqrt{2}}\overline{\Psi}_\nu\gamma_\mu(1-\gamma_5)\Psi_e\overline{\Psi}_d\gamma^\mu(1-\gamma_5)\Psi_u \qquad (2)$$

Where $G = 1.166\text{x}10^{-5}\ Gev^{-2}$ is the Fermi coupling constant.

This interaction is a current -current interaction , where

$$L_\mu=\overline{\Psi}_\nu\gamma_\mu(1-\gamma_5)\Psi_e \qquad (3a)$$

is the lepton current and

$$J_\mu=\overline{\Psi}_d\gamma_\mu(1-\gamma_5)\Psi_u \qquad (3b)$$

is the quark current.

Thus, we could write Eq. 2 as

$$\frac{G}{\sqrt{2}}\ L_\mu J^{\mu\dagger} \qquad (4)$$

This current-current form of the weak semi-leptonic interaction was first written down by Fermi [8] to explain nuclear beta decay,

$$\text{i.e.} \qquad N_i \rightarrow N_f + e^- + \overline{\nu}_e\,. \qquad (5)$$

Some further comment should be made concerning Eq. (2), namely that this is a parity non-conserving interaction. We note that the term $\overline{\Psi}\gamma^\mu\Psi$ behaves as a vector under parity, but $\overline{\Psi}\gamma_5\gamma^\mu\Psi$ behaves as a pseudovector (or axial vector). That is under a transformation of the form $(\mathbf{x},t) \rightarrow (-\mathbf{x},t)$, the space components of $\overline{\Psi}\gamma^\mu\Psi$ change sign but the time component does not. Thus, this is a vector, whereas for $\overline{\Psi}\gamma_5\gamma^\mu\Psi$, the space components are unchanged but the time component changes sign so that this quantity is an axial vector. Thus, when Eq. (2) is multiplied out, there are VV, AA, and VA terms. The AA and VV terms do not change sign under parity transformations, but the VA terms do. It is just these terms which give rise to the famous violation of parity in weak nuclear processes, the prediction of which resulted in Nobel prizes for T.D. Lee and C.N. Yang. As we stated, Eq. (2) gives the basic form for a quark-lepton interaction. The lepton part of this is now well established. The quark part, however, is never directly observed. Instead, one sees the lepton current interacting with a nuclear current, and it is not presently known how to derive the nuclear current from the quark current.

We use a method due to C.W. Kim and H. Primakoff[9,10] known as the elementary particle model. In this model the nuclei are described as particles of appropriate spin and parity and the structure of the nucleus appears in form factors much as an "elementary" particle such as a nucleon would be described. Thus, for a transition $^3He \rightarrow {}^3H$ which is of interest here:

$$\langle {}^3H | J^{\mu}(0) | {}^3He \rangle = \langle {}^3H | V^{\mu}(0) - A^{\mu}(0) | {}^3He \rangle \qquad (6)$$

where

$$\langle {}^3H | V^{\mu\dagger}(0) | {}^3He \rangle = \bar{u}_H [F_V(q^2)\gamma^{\mu} + \frac{F_M i(q^2)\sigma_{\mu\nu} q^{\nu}}{2m_p}] u_{He} \qquad (7a)$$

and

$$\langle {}^3H | A^{\mu}(0) | {}^3He \rangle = \bar{u}_H [F_A(q^2)\gamma_5\gamma^{\mu} + \frac{F_P(q^2)\gamma_5 q^{\mu}}{m_{\pi}}] u_{He} \qquad (7b)$$

The quantities $F_V(q^2)$, $F_M(q^2)$, $F_A(q^2)$, and $F_P(q^2)$ are the form factors mentioned above. They are Lorentz scalars and must be determined if we are to calculate any observable process. The writing of the hadronic current in the form $V^{\mu}(0) - A^{\mu}(0)$, (i.e., as a vector-axial vector current) was first done by Feynman and Gell-Mann[11] and is sometimes known as the V-A theory.

The form factors $F_A(q^2)$, $F_P(q^2)$,and $F_V(q^2)$ and $F_M(q^2)$ must be determined as we have stated. Fortunately, the V-A theory included what is known as the Conserved Vector Current hypothesis[11] (CVC) which makes use of the fact that the nuclei and other hadrons can be described in terms of a quantity called isospin, and which behaves very much like angular momentum but exists in a different space entirely.

Under the CVC hypothesis, the currents commute according to the relation

$$[I_i, J^{\mu}_j] = i\varepsilon_{ijk} J^{\mu}_k \qquad (8)$$

where ε_{ijk} is the totally antisymmetric Levi-Civita tensor and the weak currents may be written as

$$J^{\mu} = J_1^{\mu} + iJ_2^{\mu} \qquad (9a)$$

$$J^{\mu\dagger} = J_1^{\mu} - iJ_2^{\mu} \qquad (9b)$$

$$J^{em} = J_{\mu}^{(3)} + J_{\mu}^{(0)} \qquad (9c)$$

where Eq. (9c) and Eq. (9b) are the charge raising and charge lowering currents, respectively, and Eq. (9c) is the electromagnetic currents. The odd convention is historical from β-decay. In the above the I_j are components of the vector isospin charge. The above equations lead to the relations:

$$[I^{\dagger}, J_{\mu}^{em}] = -V_{\mu} \qquad (10)$$

where $I^{+} = I_1 + iI_2$ and

$$\langle ^3He | [I^{\dagger}, J_{\mu}^{em}] | ^3H\rangle = -\langle ^3He | V_{\mu} | ^3H\rangle . \qquad (11)$$

Thus,

$$\langle ^3H | J_{\mu}^{em} | ^3H\rangle - \langle ^3He | J_{\mu}^{em} | ^3He\rangle = -\langle ^3He | V_{\mu} | ^3H\rangle \qquad (12)$$

and taking the hermitian conjugate, one obtains

$$\langle ^3He | J_{\mu}^{em} | ^3He\rangle - \langle ^3H | J_{\mu}^{em} | ^3H\rangle = \langle ^3H | V_{\mu}^{\dagger} | ^3He\rangle . \qquad (13)$$

Writing

$$\langle ^3He | J_{\mu}^{em} | ^3He\rangle = \bar{u}_{He}[F_f^1(q^2)\gamma_{\mu} + \frac{F_f^2(q^2) i\sigma_{\mu\upsilon}q^{\upsilon}}{2m_n}]u_{He} \qquad (14a)$$

and

$$\langle {}^3H |\, J_\mu^{em} \,| {}^3H\rangle = \bar{u}_H \,[F_i^{(1)}(q^2)\,\gamma_\mu + \frac{F_i^{(2)}(q^2) i\sigma_{\mu\upsilon} q^\upsilon}{2m_n}]u_H \qquad (14b)$$

we obtain Equations (15a) and (15b) making use of Eq. (7a)

$$F_V(q^2) = F_f^{(1)} - F_i^{(1)} \qquad (15a)$$

$$F_M(q^2) = F_f^{(2)} - F_i^{(2)} \,. \qquad (15b)$$

Because $F_f^{(1,2)}$ and $F_i^{(1,2)}$ can be obtained from electron scattering experiments, the form factors $F_V(q^2)$ and $F_M(q^2)$ can thus be determined. The form factor $F_P(q^2)$ will not contribute to our process because all terms proportional to F_P are also proportional[12] to m_e^2 which is negligible. We thus need only $F_A(q^2)$. The quantity $F_A(0)$ occurs in beta decay and is known. However, we have no direct way of obtaining $F_A(q^2)$. An argument due to Kim and Primakoff[10] shows that:

$$\frac{F_A(q^2)}{F_A(0)} \cong \frac{F_M(q^2)}{F_M(0)} \,. \qquad (16)$$

This argument is based on an impulse approximation result and appears to work well for muon-capture[10] for this and many other nuclei. We use it here over a much larger range of q^2 than it has been tested. Thus, if the experiment is actually performed, this assumption will be tested. We now have all of the necessary form factors and we can calculate the cross-section.

CHAPTER 2

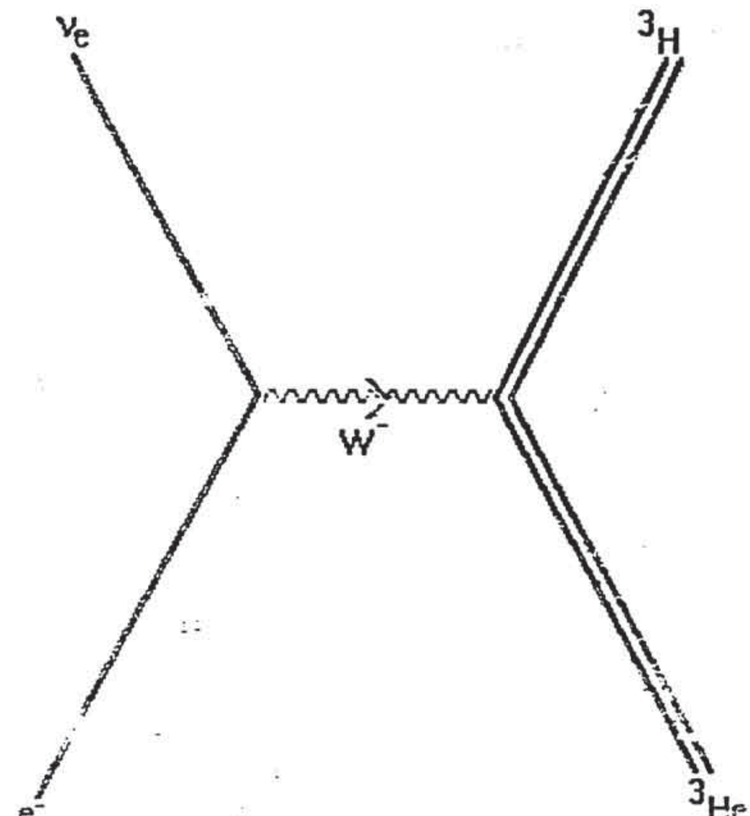
Moving Towards Obtaining A Theoretical Cross-Section For the $^3He \rightarrow {}^3H$ Transition

The Reaction which we are considering is :

$e^- + {}^3He \rightarrow {}^3H + \nu$

The Feynman diagram for this is shown below.

The electron may be seen to be giving up negative charge, which is transferred via the W boson to the helium nucleus, which thus facilitates its transformation to a tritium nucleus. (Note that the transition of the helium nucleus to tritium represents a lowering of positive charge, which is equivalent to an increase in negative charge.)



$e^- + {}^3He \longrightarrow \nu_e + {}^3H$

Consideration of the Hadronic current

We now begin our actual calculation :

The hadronic current is given by $J^{\mu} = \left(V^{\mu} - A^{\mu}\right)$

The matrix element of the vector current is given by

$$\langle f|V_{\mu}(0)|i\rangle = \overline{u}_f\left[\gamma_{\mu}F_v(q^2) + \frac{iF_m(q^2)\sigma_{\mu\nu}q^{\nu}}{2m_n}\right]u_i \qquad (17a)$$

and the axial matrix element of the current is given by

$$\langle f|A_{\mu}(0)|i\rangle = \overline{u}_f\left[\gamma_{\mu}\gamma_5 F_A(q^2) + \frac{q_{\mu}\gamma_5}{m_{\pi}}F_P(q^2)\right]u_i \qquad (17b)$$

Where (i) and (f) are the initial and final state nuclei respectively.

Thus:

$$J_{\mu} = \left(V_{\mu} - A_{\mu}\right) = \overline{u}_f\left[\gamma_{\mu}F_V + \frac{iF_m\sigma_{\mu\nu}q^{\nu}}{2m_n} - \gamma_{\mu}\gamma_5 F_A - \frac{q_{\mu}\gamma_5 F_p}{m_{\pi}}\right]u_i \qquad (18)$$

This implies that:

$$H_{\mu\nu} = \left|J_{\mu}\right|^2 = \overline{u}_f\left[\gamma_{\mu}F_V + \frac{iF_m\sigma_{\mu\nu}q^{\nu}}{2m_n} - \gamma_{\mu}\gamma_5 F_A - \frac{q_{\mu}\gamma_5 F_p}{m_{\pi}}\right]u_i \times$$

$$\overline{u}_i\gamma_0\left[\gamma_{\mu}F_V^{*} - \frac{iF_m^{*}\sigma_{\mu\nu}q^{\nu}}{2m_n} - \gamma_{\mu}\gamma_5 F_A^{*} + \frac{q_{\mu}F_p^{*}\gamma_5}{m_{\pi}}\right]\gamma_0 u_f \qquad (19)$$

Where we use the symbol $H_{\mu\nu}$ to represent the hadronic current x current tensor.

Considering the Hermitian conjugate alone, we have:

$$\bar{u}_i\gamma_0\left[\gamma_\mu F_V^* - \frac{iF_m^*\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A^* + \frac{q_\mu F_p^*\gamma_5}{m_\pi}\right]\gamma_0 u_f$$
$$= \bar{u}_i\left[\gamma_\mu F_V^* - \frac{iF_m^*\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A^* + \frac{q_\mu F_p^*\gamma_5}{m_\pi}\right]u_f \qquad (20)$$

Thus we have:

$$\left|J_\mu\right|^2 = \bar{u}_f\left[\gamma_\mu F_V + \frac{iF_m\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A - \frac{q_\mu\gamma_5 F_p}{m_\pi}\right]u_i\bar{u}_i \times$$
$$\left[\gamma_\mu F_V^* - \frac{iF_m^*\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A^* + \frac{q_\mu F_p^*\gamma_5}{m_\pi}\right]u_f \qquad (21)$$

Summing over the spins we have:

$$\left|J_\mu\right|^2 = \sum_{ss} u_f\bar{u}_f\left[\gamma_\mu F_V + \frac{iF_m\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A - \frac{q_\mu\gamma_5 F_p}{m_\pi}\right]u_i\bar{u}_i \times$$
$$\left[\gamma_\mu F_V^* - \frac{iF_m^*\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A^* + \frac{q_\mu F_p^*\gamma_5}{m_\pi}\right] \qquad (22)$$

$$= \mathrm{Tr}\left\{\begin{array}{l}\dfrac{\left(\not{p}_f + m_f\right)}{2m_f}\left[\gamma_\mu F_V + \dfrac{iF_m\sigma_{\mu\nu}q^\nu}{2m_n} - \gamma_\mu\gamma_5 F_A - \dfrac{q_\mu\gamma_5 F_p}{m_\pi}\right] \\ \times\dfrac{\left(\not{p}_i + m_i\right)}{2m_i}\left[\gamma_\mu F_V^* - \dfrac{iF_m^*\sigma_{\alpha\beta}q^\beta}{2m_n} - \gamma_\alpha\gamma_5 F_A^* + \dfrac{q_\alpha F_p^*\gamma_5}{m_\pi}\right]\end{array}\right\} \qquad (23)$$

After expanding out some of the brackets, our expression takes on the format, that is shown on the next page:

$$\frac{1}{2m_i 2m_f}\mathrm{Tr}\left\{\begin{bmatrix} \not{p}_f\gamma_\mu F_v + i\dfrac{F_M \not{p}_f \sigma_{\mu\nu} q^\nu}{2m_n} - \not{p}_f\gamma_\mu\gamma_5 F_A - \dfrac{\not{p}_f q_\mu \gamma_5}{m_\pi}F_p \\ +m_f\gamma_\mu F_v + \dfrac{im_f F_M \sigma_{\mu\nu} q^\nu}{2m_n} - m_f\gamma_\mu\gamma_5 F_A - \dfrac{m_f q_\mu \gamma_5}{m_\pi}F_p \end{bmatrix}\times\right.$$

$$\left.\begin{bmatrix} \not{p}_i\gamma_\alpha F_v^* - \dfrac{iF_M^* \not{p}_i \sigma_{\alpha\beta} q^\beta}{2m_n} - \not{p}_i\gamma_\alpha\gamma_5 F_A^* + \dfrac{\not{p}_i q_\alpha \gamma_5}{m_\pi}F_p^* \\ +m_i\gamma_\alpha F_v^* - \dfrac{im_i F_M^* \sigma_{\alpha\beta} q^\beta}{2m_n} - m_i\gamma_\alpha\gamma_5 F_A^* + \dfrac{m_i q_\alpha \gamma_5}{m_\pi}F_p^* \end{bmatrix}\right\} \quad (25)$$

$$=\frac{1}{4m_i m_f}\mathrm{Tr}\left\{\begin{bmatrix} \not{p}_f\gamma_\mu F_v + i\dfrac{F_M \not{p}_f \sigma_{\mu\nu} q^\nu}{2m_n} - \not{p}_f\gamma_\mu\gamma_5 F_A - \dfrac{\not{p}_f q_\mu \gamma_5}{m_\pi}F_p \\ +m_f\gamma_\mu F_v + \dfrac{im_f F_M \sigma_{\mu\nu} q^\nu}{2m_n} - m_f\gamma_\mu\gamma_5 F_A - \dfrac{m_f q_\mu \gamma_5}{m_\pi}F_p \end{bmatrix}\times\right.$$

$$\left.\begin{bmatrix} \not{p}_i\gamma_\alpha F_v^* - \dfrac{iF_M^* \not{p}_i \sigma_{\alpha\beta} q^\beta}{2m_n} - \not{p}_i\gamma_\alpha\gamma_5 F_A^* + \dfrac{\not{p}_i q_\alpha \gamma_5}{m_\pi}F_p^* \\ +m_i\gamma_\alpha F_v^* - \dfrac{im_i F_M^* \sigma_{\alpha\beta} q^\beta}{2m_n} - m_i\gamma_\alpha\gamma_5 F_A^* + \dfrac{m_i q_\alpha \gamma_5}{m_\pi}F_p^* \end{bmatrix}\right\} \quad (26)$$

After taking the trace, we obtain for our hadronic term $H^{\mu\nu}$, the following:

$$
\frac{1}{4m_i m_f}\left[\begin{array}{l}
4|F_v|^2\left[p_{f\mu}p_{i\alpha}-(p_f p_i)g_{\mu\alpha}+p_{f\alpha}p_{i\mu}\right]-4F_vF_A^*i\varepsilon_{\beta\mu\delta\alpha}p_f^\beta p_i^\delta+\\
\frac{2F_vF_m^*m_i}{m_n}\left[g_{\mu\alpha}(p_f q)-p_{f\alpha}q_\mu\right]-\\
\frac{|F_m|^2}{|m_n|^2}\left[\begin{array}{l}q^2\left[p_{f\mu}p_{i\alpha}+p_{f\alpha}p_{i\mu}-g_{\mu\alpha}(p_i p_f)\right]-(qp_f)\left[q_\mu p_{i\alpha}+q_\alpha p_{i\mu}\right]+\\ (qp_i)\left[2g_{\mu\alpha}(p_f q)-\left[q_\mu p_{f\alpha}\right]-\left[q_\alpha p_{f\mu}\right]\right]+q_\alpha q_\mu(p_i p_f)\end{array}\right]+\\
\frac{2F_v^*F_m m_i}{m_n}\left[g_{\mu\alpha}(p_f q)-p_{f\alpha}q_\mu\right]+\frac{2F_mF_A^*m_i}{m_n}\left[i\varepsilon_{\beta\mu\delta\alpha}p_f^\beta q^\delta\right]-\\
4F_AF_v^*i\varepsilon_{\beta\mu\delta\alpha}p_f^\beta p_i^\delta+4|F_A|^2\left[p_{f\mu}p_{i\alpha}-(p_f p_i)g_{\mu\alpha}+p_{f\alpha}p_{i\mu}\right]+\\
\frac{2F_m^*F_Am_i}{m_n}\left[i\varepsilon_{\beta\mu\delta\alpha}p_f^\beta q^\delta\right]-\frac{4F_AF_p^*m_iq_\alpha}{m_\pi}\left[p_{f\mu}\right]+\frac{2F_pF_m^*q_\mu}{m_nm_\pi}\left[\varepsilon_{\beta\gamma\alpha\tau}p_f^\beta p_i^\gamma q^\tau\right]\\
+\frac{4|F_p|^2q_\alpha q_\mu\left[p_f p_i\right]}{(m_\pi)^2}-\frac{4F_pF_A^*m_i\left[q_\mu p_{f\alpha}\right]}{m_\pi}+\frac{2F_vF_m^*m_f}{m_n}\left[p_{i\alpha}q_\mu-g_{\mu\alpha}(p_i q)\right]+\\
4m_im_f|F_v|^2g_{\alpha\mu}+\frac{2F_vF_m^*m_f}{m_n}\left[p_{i\mu}q_\alpha-g_{\mu\alpha}(p_i q)\right]+\frac{2F_mF_A^*m_f}{m_n}\left[i\varepsilon_{\beta\mu\delta\alpha}p_i^\beta q^\delta\right]-\\
\frac{m_im_f|F_m|^2}{|m_n|^2}\left(q_\mu q_\alpha-q^2g_{\mu\alpha}\right)+\frac{2F_m^*F_Am_f}{m_n}\left[i\varepsilon_{\beta\mu\delta\alpha}p_i^\beta q^\delta\right]-\frac{4F_AF_p^*}{m_\pi}\left[p_{i\mu}q_\alpha\right]-\\
4m_im_f|F_A|^2g_{\alpha\mu}+\frac{4F_pF_A^*m_f}{m_\pi}\left[q_\mu p_{i\alpha}\right]-\frac{4|F_p|^2m_im_f\left[q_\mu q_\alpha\right]}{(m_\pi)^2}
\end{array}\right]
$$

Eqn (27)

Consideration of the Leptonic current

The lepton current may be written from eqn (3a) as:

$$L_\mu = \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \qquad (28)$$

which implies;

$$|L_\mu|^2 = |\overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e|^2 \qquad (29a)$$

$$= [\overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e][\overline{u}_e \gamma_0 (1-\gamma_5) \gamma_\alpha^\dagger \gamma_0 u_\nu] \qquad (29b)$$

$$= \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e (1+\gamma_5)[\gamma_0 \gamma_\alpha^\dagger \gamma_0] u_\nu \qquad (29c)$$

$$= \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e (1+\gamma_5) \gamma_\alpha u_\nu \qquad (29d)$$

$$= u_\nu \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e (1+\gamma_5) \gamma_\alpha \qquad (29e)$$

Where $\Psi(x) = \sqrt{\dfrac{m}{E+m}} \quad u(p,s) e^{-ipx}$ (30)

Taking the absolute value squared of this expression we obtain:

$$\begin{aligned} L_{\mu\nu} = |L_\mu|^2 &= u_\nu \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e (1+\gamma_5) \gamma_\alpha \\ &= u_\nu \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e \gamma_\alpha (1-\gamma_5) \end{aligned} \qquad (31)$$

Where $L_{\mu\nu}$ is the lepton tensor.

Summing over the spins gives:

$$|L_\mu|^2 = \sum_{ss} u_\nu \overline{u}_\nu \gamma_\mu (1-\gamma_5) u_e \overline{u}_e \gamma_\alpha (1-\gamma_5) \qquad (32)$$

$$|L_\mu|^2 = \mathrm{Tr}\left[\left(\frac{p_\nu + m_\nu}{2m_\nu}\right)\gamma_\mu (1-\gamma_5)\left(\frac{p_e + m_e}{2m_e}\right)\gamma_\alpha (1-\gamma_5)\right] \qquad (33)$$

$$= \frac{1}{4m_\nu m_e} \mathrm{Tr}[(p_\nu + m_\nu)\gamma_\mu (1-\gamma_5)(p_e + m_e)\gamma_\alpha (1-\gamma_5)] \qquad (34)$$

Neglecting the lepton masses, since they are so small compared to the other momenta, energy, and masses in this reaction, (0 for the neutrino , and 0.511Mev for the electron), we have:

$$\left|L_\mu\right|^2 = \frac{1}{4m_\nu m_e}\mathrm{Tr}\left[\not{p}_\nu \gamma_\mu \left(1-\gamma_5\right)\not{p}_e \gamma_\alpha \left(1-\gamma_5\right)\right] \qquad (35)$$

Using standard techniques for evaluating traces we obtain:

$$\frac{1}{4m_\nu m_e}\mathrm{Tr}\left[\not{p}_\nu \gamma_\mu \not{p}_e \gamma_\alpha \left(1-\gamma_5\right)\left(1-\gamma_5\right)\right] \qquad (36a)$$

$$= \frac{1}{4m_\nu m_e}\mathrm{Tr}\left[\not{p}_\nu \gamma_\mu \not{p}_e \gamma_\alpha \left(1-\gamma_5\right)^2\right] \qquad (36b)$$

$$= \frac{2}{4m_\nu m_e}\mathrm{Tr}\left[\not{p}_\nu \gamma_\mu \not{p}_e \gamma_\alpha \left(1-\gamma_5\right)\right] \qquad (36c)$$

$$= \frac{1}{2m_\nu m_e}\mathrm{Tr}\left[\not{p}_\nu \gamma_\mu \not{p}_e \gamma_\alpha - \not{p}_\nu \gamma_\mu \not{p}_e \gamma_\alpha \gamma_5\right] \qquad (36d)$$

$$= \frac{1}{2m_\nu m_e}\left[4\left[\left(p_{\nu\mu} p_{e\alpha}\right) - \left(p_\nu \cdot p_e g_{\alpha\nu}\right) + \left(p_{\nu\alpha} p_{e\mu}\right) - i\varepsilon_{\beta\mu\delta\alpha} p_\nu^\beta p_e^\delta\right]\right] \qquad (37a)$$

$$= \frac{2}{m_\nu m_e}\left[\left[\left(p_{\nu\mu} p_{e\alpha}\right) - \left(p_\nu \cdot p_e g_{\alpha\nu}\right) + \left(p_{\nu\alpha} p_{e\mu}\right) - i\varepsilon_{\beta\mu\delta\alpha} p_\nu^\beta p_e^\delta\right]\right] \qquad (37b)$$

If we use the notation ($p_e = e$) and ($p_\nu = \nu$), we have:

$$L_{\mu\nu} = \left|L_\mu\right|^2 = \frac{2}{m_\nu m_e}\left[\left[\left(\nu_\mu e_\alpha\right) - (\nu \cdot e) g_{\alpha\nu} + \left(\nu_\alpha e_\mu\right) - i\varepsilon_{\beta\mu\delta\alpha} \nu^\beta e^\delta\right]\right] \qquad (38)$$

Where as we mentioned previously $L_{\mu\nu}$ is the lepton tensor.

We obtain our matrix element $|M|^2$, by contracting our hadronic term with the leptonic term,

this yields: $|M|^2 = L_{\mu\nu} H^{\mu\nu}$.

Thus we have:

$$|M|^2 = \frac{1}{m_e m_\nu}\left[\begin{array}{l}
\frac{4|F_v|^2}{m_i m_f}\left[(p_f \cdot v)(p_i \cdot e) + (p_f \cdot e)(p_i \cdot v) - [(e \cdot v) m_i m_f]\right] + \\
\frac{4|F_A|^2}{m_i m_f}\left[(p_f \cdot v)(p_i \cdot e) + (p_f \cdot e)(p_i \cdot v) + [(e \cdot v) m_i m_f]\right] + \\
\frac{2|F_M|^2}{|m_n|^2 m_i m_f}\left[\begin{array}{l}(e \cdot v)(p_i \cdot e)(p_f \cdot e) + (e \cdot v)(p_i \cdot v)(p_f \cdot v) + \\ \left[(e \cdot v)^2 m_i m_f\right]\end{array}\right] + \\
\frac{8F_v F_A}{m_i m_f}\left[(p_f \cdot v)(p_i \cdot e) - (p_i \cdot v)(p_f \cdot e)\right] + \\
\frac{4F_m F_v}{m_n m_f}\left[(e \cdot v)(p_f \cdot v) - (e \cdot v)(p_f \cdot e)\right] + \\
\frac{4F_m F_v}{m_n m_i}\left[(e \cdot v)(p_i \cdot e) - (e \cdot v)(p_i \cdot v)\right] + \\
\frac{4F_A F_m}{m_n m_f}\left[(e \cdot v)(p_f \cdot e) + (e \cdot v)(p_f \cdot v)\right] + \\
\frac{4F_A F_m}{m_n m_i}\left[(e \cdot v)(p_i \cdot e) + (e \cdot v)(p_i \cdot v)\right]
\end{array}\right]$$

Eqn (39)

We now need only to calculate the cross section and put in values for the form factors to complete our calculation.

Calculating the cross section

The differential cross section may be written*[7] as:

$$d\sigma = \frac{m_e m_\nu}{E_e E_\nu} \frac{1}{|V_1 - V_2|} |M|^2 \frac{G^2}{2} \frac{d^3\mathbf{p}_\nu}{(2\pi)^3} \frac{d^3\mathbf{p}_f}{(2\pi)^3} (2\pi)^4 \delta\left((P_f + \nu) - (P_i + e)\right) \qquad (40)$$

Where v_1 = Velocity of incoming electron $\approx$ c and v_2 = Velocity of target nucleus = 0.

Thus we have

$$d\sigma = \frac{m_e m_\nu}{E_e} \frac{M_f}{E_f} |M|^2 \left(\frac{d^3\mathbf{p}_\nu}{2E_\nu}\right) G^2 d^3\mathbf{p}_f \delta^4\left[\left(P_f + \nu\right) - \left(P_i + e\right)\right] \qquad (41)$$

We note that: $\frac{d^3\nu}{2E_\nu} = d^4\nu\delta\left(\nu^2 - \left(m_\nu\right)^2\right)$ (i)

and: $d^3P_f = \left(P_f\right)^2 dP_f d\Omega_f$. (ii)

Thus we have:

$$d\sigma = \frac{m_e m_\nu}{E_e} \frac{M_f}{} \frac{G^2}{(2\pi)^2} \int \frac{\left(P_f\right)^2 dP_f |M|^2 d\Omega_f \delta^4\nu\delta\left(\nu^2 - \left(m_\nu\right)^2\right)\delta^4\left[\left(P_f + \nu\right) - \left(P_i + e\right)\right]}{E_f}$$

Eqn (42)

We use up the $\delta^4\left[\left(P_f + \nu\right) - \left(P_i + e\right)\right]$ delta function by setting $\left(\nu = P_i + e - P_f\right)$.

Thus we have:

$$\frac{d\sigma}{d\Omega_f} = \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2 M_f}{E_e} \int \frac{dP_f \left(P_f\right)^2 |M|^2 \delta\left[\left(P_i + e - P_f\right)^2 - \left(m_\nu\right)^2\right]}{E_f} \qquad (43)$$

Because

$$E^2 = P^2 + M^2$$

thus $\quad EdE = PdP$

and $\quad P_f dP_f = E_f dE_f$.

Thus we obtain:

$$\frac{d\sigma}{d\Omega_f} = \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2 M_f}{E_e} \int \frac{|M|^2 E_f dE_f \left(P_f\right) \delta\left[\left(P_i + e - P_f\right)^2 - \left(m_\nu\right)^2\right]}{E_f} \tag{44}$$

$$= \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2 M_f}{E_e} \int \left(P_f\right) dE_f |M|^2 \delta\left(P_i + e - P_f\right)^2 . \tag{45}$$

After a little algebra we convert the delta function $\delta\left(P_i + e - P_f\right)^2$ as with any energy delta function using standard methods to obtain:

$$\frac{\partial f}{\partial E_f} = -2\left(M_i + E\right) + \left[2E\cos\theta \frac{\frac{1}{2} 2E_f}{\sqrt{\left(E_f\right)^2 - \left(M_f\right)^2}}\right] \tag{46}$$

$$= -2\left(M_i + E\right) + \frac{2EE_f}{|P_f|}\cos\theta$$

Where $f = \left(P_i + e - P_f\right)^2$ and the angle θ is defined in diagram (1) on page 29.

The scattering cross section is now given by:

$$\frac{d\sigma}{d\Omega_f} = \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2}{E} m_f \int p_f \, dE_f \, \frac{\delta\left(E_f - E_{f0}\right)|M|^2}{2\left|M_i + E - \frac{EE_f}{p_f}\cos\theta\right|} . \tag{48}$$

Using the delta function this reduces to,

$$\frac{d\sigma}{d\Omega_f} = \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2}{E} \frac{M_f p_f |M|^2}{2\left|M_i + E - \frac{EE_f}{p_f}\cos\theta\right|} . \tag{49}$$

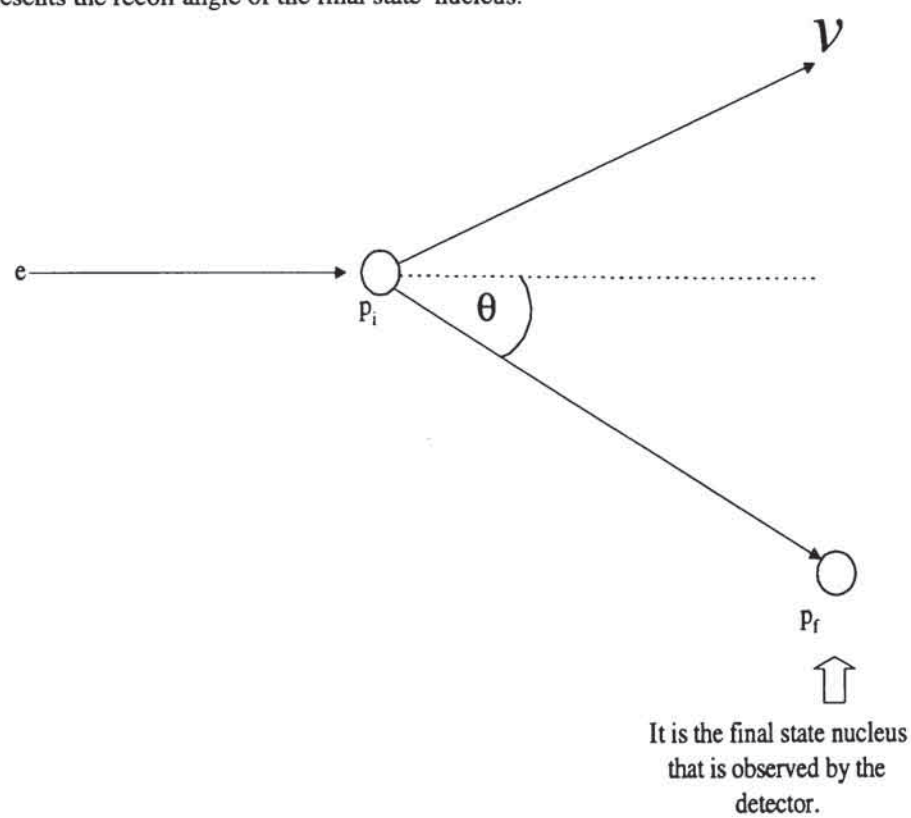
Previously we had summed over the spins, but now we need to average over the initial spins which brings in a factor of 1/2 for the hadronic term as well as a factor of 1/2 for the leptonic term, yielding an overall factor of 1/4 that must be taken into account for our cross section term. Thus we obtain:

$$\frac{d\sigma}{d\Omega_f} = \frac{m_e m_\nu}{(2\pi)^2} \frac{G^2}{E} \frac{M_f p_f |M|^2}{8\left|M_i + E - \frac{EE_f}{p_f}\cos\theta\right|} \tag{50}$$

for our differential cross section. We note that "f" here refers to the final state nucleus.

Diagram (1)

Here (θ) represents the recoil angle of the final state nucleus.



We obtain values for our form factors[15,16] by using

$F_V(q^2) = F_V(0)\cos^2(-q^2/17.1(m_\pi)^2)x(1-q^2/6.25m_\pi{}^2)^{-2}$ for $|q^2| \leq 24.5\ m_\pi{}^2$ (51a)

and $F_V(q^2) = F_V(0)\cos^2(-q^2/14.39m_\pi{}^2)x(1-q^2/4.35m_\pi{}^2)^{-2}$ $|q|^2 > 24.5m_\pi{}^2$ (51b)

where $F_v(0) = 1$. (51c)

For the F_M factor we have:

$F_M(q^2) = F_M(0)\cos^2(-q^2/28.4m_\pi{}^2)x(1-q^2/4.5m_\pi{}^2)^{-2}$ (52a)

for $|q^2| \leq 43.0m_\pi{}^2$

and

$F_M = F_M(0)\cos^2(-q^2/26.0m_\pi{}^2)x(1-q^2/3.5m_\pi{}^2)^{-2}$ (52b)

for $|q^2| > 43.0m_\pi{}^2$

where $F_M(0) = -5.44$. (52c)

For the F_A factor we have from Eqn. 16,

$F_A=F_A(0)\ FM(q^2)/F_M(0)$ (53a)

where $F_A(0) = -1.212$. (53a)

The q^2 dependence of these three form factors was obtained by M. Pourkaviani[15] using the electron scattering data of reference 16.

The Relevance Of The Researched Reaction

Before stating our results , we should ask why this process is of interest. As we mentioned earlier, at present we have no way of reliably calculating medium energy weak nuclear processess from first principles . This is of course what we would eventually wish to do. However , there is limited experimental evidence available , and when experiments are performed , it is often difficult to judge the reliability of the results.

Our method which is semi-phenomenological has given reliable results[10] for muon capture , which can be performed accurately, and seems to be in agreement with the few neutrino experiments which are available.[13] Thus we believe that it will provide useful guidance to experimentalists at electron scattering facilities, such as CEBAF (Continuous Electron Beam Accelerating Facility), or BATES, in the low q^2 region. In the higher q^2 region we must look to the experimentalists for guidance particularly for the behaviour of $F_A(q^2)$ above $q^2 \approx -m_\pi^2$. This is at present unknown and it would be very interesting to know when equation (16) breaks down.

At present relatively few experiments on weak nuclear processes have been performed. There are a large number of accurate β-decay results, but these take place at $q^2 \approx 0$. There are also a number of muon capture results available for $\mu^- + N_i \rightarrow N_f + \nu$, which are believed to be accurate to the 10 percent level. However these are all for $q^2 \approx -m_\mu^2$. Neutrino reactions give some hope for looking at higher $|q^2|$ regions but they are difficult to perform and the neutrinos are usually available over a spectrum of energy values , making the extraction of information very difficult as well.

Electron reactions offer the hope for much more controlled experiments in the higher $|q^2|$

regions. The electron energy is precisely known and can usually be varied. Furthermore, by observing the differential cross-sections at various angles while varying angles and electron energies it should be possible to obtain several different measurements for a fixed q^2 result, so that the individual form factors $F_A(q^2)$, $F_M(q^2)$, and $F_V(q^2)$ can be separately determined. If this can be done, it would be the first time that these have all been determined in a single experiment, and would therefore allow a test for $\frac{F_A(q^2)}{F_A(0)} \cong \frac{F_M(q^2)}{F_M(0)}$ as well as a test for whole nucleus CVC.

The reaction chosen here, $e^- + {}^3He \rightarrow {}^3H + \nu_e$ is particularly interesting, because the transition, ${}^3He \leftrightarrow {}^3H$, is a mirror nuclei transition. A pair of mirror nuclei are essentially the same except that a proton changes to a neutron, i.e. ${}^3He$ has two protons and one neutron, whilst ${}^3H$ has 2 neutrons and one proton, but otherwise they have essentially the same structure. The strong force which provides the basic nuclear binding does not distinguish between protons and neutrons. Thus the wave functions for ${}^3He$ and ${}^3H$ should be very similar, and the overlap will be large, leading to a relatively large cross section for the reaction. It was for this reason, that this reaction was considered[14] early as a possible reaction for CEBAF, but up till now, no actual calculations had been undertaken.

CHAPTER 3

The Results

Using the equation for the cross section discussed earlier, and our particular matrix element, $|M|^2$, we now consider the cross sectional variation with the recoil angle of the final state nucleus. We do this for the energy ranges 100 Mev to 6 Gev. The graphs illustrating this variation are shown in this chapter. In addition we also consider the variation of q^2 with the recoil angle, as well as the dependence of the cross section on the individual form factors. The results are individually labelled and we shall discuss them in the following section.

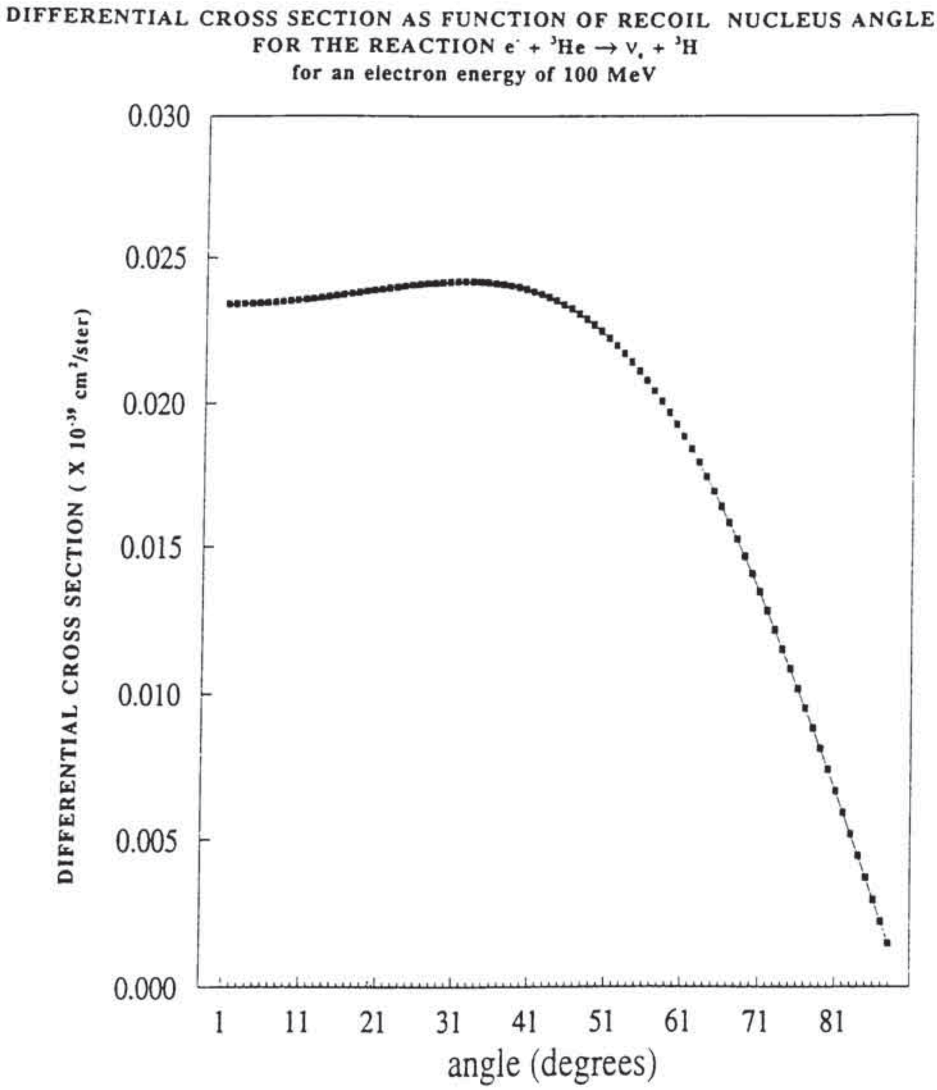

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE
FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$
for an electron energy of 100 MeV
DIFFERENTIAL CROSS SECTION (X 10^{-39} cm^2/ster)
0.030
0.025
0.020
0.015
0.010
0.005
0.000
1
11
21
31
41
51
61
71
81
angle (degrees)

FIGURE 1

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE

FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$

for an electron energy of 500 MeV

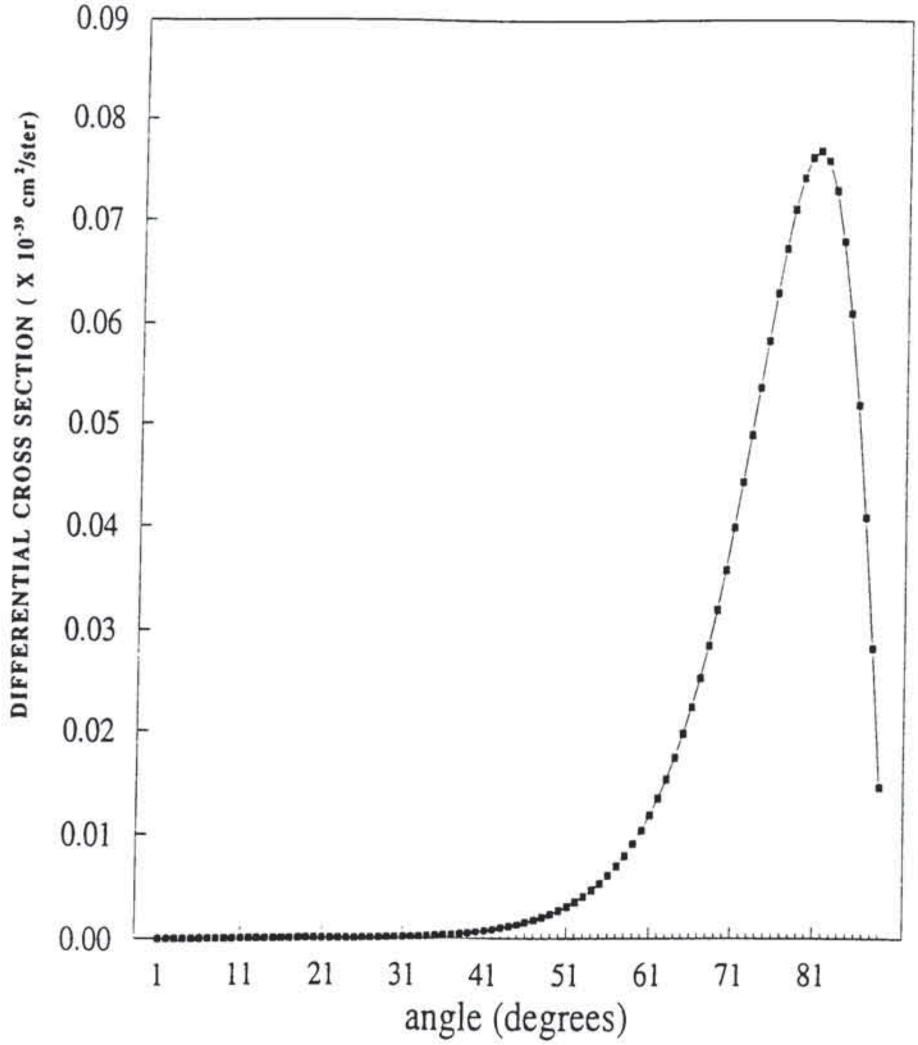


FIGURE 2

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE
FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$
for an electron energy of 1000 MeV

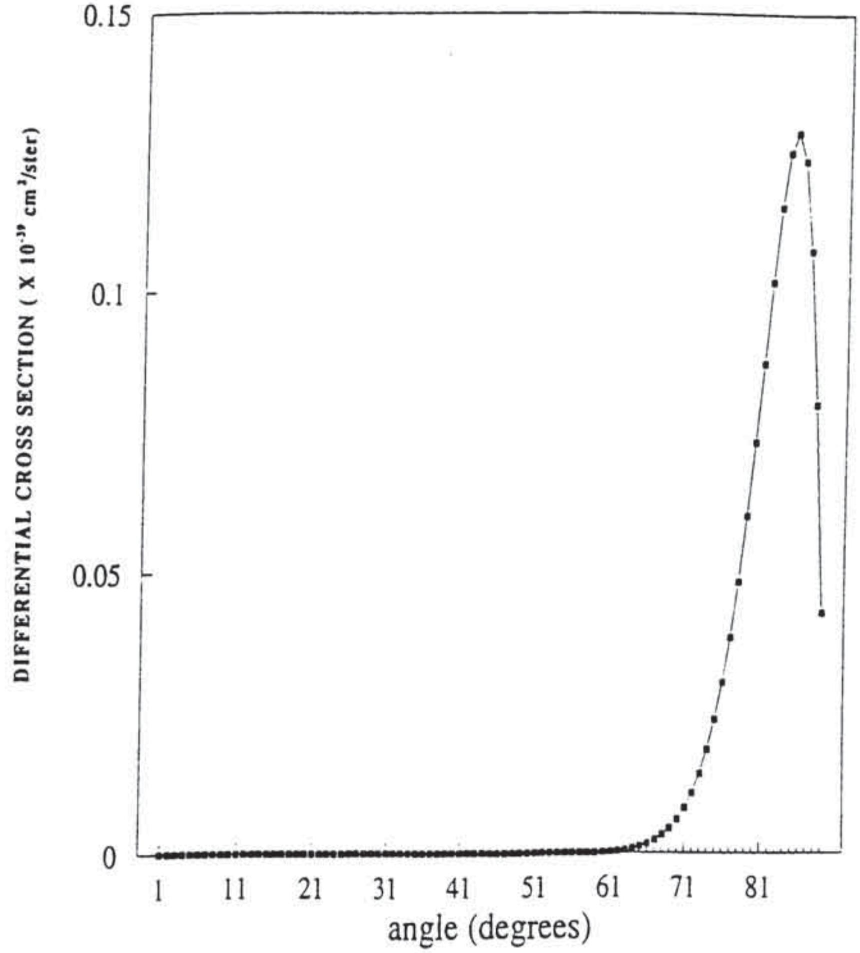


FIGURE 3

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE
FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$
for an electron energy of 2000 MeV

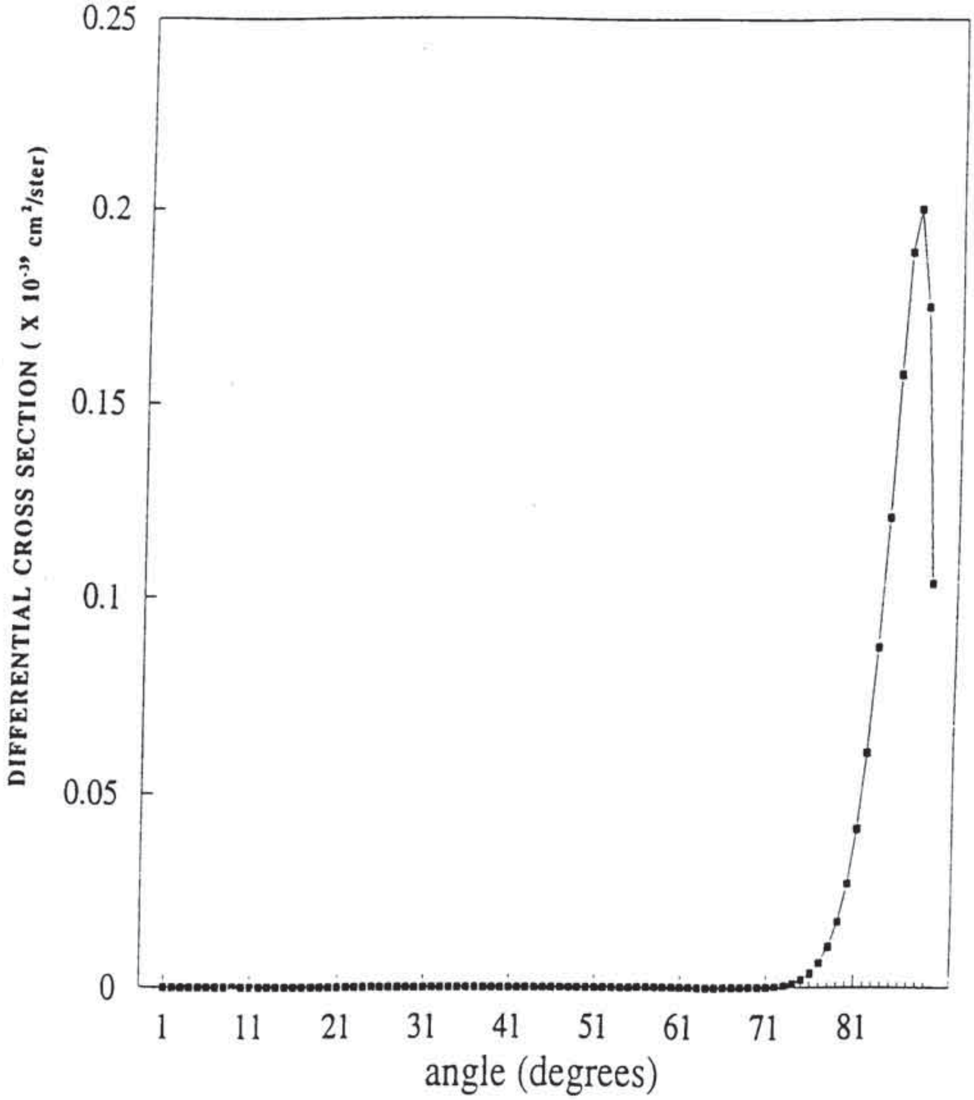


FIGURE 4

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE
FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$
for an electron energy of 4000 MeV

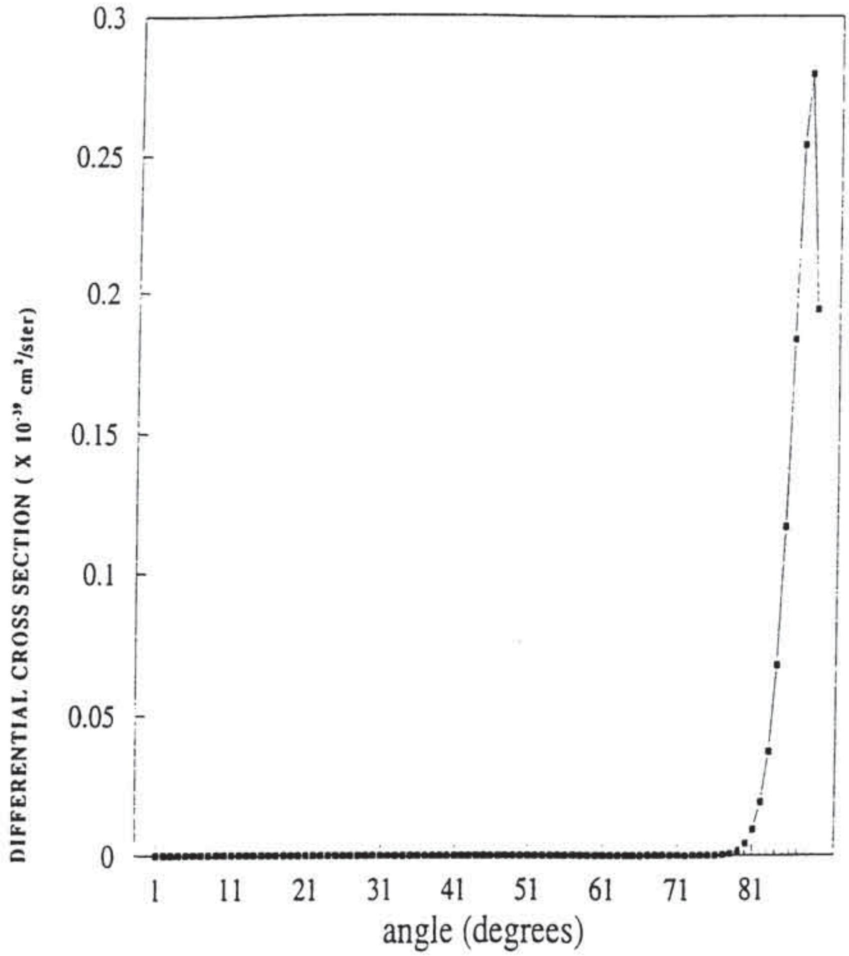


FIGURE 5

DIFFERENTIAL CROSS SECTION AS FUNCTION OF RECOIL NUCLEUS ANGLE
FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$
for an electron energy of 6000 MeV

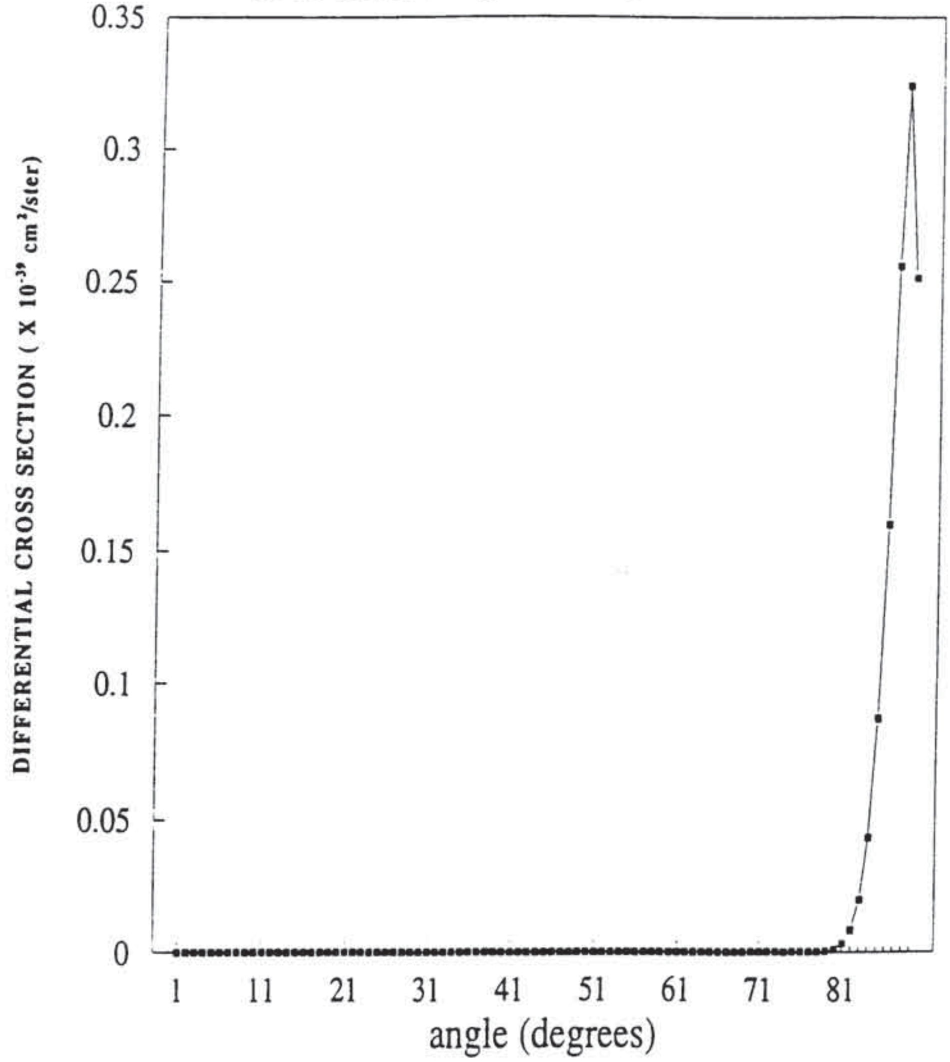


FIGURE 6

DIFFERENTIAL CROSS SECTIONS FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR ENERGIES FROM 100 MeV TO 6 GeV

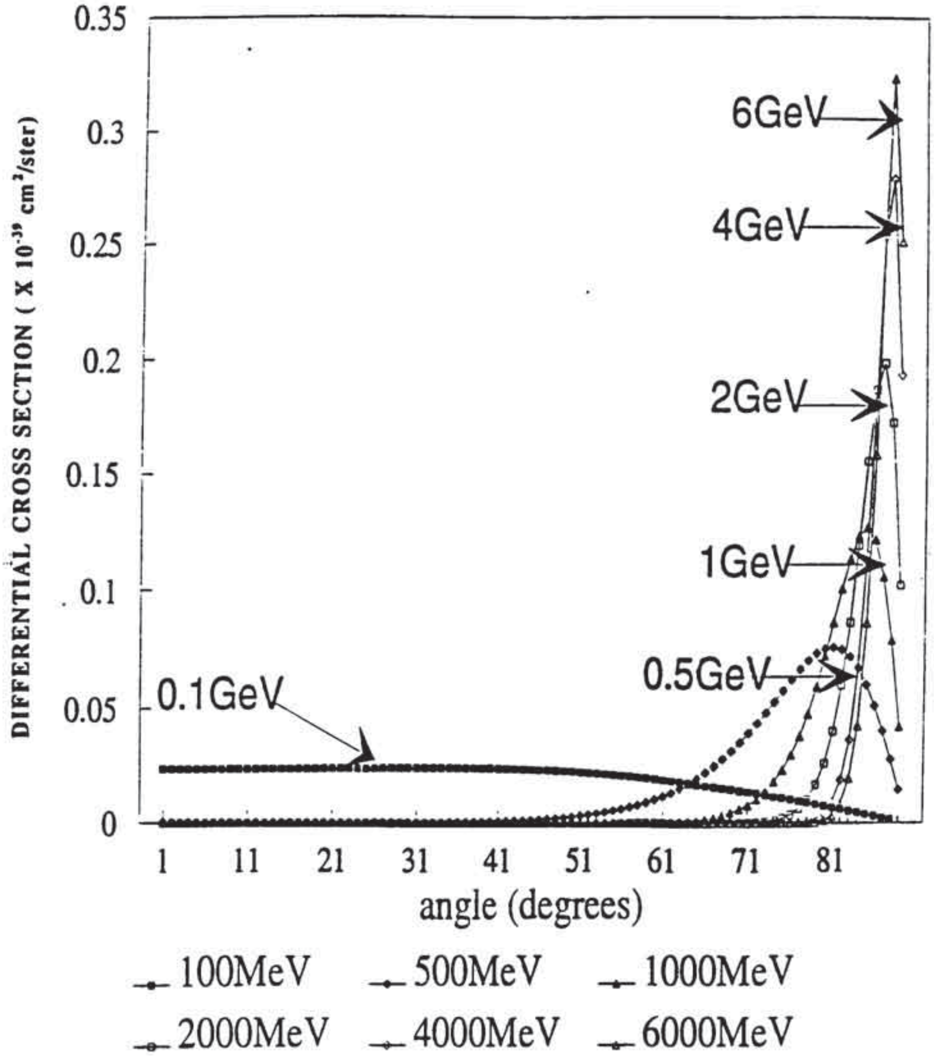


FIGURE 7

CONTRIBUTION OF THE FORM FACTOR F_M TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

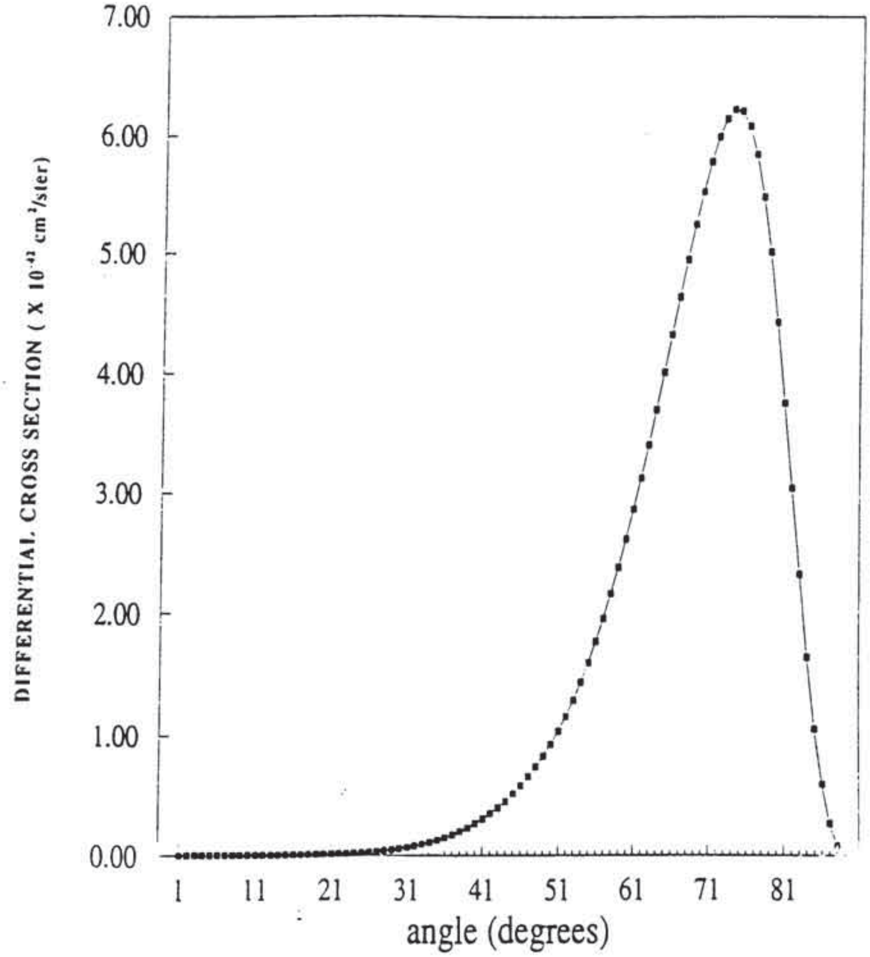


FIGURE 8

CONTRIBUTION OF THE FORM FACTOR F_M TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

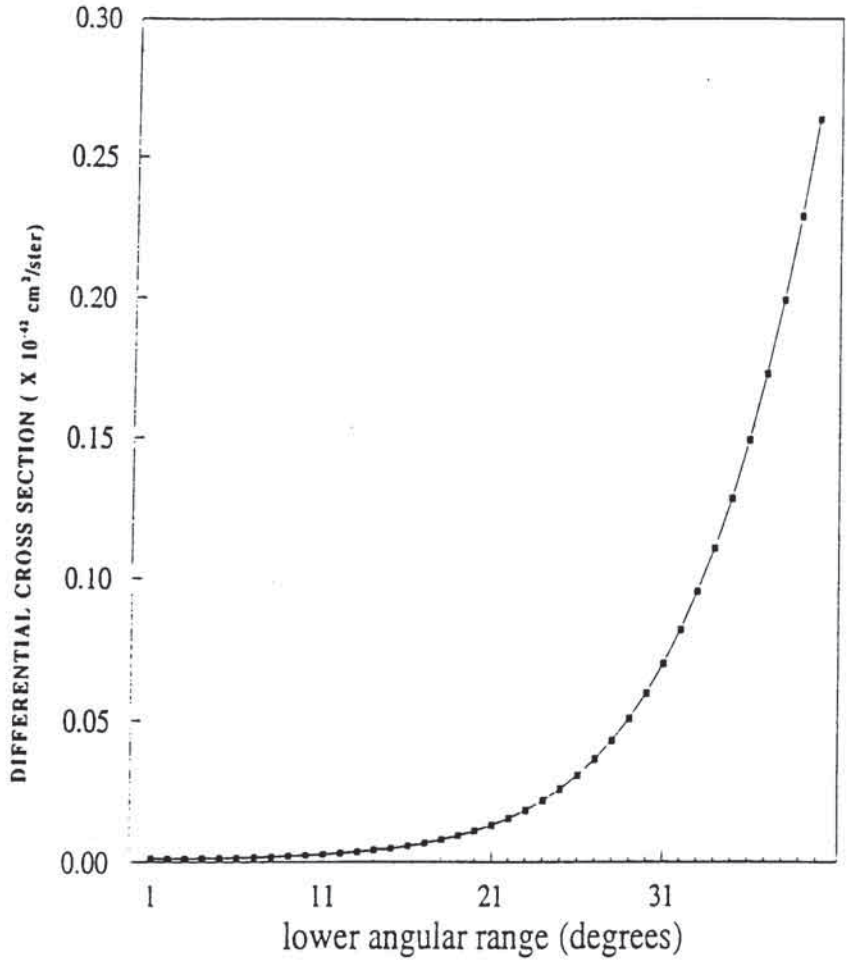


FIGURE 9

CONTRIBUTION OF THE FORM FACTOR F_M TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=2000MeV

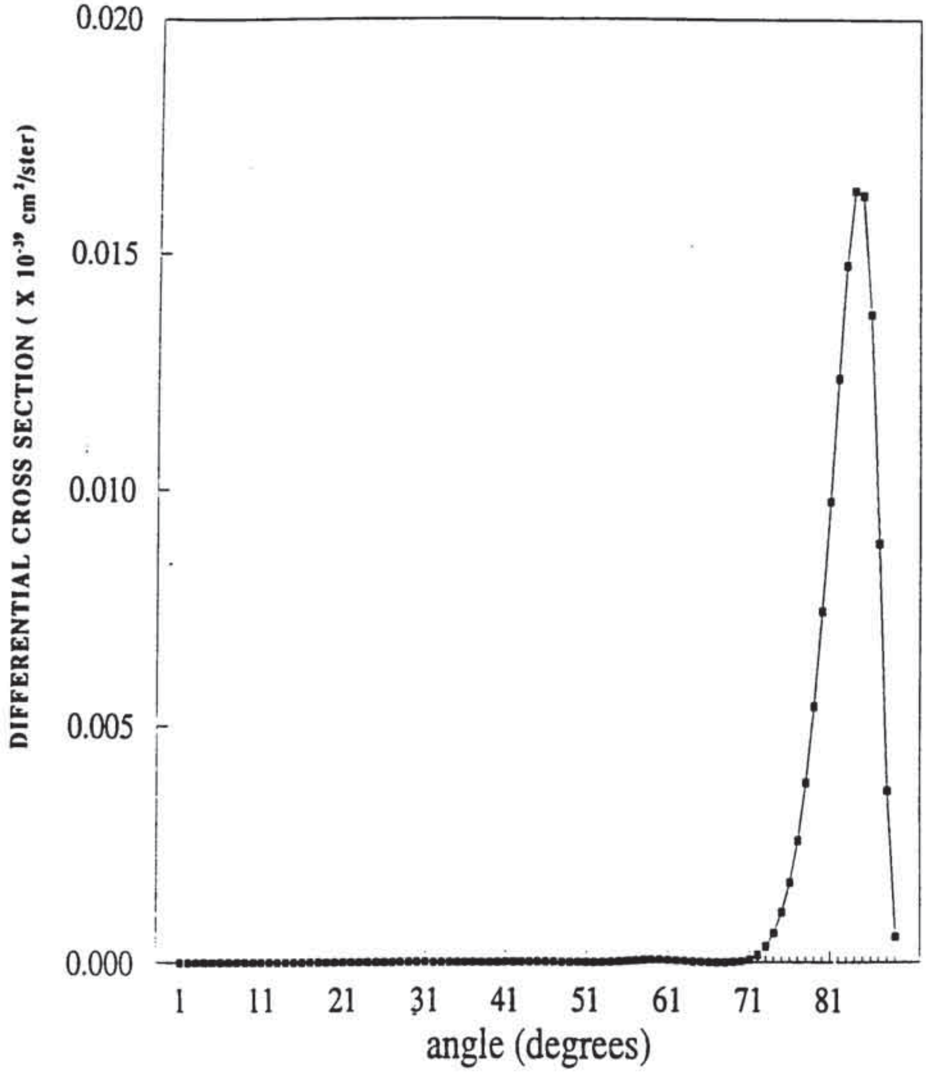


FIGURE 10

CONTRIBUTION OF THE FORM FACTOR F_M TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=2000MeV

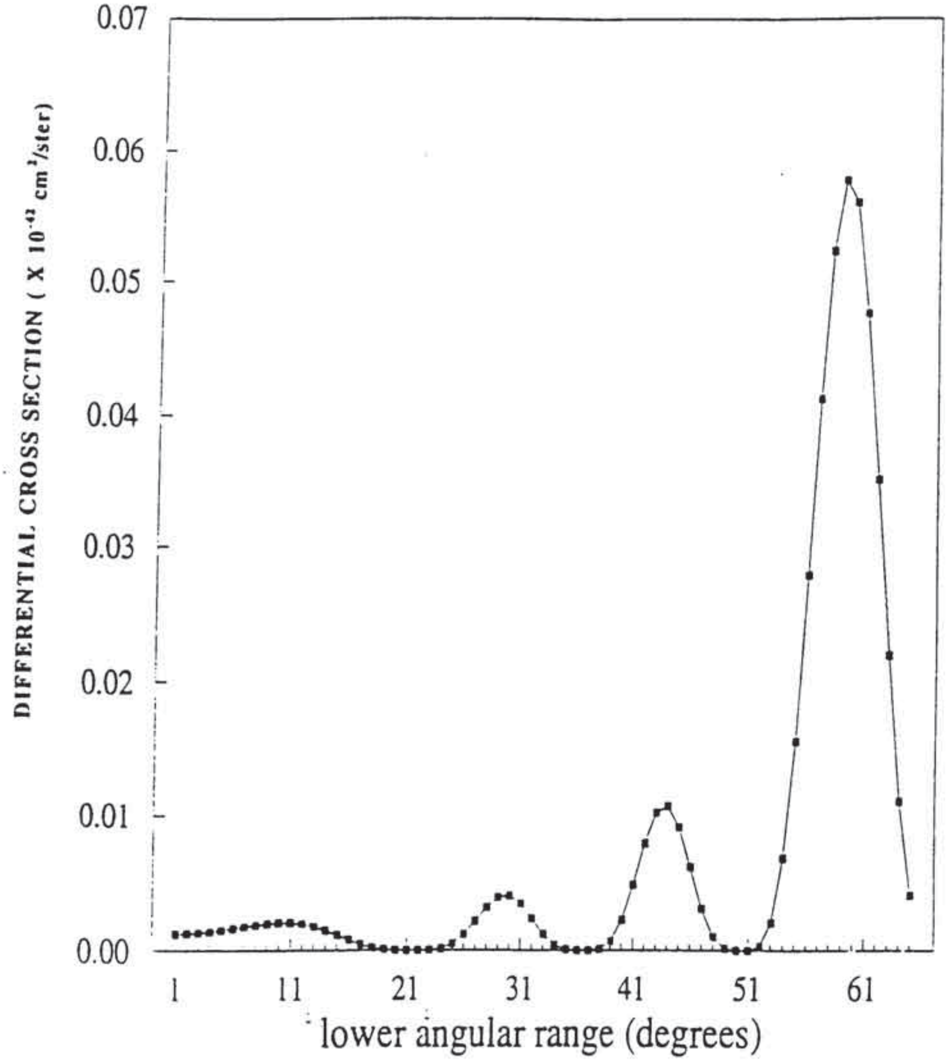


FIGURE 11

CONTRIBUTION OF THE FORM FACTOR F_V TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

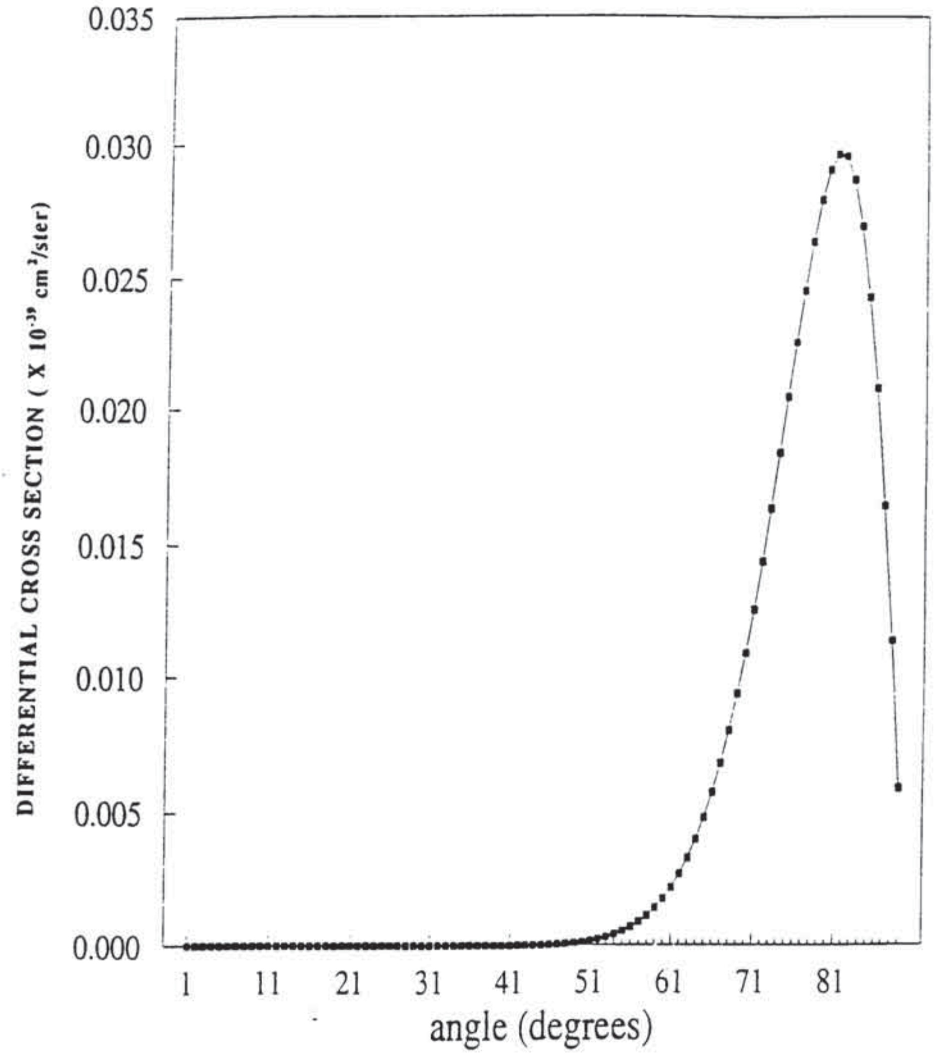


FIGURE 12

CONTRIBUTION OF THE FORM FACTOR F_V TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

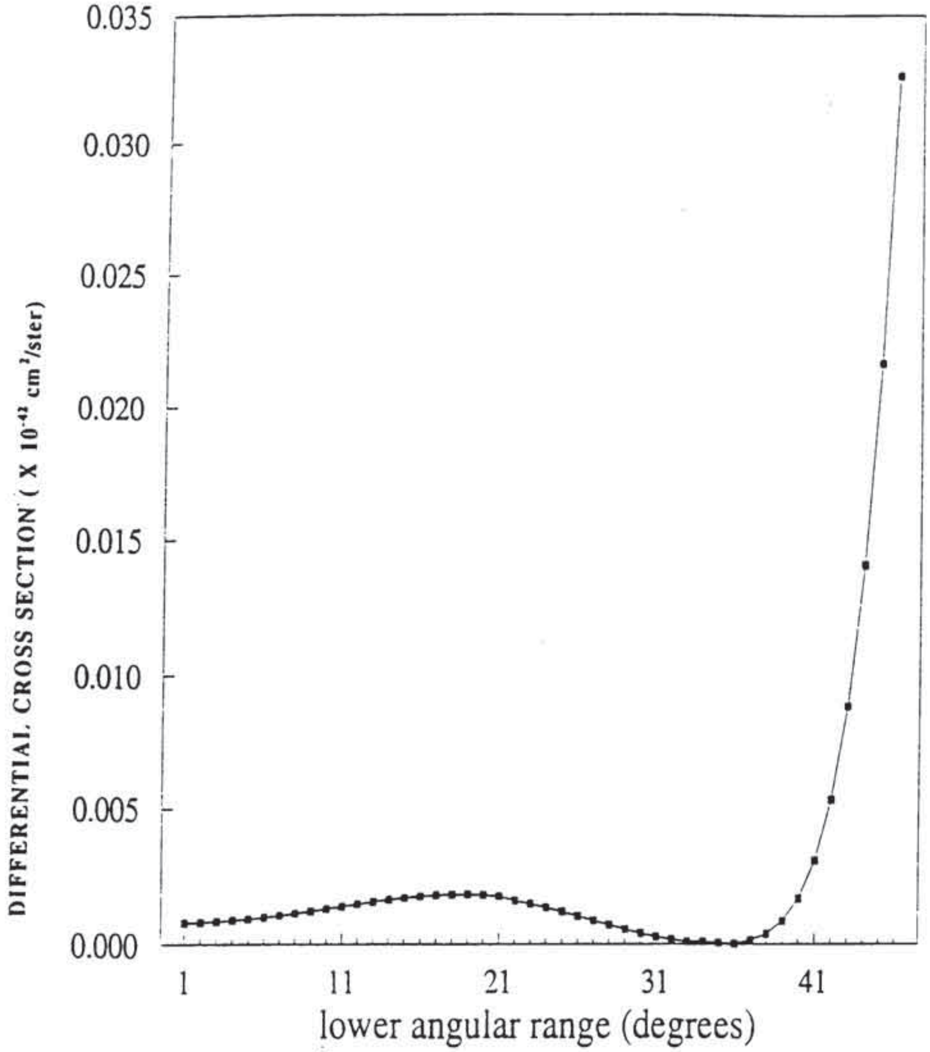


FIGURE 13

CONTRIBUTION OF THE FORM FACTOR F_V TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=2000MeV

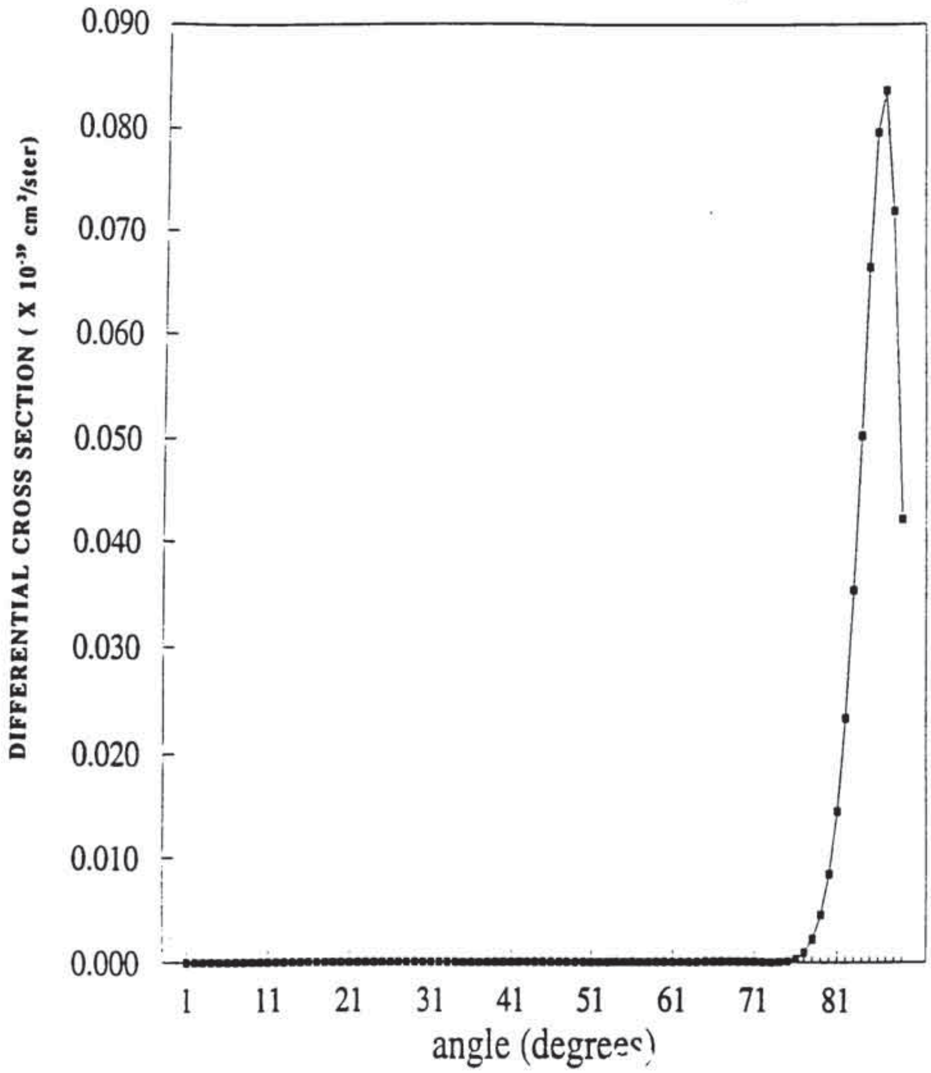


FIGURE 14

CONTRIBUTION OF THE FORM FACTOR F_V TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=2000MeV

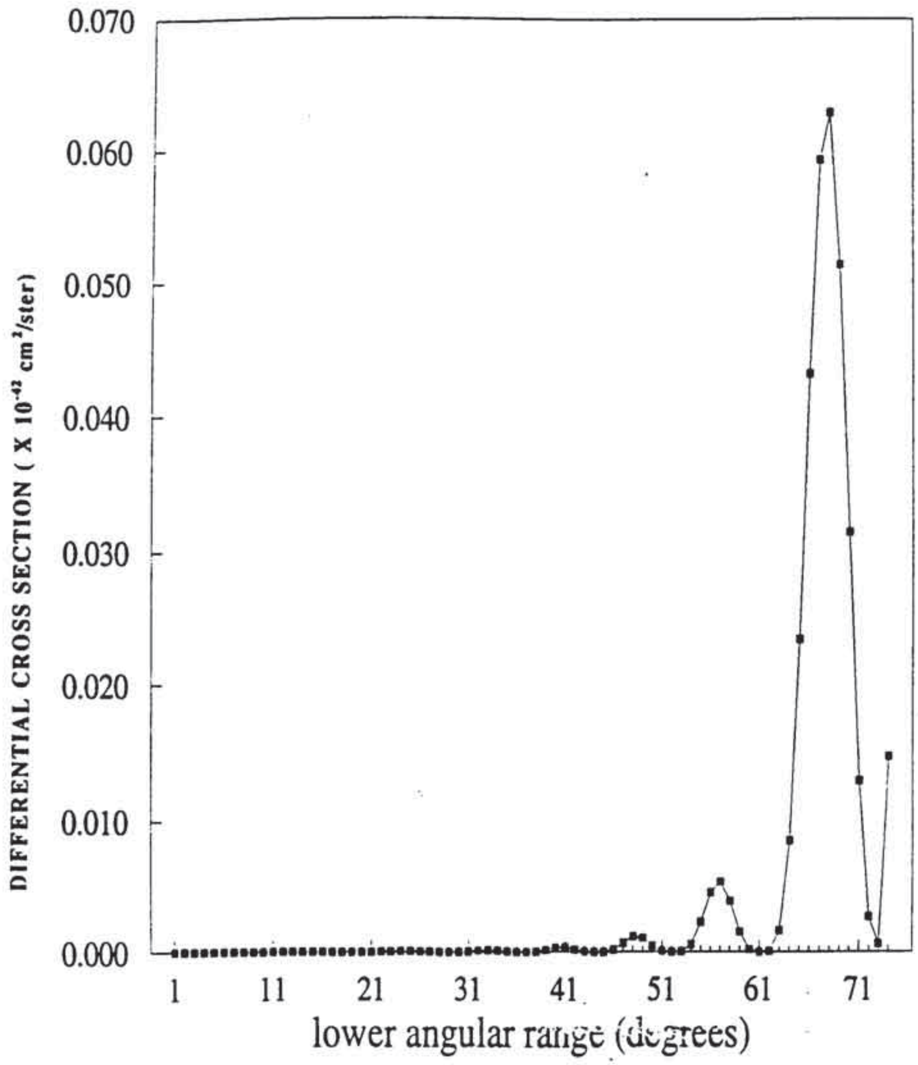


FIGURE 15

CONTRIBUTION OF THE FORM FACTOR F_A TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

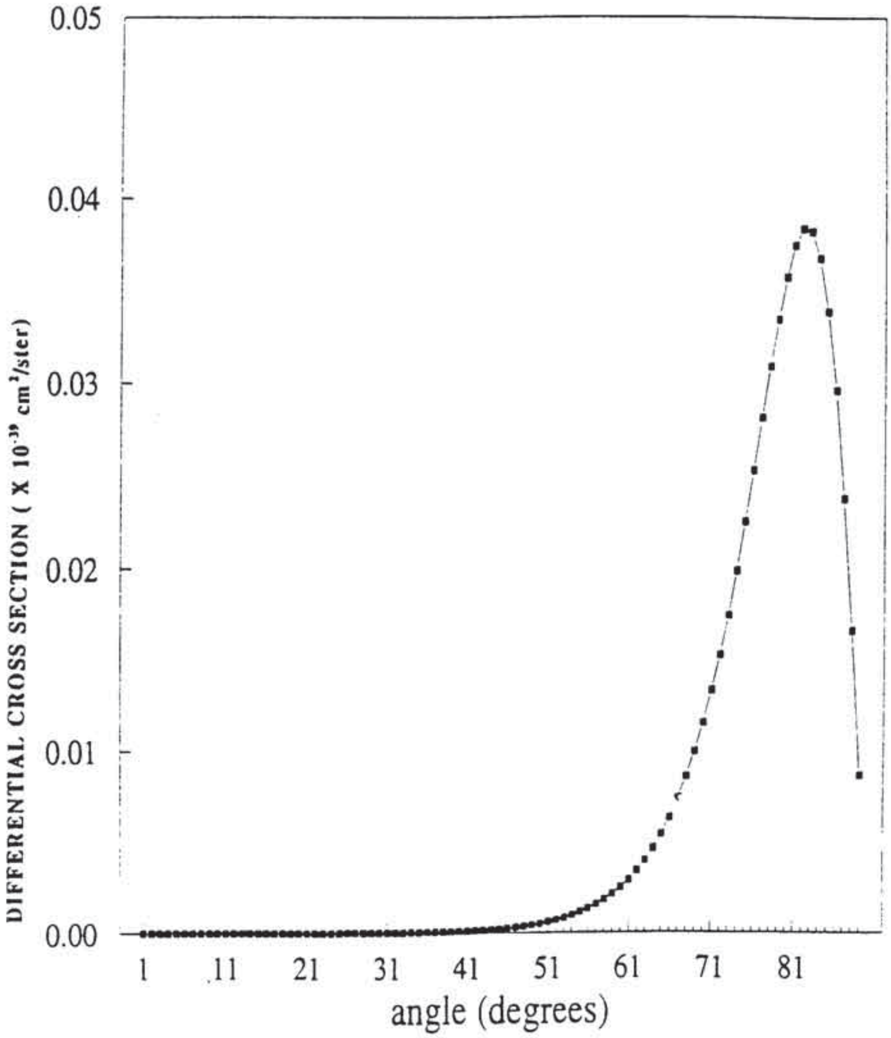


FIGURE 16

CONTRIBUTION OF THE FORM FACTOR F_A TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

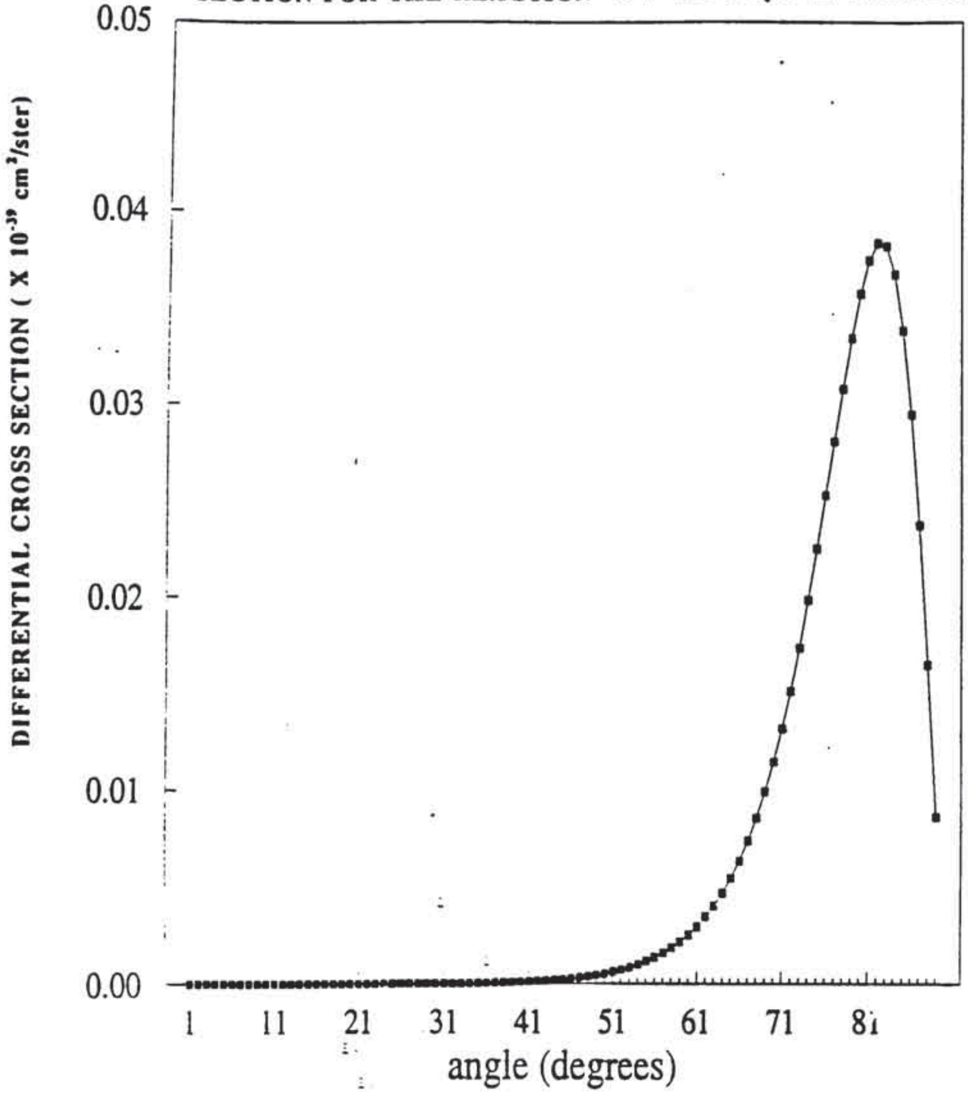


FIGURE 17

CONTRIBUTION OF THE FORM FACTOR F_A TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=2000MeV

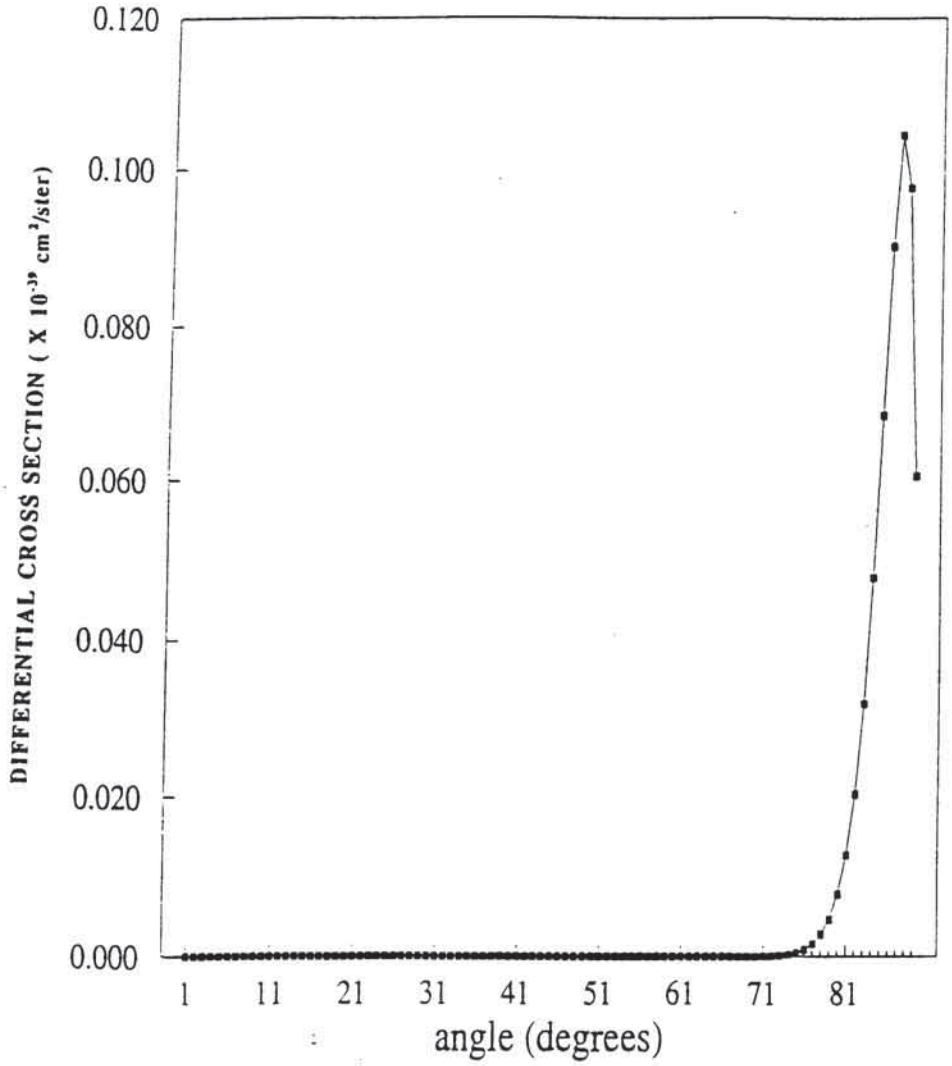


FIGURE 18

CONTRIBUTION OF THE FORM FACTOR F_A TO THE DIFFERENTIAL CROSS SECTION FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$ FOR E=500MeV

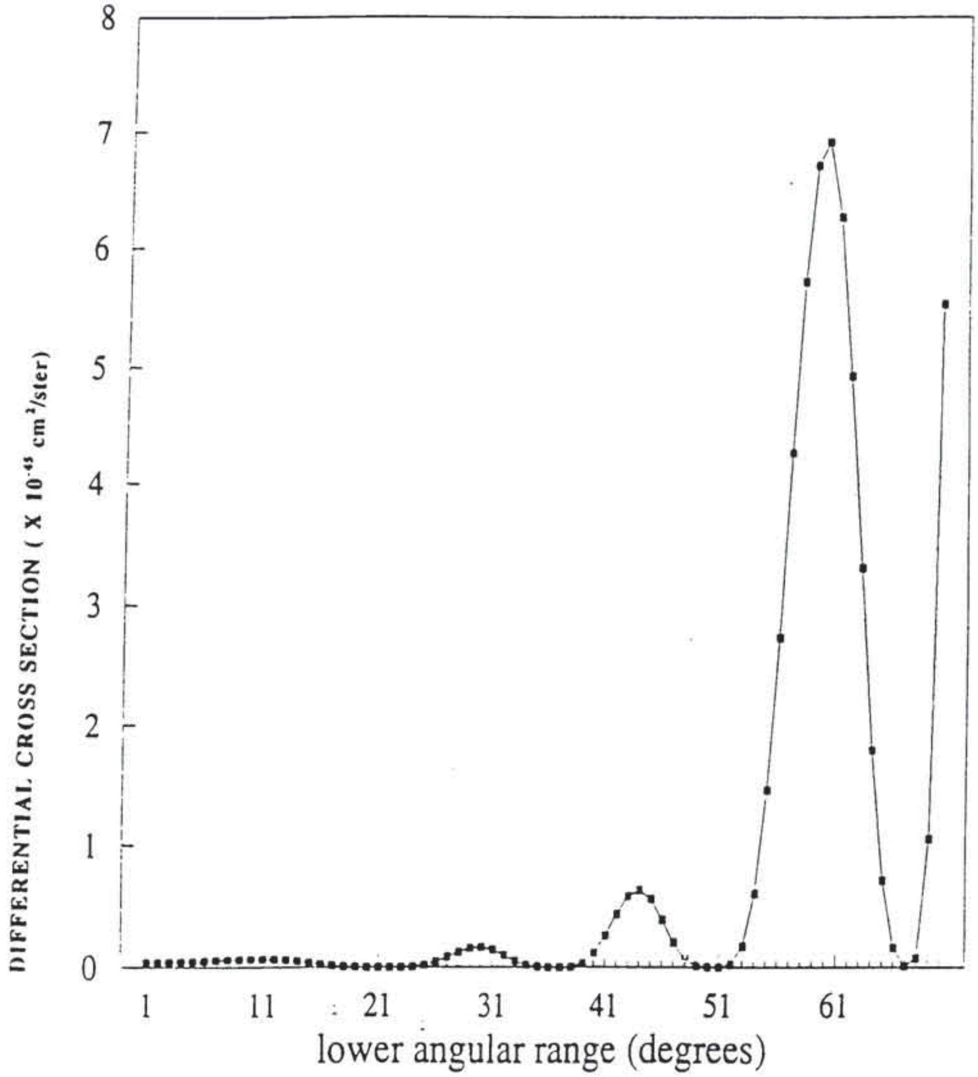


FIGURE 19

VARIATION OF $|q^2|$ WITH ANGLE AT 500 MeV AND 2000 MeV FOR THE REACTION $e^- + {}^3He \rightarrow \nu_e + {}^3H$

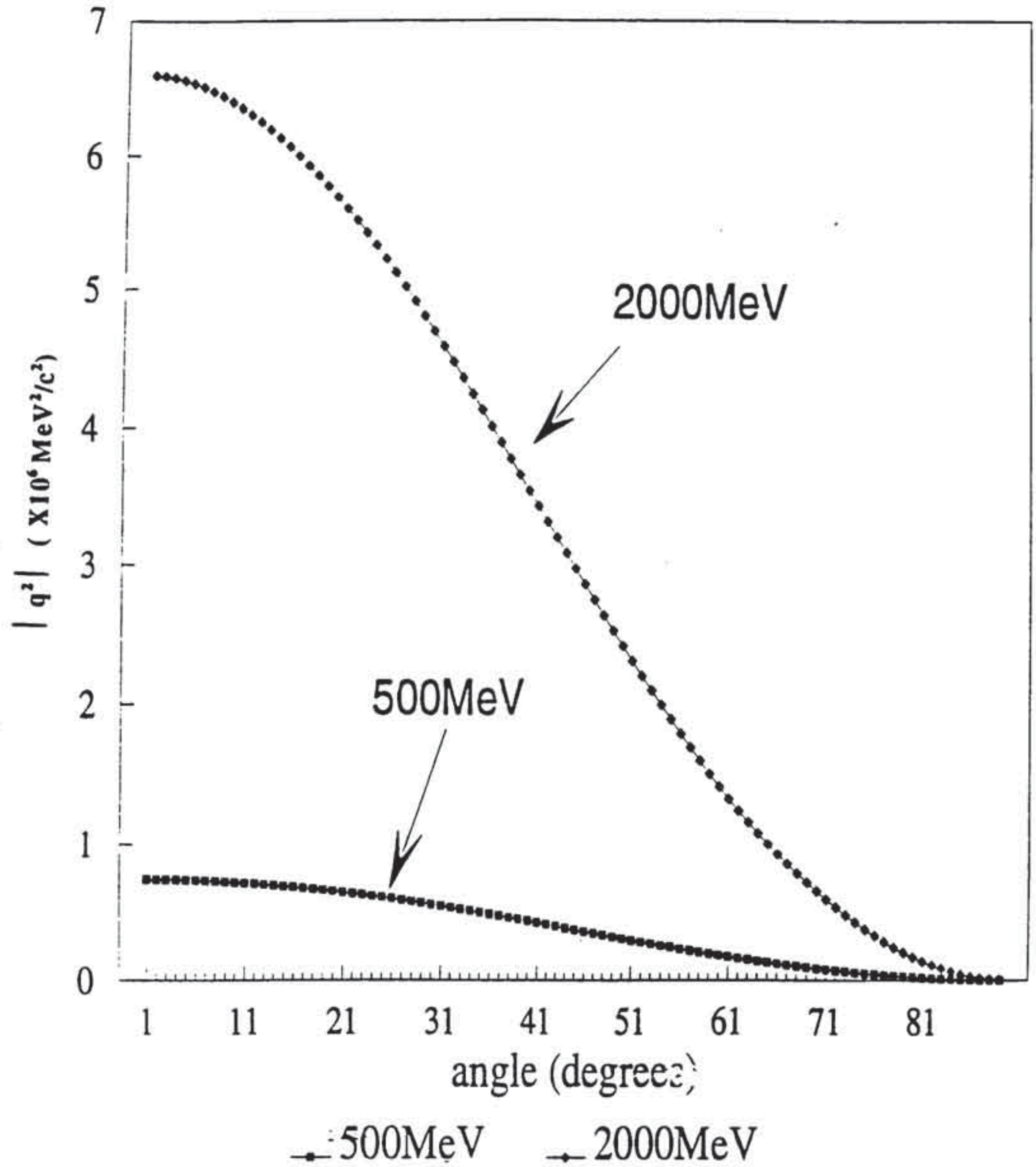


FIGURE 20

CHAPTER 4

Disscussion of Results

The results obtained are easily understood in terms of the model used here . There are three

factors that drive the cross-section. First the form factors all have factors,

of the type $\frac{1}{(1-q^2/M^2)^2}$ in them.

Now for our particular reaction $M^2 \approx 4.5(m_\pi)^2$ to $6.25(m_\pi)^2$ for smaller q^2. These masses are relatively large in comparison to nuclei such as ^{12}C where $M^2 \approx 2.86\ (m_\pi)^2$ so that the fall-off with increasing $|q^2|$ for the ^3He case is not as rapid as would be the case for larger nuclei. Second the matrix element is proportional to E_ν, the neutrino energy. Third the cross section is also proportional to $|\mathbf{p_f}|$, the magnitude of the space momemtum of the nucleus. These three considerations determine our results.

Thus for low incident electron energy, q^2 is small over the scattering range and the form factors are almost constant. The cross section is therefore relatively flat until $|q^2|$ approaches its minimum. At that point $|\mathbf{p_f}|$ is falling as the final state nucleus energy approaches its rest mass.

This can be seen most clearly in Figure 7, where the cross section for the 100 Mev case is almost flat but by 500 Mev, $|q^2|$ is large enough so that the form factors suppress the results at higher $|q^2|$. In figure 20 , the relationship between $|q^2|$ and θ is shown. The values of $|q^2|$ fall with increasing angle but reach much higher values for higher values of the initial electron energies. We should note that a low $|q^2|$ means a high out- going neutrino energy, and a low final state nucleus energy.

At higher $|q^2|$, the form factors suppress the cross section by many orders of magnitudes. This can be seen for processes in the 2 to 6 Gev range (see fig 3-6). Therefore the high $|q^2|$ part of the differential cross section is suppressed. Despite this however, E is large in the low $|q^2|$ range (which corresponds to the larger angles) resulting in E_v being large in this region, thus driving up the cross section until the falling values for $|\mathbf{p_f}|$ pull it down. This behaviour is very clearly seen in our results.

Thus the fundamental problem for an experimentalist interested in this reaction is that it is most observable at low $|q^2|$, but this is just the region in which the energy of the final state nucleus is not large. Thus although the differential cross section is large, the final state (recoil nucleus) may be difficult to observe . In addition there may be other processes leading to the same final state nucleus which are difficult to observe. However if these problems can be overcome , reactions such as the one described here may be a useful addition to the processes by which the hadronic weak interaction may be probed.

The discussion of the determination of the possible form factors will now be discussed. At lower $|q^2|$ values F_V and F_A are comparable for the E= 2 Gev case and might be simultaneously determined, but the contribution of F_M is probably too small to be observed. This can be seen in figures10, 14, and 18. At higher $|q^2|$ the contribution from $F_M(q^2)$ becomes dominant as can be seen from figures 11, 15, and 19. Unfortunately these cross sections are too small to be observed in the near future.

For 500 Mev electron scattering, the same trends are still visible. For low $|q^2|$, the cross section is dominated by $F_A(q^2)$ and $F_V(q^2)$ (see figures 8,12 and 16). But at high $|q^2|$

$F_M(q^2)$ becomes comparable to the other form factors(see figure 9, 13, and 17). However the cross sections are again too small for any possible measurements.

To sum up at low $|q^2|$ at both low and high E, it might be possible to measure these cross sections and to determine $F_A(q^2)$ and $F_V(q^2)$.

If these form factors are as expected from CVC and eq(16), then it might be possible to use muon capture results to determine $F_P(q^2)$. This form factor , the so called pseudoscalar form factor, has proved very elusive. There are several models for generating $F_P(q^2)$ but it is very difficult to obtain values for it due to the uncertainty in the other form factors. We note from above that although $F_M(q^2)$ was not obtainable directly at low $|q^2|$ its contribution to muon capture is small and if measurements confirm the CVC values for $F_V(q^2)$, the expectation would be that $F_M(q^2)$ is also given correctly by CVC.

The experiments needed to measure the cross sections calculated here are very difficult. Increasing the electron energy does drive up the cross sections, but the peaks become exceedingly narrow. It will be interesting to obtain the opinions of the experimentalists, whose task it might be to perform the necessary measurements.

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