2-19-2007

High energy three-body breakup of three-nucleon systems

Tigran Abrahamyan
Florida International University

DOI: 10.25148/etd.FI13101512

Follow this and additional works at: <https://digitalcommons.fiu.edu/etd>

Part of the [Physics Commons](#)

Recommended Citation

Abrahamyan, Tigran, "High energy three-body breakup of three-nucleon systems" (2007). *FIU Electronic Theses and Dissertations*. 1107.
<https://digitalcommons.fiu.edu/etd/1107>

This work is brought to you for free and open access by the University Graduate School at FIU Digital Commons. It has been accepted for inclusion in FIU Electronic Theses and Dissertations by an authorized administrator of FIU Digital Commons. For more information, please contact dcc@fiu.edu.

FLORIDA INTERNATIONAL UNIVERSITY

Miami, Florida

HIGH ENERGY THREE-BODY BREAKUP OF
THREE-NUCLEON SYSTEMS

A dissertation submitted in partial fulfillment of the

requirements for the degree of

DOCTOR OF PHILOSOPHY

in

PHYSICS

by

Tigran Abrahamyan

2007

To: Interim Dean Mark Szuchman
College of Arts and Sciences

This dissertation, written by Tigran Abrahamyan, and entitled High Energy Three-Body Breakup of Three-Nucleon Systems, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

John T. Landrum

Rajamani Narayanan

Joerg Reinhold

Mark I. Strikman

Misak M. Sargsian, Major Professor

Date of Defense: February 19, 2007

The dissertation of Tigran Abrahamyan is approved.

Interim Dean Mark Szuchman
College of Arts and Sciences

Dean George Walker
University Graduate School

Florida International University, 2007

DEDICATION

To my father, Valeri Abrahamyan, and my mother, Margarit Abrahamyan.

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my advisor Dr. Misak M. Sargsian for his guidance, support and encouragement throughout my graduate education. His continuous efforts to broaden my knowledge in theoretical physics are sincerely appreciated by me.

I am very grateful to all the members of my committee; Dr. John T. Landrum, Dr. Rajamani Narayanan, Dr. Joerg Reinhold and Dr. Mark I. Strikman for reading the manuscript and their comments.

I also wish to thank Dr. Richard A. Arndt from the George Washington University for the help with nucleon-nucleon partial-wave analysis numerical package.

Many thanks to all the faculty and staff of the Florida International University Physics Department for their constant support during the course of my graduate studies.

Finally, I would like to acknowledge financial support of a Florida International University Dissertation Year Fellowship and also thank all the staff of the FIU Graduate School for their help and consideration.

ABSTRACT OF THE DISSERTATION
HIGH ENERGY THREE-BODY BREAKUP OF
THREE-NUCLEON SYSTEMS

by

Tigran Abrahamyan

Florida International University, 2007

Miami, Florida

Professor Misak M. Sargsian, Major Professor

The main goal of this dissertation was to study two- and three-nucleon Short Range Correlations (SRCs) in high energy three-body breakup of ^3He nucleus in $^3\text{He}(e, e'NN)N$ reaction. SRCs are characterized by quantum fluctuations in nuclei during which constituent nucleons partially overlap with each other.

A theoretical framework is developed within the Generalized Eikonal Approximation (GEA) which upgrades existing medium-energy methods that are inapplicable for high momentum and energy transfer reactions. High momentum and energy transfer is required to provide sufficient resolution for probing SRCs. GEA is a covariant theory which is formulated through the effective Feynman diagrammatic rules. It allows self-consistent calculation of single and double re-scatterings amplitudes which are present in three-body breakup processes. The calculations were carried out in detail and the analytical result for the differential cross section of $^3\text{He}(e, e'NN)N$ reaction was derived in a form applicable for programming and numerical calculations. The corresponding computer code has been developed and the results of computation were

compared to the published experimental data, showing satisfactory agreement for a wide range of values of missing momenta.

In addition to the high energy approximation this study exploited the exclusive nature of the process under investigation to gain more information about the SRCs. The description of the exclusive ${}^3\text{He}(e, e'NN)N$ reaction has been done using the formalism of the nuclear decay function, which is a practically unexplored quantity and is related to the conventional spectral function through the integration of the phase space of the recoil nucleons. Detailed investigation showed that the decay function clearly exhibits the main features of two- and three-nucleon correlations. Four highly practical types of SRCs in ${}^3\text{He}$ nucleus were discussed in great detail for different orders of the final state re-interactions using the decay function as an unique identifying tool.

The overall conclusion in this dissertation suggests that the investigation of the decay function opens up a completely new venue in studies of short range nuclear properties.

TABLE OF CONTENTS

CHAPTER	PAGE
1 Introduction	1
2 General Discussion of Electro-Nuclear Reactions	8
3 Final State Interactions in ${}^3\text{He}(e, e'NN)N$ Reactions	11
4 Generalized Eikonal Approximation in High Energy eA Scattering	20
4.1 Reduction Theorem	21
4.2 Effective Feynman Diagram Rules	26
5 Calculation of the Scattering Amplitude of $e + {}^3\text{He} \rightarrow e' + p + p + n$ Reaction	31
5.1 Plane Wave Impulse Approximation and Pair Distortion Amplitudes .	32
5.2 Single Rescattering Amplitudes	36
5.3 Double Rescattering Amplitudes	41
6 Cross Section	45
7 Nuclear Decay Function Formalism	47
7.1 Decay Function Within Impulse Approximation	49
7.2 Decay Function Within the Distorted Wave Impulse Approximation	50
8 Technical Aspects of Numerical Estimates	53
8.1 Ground State Wave Function	53
8.2 Pair Distortion	53
8.3 Final State Interaction	55
9 Studying Short-Range Nucleon Correlation in the Nuclei	57
9.1 Two-Nucleon Correlations	58
9.2 Three-Nucleon Correlations	59
9.3 Spectral Function	60
9.3.1 Pair Distortion Effects	63
9.3.2 Final State Interaction (FSI) Effects	66
9.3.3 Summary on Numerical Studies of the Spectral Function . . .	72
9.4 The Decay Function	73
9.4.1 Pair Distortion Effects	79
9.4.2 Final State Interaction	81
9.4.3 Conclusion on Numerical Studies of the Decay Function. . . .	92

10	Three Body Forces	93
10.1	Conclusion on Three-Nucleon Forces Studies	98
11	Comparison with Experiment	100
11.1	Semiexclusive Three Body Break up of ^3He	100
11.2	GEA Prediction	101
12	Conclusions	106
	References	110
	Appendices	114
	Vita	139

LIST OF FIGURES

FIGURE		PAGE
1	High energy electro-disintegration of the three-nucleon system.	8
2	Four main contributing processes.	11
3	The $ p_{mz} $ as a function of x	13
4	The p_m dependence of the differential cross sections of $d(e, e'p)n$	14
5	The Q^2 dependence of W_1/W_2 from inclusive $d(e, e')X$ reaction. . . .	16
6	Proton-proton and proton-neutron total cross sections.	19
7	Infinite diagrams for knocked-out nucleon reinteractions.	20
8	Diagram for hadron-deuteron scattering.	21
9	Reduction theorem for hadron-deuteron scattering.	26
10	Diagram for n -fold rescattering.	27
11	Three-body break up diagrams.	31
12	Plane Wave Impulse Approximation and Pair Distortion Diagrams. . .	32
13	Single Rescattering Diagrams.	37
14	Double Rescattering Diagrams.	41
15	Total cross section for proton-neutron charge-exchange scattering. . .	51
16	Definition of the type 2N-I (a) and type 2N-II (b) correlations.	59
17	Definition of the type 3N-I (a) and type 3N-II (b) correlations.	59
18	Spectral Function for ${}^3\text{He}(e, e'N)X$ reaction.	61
19	Missing energy distribution of the Spectral Function.	62
20	Missing momentum dependence of the ratio of the Spectral Function. .	64

21	Spectral Function calculations with 3N forces included.	65
22	Angular dependence of transparency T	67
23	Missing energy dependence of transparencies.	70
24	E_m dependence of the Spectral Function for 2N-II SRCs.	71
25	E_m dependence in parallel and anti-parallel kinematics.	73
26	Dependence of the Decay Function on the momenta of the recoil pp -pair.	74
27	Dependence of the Decay Function on the momenta of the recoil pn -pair.	75
28	The (E_m, p_{r2}) distribution of the Decay Function.	77
29	Dependence of the Decay Function on θ_{23} and E_m	79
30	Signatures of the type 3N-I and type 3N-II correlations.	80
31	Pair Distortion effects for different SRCs.	82
32	Dependence of the Decay Function on the θ_{r2} for the type 2N-I SRC.	83
33	Dependence of the Decay Function on the p_{r2} for the type 2N-I SRC.	84
34	Dependence of the Decay Function on the θ_{r2} for the type 2N-II SRC.	86
35	Dependence of the Decay Function on the p_{r2} for the type 2N-II SRC.	87
36	Dependence of the Decay Function on the p_{r2} for the type 3N-I SRC.	89
37	Dependence of the Decay Function on the θ_{r2} for the type 3N-I SRC.	90
38	Dependence of the Decay Function on the θ_{r2} for the type 3N-II SRC.	91
39	Differential cross section for 65 MeV proton-deuteron scattering.	94
40	Dependence of the Decay Function on p_{r2} for the type 3N-II SRC.	95
41	Three-nucleon forces in the Decay Function distribution for 3N-II SRC.	96
42	Three-nucleon forces in the Decay Function distribution for 2N-I SRC.	97

43	Cross section results for the $p_m = 270$ MeV/c	103
44	Cross section results for the $p_m = 340$ MeV/c	104
45	Cross section results for the $p_m = 440$ MeV/c	104
46	Cross section results for the $p_m = 620$ MeV/c	105
47	Cross section results for the $p_m = 800$ MeV/c	105
48	Poles on the complex plain.	123
49	Integral over $d\mu = d(\cos \theta)$	124
50	Proton-proton total cross section.	126
51	Neutron-proton (proton-neutron) total cross section.	128
52	The ratio of the real to imaginary part of the proton-proton amplitude.	130
53	The ratio of the real to imaginary part of the proton-neutron amplitude.	132
54	Diffraction slope parameter for proton-proton forward scattering. . . .	134
55	Diffraction slope parameter for proton-neutron forward scattering. . .	136

LIST OF SYMBOLS

- $(E_e, \mathbf{k}_e)$: four-momentum of incoming electron
- $(E_{e'}, \mathbf{k}_{e'})$: four-momentum of scattered electron
- $(q_0, \mathbf{q}) \equiv (E_e - E_{e'}, \mathbf{k}_e - \mathbf{k}_{e'})$: four-momentum of virtual photon
- $Q^2 \equiv \mathbf{q}^2 - q_0^2$: negative square of four-momentum of virtual photon
- $\psi_{^3He}(p_1, p_2, p_3)$: wave function of ^3He nucleus with preexisting momenta of constituent nucleons p_1, p_2 and p_3
- $(E_f, \mathbf{p}_f)$: final four-momentum of knocked-out nucleon
- $(E_{r2}, \mathbf{p}_{r2})$: final four-momentum of the first spectator nucleon
- $(E_{r3}, \mathbf{p}_{r3})$: final four-momentum of the second spectator nucleon
- $\mathbf{p}_m = \mathbf{p}_f - \mathbf{q}$: missing momentum of the reaction
- $E_m = m + E_{A-1} - M_A$: missing energy of the reaction (m is the mass of the nucleon, M_A — mass of the nucleus and E_{A-1} is the energy of residual $A - 1$ nuclear system)
- $x = Q^2/2mq_0$: Bjorken scaling variable
- $\mathbf{p}_{23} = \frac{\mathbf{p}_2 - \mathbf{p}_3}{2}$: relative momentum of p_2 and p_3 nucleons
- $\Gamma_A(p_1, p_2, p_3)$: nucleus A to constituent nucleons transition vertex function
- $\Gamma_{\gamma^* N}^\mu$: vertex function of electron-nucleon interaction
- $\Gamma_{NN, int}^+(p_2, p_3)$: reinteraction vertex function of two spectator nucleons
- $\Gamma_{NN}^+(p_2, p_3)$: generalized reinteraction vertex function of two spectator nucleons which includes also the part corresponding to two outgoing plane waves for the recoil nucleons
- $(E_{p'_i}, \mathbf{p}'_i)$: intermediate four-momentum of rescattered nucleon after the scattering, $i = 1, 2, 3$

1 Introduction

Presently, our knowledge of the microscopic structure of nuclei is limited mainly to the dynamics of single-nucleon states characterized by the momenta $\leq 200 - 250$ MeV/c and excitation energies ≤ 100 MeV (significantly smaller than the nucleon mass, ~ 940 MeV). The dynamics of the nuclear structure at very short distances in which nucleons may be significantly overlapped is practically unexplored. Such configurations, called Short-Range Correlations (SRC), may provide instantaneous hadronic densities up to eight times larger than average nuclear densities, comparable to those that may occur in neutron stars. The probability for such fluctuations is not negligible. Both experimental high energy data and theoretical calculations of nuclear wave functions with realistic NN potentials indicate that in heavy nuclei, nucleons partially overlap $\sim 25\%$ of a time.

Normally nucleons are closely packed but overlap very little. The proton electromagnetic radius is ~ 0.86 fm, and in the ground state of infinite nuclear matter the center-to-center (average) distance between nearby nucleons is about 1.7 fm. However, quantum fluctuations may occur, during which nucleons may find themselves in SRC. Well-designed experiments may expose the physics occurring when nucleons occasionally encounter each other at smaller than average distances. For example, if such a fluctuation reduces the center-to-center separation to 1 fm, then there is a significant region of overlap. Using the empirical uniform nucleon charge distribution, $\rho(r) = \frac{Q_0^3}{8\pi} \exp^{-Q_0 r}$ with $Q_0 = 4.27 \text{ fm}^{-1}$, the density in the region of overlap is about four to five times that of normal nuclear matter, $\rho_0 = 0.17 \text{ nucleon/fm}^3$.

Traditional nuclear physics describes nuclear properties in terms of protons and neutrons. In an attempt to understand the physics of the nucleus, a full theoretical explanation would need a solution to a quantum mechanical many-body problem, with not only a set of coupled equations describing the mutual interactions of the nucleons, but also the short range correlations of all these nucleons as they act on others. However, so far this kind of SRC is not well understood. In the most comprehensive approach the nuclear wave functions are calculated using NN interaction potentials which may be constructed from phase space analysis of nucleon-nucleon scattering data. There are several such potentials: CD Bonn [1], Argonne V18 [2], Nijmegen-93 [3], etc. However these potentials do not solve fully the problem of 2N SRC, since they are not well defined at distances < 1.2 fm. Even more, they tell nothing about the three-nucleon (3N) SRCs. The fact that 3N forces are important was first discussed by Wigner [4]. For example, with all the above mentioned modern NN interaction potentials the calculated binding energy of the triton is underestimated by roughly $0.5 - 1$ MeV. Unless our understanding of these NN forces is badly deficient, the correct estimate requires the existence of additional three-nucleon forces. There are several 3N models available for theoretical calculations such as the Tucson-Melbourne [5] and the Urbana IX [6] potentials.

Short range nucleon correlations are the best testing grounds for the dynamics of both 2N and 3N forces. The main goal of our research is to study two- and three-nucleon SRCs in high energy electro-nuclear processes. The electro-nuclear reactions are best suited for high momentum and energy transfer processes since the electron

coupling with the virtual photon can be precisely calculated in QED. Additionally, high momentum and energy transfer reactions may provide sufficient resolution to probe the microscopic structure of SRCs.

In our research we are particularly interested in the following region of four momentum transfer: $1.5 \leq Q^2 \leq 4 \text{ GeV}^2/c^2$. Here, the lower limit is defined by the condition that the knocked-out hadronic system is energetic enough such that high energy theoretical approaches become applicable. The upper limit is defined from the conditions that color coherence effects are small and the produced hadronic state represents a single state (e.g. nucleon) but not the superposition of different hadronic states – wave packets (see e.g. [7]). Thus we have a unique kinematic window where the theoretical methods of high energy scattering may be applied to the rather simple dynamical system consisting of nucleons only.

In addition to the increase of the transferred momenta and energies, the increase of the degrees of exclusiveness of electro-nuclear reactions, when more products are registered in the final state of the reaction allows us to gain more information about the microscopic structure of the short-range nucleon correlations. The advantage of considering the Helium isotope, ^3He , related to the fact that quantum mechanical methods of solving the three-body problems for structureless particles interacting with a given local potential is presently well-established (see e.g. [8]). As a result one has good control of the baseline calculations.

The combination of two factors: high energy and momentum transfer and exclusiveness of the reaction makes the three-body electro-nuclear breakup reactions a

unique laboratory for studying various nuclear properties, particularly Short Range Correlations in nuclei.

Since nuclei are rather loosely bound systems the overlap of nucleons happens in stages, in which first two nucleons overlap yielding a two nucleon SRC. At higher density fluctuations it is possible that overlap of three-nucleon correlations may occur. The latter happens with significantly less probability. The recent estimates [9] suggest one to two orders of magnitude decrease in the probability of 3N SRC as compared with 2N SRC. For example, for the ^4He absolute per-nucleon probability of 2N SRC found to be 15.4% whereas 3N SRC occurs in 0.42% of cases. For ^{53}Fe these values compare as 23.9% and 0.83%, respectively. For the ^3He Ref. [9] cites probability of 8.0% for 2N SRC and 0.18% for 3N SRC. If some properties of two-nucleon correlations can, in principle, be accessed from phase-shifts of nucleon-nucleon scattering (i. e. the dominance of the tensor force, etc.) the 3N SRC are genuinely three-nucleon bound problem. One of the aims of present research was to establish theoretical framework for systematic studies of such configurations being probed in electronuclear reactions.

The observation of SRCs in nuclei has long been considered one of the most significant aims of nuclear physics. The problem has been a lack of experimental data at high-momentum transfer kinematics which could decisively discriminate between uncorrelated long-range effects and SRCs in the ground state of the nucleus. This is apparent in the case of dominance of the Impulse Approximation channel when no distortion happens due to long-range multi-step effects such as final state interactions (FSIs), meson exchange currents (MECs) and intermediate Δ -isobar contributions

(IC). However, FSIs, MECs and ICs can considerably alter the above mentioned clarity of the Impulse Approximation. Therefore, for studying SRCs we must consider the several distinct processes as:

- The Impulse Approximation (IA) channel, in which the virtual photon knocks-out the bound nucleon that propagates to the final state without further interactions,
- The Final State Interaction (FSI) channel, in which the knocked-out nucleon reinteracts with the residual hadronic system,
- Channels with Meson Exchange Currents (MEC), in which the virtual photon interacts with the exchanged (between two-nucleon system) mesons,
- The Isobar Current contribution channel (IC), in which the virtual photon produces a Δ -isobar which reinteracts with the residual nuclear system producing the final hadronic state.

Final state reinteractions in hadron induced nuclear reactions at higher energies ($E_h > 1 \text{ GeV}$) are often described within the approximation of the additivity of phases, acquired in the sequential rescatterings of high-energy projectiles off the target nucleons (Glauber model, [10]). This approximation made it possible to describe the data on elastic hadron-nucleus scattering at hadron energies $1 \text{ GeV} < E_h < 10 - 15 \text{ GeV}$.

There are many theoretical works that have appeared in the last several years where the same principle of Glauber rescattering has been applied to the description

of the cross sections of $A(e, e'N)(A - 1)$ reactions (see [11]–[20]). Many of these works discussed the reactions in which the cross sections were integrated over the excitation energies of the residual nuclear system and the kinematics were restricted to small momenta of the residual system, $\vec{p}_{A-1} \leq p_F$, where $p_F \approx 250 \text{ MeV}/c$ is the characteristic Fermi momentum of the nuclear system. In all these cases the Glauber approximation, which considers the nucleons in the nuclei as stationary scatterers, was a good approximation.

However, the Glauber approximation cannot be applied to the class of eA reactions in which the main emphasis is given to the high momentum of the bound nucleon and high excitation energies of the residual nuclei. This situation is especially important in studies of SRCs in which large values of missing momentum and excitation energy are involved.

These issues are addressed by the Generalized Eikonal Approximation (GEA) [21] which provides the theoretical framework to take into account the pre-existing momenta of the nucleon in SRC as well as the large values of excitation energies. GEA allows for the regrouping of a potentially infinite number of channels in hadronic reinteractions into the finite set of covariant Feynman diagrams using special rules [22]. Recently calculations within GEA produced very reasonable agreement with the JLAB data on deuteron electrodisintegration [23].

The main research topics of the present work are:

a) The development of GEA for calculation of the high energy breakup electro-nuclear reactions, involving ^3He nucleus.

- b) Studies of 2N and 3N Short Range Correlations by applying the Decay Function Formalism.
- c) The suggestion of certain new approaches in studies of three-nucleon forces.

This dissertation organized as follows. General discussion of electro-nuclear reactions is presented in Chapter 2. Next, in Chapter 3 we consider Final State Interactions in three-body breakup of ^3He nucleus and continue with Generalized Eikonal Approximation, presented in Chapter 4. Chapter 5 is devoted to the calculation of scattering amplitudes and results are summed up in Chapter 6, where the analytical form of cross section is presented. Chapter 7 concerns itself with the Decay Function formalism while the technical aspects of numerical estimates presented in Chapter 8. Detailed studies of Short Range Correlations within the Spectral Function and the Decay Function formalisms are presented in Chapter 9. We present some new approaches in studies of three-nucleon forces in Chapter 10, while in Chapter 11 contains comparison of our cross section calculations with the high energy three-body breakup double coincidence experiment data from Jefferson Lab Hall A. Conclusion in Chapter 12 wraps up the main body of the dissertation, with some relevant appendices attached at the end of this work.

2 General Discussion of Electro-Nuclear Reactions

In this dissertation we are considering exclusive electro-disintegration of three-body systems, on the specific example of the ^3He nucleus. The corresponding reaction is given by:

$$e + {}^3\text{He} \rightarrow e' + p + p + n, \quad (1)$$

Figure 1: High energy electro-disintegration of the three-nucleon system.

and shown schematically in Fig. 1. The discussion throughout assumes that high momentum $q \equiv k_e - k_{e'} = (q_0, \mathbf{q})$ is transferred to the ^3He nucleus. The main feature of the final state in these reactions is that it contains a fast nucleon that carries almost the entire momentum of the virtual photon $\mathbf{p}_f \approx \mathbf{q}$, with $|\mathbf{q}| = \sqrt{Q^2 + q_0^2} \sim$ several GeV/c . The two other hadrons (p_{r2} and p_{r3}) in the final state have relatively low momenta ($p_{r2,r3} \sim$ hundreds MeV/c).

Hereafter, the z direction is defined by $\mathbf{q}$ and the scattering plane is the plane of $\mathbf{q}$ and $\mathbf{k}_{e'}$ vectors.

Thus the major kinematic requirements are the following:

$$\begin{aligned}
(a) \quad & 4 \text{ GeV}^2/c^2 \geq Q^2 \geq 1.5 \text{ GeV}^2/c^2, \\
(b) \quad & \mathbf{p}_f \approx \mathbf{q}, \\
(c) \quad & p_f \gg p_m, p_{r2}, p_{r3} \sim \text{few hundreds MeV}/c.
\end{aligned} \tag{2}$$

where $\mathbf{p}_m = \mathbf{p}_f - \mathbf{q}$ is the missing momentum of the reaction. We define the missing energy as $E_m = m + E_{A-1} - M_A - \frac{p_m^2}{M_{A-1}}$, which characterizes the excitation of the residual nuclear system (m is the mass of the nucleon and E_{A-1} is the energy of the residual $A - 1$ nuclear system).

The lower limit in Eq.(2(a)) is the main requirement that provides a high-momentum transfer in the electrodisintegration. This condition together with Eqs.(2(b)) and (c) allows us to identify the p_f as a knocked-out nucleon, while p_{r2} and p_{r3} can be considered as recoil nucleons which do not interact directly with the virtual photon. The upper limit in Eq.(2(a)) is defined from the condition that the color coherence effects are small and the produced hadronic state represents a single state (i. e. nucleon) but not a superposition of different hadronic states in the form of a wave packet [7]. Also, due to the conditions established by Eq.(2(c)), the nucleons are the basic degrees of freedom describing the interacting nuclear system. From a technical point-of-view, this means that for the set of noncovariant diagrams, comprising the covariant scattering amplitude, one can neglect the noncovariant diagrams containing

non-nucleonic degrees of freedom (e.g. negative energy projections of bound nucleon spinors or vacuum fluctuations). In this case one can reduce the nuclear vertices to the nonrelativistic nuclear wave functions of the nuclei.

Another consequence of our choice of these kinematics is the availability of certain small parameters. If one constructs the $\pm$ components of four-momenta (which correspond to the energy and longitudinal momentum of the particle in the light-cone reference frame) $p_{\pm} = p_0 \pm p_z$, where z direction is defined by the direction of virtual photon momentum $\mathbf{q}$, then one observes that the condition of Eq.(2) corresponds to the smallness of the following combinations:

$$\frac{q_-}{q_+} \approx \frac{mx}{2q} \ll 1 \quad \text{and} \quad \frac{p_{f-}}{p_{f+}} \approx \frac{m^2}{4p_f^2} \ll 1, \quad (3)$$

were $x = Q^2/2mq_0$. The availability of these *small parameters* is one of the important features of high energy scattering as compared to the low and intermediate energy reactions. We will see below how these conditions will simplify the theoretical treatment of exclusive three-nucleon electrodisintegration reactions.

3 Final State Interactions in ${}^3\text{He}(e, e'NN)N$

Reactions

The first exclusive electro-nuclear reactions which have been studied at intermediate energies confirmed the expectations that the complexity of the hadronic system significantly restricts the unambiguous treatment of the underlying dynamics of the reaction [24, 25, 26]. In general, four main processes are contributing to the three-nucleon breakup of ${}^3\text{He}$ (Fig. 2): Impulse approximation (IA) channel (Fig. 2a), Final State Interaction (FSI, Fig. 2b), the Meson Exchange Currents (MEC, Fig. 2c) and the Isobar Current contribution (IC, Fig. 2c):

Figure 2: Impulse Approximation (a), final state interaction (b), meson exchange (c), and Δ -isobar contribution (d) diagrams.

There are several reasons for the dominance of FSI, MEC and IC diagrams at low and intermediate energies (small Q^2) in the kinematics with the large missing momentum, p_m , and missing energy E_m . While at large p_m and E_m the IA amplitude is defined by the nuclear wave function at short inter-nucleon distances, the FSI, MEC and IA amplitudes are defined by the nuclear wave functions of the average configurations. The dominance of MEC and IC amplitudes follows also from the

kinematical considerations that at small Q^2 high missing momenta p_m in exclusive $A(e, e'N)X$ reactions can be observed only at $x < 1$. It can be seen from Fig. 3, where we calculated the $|p_{mz}|$ dependence on x for reaction of Eq.(1) for the isolated two nucleon kinematics that at $Q^2 < 1 \text{ GeV}^2/c^2$ only $x < 1$ is appropriate for detection of large $|p_{mz}| \geq 300 \text{ MeV}/c$. For this case the final mass, W , produced on stationary nucleon, exceeds the mass of nucleon m , since it can be shown that:

$$W^2 = -Q^2 + 2q_0m + m^2 = -Q^2 + \frac{Q^2}{x} + m^2 = Q^2 \left(\frac{1-x}{x} \right) + m^2. \quad (4)$$

Therefore, one expects an enhanced contribution from inelastic channels, such as the Δ -excitation or MEC.

What concerns the final state interactions at small Q^2 , they are dominated by S -wave scattering and have broad angular distributions. Thus there exist no clear criteria how to isolate or suppress FSI with respect to IA.

In general terms the dominance of FSI, MEC and IA amplitudes means the impossibility to probe small space-time intervals in the nucleus using probes of larger size ($1/q \geq 1 \text{ fm}$).

In Fig. 4 we present the typical intermediate energy measurement, which demonstrates how large MEC and IC contributions are in the cross section at large missing momenta and low Q^2 . The calculations in the kinematic region of these experiments show that at large p_m MEC and IC significantly dominate the PWIA contribution.

With an increase in energy the situation changes qualitatively. It may sound paradoxical, but at high energy transfer, when the wavelength of the probe becomes much smaller than the sizes of interacting particles the situation becomes simpler.

Figure 3: The $|p_{mz}|$ as a function of x , for different values of Q^2 , for quasielastic ${}^3\text{He}(e, e'NN)N$ reaction. The lines between $Q^2 = 0.5 \text{ GeV}^2$ and $Q^2 = 4.5 \text{ GeV}^2$ curves correspond to the values of $Q^2 = 1.5, 2.5, 3.5 \text{ GeV}^2$.

Contribution of Meson Exchange Currents: The major problem we face in the estimation of MECs in $A(e, e'N)$ processes is that with an increase of energies the virtuality of the exchanged mesons grows proportional to Q^2 ($\gg m_{meson}^2$). Even though the idea of deeply virtual exchanged mesons is highly complicated or may even be meaningless (see discussion in Ref. [28]), one can still estimate its Q^2 -dependence as compared to the SRC contribution. Turning to more quantitative discussion, in a kinematic setting typical for studies of SRCs, in which the knocked-out nucleon carries almost

Figure 4: The p_m dependence of the differential cross sections of $d(e, e'p)n$ reactions at $Q^2 = 0.13 - 0.33 \text{ GeV}^2$. The data are from Ref. [25]. Solid and dashed lines correspond to the calculation of Arenhövel, Ref. [27], with and without MEC and IC contributions.

the entire momentum of the virtual photon (while the missing four-momentum of the recoil system does not change with Q^2), the Q^2 -dependence of the MEC amplitude can be estimated as follows:

$$\begin{aligned}
 A_{MEC}^\mu &\sim \int d^3p \cdot \Psi(p) \frac{J_m^\mu(Q^2)}{(Q^2 + m_{\text{meson}}^2)} \Gamma_{MNN}(Q^2) \\
 &\propto \int d^3p \cdot \Psi(p) \left(\frac{1}{(Q^2 + m_{\text{meson}}^2)^2 (1 + Q^2/\Lambda^2)^2} \right), \quad (5)
 \end{aligned}$$

where $J_m^\mu(Q^2)$ is the meson electromagnetic current proportional to the elastic form factor of the meson $\sim \frac{1}{Q^2 + m_{\text{meson}}^2}$, and $m_{\text{meson}}^2 \approx 0.71 \text{ GeV}^2/c^2$. For the meson-nucleon

vertices, $\Gamma_{MNN}(Q^2)$, we assume a dipole dependence with the characteristic parameter $\Lambda \sim 0.8 - 1 \text{ GeV}^2/c^2$.

Since the leading Q^2 -dependence in the SRC contribution comes from the nucleon elastic form factors, Eqn. (5) will result in an additional $\sim (1 + Q^2/\Lambda^2)^{-2}$ suppression of the MEC amplitude as compared to the SRC contribution. Note that this gives an upper limit to the MEC contribution, since for large Q^2 the quark counting rule predicts stronger Q^2 suppression for $\Gamma_{MNN}(Q^2) \sim \frac{1}{Q^6}$ [29]. Thus, one expects that MEC contributions will be strongly suppressed as soon as Q^2 is greater than m_{meson}^2 and Λ^2 , both $\sim 1 \text{ GeV}^2$. This conclusion is relevant only for small angle nucleon knock-out kinematics (optimal for studies of SRCs). In the large angle kinematics quark-exchange mechanism may become important similar to the large angle $\gamma d \rightarrow pn$ reactions [30].

As an experimental indication of MEC suppression at high Q^2 in Fig. 5 we demonstrate the Q^2 dependence of the ratios of inclusive structure functions W_1/W_2 for different values of E_{pn} . Note that $E_{pn} = 0$ corresponds to the deuteron threshold in which two nucleons have vanishing relative momentum. In that case MEC should be especially enhanced. The figure clearly indicates that MEC contribution decreases with the increase of Q^2 and additionally it indicates that starting at $E_{pn} \geq 50 \text{ MeV}$ MEC contribution becomes small at $Q^2 \geq 1.5 \text{ GeV}^2/c^2$ region. *The Δ -isobar contribution:* For the case of IC contribution, similar to the consideration of MEC, we are interested mainly in the energy dependence of scattering amplitude as compared to the IA amplitude, A_0 . For this it is sufficient to observe that

Figure 5: The Q^2 dependence of the ratios of structure functions W_1/W_2 from inclusive $d(e, e')X$ reaction. $E_{pn} = (W_{pn} - m_D - \epsilon_d^{\text{bound}})$, where $W_{pn}^2 = W_{\gamma^*D}^2 = (q^\mu + p_D^\mu)^2$, is the C.M. energy of the proton and neutron in the final state of the reaction. Solid lines are PWIA predictions within light cone dynamics in a collinear approach.

one can estimate the scattering amplitude of Fig. 2d as follows:

$$A_{IC}^\mu \sim i \int \psi_D(p_{mt} - k_t, p_{mz} - \frac{m_\Delta^2 - m^2}{2q}) J_{\gamma^*N\Delta}(Q^2) A_{\Delta,N \rightarrow NN}(k_t) d^2k_t, \quad (6)$$

where $J_{\gamma^*N\Delta}(Q^2)$ and $A_{\Delta,N \rightarrow NN}$ are electromagnetic $N \rightarrow \Delta$ and hadronic $N\Delta \rightarrow NN$ transition amplitudes respectively. Comparing A_{IC} with Impulse Approximation amplitude, A_{IA} , given as

$$A_{IA} \approx \psi_{3He}(\tilde{p}_1, p_2, p_3) J_{eN}(Q^2), \quad (7)$$

one first observes that the $x < 1$ and $x > 1$ regions at same $|p_m|$ have a different contributions from the intermediate Δ excitations. It can be seen from Eq.(6) that the amplitude of IC contribution is proportional to:

$$\psi_{3He}(\tilde{p}_1, p_2, p_3), \quad (8)$$

where $\tilde{p}_1 = (p_{mt} - k_t, p_{mz} - \frac{m_\Delta^2 - m^2}{2q})$ with p_{mt} and p_{mz} are the transverse and longitudinal components of the measured missing momentum. The Eq.(6) shows that one has more IC contribution in $x < 1$ region than in the region where $x > 1$ since in the first case $p_{mz} - \frac{M_\Delta^2 - M_N^2}{2q} < p_{mz}$, while for $x > 1$, $|p_{mz} - \frac{M_\Delta^2 - M_N^2}{2q}| > |p_{mz}|$.

Therefore the access to the $x > 1$ region at high Q^2 (Fig. 3) allows us to separate regions where the IC contribution is suppressed kinematically as compared to the IA contribution.

Another possible suppression comes from the comparison of elastic $J_{\gamma^*N}^\mu$ and Δ transition $J_{\gamma^*N\Delta}$ currents. Such a suppression at high Q^2 is expected from the perturbative QCD arguments, since Δ represents helicity-flip transition which is suppressed in the domain of pQCD. The hypothesis on smooth matching of nonperturbative and perturbative regimes of QCD suggests that the suppression of helicity flip amplitudes should be observed already at relatively high $Q^2 \geq few \text{ GeV}^2/c^2$. Indeed the experimental data on elastic and transition form factors suggest such a suppression [31].

And finally the additional suppression of the IC contribution comes from the energy dependence of $A_{\Delta, N \rightarrow NN}$ amplitude. To estimate the energy dependence of this amplitude one observes that the Feynman amplitude of scattering process which pro-

ceeds by an exchange of a particle with spin J behaves as $A \sim s^J$ [32, 33], in which s is the square of the center of mass energy of the final NN system, ($s \approx (\frac{2}{x} - 1)Q^2 + 2m^2$). In the QCD description, in which hadrons are bound states of quarks and gluons J is the Regge trajectory with quantum numbers permitted for a given process. Some trajectories are known experimentally: $J_\pi(t) \approx \alpha't$, $J_\rho(t) \approx 0.5 + \alpha't$, where $\alpha' \approx 1 \text{ GeV}^{-2}$ and t is momentum transfer (see e.g. [34]).

Since the transition in $A_{\Delta,N \rightarrow NN}$ is dominated by pion ($J = 0$) or ρ -type reggeon ($J=1/2$) exchange the $\Delta N \rightarrow NN$ transition amplitude will be suppressed at least by factor of $\frac{1}{\sqrt{Q^2}}$ as compared to the elastic $NN \rightarrow NN$ amplitude, which enters in the FSI diagram on Fig. 2(b). Thus IC amplitude will be suppressed additionally at least by factor of $\frac{1}{\sqrt{Q^2}}$. This is the upper limit in the estimation of IC contribution, since the data on $NN \rightarrow N\Delta$ scattering indicate that the ρ regime is unimportant up to very high, $\sim 100 \text{ GeV}$ energy range, thus in the Q^2 range of our interest more rapid suppression is expected with the increase of Q^2 .

Final State Interaction: With an increase of energy the characteristics of soft (small angle) hadron-hadron interactions in the amplitude of Fig. 2(b) simplify in several ways. The *first* major qualitative change is the emergence of a practically energy independent total cross section for hadron-hadron interactions at lab momenta $\geq 1 - 1.5 \text{ GeV/c}$ (total cross sections are constant up to momenta of 400 GeV/c , see Fig. 6. Both pp and pn total cross sections level out and become practically energy independent at lab momenta greater than a few GeV/c . It is easy to observe, that this simplifies tremendously the description of the final state interactions since the

Figure 6: Proton-proton and proton-neutron total cross sections as a function of incoming proton momenta in Lab. frame. Solid lines are fits from this work (see Appendix C for the corresponding Fortran listings and plots) to the world data, [40].

small angle NN scattering is proportional to σ_{NN}^{total} . The additional simplification associated with the increase of energy is that at small angles the rescattering amplitude is predominantly imaginary and conserves the helicity of interacting particles.

4 Generalized Eikonal Approximation in High Energy eA Scattering

As it was mentioned above the *first* major qualitative change which simplifies tremendously the description of the final state interactions is the emergence of a practically energy independent total cross section of hadron-hadron interactions at lab momenta $\geq 1 - 1.5$ GeV/c. The *second* simplification is related to the eikonal nature of interaction of high energy knocked-out nucleon off the spectator nucleons in the nuclei.

Figure 7: Infinite diagrams for knocked-out nucleon reinteractions with the spectator nucleons in the nucleus.

To understand the nature of this simplification it is worth noting that, in general, there are an infinite number of ways by which the knocked-out nucleon can reinteract with the spectator nucleons in the nucleus (See Fig. 7). However the following theorem can significantly simplify the situation.

4.1 Reduction Theorem

Let us consider the following **theorem**: *High energy particles propagating in the nuclear medium can not interact with a specific bound nucleon for a second time after interacting with another bound nucleon.* In other words, all those rescatterings which have segments similar to Fig. 8 are suppressed.

Figure 8: Diagram for hadron-deuteron scattering.

We will prove the above statement for the case of the amplitude of the diagram of Fig. 8. This proof can be easily generalized to all other cases as in Fig. 7. The invariant amplitude of the diagram in Fig. 8 can be presented as follows:

$$\begin{aligned}
 A_{pd \rightarrow ppn} = & - \int \frac{d^4 p_{s1}}{i(2\pi)^4} \frac{d^4 p_{s2}}{i(2\pi)^4} \frac{T_3(p_s - p_{s2}) T_2(p_4 - (p_D - p_{s1})) T_1(p_{s2} - p_{s1})}{D(p_3 + p_s - p_{s2}) D(p_1 + p_{s1} - p_{s2})} \\
 & \times \frac{\Gamma_{DNN}(p_D - p_{s1}, p_{s1})}{D(p_{s2}) D(p_{s1}) D(p_D - p_{s1})}, \quad (9)
 \end{aligned}$$

where $D(p) = -(p^2 - m^2 + i\epsilon)$. In this equation Γ_{DNN} denotes deuteron to two nucleons' transition vertex, while $T_{1,2,3}$ characterize the strength of NN scattering and are functions of scalar products of momenta of the interacting particles. We neglect the spins since they are not relevant for our discussion.

It is convenient for this section to use a light-cone $(p_-, p_+, p_\perp)$ representation for four-momentum $p^\mu = (p_0, p_z, p_\perp)$, constructing the $\pm$ components as:

$$p_\pm = p_0 \pm p_z. \quad (10)$$

The scalar product of two four-vectors is then given by

$$p_1 \cdot p_2 \equiv p_{10}p_{20} - p_{1z}p_{2z} - p_{1\perp}p_{2\perp} = \frac{1}{2}p_{1+}p_{2-} + \frac{1}{2}p_{1-}p_{2+} - p_{1\perp}p_{2\perp}. \quad (11)$$

We observe that the kinematics for the scattering corresponding to Fig. 8 is such that

$$p_{1+} \sim p_{3+} \gg m \quad \text{and} \quad p_{1-} \sim p_{3-} \ll m, \quad (12)$$

i.e. the propagating nucleon in the top of Fig. 8 is very energetic. Meanwhile

$$p_{4+} \sim p_{4-} \sim p_{s+} \sim p_{s-} \sim m, \quad (13)$$

i.e. recoiled nuclear system is non relativistic.

Next we introduce momenta:

$$k_1 = p_{s2} - p_{s1}, \quad k_2 = p_s - p_{s2} \quad \text{and} \quad K = k_1 + k_2. \quad (14)$$

This results in replacement

$$p_{s2} = p_s - k_2, \quad (15)$$

$$p_{s1} = p_{s2} - k_1 = p_s - k_1 - k_2 = p_s - K. \quad (16)$$

Because $K_0 = (K_+ + K_-)/2$ and $K_z = (K_+ - K_-)/2$:

$$d^4K = d^2K_\perp dK_0 dK_z = d^2K_\perp \left| \frac{\partial(K_0, K_z)}{\partial(K_+, K_-)} \right| dK_+ dK_- = \frac{1}{2} d^2K_\perp dK_+ dK_-. \quad (17)$$

Using above definitions we obtain from Eq.(9)

$$\begin{aligned}
A_{pd \rightarrow ppn} &= \frac{1}{4} \int \frac{d^2 K_{\perp} d^2 k_{2\perp}}{(2\pi)^4} \frac{dK_- dk_{2-}}{(2\pi)^2} \frac{dK_+ dk_{2+}}{(2\pi)^2} \\
&\times \frac{T_3(k_2) T_2(p_4 - (p_D - p_s + K)) T_1(K - k_2)}{D(p_3 + k_2) D(p_1 + k_2 - K)} \\
&\times \frac{\Gamma_{DNN}(p_D - p_s + K; p_s - K)}{D(p_s - k_2) D(p_s - K) D(p_D - p_s + K)}. \tag{18}
\end{aligned}$$

First we integrate over dK_+ . As we consider the K_+ complex plane one can clearly see that poles of propagators $D(p_s - K)$ and $D(p_1 + k_2 - K)$ lay in the upper half-plane whereas $D(p_D - p_s + K)$ has pole in the lower half-plane. Therefore we can close the contour of integration in the upper plane. As we write:

$$\begin{aligned}
-D(p_1 + k_2 - K) &= (p_1 + k_2 - K)_- (p_1 + k_2 - K)_+ - \\
&- (p_1 + k_2 - K)_{\perp}^2 - m^2 + i\varepsilon = -(p_1 + k_2 - K)_- \times \\
&\times \left[K_+ - \left((p_1 + k_2)_+ - \frac{(p_1 + k_2 - K)_{\perp}^2 + m^2 - i\varepsilon}{(p_1 + k_2 - K)_-} \right) \right], \tag{19}
\end{aligned}$$

we observe that pole integration of this propagator will return negligible result. Indeed, application of the residue theorem will replace K_+ with the quantity proportional to p_{1+} . The $D \rightarrow NN$ vertex function therefore will look like $\sim \Gamma_{DNN}(p_{1+})$. After non-relativistic reduction of the vertex function the remaining integrand will contain deuteron wave function with the argument $p_{1+} \gg m$. For such a high momenta $\psi_D \sim 0$ and corresponding integral is strongly suppressed.

Now we turn to the last K_+ pole and get

$$\begin{aligned}
-D(p_s - K) &= (p_s - K)_- (p_s - K)_+ - (p_s - K)_{\perp}^2 - m^2 + i\varepsilon = \\
&= -(p_s - K)_- \left[K_+ - \left(p_{s+} - \frac{(p_s - K)_{\perp}^2 + m^2 - i\varepsilon}{(p_s - K)_-} \right) \right], \tag{20}
\end{aligned}$$

so integration over dK_+ will merely leave us with the factor of $2\pi i/(p_s - K)_-$ at

$$K_+ = \left(p_{s+} - \frac{(p_s - K)_\perp^2 + m^2}{(p_s - K)_-} \right) \lesssim m. \quad (21)$$

Using the same approach we can now integrate over dk_{2+} by closing contour in the upper half plane with the single pole corresponding to propagator $D(p_s - k_2)$ (other two propagators containing k_{2+} have poles in lower half-plane). As before, one can write

$$\begin{aligned} -D(p_s - k_2) &= (p_s - k_2)_-(p_s - k_2)_+ - (p_s - k_2)_\perp^2 - m^2 + i\varepsilon = \\ &= -(p_s - k_2)_- \left[k_{2+} - \left(p_{s+} - \frac{(p_s - k_2)_\perp^2 + m^2 - i\varepsilon}{(p_s - k_2)_-} \right) \right] \end{aligned} \quad (22)$$

and effectively replace dk_{2+} integration with the factor of $2\pi i/(p_s - k_2)_-$ at

$$k_{2+} = \left(p_{s+} - \frac{(p_s - k_2)_\perp^2 + m^2}{(p_s - k_2)_-} \right) \lesssim m. \quad (23)$$

The overall result reads:

$$\begin{aligned} A_{pd \rightarrow ppn} &= \frac{-1}{4} \int \frac{d^2 K_\perp d^2 k_{2\perp}}{(2\pi)^4} \frac{dK_- dk_{2-}}{(2\pi)^2} \frac{T_3(k_2) T_2(p_4 - (p_D - p_s + K)) T_1(K - k_2)}{D(p_3 + k_2) D(p_1 + k_2 - K)} \\ &\quad \frac{\Gamma_{DNN}(p_D - p_s + K; p_s - K)}{(p_s - k_2)_- (p_s - K)_- D(p_D - p_s + K)}. \end{aligned} \quad (24)$$

Next, to integrate over dK_- and dk_{2-} one observes that in the high energy limit when $p_{1+}, p_{3+} \gg m$ and $p_{1-}, p_{3-} \ll m$ the fast nucleon's propagator can be expressed as follows:

$$\begin{aligned} -D(p_3 + k_2) &= (p_3 + k_2)_+(p_3 + k_2)_- - (p_3 + k_2)_\perp^2 - m^2 + i\varepsilon \approx p_{3+}(k_{2-} + i\varepsilon) \\ -D(p_1 + k_2 - K) &= (p_1 + k_2 - K)_+(p_1 + k_2 - K)_- - (p_1 + k_2 - K)_\perp^2 - \\ &\quad -m^2 + i\varepsilon \approx p_{1+}(k_{2-} - K_- + i\varepsilon). \end{aligned} \quad (25)$$

Inserting these expressions into Eq.(24) one obtains:

$$A_{pd \rightarrow ppn} = \frac{-1}{4} \int \frac{d^2 K_{\perp} d^2 k_{2\perp}}{(2\pi)^4} \frac{dK_{-} dk_{2-}}{(2\pi)^2} \frac{T_3(k_2) T_2(p_4 - (p_D - p_s + K)) T_1(K - k_2)}{p_{3+}(k_{2-} + i\epsilon) p_{1+}(k_{2-} - K_{-} + i\epsilon)} \frac{\Gamma_{DNN}(p_D - p_s + K; p_s - K)}{(p_s - k_2)_{-} (p_s - K)_{-} D(p_D - p_s + K)}. \quad (26)$$

Analyzing the range of the integration over dK_{-} and dk_{2-} we first observe that the soft rescattering amplitudes depend predominantly on $q_{\perp}$, i.e. $T_i(q) \sim T_i(q_{\perp})$ and does not contain any singularities associated with K_{-} and k_{2-} . Other apparent singularities may come from $(p_s - k_2)_{-} = 0$ and $(p_s - K)_{-} = 0$. However according to the redefinitions of Eq.(14) they correspond to $p_{s2-} = 0$ and $p_{s1-} = 0$. The latter conditions represent a bound nucleon with infinite virtuality $p_{s1+,2+} \sim \frac{m^2}{p_{s1-,2-}}$ which is suppressed by wave function of the bound nucleon. Therefore the structure of dK_{-} and dk_{2-} integrations will be defined only by two denominators of a fast propagating nucleon, i.e. by,

$$\int \frac{dk_{2-} dK_{-}}{(k_{2-} + i\epsilon)(k_{2-} - K_{-} + i\epsilon)} = 0. \quad (27)$$

The above integral is zero since both poles over the k_{2-} are on the same side of the complex k_{2-} semi-plane and one can close the contour of integration on the side where no singularities exist. Thus this contribution results $A_{pd \rightarrow ppn} = 0$.

This result allows us to reduce potentially an infinite number of rescattering diagrams as in Fig. 7 to a finite set of diagrams. The only diagrams that survive are those in which a propagating fast nucleon interacts first with one target nucleon then the next one and so on, making rescatterings strictly sequential. In addition, the other consequence of this theorem is that, if the virtuality of a bound nucleon,

which is interacting with the propagating (energetic) nucleon, can be neglected, the sum of the interaction amplitudes with a given nucleon can be replaced by the invariant NN scattering amplitude, F_{NN} as in Fig. 9. The later can be replaced by the phenomenological NN scattering amplitude taken from the NN scattering data.

Figure 9: Reduction theorem for hadron-deuteron scattering.

Thus we will end up with the finite set of scattering diagrams for which Feynman diagram rules can be identified.

The above result represents the realization of the eikonal approximation. However the major difference from the conventional semiclassical approximation is that the present approach does not require the spectator nucleons, to be stationary scatterers [10]. Henceforth, we will refer the present approach as generalized eikonal approximation (GEA).

4.2 Effective Feynman Diagram Rules

We will now define the effective Feynman diagram rules, within GEA, for the scattering amplitude of knocked-out nucleon to undergo n rescatterings off the nucleons of $(A - 1)$ residual system. The case $n = 0$ corresponds to the plane wave impulse approximation in which the knocked-out nucleon does not interact with residual nu-

cleus. Since we are restricted by the kinematics of Eq. (2) we systematically neglect the diffractive excitation of the nucleons in the intermediate states, which is a small correction for the knock-out nucleon (projectile) energies ≤ 10 GeV. For the same reason the color transparency phenomenon is still a small correction and intermediate hadronic states can be treated as a nucleon states.

Figure 10: Diagram for n -fold rescattering.

According to the above discussion the n -fold rescattering amplitude will be represented through n vertex amplitude in which the each vertex corresponds to one NN scattering - see Fig. 10. We can formulate the following Feynman rules of calculation of the diagram of Fig. 10 (see also Ref. [21]).

- We assign the vertex functions $\Gamma_A(p_1, \dots, p_A)$ and $\Gamma_{A-1}^\dagger(p'_2, \dots, p_A)$ to describe the transitions between "*nucleus A*" to "*A nucleons*" with momenta $\{p_n\}$, $\{p'_n\}$ and "*(A - 1) nucleons*" with momenta $\{p'_n\}$ to "*(A - 1) nucleon final state*" respectively.
- For γ^*N interaction we assign vertex, $\Gamma_{\gamma^*N}^\mu$.
- For each NN interaction we assign the vertex function $F_k^{NN}(p_{k+1}, p'_{k+1})$. This

vertex function are related to the amplitude of NN scattering as follows:

$$\begin{aligned} \bar{u}(p_3)\bar{u}(p_4)F^{NN}u(p_1)u(p_2) &= \\ &= \sqrt{s(s-4m^2)}f^{NN}(p_3,p_1)\delta_{\lambda,\lambda'} \approx sf^{NN}(p_3,p_1)\delta_{\lambda,\lambda'}, \end{aligned} \quad (28)$$

where s is the total invariant energy of two interacting nucleons with momenta p_1 and p_2 and

$$f^{NN} = \sigma_{tot}^{NN}(i + \alpha^{NN})e^{-\frac{B^{NN}}{2}(p_3-p_1)_\perp^2}, \quad (29)$$

where σ_{tot}^{NN} , α^{NN} and B^{NN} are known experimentally from NN scattering data.

The transverse components in the exponent are defined with respect to the direction of $\mathbf{p}_1$. The vertex functions are accompanied with δ -function of energy-momentum conservation.

- For each intermediate nucleon with four momentum p we assign the propagator $D(p)^{-1} = -(\hat{p} - m + i\epsilon)^{-1}$. Following to Ref. [35] we choose the “minus” sign for the nucleon propagators to simplify the calculation of the overall sign of the scattering amplitude.
- The factor $n!(A-n-1)!$ accounts for the combinatorics of n - rescatterings and $(A-n-1)$ spectator nucleons.
- For each closed contour one gets the factor $\frac{1}{i(2\pi)^4}$ with no additional sign.

Using the above defined effective Feynman diagram rules one obtains the following expression for the manifestly covariant scattering amplitude $F_{A,A-1}^{(n)}(q, p_f)$ of Fig. 10:

$$\begin{aligned}
F_{A,A-1}^{(n)}(q, p_f) = & \sum_h \frac{1}{n!(A-n-1)!} \prod_{i=1}^A \prod_{j=2}^A \int d^4 p_i d^4 p'_j \frac{1}{[i(2\pi)^4]^{A-2+n}} \\
& \delta^4\left(\sum_{i=1}^A p_i - \mathcal{P}_A\right) \delta^4\left(\sum_{j=2}^A p'_j - \mathcal{P}_{A-1}\right) \prod_{m=n+2}^A \delta^4(p_m - p'_m) \times \\
& \frac{\bar{u}(p_f) \chi_{A-1}^\dagger \Gamma_{A-1}^\dagger(p'_2, \dots, p'_{n+1}, p_{n+2}, \dots, p_A)}{D(p'_2) \dots D(p'_{n+1})} \frac{f_n^{NN}(p_{n+2}, p'_{n+2}) \dots f_1^{NN}(p_2, p'_2)}{D(l_1) \dots D(l_k) \dots D(l_{n-1})} \\
& \frac{\Gamma_{\gamma^* N}^h(Q^2)}{D(p_1 + q)} \frac{\Gamma_A(p_1, \dots, p_A) \chi_A}{D(p_1) D(p_2) \dots D(p_{n+1}) D(p_{n+2}) \dots D(p_A)}. \tag{30}
\end{aligned}$$

Here $\mathcal{P}_A$ and $\mathcal{P}_{A-1}$ are the four momenta of the target nucleus, and final $(A-1)$ system, p_j and p'_j are nucleon momenta in the nucleus A and residual $(A-1)$ system respectively. The sum, $\sum_h$, in Eq.(30) goes over virtual photon interactions with different nucleons, where $\Gamma_{\gamma^* N}^{\mu(h)}$ are electromagnetic vertices. $-D(p_k)^{-1}$ is the propagator of a nucleon with momentum p_k and $-D(l_k)^{-1}$ is the propagator of the struck nucleon in the intermediate state, with momentum $l_k = q + p_1 + \sum_{i=2}^k (p_i - p'_i)$ between $k-1$ -th and k -th rescatterings. The intermediate spectator states in the diagram of Fig. 10 are expressed in terms of nucleons but not nuclear fragments because the closure over various nuclear excitations in the intermediate state is used. The possibility to use closure is related to the fact that the typical scale characteristic for high energy phenomena is significantly larger than the energy scale of nuclear excitations for the nucleons in the intermediate state.

After evaluation of the intermediate state nucleon propagators, the covariant amplitude will be reduced to a set of time ordered non covariant diagrams. This will help to establish the correspondence between the nuclear vertex functions and the nuclear wave functions. Particularly in the nonrelativistic limit the momentum space

wave function is defined through the vertex function as follows [36]:

$$\psi_A(p_1, p_2, \dots, p_A) = \frac{1}{\left(\sqrt{(2\pi)^3 2m}\right)^{A-1}} \frac{\bar{u}(p_1)\bar{u}(p_2)\dots\bar{u}(p_A)\Gamma_A(p_1, p_2, \dots, p_A)}{p_1^2 - m^2} \chi^A, \quad (31)$$

normalized as:

$$\int |\psi_A(p_1, p_2, \dots, p_A)|^2 \delta^3\left(\sum p_i - p_A\right) \prod_{i=1}^A d^3 p_i = N \quad (32)$$

where $N = A$ for bound states and $N = \prod_{i=1}^A \delta^3(p_i - p'_i)$ for A body continuum state.

Note that to apply the relativistic normalization for the spinors ($\bar{u}u = 2m$) the $\sqrt{(2\pi)^3 2m}^{-1}$ factor should be associated with the plane wave single nucleon wave function.

5 Calculation of the Scattering Amplitude of

$e + {}^3\text{He} \rightarrow e' + p + p + n$ Reaction

From the discussions of Sec. 2 we conclude that only Impulse Approximation and Final State Interactions diagrams should be considered in high energy limits of eA reactions. For three body electrodisintegration this corresponds to the diagrams presented in Fig. 11:

Figure 11: Three-body break up diagrams: a) Plane Wave Impulse Approximation, where all three nucleons propagate as free waves; b) Impulse Approximation with the Pair Distortion; c) Single Rescattering (both with and without Pair Distortion); d) Double Rescattering (both with and without Pair Distortion).

5.1 Plane Wave Impulse Approximation and Pair Distortion Amplitudes

The contribution to the electromagnetic transition amplitude A^μ , in which the knocked-out nucleon does not interact with other nucleons, corresponds to the impulse approximation (IA). In this case the wave function of the knocked out nucleon is a plane wave.

Figure 12: (a) Plane Wave Impulse Approximation Diagram; (b) Impulse Approximation with the Pair Distortion

The IA term of the scattering amplitudes are described by the Feynman diagrams of Fig. 12. Fig 12 (a) and correspond to the Plane Wave Impulse Approximation (PWIA), where all three nucleons propagate as free waves, whereas Fig 12 (b) depicts the reinteraction between the pair of the slow spectator nucleons which distorts the plane wave of the outgoing recoil nucleons. Following Ref. [37] we will refer to this term as a pair distortion contribution. Using the diagrammatic rules summarized in Section 4.2 and identifying knocked-out and two recoil nucleons in the initial state by

indexes 1, 2 and 3, respectively, for the PWIA term one obtains:

$$A_0^\mu = \bar{u}(p_{r2})\bar{u}(p_{r3})\bar{u}(p_f) \cdot \Gamma_{\gamma^*N}^\mu \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \cdot \Gamma_{^3\text{He}NNN}(p_1, p_{r2}, p_{r3}) \chi^A, \quad (33)$$

where $\mathbf{p}_1 = \mathbf{p}_m \equiv \mathbf{p}_f - \mathbf{q}$ and $p_3 = p_A - p_1 - p_2$. For Pair Distortion term we have:

$$\begin{aligned} A_{0s}^\mu &= - \int \frac{d^4 p_2}{i(2\pi)^4} \bar{u}(p_{r2})\bar{u}(p_{r3})\bar{u}(p_f) \cdot \Gamma_{NN,int}^+(p_2, p_3) \cdot \Gamma_{\gamma^*N}^\mu \cdot \frac{\hat{p}_3 + m}{p_3^2 - m^2 + i\varepsilon} \times \\ &\times \frac{\hat{p}_2 + m}{p_2^2 - m^2 + i\varepsilon} \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \cdot \Gamma_{^3\text{He}NNN}(p_1, p_2, p_3) \chi^A. \end{aligned} \quad (34)$$

It's worth to note that since both $\Gamma_{^3\text{He}NNN}$ and $\Gamma_{\gamma^*N}^\mu$ are relativistic covariant vertices therefore in the time ordered expansion they contain both impulse approximation and vacuum fluctuation diagrams. This implies that $\Gamma_{\gamma^*N}^\mu$ may represent, for example, $\gamma^* \rightarrow \bar{N}N$ and $\Gamma_{^3\text{He}NNN}$ also describe the processes like $\bar{N}^3\text{He} \rightarrow NN$. However, if we restrict by the kinematics defined in Eq.(2), then in the set of non-covariant diagrams, comprising the covariant diagram of Fig. 12, one can neglect the diagrams which correspond to the vacuum fluctuations. Overall in discussed kinematics the relativistic effects in the nuclear wave function are a small correction and the impulse approximation(IA) can be calculated via nonrelativistic reduction of the covariant nuclear vertices in Eqs.(33) and (34). Such a reduction corresponds to taking the residue over dp_2^0 , at the nearest nucleon pole in the spectator nucleon propagator. The restriction by the nearest pole in the nucleon propagators follows from the observation that this is the only pole not associated with $N\bar{N}$ production in our kinematics, where nuclear excitations are small as compared to the scale of energies characteristic for the nucleon excitations. After such integration one is left with the time ordered diagram corresponding to the IA, where the virtual photon knocks

out the target nucleon with momentum p_1 . Such integration effectively corresponds to the replacement:

$$\int \frac{dp_2^0}{p_2^2 - m^2 + i\varepsilon} = -\frac{i2\pi}{2E_2} \approx -\frac{i2\pi}{2m}. \quad (35)$$

The condition that the internal momenta of the nucleons remain small ($\mathbf{p}_{\mathbf{m},2,3}^2 \ll m^2$) also allows one to use the closure relation for on-mass shell nucleons to express the numerator of the bound nucleon propagator as follows:

$$\hat{p} + m = \sum_s u(p, s) \bar{u}(p, s). \quad (36)$$

Using Eqs.(35) and (36) in Eq.(33) and Eq.(34) one derives the following:

$$\begin{aligned} A_0^\mu &= \sum_{s_1} \bar{u}(p_f, s_f) \Gamma_{\gamma^* N}^\mu u(p_1, s_1) \bar{u}(p_1, s_1) \bar{u}(p_{r2}, s_{r2}) \bar{u}(p_{r3}, s_{r3}) \times \\ &\times \Gamma_{3\text{He}NNN}(p_1, p_{r2}, p_{r3}) \chi^A, \end{aligned} \quad (37)$$

$$\begin{aligned} A_{0s}^\mu &= \sum_{s_1 s_2 s_3} \int \frac{d^3 p_2}{2m(2\pi)^3} \frac{\bar{u}(p_{r2}, s_{r2}) \bar{u}(p_{r3}, s_{r3}) \Gamma_{NN, int}^+(p_2, p_3) u(p_3, s_3) u(p_2, s_2)}{p_3^2 - m^2 + i\varepsilon} \times \\ &\times \bar{u}(p_f, s_f) \Gamma_{\gamma^* N}^\mu u(p_1, s_1) \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2) \bar{u}(p_3, s_3) \Gamma_{3\text{He}NNN}(p_1, p_2, p_3) \chi^A}{p_1^2 - m^2 + i\varepsilon}. \end{aligned} \quad (38)$$

Using Eq.(31) and introducing the electromagnetic nucleon current,

$$j^\mu(p_f, s_f; p_m, s_1) = \bar{u}(p_f, s_f) \Gamma_{\gamma^* N}^\mu u(p_m, s_1), \quad (39)$$

the sum of the A_0^μ and A_{0s}^μ can be represented as:

$$\begin{aligned} A_0^\mu + A_{0s}^\mu &= \sqrt{2E_{r2}2E_{r3}}(2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \int d^3 p_2 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p_2, s_2, t_2; p_3, s_3, t_3) \\ &\times j_{t1}^\mu(p_m + q, s_f; p_m, s_1) \Psi_A^{s_A}(p_m, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \end{aligned} \quad (40)$$

where $s_A, s_1, s_2, s_3, s_f, s_{r2}, s_{r3}$ describe the spin projections of the ${}^3\text{He}$ -nucleus, the initial nucleons and the final nucleons respectively. We represent the isospin projections of nucleons by $t_1, t_2, t_3, t_f, t_{r2}, t_{r3}$. In the above equation, $\Psi_A^{s_A}$ is the ground state wave function of the ${}^3\text{He}$ nucleus with polarization vector $\mathbf{s}_A$ and Ψ_{NN} represents the continuum NN wave function. One can simplify further Eq.(40) using the fact that Ψ_{NN} is a function only of the relative three-momenta of spectator nucleons and the spins. This allows us to replace the d^3p_2 integration by d^3p_{23} . To do this one can introduce $d^3p_3\delta^3(p_{r2}+p_{r3}-p_2-p_3)$ in Eq.(40), then replace $d^3p_2d^3p_3$ by $d^3p_{23}d^3P_{cm23}$, with $\mathbf{p}_{23} = \frac{\mathbf{p}_2-\mathbf{p}_3}{2}$ and $\mathbf{P}_{cm23} = \mathbf{p}_2 + \mathbf{p}_3$ and integrate out the δ function through d^3P_{cm23} . This yields:

$$A_0^\mu + A_{0s}^\mu = \sqrt{2E_{r2}2E_{r3}}(2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \int d^3p_{23} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3) \\ \times j_{t1}^\mu(p_m + q, s_f; p_m, s_1) \Psi_A^{s_A}(p_m, s_1, t_1; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} - p_{23}, s_3, t_3). \quad (41)$$

For the case of the reaction of Eq.(1), Ψ_{NN} is a continuum NN wave function which can be represented through the Lippmann-Schwinger equation as follows (see Appendix B):

$$\Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3) = \langle s_{r2}, t_{r2}; s_{r3}, t_{r3} \mid s_2, t_2; s_3, t_3 \rangle + \\ + \frac{1}{2\pi^2} \frac{\langle s_{r2}, t_{r2}; s_{r3}, t_{r3} \mid f_{NN}^{\text{off shell}}(\mathbf{p}_{r23}, \mathbf{p}_{23}) \mid s_2, t_2; s_3, t_3 \rangle}{\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2 - i\varepsilon}, \quad (42)$$

where $\mathbf{p}_{23} = \frac{\mathbf{p}_2-\mathbf{p}_3}{2}$, $\mathbf{p}_{r23} = \frac{\mathbf{p}_{r2}-\mathbf{p}_{r3}}{2}$ and $f_{NN}^{\text{off shell}}$ is a half-off-shell non-relativistic amplitude of NN scattering (see e.g. Ref. [38]). Terms at the right-hand side of Eq.(42) characterize two distinctive dynamics of production of the recoil NN state. If only the first term of Eq.(42) is kept in Eq.(41), this will lead to the approximation

in which all three final nucleons propagate as plane waves, corresponding to PWIA.

For this case we have:

$$\langle s_{r2}, t_{r2}; s_{r3}, t_{r3} \mid s_2, t_2; s_3, t_3 \rangle = \delta^3(\mathbf{p}_{23} - \mathbf{p}_{r23}) \delta^{s_2, s_{r2}} \delta^{s_3, s_{r3}} \delta^{t_2, t_{r2}} \delta^{t_3, t_{r3}}. \quad (43)$$

The second term in Eq.(42) describes a reinteraction between the pair of the slow nucleons which distorts the plane wave of the outgoing recoil nucleons, which we referred as Pair Distortion Term.

Here we present the final form of the PWIA term for further references:

$$A_0^\mu = \sqrt{2E_{r2}2E_{r3}}(2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \times j_{t1}^\mu(p_m + q, s_f; p_m, s_1) \Psi_A^{sA}(p_m, s_1, t_1; -\frac{p_m}{2} + p_{r23}, s_2, t_2; -\frac{p_m}{2} - p_{r23}, s_3, t_3). \quad (44)$$

5.2 Single Rescattering Amplitudes

The diagrams in Fig.13(a) and 13(b) describe the process where the fast (p_f) knocked-out nucleon rescatters off one of the spectator nucleons. The general expression for the amplitude corresponding to the diagram in Fig. 13(a) is given by $n = 1$ term of Eq.(30) as follows:

$$\begin{aligned} A_{1a}^\mu &= - \int \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_3}{i(2\pi)^4} \bar{u}(p_{r3}) \bar{u}(p_{r2}) \bar{u}(p_f) \frac{\Gamma_{NN}^+(p'_2, p_3)(\hat{p}'_2 + m)}{p_2'^2 - m^2 + i\varepsilon} \times \\ &\times \frac{F_{NN}^a(p'_2 - p_2)(\hat{p}_1 + \hat{q} + m)}{(p_1 + q)^2 - m^2 + i\varepsilon} \cdot \Gamma_{\gamma^* N}^\mu \cdot \frac{\hat{p}_3 + m}{p_3^2 - m^2 + i\varepsilon} \times \\ &\times \frac{\hat{p}_2 + m}{p_2^2 - m^2 + i\varepsilon} \cdot \frac{\hat{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \cdot \Gamma_{3\text{He}NN}(p_1, p_2, p_3) \chi^A, \end{aligned} \quad (45)$$

where the generalized NN interaction vertex Γ_{NN}^+ includes also the part corresponding to two outgoing plane waves for the recoil nucleons. Using the same arguments as in the previous subsection we evaluate $d^0 p_2$ and $d^0 p_3$ integrals through

Figure 13: Single Rescattering Diagrams.

the residues at positive energy poles in the recoil nucleon propagators. This yields a replacement of $\int d_{2,3}^0 \frac{1}{p_{2,3}^2 - m^2 + i\epsilon} \approx -i \frac{2\pi}{2m}$ and reduces the covariant Feynman diagram of Eq.(45) to the time-ordered noncovariant diagram in which nucleon “1” is first struck by a virtual photon and then rescatters off the spectator nucleon, “2”. The rescattered nucleon “2” subsequently combines with the nucleon “3” into the NN continuum state. Now, we can use Eq.(36) for the intermediate nucleons. Furthermore, relating the nuclear vertex functions through the non-relativistic reduction to the nuclear wave functions according to Eq.(31) and the NN rescattering vertex functions, F_{NN}^a to the NN high energy soft scattering amplitude, f_{NN} , according to Eq.(28), one obtains:

$$\begin{aligned}
A_{1a}^\mu &= -F \sum_{\text{spins}} \sum_{t_1, t_2', t_2, t_3} \frac{1}{2m} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p_2', s_2', t_2'; p_3, s_3, t_3) \times \\
&\times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)} f_{NN}(p_2', s_2', t_2', p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_1', t_1)}{(p_1 + q)^2 - m^2 + i\varepsilon} \\
&\times j_{t1}^\mu(p_1 + q, s_1'; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \tag{46}
\end{aligned}$$

where $F = \sqrt{2E_{r2}2E_{r3}}(2\pi)^3$, $p_1 = p_A - p_2 - p_3$, $p_2' = p_{r2} + p_{r3} - p_3$ and $s_2^{NN} = (p_1 + q + p_2)^2$.

Now we analyze the propagator of the knocked-out nucleon:

$$(q + p_1)^2 - m^2 + i\epsilon = 2q \left[\frac{2mq_0 - Q^2}{2q} - p_{1z} + i\epsilon \right]. \tag{47}$$

The factor $\frac{2mq_0 - Q^2}{2q}$ is fixed by external (measurable) kinematical variables such as Q^2 , q_0 , $\mathbf{p}_f$ and $\mathbf{p}_{r2}$. Using the condition of the quasi-elastic scattering for reaction (1), $(q + p_A - (p_{r2} + p_{r3}))^2 = m^2$, one obtains:

$$\begin{aligned}
m^2 &= (q + P_{3\text{He}} - p_{r2} - p_{r3})^2 = \\
&= M_{3\text{He}}^2 - 2P_{3\text{He}}(p_{r2} + p_{r3}) + 2m^2 + 2q(P_{3\text{He}} - p_{r2} - p_{r3}) - Q^2, \tag{48}
\end{aligned}$$

so we can now write:

$$\frac{2mq_0 - Q^2}{2q} = p_{mz} + \frac{q_0}{q}(T_{r2} + T_{r3} + |\epsilon_A|) + \frac{m^2 - \tilde{m}^2}{2q} \approx p_{mz} + \Delta^0, \tag{49}$$

where $p_{mz} = p_{fz} - q$. T_{r2} and T_{r3} are the kinetic energies of recoil nucleons, $|\epsilon_A|$ is the modulus of the binding energy of the target and $\tilde{m}^2 = [p_A - (p_{r2} + p_{r3})]^2$. In the last step in Eq.(49) we neglected the $(m^2 - \tilde{m}^2)/(2q)$ term, since for the fixed decay kinematics it vanishes with an increase of q . We have also introduced the new parameter, Δ^0 :

$$\Delta^0 = \frac{q_0}{q}(T_{r2} + T_{r3} + |\epsilon_A|). \tag{50}$$

This new parameter is in fact the effective longitudinal momentum transferred in the NN rescattering. Note that Δ^0 , which is absent in the conventional eikonal approximation, is important at large values of recoil nucleon energies $T_{r2}, T_{r2} \sim m$. Substituting Eqs. (47) and (49) into Eq.(46) for the scattering amplitude described by Fig.13, one obtains:

$$\begin{aligned}
A_{1a}^\mu &= -\frac{F}{2} \sum_{s_{1'}, s_{2'}, s_1, s_2, s_3} \sum_{t_1, t_2, t_3} \int \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p_2', s_2', t_2'; p_3, s_3, t_3) \\
&\times \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}}{2qm} \frac{f_{NN}(p_2', s_2', t_2', p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_{1'}, t_1)}{p_{mz} + \Delta^0 - p_{1z} + i\varepsilon} \\
&\times j_{t_1}^\mu(p_1 + q, s_{1'}; p_1, s_1) \cdot \Psi_A^{s_A}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3). \tag{51}
\end{aligned}$$

Two important features of soft NN scattering allow us to simplify Eq.(51):

- In the high-energy regime the soft, low t , NN amplitude, which dominates in Eq.(51) conserves the helicities of nucleons, which are the spin projections over the momenta of the particles.. Starting at $Q^2 \geq 1 \text{ GeV}^2/c^2$ the approximation of helicity conservation is accurate on the level of 2-3% [39] (for recent analysis see Ref. [40]). It is even smaller factor for the observables of reaction (1) in which no polarization degrees of freedom are fixed. Because of this we've been able to take an average over the initial spin states and sum over the final spin states, which is represented by the label "spins" in the Eq.(46).
- One can also observe, that the soft amplitude is a function of the transverse component of the transferred momentum, $(p_2' - p_2)_\perp$ only. Since $\hat{z} \parallel \mathbf{q}$, the helicity conserving argument implies the conservation of the polarization projections of the interacting nucleons in the z direction.

These simplifications yield the equation,

$$\begin{aligned} f_{NN}(p'_2, s'_2, t'_2, p_f, s_f, t_f; |p_2, s_2, t_2, p_1 + q, s_1, t_1) = \\ = f_{NN}^{t'_2, t_f | t_2, t_1}(p'_{2\perp} - p_{2\perp}) \delta^{s_f, s'_1} \delta^{s_2', s_2}. \end{aligned} \quad (52)$$

Using this relation and defining the transferred momentum in f_{NN} as $k \equiv p'_2 - p_2 = p_1 - p_m$ we can rewrite Eq.(51) as follows:

$$\begin{aligned} A_{1a}^\mu &= -\frac{F}{2} \sum_{s_1, s_2, s_3} \sum_{t_1, t'_2, t_2, t_3} \int \frac{d^3k}{(2\pi)^3} d^3p_3 \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t'_2; p_3, s_3, t_3) \times \\ &\times \frac{\chi_1(s_2^{NN}) f_{NN}^{t'_2, t_f | t_2, t_1}(k_\perp)}{\Delta^0 - k_z + i\varepsilon} \cdot j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \times \\ &\times \Psi_A^{s_A}(p_m + k, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \end{aligned} \quad (53)$$

where $\chi_1(s_2^{NN}) = \frac{\sqrt{s_2^{NN}(s_2^{NN} - 4m^2)}}{2qm}$.

The contribution of the second diagram in Fig. 13 can be calculated by interchanging the momenta of “2” and “3” nucleons in Eq.(53). This allows us to obtain the following expression for the amplitude describing the struck nucleon rescattering off the second recoil nucleon:

$$\begin{aligned} A_{1b}^\mu &= -\frac{F}{2} \sum_{s_1, s_2, s_3} \sum_{t_1, t'_3, t_3, t_2} \int \frac{d^3k}{(2\pi)^3} d^3p_2 \Psi_{NN}^{\dagger p_{r3}, s_{r3}, t_{r3}; p_{r2}, s_{r2}, t_{r2}}(p'_3, s_3, t'_3; p_2, s_2, t_2) \times \\ &\times \frac{\chi_1(s_3^{NN}) f_{NN}^{t'_3, t_f | t_3, t_1}(k_\perp)}{\Delta^0 - k_z + i\varepsilon} \cdot j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \times \\ &\times \Psi_A^{s_A}(p_m + k, s_1, t_1; p_3, s_3, t_3; p_2, s_2, t_2). \end{aligned} \quad (54)$$

Changing the integration variables d^3p_2 and d^3p_3 to d^3p_{23} (similar to Sec. 5.1) for the complete single rescattering amplitude, $A_1^\mu = A_{1a}^\mu + A_{1b}^\mu$, one obtains:

$$\begin{aligned}
A_1^\mu &= -\frac{F}{2} \sum_{s_1, s_2, s_3} \sum_{t_2', t_3'} \sum_{t_1, t_2, t_3} \int \frac{d^3 k d^3 p_{23}}{(2\pi)^3} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2'; s_3, t_3') \times \\
&\times j_{t_1}^\mu(p_1 + q, s_f; p_1, s_1) \times \\
&\times \left[\frac{\chi_1(s_2^{NN}) f_{NN}^{t_f, t_2', t_1, t_2}(k_\perp) \delta^{t_3', t_3}}{\Delta^0 - k_z + i\varepsilon} \Psi_A^{s_A}(p_m + k; -\frac{p_m}{2} + p_{23} - k; -\frac{p_m}{2} - p_{23}) \right. \\
&\quad \left. + \frac{\chi_1(s_3^{NN}) f_{NN}^{t_f, t_3', t_1, t_3}(k_\perp) \delta^{t_2', t_2}}{\Delta^0 - k_z + i\varepsilon} \Psi_A^{s_A}(p_m + k; -\frac{p_m}{2} + p_{23}; -\frac{p_m}{2} - p_{23} - k) \right].
\end{aligned} \tag{55}$$

5.3 Double Rescattering Amplitudes

This section is concerned with the double rescattering amplitude. The diagrams of Fig.14(a) and 14(b) describe the process where the fast (p_f) knocked-out nucleon rescatters sequentially off both spectator nucleons. The general expression for the amplitude corresponding to the diagram in Fig. 14(a) is given by $n = 2$ term of Eq.(30) as follows:

Figure 14: Double Rescattering Diagrams.

$$\begin{aligned}
A_{2a}^\mu &= - \int \frac{d^4 p'_3}{i(2\pi)^4} \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_3}{i(2\pi)^4} \bar{u}(p_{r3}) \bar{u}(p_{r2}) \bar{u}(p_f) \frac{\Gamma_{NN}^+(p'_2, p'_3)(\hat{p}'_2 + m)(\hat{p}'_3 + m)}{(p'^2_2 - m^2 + i\varepsilon)(p'^2_3 - m^2 + i\varepsilon)} \times \\
&\times \frac{F_{NN}^b(p'_3 - p_3)(\hat{p}_1 + \hat{q} + \hat{p}_2 - \hat{p}'_2 + m)}{(p_1 + q + p_2 - p'_2)^2 - m^2 + i\varepsilon} \times \\
&\times \frac{F_{NN}^a(p'_2 - p_2)(\hat{p}_1 + \hat{q} + m)}{(p_1 + q)^2 - m^2 + i\varepsilon} \cdot \Gamma_{\gamma^* N}^\mu \cdot \frac{\hat{p}_3 + m}{p^2_3 - m^2 + i\varepsilon} \times \\
&\times \frac{\hat{p}_2 + m}{p^2_2 - m^2 + i\varepsilon} \cdot \frac{\hat{p}_1 + m}{p^2_1 - m^2 + i\varepsilon} \cdot \Gamma_{\text{He}NNN}(p_1, p_2, p_3) \chi^A, \tag{56}
\end{aligned}$$

where p_2 and p_3 are the momenta of the spectator nucleons before rescattering; $p_1 = p_A - p_2 - p_3$. Here Γ_{NN}^+ is the generalized NN interaction vertex, which includes also the part corresponding to two outgoing plane waves for the recoil nucleons.

Using the same approximation we used for the IA and single rescattering amplitudes we estimate the integrals over $d^0 p_{3',3,2}$ through the positive energy poles of the nucleon propagators with momenta p'_3 , p_3 and p_2 , respectively. These integrations result in the replacement of $\int \frac{d^0 p_j}{2\pi i(p_j^2 - m^2 + i\varepsilon)} \rightarrow -\frac{1}{2E_j} \approx -\frac{1}{2m}$, ($j = 2, 3, 3'$).

Applying the closure condition of Eq.(36) and using the reduction (Eq.(31)) of nuclear vertices to the nonrelativistic nuclear wave functions, one obtains:

$$\begin{aligned}
A_{2a}^\mu &= \frac{F}{(2m)^2} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_{1'}, t_{2'}, t_{3'}} \int \frac{d^3 p'_3}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \times \\
&\times \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t_{2'}; p'_3, s_3, t_{3'}) \times \\
&\times \frac{\sqrt{s_{b3}^{NN}(s_{b3}^{NN} - 4m^2)} f_{NN}^{t_{3'}, t_f | t_3, t_{1'}}(p'_{3\perp} - p_{3\perp})}{(p_1 + q + p_2 - p'_2)^2 - m^2 + i\varepsilon} \times \\
&\times \frac{\sqrt{s_{a2}^{NN}(s_{a2}^{NN} - 4m^2)} f_{NN}^{t_{2'}, t_{1'} | t_2, t_1}(p'_{2\perp} - p_{2\perp})}{(p_1 + q)^2 - m^2 + i\varepsilon} \times \\
&\times j_{t1}^\mu(p_1 + q, s_f; p_1, s_1) \cdot \Psi_A^{sA}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3), \tag{57}
\end{aligned}$$

where s_{b3}^{NN} and s_{a2}^{NN} are total invariant energies of nucleons coupling at the vertices F_{NN}^b and F_{NN}^a , respectively.

Let us now consider the denominators of the knocked-out nucleon in the intermediate states. For $(p_1 + q)^2 - m^2 + i\varepsilon$, similar to the case of single rescattering, one gets

$$(p_1 + q)^2 - m^2 = 2q(\Delta^0 + p_{mz} - p_{1z} + i\epsilon) , \quad (58)$$

where Δ^0 is defined according to Eq.(50).

For the denominator, $(p_1 + q + p_2 - p'_2)^2 - m^2 + i\varepsilon$ in Eq.(57) using energy-momentum conservation for the knocked-out nucleon in the intermediate state after one rescattering given by relation

$$p_1 + q + p_2 - p'_2 = p_f + p'_3 - p_3, \quad (59)$$

we can write:

$$\begin{aligned} (p_1 + q + p_2 - p'_2)^2 - m^2 + i\epsilon &= (p_f + p'_3 - p_3)^2 - m^2 + i\epsilon \\ &= 2p_{fz} \left[\frac{E_f}{p_{fz}}(E'_3 - E_3) - (p'_{3z} - p_{3z} - \frac{p_{f\perp}}{p_{fz}}(p'_{3\perp} - p_{3\perp}) + i\epsilon \right] \\ &= 2p_{fz} [\Delta_3 - (p'_{3z} - p_{3z}) + i\epsilon] , \end{aligned} \quad (60)$$

where $\Delta_3 = \frac{E_f}{p_{fz}}(E'_3 - E_3) - \frac{p_{f\perp}}{p_{fz}}(p'_{3\perp} - p_{3\perp})$. Let us define $\chi_1(s_{a2}^{NN}) = \frac{\sqrt{s_{a2}^{NN}(s_{a2}^{NN} - 4m^2)}}{2qm}$ with $s_{a2}^{NN} = (p_1 + q + p_2)^2$ as an invariant mass and $\chi_2(s_{b3}^{NN}) = \frac{\sqrt{s_{b3}^{NN}(s_{b3}^{NN} - 4m^2)}}{2p_{fz}m}$ with $s_{b3}^{NN} = (p_1 + q + p_2 - p'_2 + p_3)^2$. Substituting Eqs.(58) and (60) , similar to what we did in the case of single rescattering amplitude, into Eq.(56), one obtains the following unfactorized (in terms of current j^μ) final expression for the double rescattering amplitude associated with the first diagram of Fig. 14, where the struck

nucleon first interacts with the first recoil nucleon and then with the second one:

$$\begin{aligned}
A_{2a}^\mu &= \frac{F}{4} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_1', t_2', t_3'} \int \frac{d^3 p'_3}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} d^3 p_3 \times \\
&\times \Psi_{NN}^{\dagger p_{r2}, s_{r2}, t_{r2}; p_{r3}, s_{r3}, t_{r3}}(p'_2, s_2, t_2'; p'_3, s_3, t_3') \times \\
&\times \frac{\chi_2(s_{b3}^{NN}) f_{NN}^{t_{3'}, t_f | t_3, t_1'}(p'_{3\perp} - p_{3\perp})}{\Delta_3 + p'_{3z} - p_{3z} + i\varepsilon} \frac{\chi_1(s_{a2}^{NN}) f_{NN}^{t_2', t_1' | t_2, t_1}(p'_{2\perp} - p_{2\perp})}{\Delta^0 + p_{mz} - p_{1z} + i\varepsilon} \times \\
&\times j_{t1}^\mu(p_1 + q, s_f; p_1, s_1) \cdot \Psi_A^{sA}(p_1, s_1, t_1; p_2, s_2, t_2; p_3, s_3, t_3). \tag{61}
\end{aligned}$$

To complete the calculation of the double rescattering amplitude one should calculate also the amplitude corresponding to the diagram of Fig. 14(b). This amplitude is obtained by interchanging momenta of nucleons “2” and “3” in Eq.(61). Furthermore, it is more convenient to express the integrand of the double rescattering amplitude through the momentum transfers in the NN rescattering amplitude $k_2 = p'_2 - p_2$ and $k_3 = p'_3 - p_3$. Using these variables, and changing the $d^3 p'_3$ integration to $d^3 p'_{23}$ (similar to what was done in Sec. 5.2), one obtains:

$$\begin{aligned}
A_2^\mu &= A_{2a}^\mu + A_{2b}^\mu = \frac{F}{4} \sum_{s_1, s_2, s_3} \sum_{t_1, t_2, t_3, t_1', t_2', t_3'} \int d^3 p'_{23} \frac{d^3 k_3}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \\
&\times \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p'_{23}, s_2, t_2'; s_3, t_3') \times \\
&\times \left[\frac{\chi_2(s_{b3}^{NN}) f_{NN}^{t_{3'}, t_f | t_3, t_1'}(k_{3\perp})}{\Delta_3 - k_{3z} + i\varepsilon} \frac{\chi_1(s_{a2}^{NN}) f_{NN}^{t_2', t_1' | t_2, t_1}(k_{2\perp})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} + \right. \\
&+ \left. \frac{\chi_2(s_{b2}^{NN}) f_{NN}^{t_2', t_f | t_2, t_1'}(k_{2\perp})}{\Delta_2 - k_{2z} + i\varepsilon} \frac{\chi_1(s_{a3}^{NN}) f_{NN}^{t_3', t_1' | t_3, t_1}(k_{3\perp})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} \right] \times \\
&\times j_{t1}^\mu(p_m + k_2 + k_3 + q, s_f; p_m + k_2 + k_3, s_1) \\
&\times \Psi_A^{sA}(p_m + k_3 + k_2, s_1, t_1; -\frac{p_m}{2} - k_2 + p'_{23}, s_2, t_2; -\frac{p_m}{2} - k_3 - p'_{23}, s_3, t_3). \tag{62}
\end{aligned}$$

6 Cross Section

Based on above derived scattering amplitudes, we can now calculate the differential cross section of the reaction Eq.(1) for an unpolarized case:

$$d^{12}\sigma = \frac{1}{4j_A}(2\pi)^4\delta^4(k_e + P_A - k'_e - p_f - p_{r2} - p_{r3})\frac{1}{4}A \sum_{nucleons} \sum_{spins} |M_{fi}|^2 \\ \times \frac{d^3k'_e}{(2\pi)^3 2E'_e} \frac{d^3p_f}{(2\pi)^3 2E_f} \frac{d^3p_{r2}}{(2\pi)^3 2E_{r2}} \frac{d^3p_{r3}}{(2\pi)^3 2E_{r3}}, \quad (63)$$

where $j_A = \sqrt{(k_e P_A)^2 - m_e^2 M_A^2}$. Here we sum over final and average over initial spins. The factor $1/4$ comes from the averaging over the initial polarizations of the electron and the nucleus. Since one of the recoil nucleons is not observed, one eliminates this degree of freedom by integrating over d^3p_{r3} . Thus integrated differential cross section is

$$d^9\sigma = \frac{1}{4j_A}(2\pi)^4\delta(E_e + M_A - E'_e - E_f - E_{r2} - E_{r3})\frac{1}{4}A \sum_{nucleons} \sum_{s_e, s_A} \sum_{s'_e, s_f, s_{r2}, s_{r3}} |M_{fi}|^2 \\ \frac{d^3k'_e}{(2\pi)^3 2E'_e} \frac{d^3p_f}{(2\pi)^3 2E_f} \frac{d^3p_{r2}}{(2\pi)^3 2E_{r2}} \frac{1}{(2\pi)^3 2E_{r3}}, \quad (64)$$

where $\mathbf{p}_{r3} = \mathbf{k}_e - \mathbf{k}'_e - \mathbf{p}_f - \mathbf{p}_{r2}$. In Eqs. (63) and (64) the transition matrix, M_{fi} , represents the convolution of the electron and nuclear currents, in which the nuclear current within GEA represents the sum of the PWIA (Eq. (45)), IA (Eq. (41)), single (Eq. (56)) and double (Eq. (63)) rescattering amplitudes,

$$M_{fi} = -4\pi\alpha \frac{1}{q^2} j_\mu^e \cdot (A_0^\mu + A_{0s}^\mu + A_1^\mu + A_2^\mu), \quad (65)$$

where A_0 , A_{0s} , A_1 and A_2 are derived in Sec. 5 and represent PWIA, Pair distortion, single and double rescattering amplitudes, respectively. As we plug Eqs. (45), (41),

(56) and (63) in Eq.(65) and than Eq.(64) we obtain in factorized approximation the final result for the differential cross section of exclusive electrodisintegration of ^3He :

$$\begin{aligned}
\frac{d\sigma}{dE'_e d\Omega'_e d^3p_f d^3p_{r2}} &= \frac{j_N}{j_A} \cdot A \sum_N \sigma_{eN}(p_f, k_e, k'_e) \cdot \frac{1}{2} \sum_{s_A, s_m, s_{r2}, s_{r3}} \\
&\times \left| \sum_{s_2, s_3} \sum_{t_2, t_3} \left\{ \int d^3p_{23} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3) \right. \right. \\
&\times \Psi_A^{s_A}(p_m, s_m, t_f; p_2, s_2, t_2; p_3, s_3, t_3) \\
&- \frac{1}{2} \int \frac{d^3k d^3p_{23}}{(2\pi)^3} \frac{\Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3)}{\Delta^0 - k_z + i\varepsilon} \\
&\times \left[\chi_1(s_2^{NN}) f_{NN}^{t_f, t_2 | t_f, t_2}(k_\perp) \right. \\
&\cdot \Psi_A^{s_A}(p_m + k, s_m, t_f; -\frac{p_m}{2} + p_{23} - k, s_2, t_2; -\frac{p_m}{2} - p_{23}, s_3, t_3) \\
&+ \Psi_A^{s_A}(p_m + k, s_m, t_f; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} - p_{23} - k, s_3, t_3) \\
&\cdot \left. \chi_1(s_3^{NN}) f_{NN}^{t_f, t_3 | t_f, t_3}(k_\perp) \right] \\
&+ \frac{1}{4} \int d^3p'_{23} \frac{d^3k_3}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p'_{23}, s_2, t_2; s_3, t_3) \\
&\times f_{NN}^{t_3, t_f | t_3, t_f}(k_{3\perp}) f_{NN}^{t_2, t_f | t_2, t_f}(k_{2\perp}) \\
&\times \left[\frac{\chi_2(s_{b3}^{NN})}{\Delta_3 - k_{3z} + i\varepsilon} \frac{\chi_1(s_{a2}^{NN})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} + \frac{\chi_2(s_2^{NN})}{\Delta_2 - k_{2z} + i\varepsilon} \frac{\chi_1(s_{a3}^{NN})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} \right] \\
&\times \Psi_A^{s_A}(p_m + k_3 + k_2, s_m, t_f; -\frac{p_m}{2} - k_2 + p'_{23}, s_2, t_2; -\frac{p_m}{2} - k_3 - p'_{23}, s_3, t_3) \left. \right\} \Big|^2 \\
&\times \delta(E_m - T_{r2} - T_{r3} - |\epsilon_A| + \frac{p_m^2}{4m}). \tag{66}
\end{aligned}$$

7 Nuclear Decay Function Formalism

During the last two decades there were extensive studies of semi-inclusive $(e, e'N)$ reactions, both experimental (see, for instance, [41] and references within, [42]) and theoretical ([43]). In these reactions scattered electrons are detected with knocked-out nucleon in coincidence. No other particles are detected in the final state of the reaction. The main quantity which was discussed in these studies was the Spectral Function $S^N(E_m, \mathbf{p}_m)$. In the impulse approximation picture the Spectral Function describes the probability of finding a struck-nucleon in the nucleus initially having missing momentum, $\mathbf{p}_m$, and missing energy, $E_m = q_0 - T_f - \frac{p_m^2}{2(A-1)m}$. The missing energy characterizes the excitation energy of the final $A - 1$ system. The cross section of semi-inclusive $(e, e'N)$ reaction is expressed through the Spectral Function as follows:

$$\frac{d\sigma^N}{dE_e' d\Omega_{e'} dp_f} = K(Q^2, \mathbf{p}_m) \sigma_{eN} S^N(E_m, \mathbf{p}_m), \quad (67)$$

where $K(Q^2, \mathbf{p}_m)$ is a kinematic factor and σ_{eN} is a cross-section of electron scattering off the free nucleon. The FSI effects in this case are included in the Spectral Function. The approximation of Eq.(67) referred as the Distorted Wave Impulse Approximation (DWIA).

For the case of the reaction Eq.(1) one need to observe that Eq.(67) is inadequate, since in addition to knocked-out nucleon a recoil nucleon is detected in the final state of the reaction. For this reason we generalize the Spectral Function approach for the case of the exclusive reaction (1), as well as for any semi-exclusive reactions involving

two-nucleon emission by introducing a nuclear Decay Function which can be formally defined as [44, 45]:

$$D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}) = \sum_f |\langle A-1 | a^\dagger(\mathbf{p}_{r2}) a(\mathbf{p}_m) | \Psi_A \rangle|^2 \delta(E_m - (\epsilon_{A-2}^f + T_{r2} - \epsilon_A - \frac{p_m^2}{2(A-1)m})), \quad (68)$$

where one sums over the states, f , of $A-2$ residual nucleus and $\epsilon_{A-2}^f = E_{A-2}^f - (A-2)m$, where E_{A-2}^f is the total energy of $A-2$ state. It follows from this definition that the Decay Function characterizes the joint probability to find a struck nucleon in the nucleus with momentum $\mathbf{p}_m$, missing energy E_m and the recoil nucleon with momentum $\mathbf{p}_{r2}$ in the decay products of the residual $A-1$ nucleus. Note that for $A \geq 4$, Eq.(69) assumes the sum over all possible (bound and continuum) ($A-2$) substates, provided the total excitation energy of $A-1$ final state is E_m . The general properties of the Decay Function are discussed in [45] where the qualitative features are studied within the two-nucleon correlation model. The following sum rule and normalization condition follow from Eq.(69) (see [45]):

$$\begin{aligned} \int d^3 p_{r2} D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}) &= S^N(E_m, \mathbf{p}_m); \\ \sum_N \int dE_m d^3 p_m S^N(E_m, \mathbf{p}_m) &= 1. \end{aligned} \quad (69)$$

$S^N(E_m, \mathbf{p}_m)$ is a conventional Spectral Function of semi-exclusive reaction and in impulse approximation picture characterizes the joint probability of finding a struck-nucleon in the nucleus initially having missing momentum $\mathbf{p}_m$ and missing energy $E_m = q_0 - T_f - \frac{p_m^2}{2(A-1)m}$. The missing energy describes the excitation energy of the final $A-1$ system.

7.1 Decay Function Within Impulse Approximation

Within the impulse approximation the cross section of the reaction with two-nucleon emission can be expressed through the Decay Function as follows:

$$\frac{d\sigma}{dE'_e d\Omega'_e d^3p_f d^3p_{r2}} = \frac{j_N}{j_A} \cdot A \sum_N \sigma_{eN}(p_f, k_e, k'_e) \cdot D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}), \quad (70)$$

where $E_m = T_{r2} + T_{r3} + |\epsilon_A| - T_{A-1}$, with T_{A-1} being the kinetic energy of the center mass of the residual $A - 1$ system. j_N is the flux calculated for the moving nucleon with momentum $\mathbf{p}_m$, and σ_{eN} represents the cross section of electron - off-shell nucleon scattering. We need to note, that in some cases σ_{eN} is defined without the flux factor (see e.g. [47]). In these cases factor $\frac{j_N}{j_A}$ should not be included in Eq.(70).

It is straightforward to calculate the Decay function of reaction (1). Using Eq.(41) one obtains [45]:

$$D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}, t_{r2}) = \frac{1}{2} \sum_{s_A, s_m, s_{r2}, s_{r3}} \left| \sum_{s_2, s_3} \sum_{t_2, t_3} \int d^3p_{23} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3) \right. \\ \left. \times \Psi_A^{s_A}(p_m, s_m, t_m; p_2, s_2, t_2; p_3, s_3, t_3) \right|^2 \cdot \delta(E_m - T_{r2} - T_{r3} - |\epsilon_A| + \frac{p_m^2}{4m}). \quad (71)$$

Note that hereafter we use the ground state wave Ψ^{s_A} normalized to 1.

To understand the role of the pair distortion and FSI in the final state of the residual nucleus we will compare D^N with the PWIA version of the Decay function (D_{PWIA}^N) which will allow to estimate which kinamtics can suppress or boost the Final State Interaction or Pair Distortion effects.

7.2 Decay Function Within the Distorted Wave

Impulse Approximation

We also can generalize the definition of the Decay Function to include the single and double rescatterings of the knock-out nucleon with the residual nucleons. Such a generalization is usually meaningful within an approximation in which electromagnetic current of the knocked-out nucleon is factorized from rescattering integrals. Two conditions should be met in order for this factorization to be justified:

- First, one should be able to neglect the charge-exchange part of the amplitude for high energy small angle NN rescatterings. This follows, first, from the fact that at sufficiently high energies the energy dependence of the small angle hadronic scattering amplitude scales as $\sim s^{J-1}$, see Fig. 15 where s is the invariant energy of rescattering NN system and J corresponds to the spin of the exchanged particle [32]. Since, at the energies considered, charge-exchange requires predominantly spin-0 exchange in the t -channel as compared to spin-one exchange in the diagonal channel, one expects strong s suppression of the charge-exchange amplitude as compared to the elastic NN amplitude. Analysis of the existing data on NN scattering demonstrates (see e.g. [48]) that starting at $Q^2 \geq 2 \text{ GeV}^2/c^2$ the charge-exchange part amounts only a third of the forward scattering NN amplitude and decreases linearly with an increase of Q^2 . Moreover, this contribution is further suppressed due to the fact that the charge-exchange amplitude is predominantly real and does not interfere with

the dominant part of the FSI amplitudes which are predominantly imaginary. Taking into account also the larger slope factor of the t dependence of charge-exchange amplitude as compared to the forward scattering, one estimates only a few percent overall contribution to the NN rescattering amplitude at $Q^2 \geq 2$ GeV^2/c^2 . This contribution decreases sharply with an increase of Q^2 .

Figure 15: Total cross section for proton-neutron charge-exchange scattering over the diagonal scattering total cross section vs. s , square of the center-of-mass energy of the reaction.

- Secondly, the transferred momentum, q , should be large enough that one can neglect the k , k_2 and k_3 dependencies in the electromagnetic current. Such an approximation is justified by the kinematic condition of Eq.(2) and by the fact that the characteristic average momenta transferred during NN rescattering

are restricted by the slope of the exponent in Eq.(29) , i.e. $\langle k^2 \rangle, \langle k_2^2 \rangle, \langle k_3^2 \rangle \leq \frac{2}{B} \sim 250 \text{ (MeV/c)}^2$.

Keeping only the diagonal part in the NN rescattering amplitude and factorizing the electromagnetic current of knocked-out nucleon, we arrive at an expression similar to Eq.(70) in which we refer D as a Decay Function calculated within the distorted wave impulse approximation, D_{DWIA} (see [45]). The final result is:

$$\begin{aligned}
D_{DWIA}^N(Q^2, \mathbf{p}_m, E_m, \mathbf{p}_{r2}, t_{r2}) &= \frac{1}{2} \sum_{s_A, s_m, s_{r2}, s_{r3}} \\
&\times \left| \sum_{s_2, s_3} \sum_{t_2, t_3} \left\{ \int d^3 p_{23} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3) \right. \right. \\
&\times \Psi_A^{s_A}(p_m, s_m, t_f; p_2, s_2, t_2; p_3, s_3, t_3) \\
&- \frac{1}{2} \int \frac{d^3 k d^3 p_{23}}{(2\pi)^3} \frac{\Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p_{23}, s_2, t_2; s_3, t_3)}{\Delta^0 - k_z + i\varepsilon} \\
&\times \left[\chi_1(s_2^{NN}) f_{NN}^{t_f, t_2 | t_f, t_2}(k_\perp) \right. \\
&\cdot \Psi_A^{s_A}(p_m + k, s_m, t_f; -\frac{p_m}{2} + p_{23} - k, s_2, t_2; -\frac{p_m}{2} - p_{23}, s_3, t_3) \\
&+ \Psi_A^{s_A}(p_m + k, s_m, t_f; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} - p_{23} - k, s_3, t_3) \\
&\cdot \left. \chi_1(s_3^{NN}) f_{NN}^{t_f, t_3 | t_f, t_3}(k_\perp) \right] \\
&+ \frac{1}{4} \int d^3 p'_{23} \frac{d^3 k_3}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \Psi_{NN}^{\dagger p_{r23}, s_{r2}, t_{r2}; s_{r3}, t_{r3}}(p'_{23}, s_2, t_2; s_3, t_3) \\
&\times f_{NN}^{t_3, t_f | t_3, t_f}(k_{3\perp}) f_{NN}^{t_2, t_f | t_2, t_f}(k_{2\perp}) \\
&\times \left[\frac{\chi_2(s_{b3}^{NN})}{\Delta_3 - k_{3z} + i\varepsilon} \frac{\chi_1(s_{a2}^{NN})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} + \frac{\chi_2(s_2^{NN})}{\Delta_2 - k_{2z} + i\varepsilon} \frac{\chi_1(s_{a3}^{NN})}{\Delta^0 - k_{2z} - k_{3z} + i\varepsilon} \right] \\
&\times \Psi_A^{s_A}(p_m + k_3 + k_2, s_m, t_f; -\frac{p_m}{2} - k_2 + p'_{23}, s_2, t_2; -\frac{p_m}{2} - k_3 - p'_{23}, s_3, t_3) \Big\} \Big|^2 \\
&\times \delta(E_m - T_{r2} - T_{r3} - |\epsilon_A| + \frac{p_m^2}{4m}). \tag{72}
\end{aligned}$$

8 Technical Aspects of Numerical Estimates

8.1 Ground State Wave Function

In our numerical analysis we use the ${}^3\text{He}$ ground state wave functions calculated by the Bochum group [49] which solved the Faddeev equation for different sets of realistic NN potentials. One of the available NN potentials is Argonne V18 [2], which has a dominant charge-independent piece plus additional charge-dependent and charge-symmetry-breaking terms (total of 18 terms), including a complete electromagnetic interaction. Another potential is the high-precision, charge-dependent Bonn nucleon-nucleon potential (CD-Bonn) [1]. Authors in Ref. [49] consider also the different models of 3N interactions such as the Tucson-Melbourne [5] and the Urbana IX [6] potentials with the main motivation to describe the binding energy of the $A = 3$ system.

8.2 Pair Distortion

To estimate the reinteraction of two outgoing slow nucleons in Eq.(42) we use the SAID group's parameterization of the NN scattering amplitudes based on the partial wave analysis of the NN scattering data [40]. This parameterization successfully describes the NN scattering data at a low to intermediate momentum range ($p_{LAB} \leq 1.3/3$ GeV/c for pn/pp scattering).

In Appendix B we show the derivation of the nucleon-nucleon scattering amplitude treating elastic scattering of two particles as a scattering of one particle with the

reduced mass in the field of a fixed force center. In order to perform the numerical integrations in Eq.(41) for the case of amplitude A_{0s}^μ we therefore will use the results of Appendix B and rewrite Eq.(41) in the following form:

$$\begin{aligned}
A_{0s}^\mu &= \sqrt{2E_{r2}2E_{r3}}(2\pi)^3 \sum_{s_1, s_2, s_3, t_2, t_3} \\
&\times \int d^3p_{23} \frac{1}{2\pi^2} \frac{\langle s_{r2}, t_{r2}; s_{r3}, t_{r3} \mid f_{NN}^{\text{off shell}}(\mathbf{p}_{r23}, \mathbf{p}_{23}) \mid s_2, t_2; s_3, t_3 \rangle}{\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2 - i\varepsilon} \\
&\times j_{t1}^\mu(p_m + q, s_f; p_m, s_1) \cdot \Psi_A^{s_A}(p_m, s_1, t_1; -\frac{p_m}{2} + p_{23}, s_2, t_2; -\frac{p_m}{2} - p_{23}, s_3, t_3).
\end{aligned} \tag{73}$$

To proceed further we recall the identity which holds under the sign of integral:

$$\frac{1}{\alpha - \alpha_c + i\varepsilon} = \text{P.V.} \frac{1}{\alpha - \alpha_c} - i\pi\delta(\alpha - \alpha_c), \tag{74}$$

where P.V., as always, denotes the Cauchy principal value integral. With the help of this we can replace the denominator in Eq.(74) as follows:

$$\frac{1}{\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2 - i\varepsilon} = \underbrace{i\pi\delta(\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2)}_{\text{pole term}} + \underbrace{\text{P.V.} \frac{1}{\mathbf{p}_{23}^2 - \mathbf{p}_{r23}^2}}_{\text{P.V.term}}. \tag{75}$$

We will restrict our calculations with the pole term only. This corresponds to the scattering problem of unbound nucleons. Within the framework of considered kinematics, contribution of P.V. term plays the role of a small correction. Rewriting $d^3p_{23} = 1/2|\mathbf{p}_{23}|dp_{23}^2d\Omega_{23}$ and performing delta-function integration one has:

$$\begin{aligned}
A_{0s}^{\mu(\text{pole})} &= \pi^2 \sqrt{2E_{r2}2E_{r3}} \sum_{s_1, s_2, s_3, t_2, t_3} \int d\Omega_{23} 2i|\mathbf{p}_{r23}| f_{NN}^{\text{on shell}}(\mathbf{p}_{r23}, \mathbf{p}'_{r23}) \times \\
&\times j_{t1}^\mu(p_m + q, p_m) \cdot \Psi_{3\text{He}}(p_m; p'_{r23} - \frac{p_m}{2}; -p'_{r23} - \frac{p_m}{2}),
\end{aligned} \tag{76}$$

where, as a result of delta-function integration, we have a new vector, $\mathbf{p}'_{r23}$, which has an absolute value of $\mathbf{p}_{r23}$ but points in the direction of $\mathbf{p}_{23}$.

Thus, the final expression for pair distortion amplitude can be put in the following form, suitable for computer programming and numerical estimates:

$$\begin{aligned}
A_{0s}^{\mu(\text{pole})} &= \pi^2 \sqrt{2E_{r2}2E_{r3}} \sum_{s_1, s_2, s_3, t_2, t_3} j_{t1}^\mu(p_m + q, p_m) \int d\Omega_{23} \tilde{f}_{NN}^{\text{on shell}}(\mathbf{p}_{r23}, \mathbf{p}'_{r23}) \times \\
&\times \Psi_{3\text{He}}(p_m; p'_{rs} - \frac{p_m}{2}; -p'_{rs} - \frac{p_m}{2}), \tag{77}
\end{aligned}$$

where NN scattering amplitude can be expressed through partial waves phase shifts δ_l using the well-known expansion

$$\begin{aligned}
\tilde{f}_{NN}^{\text{on shell}}(\mathbf{p}_{r23}, \mathbf{p}'_{r23}) &\equiv 2i|\mathbf{p}_{r23}| \cdot f_{NN}^{\text{on shell}}(\mathbf{p}_{r23}, \mathbf{p}'_{r23}) = \\
&= \sum_{l=0}^{\infty} (2l+1)(\exp(2i\delta_l) - 1) P_l \left(\frac{\mathbf{p}_{r23}}{|\mathbf{p}_{r23}|} \cdot \frac{\mathbf{p}'_{r23}}{|\mathbf{p}'_{r23}|} \right). \tag{78}
\end{aligned}$$

8.3 Final State Interaction

To estimate the single and double rescattering contributions in the total scattering amplitude, we use the following conventional parameterization for high energy small angle NN scattering:

$$f^{NN} = \sigma_{tot}^{NN}(s)(i + \alpha^{NN}(s))e^{\frac{B^{NN}(s)}{2}t} \delta^{h_1, h'_1} \delta^{h_2, h'_2}, \tag{79}$$

Here $\sigma_{tot}^{NN}(s)$ represents the total cross section of nucleon-nucleon scattering, $\alpha^{NN}(s)$ is the ratio of the real to imaginary part of the NN scattering and $B^{NN}(s)$ is the diffraction slope parameter for the NN forward scattering. We have parameterized [50] σ_{tot}^{NN} , $\alpha^{NN}(s)$ and $B^{NN}(s)$ for both pn and pp scatterings (see also Appendix C). We use practically all published data on small angle nucleon-nucleon scattering. The Fortran listing of our parametrizations can be found in the Appendix C.

In performing the numerical integrations in Eqs.(56) and (63) we represent the knocked-out nucleon propagator in the integral as a sum of the pole and the principal value (P.V.) terms

$$\frac{1}{\Delta - k_z + i\epsilon} = -i\pi\delta(\Delta - k_z) + P.V.\frac{1}{\Delta - k_z}, \quad (80)$$

where k_z and Δ characterize the transferred longitudinal momenta due to rescattering. Note that strictly speaking one can use the parameterization of the free (on-shell) NN scattering amplitude in Eq.(79) for the pole term of Eq.(80) only. For the P.V. term, the NN rescattering amplitude is off-shell. However, we use the parameterization in Eq.(79) for P.V. term too since, in the kinematics of interest, the P.V. term is only a small correction as compared to the pole contribution of Eq.(80).

9 Studying Short-Range Nucleon Correlation in the Nuclei

The published definitions of Short Range Correlations (SRC) vary, frequently referring to the difference between a mean field wave function and an exact wave function. This dissertation will use an experimental definition of an SRC as an NN pair with large relative momentum and small total momentum. In other words we are interested in the short range interactions between the partners of NN pair, while the total system is in its ground state. The short range interactions manifest themselves as the momentum and angular correlations. For the case of three body break up these short range NN interactions can be distorted due to the long range reinteractions of the third nucleon off the NN correlation.

We also need to point out what is not considered as a correlation. When the system is viewed as an assembly of independent particles moving in an average potential one does not speak about correlations: if the particles move independently they are not correlated by definition. Also one may point to the Pauli blocking as an example of correlation: indeed, each particle feels the presence of others due to the simple fact that it is not allowed to occupy a single particle state already occupied by another particle. However this correlation has statistical origin and therefore is not the dynamic correlation, which we are interested to study in this dissertation.

The dynamical correlations result from different range properties of the NN potential. There are long range, intermediate range and short range correlations of

dynamical origin. Short range correlations reflect the repulsive nature of the NN potential at small distances. SRC affect the reactions where the momentum transferred from the probe to the system is high enough to resolve the dynamical properties of the particles when they are close to each other.

In the following numerical analyzes our main goal is to identify the strategies which are best suited for investigation of two- and three- nucleon short range correlations in the three-body breakup reactions.

9.1 Two-Nucleon Correlations

Here we are particularly interested in studies of the following configurations:

- **Type 2N-I** correlations from which a nucleon is knocked-out by virtual photon, while the third nucleon moves in the mean field of 2N SRC Fig. 16(a). For the idealized case of third nucleon at rest one obtains a correlation relation:

$$E_m^{(2N-I)} = \sqrt{m^2 + p_m^2} - m - T_{A-1}, \quad (81)$$

where $T_{A-1} = \sqrt{4m^2 + p_m^2} - 2m$. Note that although we apply the approximation, $p^2/m^2 \ll 1$ in calculation of the scattering amplitudes, we still use the relativistic form of the kinetic energy to preserve the relativistic kinematics.

- **Type 2N-II** correlations in which virtual photon strikes a third isolated nucleon while 2N SRC breaks up at the final state of the reaction, Fig. 16(b). In this case the expected correlation between E_m and $\mathbf{p}_{\mathbf{r}2} \approx -\mathbf{p}_{\mathbf{r}3}$ for the case of $p_m \approx 0$ is:

$$E_m^{(2N-II)} = \sqrt{m^2 + p_{r2}^2} + \sqrt{m^2 + p_{r3}^2} - 2m. \quad (82)$$

Figure 16: Definition of the type 2N-I (a) and type 2N-II (b) correlations.

Figure 17: Definition of the type 3N-I (a) and type 3N-II (b) correlations.

9.2 Three-Nucleon Correlations

Here we focus on two particular situations:

- **Type 3N-I** correlations, in which case the high missing momentum of the knocked-out nucleon is balanced by two nucleons which share almost equally the missing momentum, p_m , as it depicted on Fig. 17(a). This corresponds to the minimal missing mass of recoil 2N system with missing energy:

$$E_m^{(2N-I)} \approx |\epsilon_A|. \quad (83)$$

- **Type 3N-II** correlations, in which case the nucleon is knocked-out from the symmetric configurations where all three nucleons have the same momentum p_m , as

it schematically depicted on Fig. 17(b). This corresponds to a significantly higher value of E_m as compared to the the above discussed 2N and 3N SRC cases:

$$E_m^{(3N-II)} = 2\sqrt{m^2 + p_m^2} - 2m - T_{A-1}. \quad (84)$$

9.3 Spectral Function

First we start with the calculation of the Spectral Function as it is defined in Eq.(90). This quantity is relevant primarily for semi-exclusive ($e, e'N$) reactions, in which only the scattered electron and knocked-out nucleon are registered. Fig. 18 shows the Spectral Functions for the cases of knocked-out proton (a) and knocked-out neutron (b) calculated with the ${}^3\text{He}$ ground state wave function based on the Urbana-IX potential [49, 51]. The hatched surface represents the PWIA prediction while dotted contours show the effect of pair distortion.

The correlation feature of Eq.(81) is reflected in the emergence of the broad peak in E_m distribution at $p_m \geq 300$ MeV/c, while the signature of 2N-II correlations is seen in the minimum of the E_m distribution at $p_m \approx 0$ for the case of knocked-out neutron with two recoil protons. This minimum reflects the fact that the relative momentum distribution in the pp pair has a node in the S -state at ≈ 420 MeV/c. These results are in agreement with previous analyses of Spectral Function [52].

The emergence of these correlations at $p_m \geq 300$ GeV/c are clearly seen in Fig.(19), in which solid and doted lines represent calculations within PWIA and IA approximations respectively. The positions of the peaks can be related to the dominance of the type 2N-I SRCs in the high missing momentum part of the nuclear

Figure 18: Spectral Function for ${}^3\text{He}(e, e'N)X$ reaction for the struck proton (a) and the neutron (b) calculated within the PWIA and IA. The lines at the sides of the histograms shows the effects due to pair distortion.

wave function [53, 54] according to Eq.(81) (i.e. $E_m^{Peak} \approx E_m^{2N-I}$). The solid arrows in the figure correspond to the prediction of Eq.(81) corresponding to the scattering off a quasi-free and stationary two-nucleon correlation. This situation is reflected in the fact that, at large nucleon momenta p_m , the Spectral Function has similar functional dependence on p_m as the stationary two nucleon correlation [53]. It is worth noting that there were several experimental indications [24, 26] of $E_m - p_m$ correlations according to Eq.(81). However, the lower values of Q^2 in these experiments did not

allow direct discrimination of the short-range two-body forces from the long-range two-body currents corresponding to the meson-exchange contributions.

Figure 19: Missing energy distribution of the Spectral Function for the removal of the proton (a) and neutron (b) at different values of missing momenta. Curves counted from the above at $E_m = 0$ correspond to the missing momenta 0 to 700 MeV/c with 100 MeV/c increments. Solid and dotted lines correspond to PWIA and IA predictions respectively. Arrows correspond to the prediction of Eq.(81)- scattering off a quasi-free stationary two-nucleon correlation.

The type 2N-II SRC can be identified clearly only for the Spectral Function involving two protons in the recoil kinematics, Fig. 19(b). As Figs.18 and 19

demonstrate, no clear signatures are seen for 3N correlations. One expects the type 3N-I correlations to dominate at the left corner of the E_m distribution starting at $p_m \geq 400 - 500$ MeV/c, while the type 3N-II correlations dominate at the right (higher value) corner of the E_m distribution starting at $p_m \sim 400$ MeV/c. Fig. 19 also demonstrates a strong dependence of pair-distortion effects on considered values of E_m and p_m .

9.3.1 Pair Distortion Effects

The pair distortion effects can be assessed quantitatively in Fig. 20 which shows the missing momentum, p_m , distribution of the ratios of the Spectral Functions calculated within the PWIA and IA approximations for different values of E_m . These calculations demonstrate that the pair distortion strongly suppresses (by a factor of five) the Spectral Function at the kinematic region of small p_m and E_m . This reflects the fact that in this region two spectator nucleons have vanishing relative momentum at which the interaction cross section is very large. For the same reason, one observes significant pair distortion effects in the kinematics (see Eq.(83)) favorable for studies of the type 3N-I correlations (high p_m part of the solid curve in Fig.(20)). Note that in this case pair distortion is larger for the pn recoil case than for the pp case.

Pair distortion is also large in the kinematics of very large $E_m > E_m^{2N-I,II}$ (dotted and dash doted lines in Fig. 20) where one expects the dominance of the type 3N-II correlations. The large pair distortion effects in the kinematics of 3N correlations can be understood qualitatively. The pair distortion effectively represents the three-

Figure 20: Missing momentum dependence of the ratio of Spectral Function calculated within the IA and PWIA approximation. Solid lines correspond to $E_m = 0$. For the dashed lines, E_m is defined according to Eq.(81). For the dashed-dotted lines, E_m defined according to Eq.(84) and for dotted lines, $E_m = 350$ MeV.

nucleon correlation, in which the initial short range NN correlation between knocked-out nucleon and one of the spectator nucleons is combined with the final state NN reinteraction between two recoil nucleons.

For the type 2N-I correlations, Fig. 20 (dashed lines) reveals modest pair distortion effects starting at $p_m \geq 400$ MeV/c. At the same time, Fig. 20 demonstrates more pair distortion for the type 2N-II correlations. This result can also be understood qualitatively. For the type 2N-I correlations, one of the recoil nucleons is initially in the SRC, while the second one is separated from the SRC with relatively

small momentum. As a result, once one of the nucleons is instantaneously removed from the SRC, the two recoil nucleons in the final state will be spatially separated, thus reducing the probability for their interaction.

Figure 21: Missing momentum dependence of the Spectral Function at different values of missing momenta. The curves counted from the upper left to the lower left of the figures correspond to the missing momenta 100, 200, 400 and 700 MeV/c. Dotted lines – PWIA prediction with only NN potential considered, dashed line – PWIA with additional 3N forces included, solid line – IA with the same 3N forces included.

The situation is different for the type 2N-II correlations in which two recoil nucleons are in the SRC and are spatially close to each other. This will result in much larger pair distortion effects than for the type 2N-I correlations. The observed feature is consistent with the qualitative arguments of Ref. [53, 46] that to break the SRC

and release a spectator from it with minimal distortion it is preferable to knock-out a nucleon directly from the SRC.

The next question we address is whether the Spectral Function can provide an effective framework for studies of the implication of three-nucleon forces (3NFs) in the ground state wave function of ${}^3\text{He}$. Qualitatively, one expects that the onset of 3NFs to arise in the kinematics dominated by three-nucleon correlations. This expectation is confirmed in Fig. 21 where the main 3NF effects are seen at $E_m = |\epsilon_A|$ and $E_m > E_m^{2N-I,II}$. Here we consider the difference in the predictions of the Spectral Functions calculated based on the ${}^3\text{He}$ wave functions which are calculated with and without explicit inclusion of 3NFs (see e.g. [49]). Predictions are made within PWIA are denoted by dotted and dashed lines in Fig. 21.

However, as it was discussed above, in the same region of E_m and p_m we predict sizable effects due to pair distortion, which effectively imitates a three-nucleon correlation. As Fig. 21 demonstrates pair distortion (solid lines) will considerably diminish signatures related to 3NF effects in the Spectral Function.

9.3.2 Final State Interaction (FSI) Effects

The inclusion of the Final State Interactions of the knocked-out nucleon with spectator nucleons removes the isotropy of Spectral Function with respect to the direction of the virtual photon, $\mathbf{q}$. To asses the FSI effects quantitatively, we analyze the ratio, T_S , defined as follows:

$$T_S = \frac{S_{DWIA}^N(Q^2, q, E_m, p_m)}{S_{IA}^N(E_m, p_m)}, \quad (85)$$

Figure 22: Angular dependence of transparency T as defined in Eq.(85) for the reaction with struck proton at different values of missing momenta. The missing energy here is defined according to the type 2N-I correlation condition of Eq.(81). Solid line – only single rescattering amplitude is included in FSI, dashed line – both single and double rescattering amplitudes are included in FSI.

where subscripts *DWIA* and *IA* denote the Spectral Functions calculated with and without FSI effects. Here $T_s \approx 1$ corresponds to the small effects of the FSI, while $T > 1$ or $T < 1$ will indicate the dominance of the absorption or the rescattering effects due to the FSI. Based on the analysis of the analytic properties of rescattering amplitudes in GEA [21], it is possible to identify the kinematic regions in which one can isolate the FSI or the SRC as the dominant term in the scattering amplitude. In particular, similar to the reaction of exclusive electro-disintegration of the deuteron

[23], one expects the FSI to dominate in nearly transverse kinematics in which $\alpha_m = 1$ and $p_m \geq 100$ MeV/c. Here, α_m in the IA describes the momentum fraction of the nucleus carried out by a nucleon “1” in the infinite momentum frame of the nucleus. It is defined as follows:

$$\alpha_m = \frac{p_{m-}}{p_{A-}} \approx \frac{p_{f-} - q_-}{m} \big|_{Lab \text{ Frame}}, \quad (86)$$

where the “minus” components of momenta are defined by $k_- = k_0 - k_z$.

Fig. 22 represents the θ_m (α_m) dependence of T_S at different values of p_m calculated at $Q^2 = 4$ GeV²/c². The missing energy in these calculations is chosen for the type 2N-I correlation (Eq.(81)). The figure demonstrates the presence of significant FSI effects in the near-transverse kinematics, $\alpha_m \approx 1$, with the effects diminishing at parallel ($\theta_m = 0^\circ$) and anti-parallel ($\theta_m = 180^\circ$) kinematics. (Everywhere in the text parallel/anti-parallel kinematics correspond to $\theta_m = 0^\circ/180^\circ$.)

The missing energy E_m , gives us an additional degrees of freedom to “manipulate” the relative strength of the FSI and SRC effects in the different kinematics of electro-disintegration. This is especially important for isolating SRC structures in the Spectral Function as it is measured in $A(e, e'N)X$ reaction. In the beginning of Sec. 9.3, we identified several kinematic regions in which the strength of the Spectral Function is largely determined by SRCs. In all these cases, the initial momenta of at least two nucleons in the nucleus exceeds 300 – 400 MeV/c. The final state reinteraction of the knocked-out nucleon with the spectator nucleons can largely destroy this picture. For example, as can be seen from Eqs.(56) and (63) due to the integration in the rescattering loops, it is impossible to ensure the appearance of large values of

internal momenta in the ground state wave function of the nucleus in a straightforward way. Thus, the condition $p_m \geq 300$ MeV/c or $E_m \sim 100$ MeV may not ensure the dominance of the high-momentum component in the ground state wave function of the nucleus. This situation may significantly affect the identification of the type 2N and 3N correlations.

The problem of suppression of the FSI in probing the SRC in the $A(e, e'N)X$ reaction was addressed within the GEA in Ref. [21]. It was observed that a trivial condition, $p_m \geq k_F$, is not sufficient to probe the SRC component of the ground state nuclear wave function. One needs to impose the following additional condition on the effective longitudinal momentum, p_Z , entering the single rescattering amplitude,

$$p_Z \equiv p_{mz} + \frac{q_0}{q} \left(E_m + \frac{p_m^2}{4m} \right) > k_F. \quad (87)$$

With this, one will ensure that rescatterings happen within the SRC. Note that “ p_Z ”s in Eq.(87) correspond to the pole values of fast nucleon propagators in the rescattering amplitude (see e.g. Eq.(45))

In Fig. 23, we consider the $E_m(p_Z)$ dependence of T_S for parallel (a) and anti-parallel (b) kinematics for values of p_m equal to 400, 500 and 600 MeV/c. In Fig. 23(a), the center of mass momentum of the two recoil nucleons is in the direction backward to $\mathbf{q}$, while in Fig. 23(b), the recoil system is produced in the forward direction. One can see from this figure that the FSI indeed decreases with increase of $|p_Z|$. This indicates that the FSI is increasingly confined within the SRC. An interesting feature of these results is that while the FSI contribution keeps decreasing with increase of $|p_Z|$ in the kinematic region relevant for the type 2N-I and 3N-II

Figure 23: Missing energy dependence of transparencies for the knocked-out proton reaction at parallel, $\theta_m = 0^\circ$, (a) and anti-parallel, $\theta_m = 180^\circ$, (b) kinematics. The solid, dashed and dotted curves corresponds to the calculation with missing momenta equal to 400, 500 and 600 MeV/c respectively.

correlations, it grows sharply in the region associated with the type 3N-I correlations. Both trends can be understood qualitatively. When $|p_Z| \leq k_F$, one has dominant contributions from the FSI involving uncorrelated nucleons which have a larger probability amplitude in the ground state wave function of ${}^3\text{He}$. Once $|p_Z| > k_F$, the FSI takes place predominantly within the 2N correlation. In the latter case, for the type 2N-I SRC, the third nucleon has a small momentum and is spatially separated from the NN SRC. Thus, it does not contribute substantially to the FSI.

For the type 3N-I correlations both spectator nucleons contribute to the FSI since they are both correlated with the knocked-out nucleon. This results in larger FSI

Figure 24: Missing energy dependence of the Spectral Function for the type 2N-II correlation condition at missing momentum $p_m = 0$. Dotted, dashed-dotted, dashed and solid lines correspond to PWIA, IA, IA+FSI1 and full (IA+FSI) calculations respectively.

effects as compared to the IA contribution. These calculations indicate that (except for the case of the type 3N-II SRC in the parallel kinematics) the FSI generally dominates in the kinematics where one expects an enhanced contribution from 3N SRCs in the wave function of 3He .

For the type 2N-II correlations, the FSI takes place between the knocked-out nucleon, which is initially almost at rest and spatially separated from the 2N SRC (see Fig. 16(b)). As a result, one expects a diminished FSI contribution in whole range of E_m . Such behavior can be seen in Fig. 24 for the calculated E_m dependence

of the Spectral Function at $p_m = 0$. This calculation shows that the type 2N-II SRC for the knocked-out neutron reaction attains the characteristic minimum in E_m distribution although smeared out strongly due to rescatterings.

In Fig. 25 we summarize the predictions for the E_m dependence for the Spectral Function at different values of missing momenta p_m calculated for parallel (a) and anti-parallel (b) kinematics. These calculations demonstrate that the observed $E_m - p_m$ correlation within the IA for the type 2N-I SRC generally survives the FSI for both parallel and anti-parallel kinematics. For 3N correlations, the type 3N-II SRC survives FSI for parallel kinematics while the type 3N-I SRC is strongly affected by FSI.

9.3.3 Summary on Numerical Studies of the Spectral Function

Summarizing our consideration of the Spectral Function we can conclude that the $A(e, e'N)X$ reaction is best suited for studies of 2N correlations only. Type 2N-I SRC survives both pair distortion and the FSI while the type 2N-II attains its specific feature for neutron-knock out reactions at $p_m \leq 100$ MeV/c. As a note of caution, we point out that the minimum in the E_m distribution is very narrow. Hence its experimental observation will require resolution in E_m on the order of a few MeV.

Our calculations show that, in general, 3N correlations are strongly affected by pair distortion and FSI effects. Pair distortion, having a qualitative features of 3N correlation, strongly affects both the types of 3N correlations, while the FSI contributes strongly in the type 3N-I SRC and has a diminished impact on the type 3N-II SRCs only at parallel kinematics.

Figure 25: Missing energy dependence of the Spectral Function for the knocked-out proton reaction in parallel (a) and anti-parallel (b) kinematics at different values of missing momenta. The upper, middle and lower curves at the left corner of the figure correspond to the missing momenta of 400, 500 and 600 MeV/c respectively. Dashed line – IA prediction, solid line IA+FSI prediction.

Finally, for near transverse kinematics starting at $p_m \geq 400$ MeV/c, the $A(e, e'N)X$ reactions represent an ideal tool for studying the structure of FSIs.

9.4 The Decay Function

The Decay Function is practically an unexplored quantity and the experiments that will allow us to extract it from the exclusive cross section data are only emerging [55]. Our main motivation in these numerical analyses is to highlight those significant

Figure 26: Decay Function vs. momenta of the two recoil protons p_{r2} and p_{r3} at different cuts of missing momentum of the knocked-out neutron. (a) $p_m > 150$ MeV/c, (b) $p_m > 300$ MeV/c, (c) $p_m > 400$ MeV/c and (d) $p_m > 700$ MeV/c.

features of the Decay Function that are related to the short-range structure of the ground state nuclear WF and to the structure of reinteractions between the nucleons in the final state of the reaction.

We will consider the partially integrated Decay Function, which will allow us to remove the δ -function in Eq.(71). Namely, we consider [45]:

$$D_{int}^N(\mathbf{p}_m, E_m, p_{r2}, t_{r2}) = \int D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}, t_{r2}) p_{r2}^2 d\Omega_{r2}, \quad (88)$$

where $D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2})$ is defined in Eqs.(71). This integration takes into account the fact that one of the components of $\mathbf{p}_m$ or $\mathbf{p}_{r2}$ is not independent and is fixed from the

energy conservation condition for the quasi-elastic disintegration of ${}^3\text{He}$. Therefore, D_{int} represents a quantity which could be extracted directly from the ${}^3\text{He}(e, e'NN)N$ experiment. Furthermore, we will refer to D_{int} as a Decay Function.

Figure 27: The same as in Fig. 26 for proton and neutron recoil with momenta p_{r2} and p_{r3} respectively.

We start by analyzing the Decay function in the PWIA, focusing on those features that are related to SRC signatures of the ground state wave function which are described in Figs.16 and 17. As we observed in the previous section, the type 2N-I SRCs exhibited measurable (though broad) correlation between E_m and p_m (starting at $p_m \geq 300$ MeV/c), with the peak of the E_m distribution defined by Eq.(81).

These correlations could be understood qualitatively as a result of strong short range interactions between the struck nucleon (with momentum p_m) and one of the recoil nucleons in the nucleus with the third nucleon having a relatively small momentum in the mean field of the 2N SRC pair. To check this picture, we calculate the Decay Function distribution as a function of p_{r2} and p_{r3} imposing different cuts on the missing momentum, p_m .

Figs.26 and 27 present the results of the calculation of the Decay Function of the reactions corresponding to the knock-out of the neutron and proton respectively. The (a), (b), (c) and (d) parts correspond to the p_m cuts at $p_m > 150, 300, 400$ and 700 MeV/c respectively. The calculations clearly show emerging the type 2N-I correlations between the knocked-out nucleon and one of the spectator nucleons with an increase of p_m . These correlations are dominating the landscape of the (p_{r2}, p_{r3}) distribution once $p_m \geq 300$ MeV/c. Fig. 27 shows also that the pn SRCs are significantly larger than the pp correlations. Thus, measuring the Decay Function will allow a direct check of the relative strength of pp and pn correlations. We conclude, from the discussions of Figs.26 and 27, that the Decay Function is strikingly more sensitive to the SRC than the $E_m - p_m$ correlation observed in the Spectral Function.

Next we analyze the features of the Decay Function related to the type 2N-II correlations. From the consideration of the Spectral Function in the previous section, we learned that reactions corresponding to the knock-out of the neutron are best suited for studies of the type 2N-II correlations since, at small $p_m < 100$ MeV/c

Figure 28: The (E_m, p_{r2}) distribution of the Decay Function for the neutron knockout reaction. The missing momentum is restricted to $p_m < 50$ MeV/c.

and large E_m , the spectral function exhibits a minimum associated with the node in the relative momentum distribution of the recoil pp pair in the S -state. In Fig. 28 we show the (E_m, p_{r2}) distribution of the Decay Function which demonstrates strong correlations between the minimum in the E_m distribution and the value of recoil nucleon momenta at small p_m (< 50 MeV/c in the calculation).

Turning now to the three-nucleon correlations, we observe that for both types of 3N SRCs (Fig. 17) the two recoil nucleons have comparable momenta. This situation corresponds kinematically to the area around the saddle in Figs.26 and 27. One can

check whether the expectation that type 3N-I and 3N-II correlations will dominate in the dynamics of three-nucleon correlations is justified. For this we observe that the configurations of Fig. 17 are characterized by a distinctive angular relations between the two recoil nucleons: for the type 3N-I SRC two recoil nucleons will be produced at small relative angle (almost parallel) while for the type 3N-II SRC the relative angle of the recoil nucleon momenta is $\approx 120^\circ$. In Fig. 29 we analyze the dependence of the Decay Function on the relative angle between the recoil nucleons, θ_{23} , and the missing energy, E_m , for different values of missing momentum, p_m .

Fig. 29(c) and (d) clearly show an emergence of peaks at small E_m and θ_{23} (the type 3N-I SRCs) and at large E_m and $\theta_{23} \approx 120^\circ$ (the type 3N-II SRCs). The appearance of the peaks is more clearly seen in Fig. 30 where Fig. 29 (d) is demonstrated from different viewpoints. Fig. 30 shows also the peak at $\theta_{23} \approx 180^\circ$ and at moderate values of E_m which corresponds to the type 2N-I correlation in which only one of the recoil nucleons is correlated with the knocked-out nucleon and is produced in the direction opposite to the direction of $\mathbf{p}_m$.

Moving to a more quantitative discussion of correlation properties of the Decay Function, in the following subsections we study the following question: To what degree do the detected properties of the recoil nucleon in coincidence with the knocked-out nucleon reflect the properties of preexisting short range configurations in the ground state nuclear wave function?

Figure 29: The dependence of the Decay Function of the reaction ${}^3\text{He}(e, e'pp)n$ on the relative angle between the recoil pn nucleons, θ_{23} , and the missing energy, E_m , for different cuts of missing momentum, p_m . (a) – no cuts on p_m , (b) – $p_m > 300$ MeV/c, (c) – $p_m > 500$ MeV/c, (d) – $p_m > 700$ MeV/c.

9.4.1 Pair Distortion Effects

In Fig. 31 we discuss the effects of the reinteraction between recoil nucleons (pair distortion) in the p_{r2} -momentum distribution of the Decay Function for all four types of correlations. Herewith, we will label calculations by (N_f, N_{r2}) , in which N_f and N_{r2} describe the type of the knocked-out and recoil nucleons with momenta $\mathbf{p}_f = \mathbf{p}_m + \mathbf{q}$ and $\mathbf{p}_{r2}$ respectively. In Fig. 31(a) one observes a significant difference in the yields of

the recoil proton or neutron for a reaction in which one of the protons is knocked-out from the type 2N-I correlations. Fig. 31(b) shows the momentum distribution of the recoil proton when proton (labeled “pp” in the figure) or neutron (labeled “np”) is knocked-out in the kinematics of type 2N-II correlations.

Figure 30: Reproduction of Fig. 29d from different viewpoints to emphasize the signatures of the type 3N-I (a) and type 3N-II (b) correlations.

This figure demonstrates significant pair distortion effects as compared to the type 2N-I correlations. Figs.31(c) and (d) demonstrate the momentum distributions from the type 3N-I and II SRC's respectively. As in the case of the Spectral Function, one observes that, in general, the pair distortion interferes significantly with the three-nucleon correlations. It is worth noting, however, that, due to the depleted interaction strength in the pp channel at relative momenta ~ 400 MeV/c, pair distortion effects

are suppressed in the type 3N-II correlations for the neutron knock-out reaction in a recoil momentum range of $300 - 550 \text{ MeV}/c$ (see Fig. 31(d)). We will discuss in Sec.10 how this observation could be used to explore the type 3N-II kinematics for investigation of three-nucleon forces in ${}^3\text{He}$.

Also, Fig. 31 reveals an additional feature that allows us to discriminate between 2N and 3N SRC signals. This feature is the relative abundance of pp and pn pairs in different correlations. For 2N correlations one observes significantly smaller yield associated with pp correlations as compared to pn correlations while for 3N correlations these yields are comparable.

9.4.2 Final State Interaction

The inclusion of the final state interaction of the knocked-out nucleon with residual nucleons removes the isotropy of the Decay Function with respect to the momentum vector of the virtual photon, $\mathbf{q}$. Note that we have chosen $\hat{z}$ to be parallel to $\mathbf{q}$. Therefore, staying in the framework of consideration of the type 2N and 3N correlations, we investigate FSI effects in angular and momentum dependences of the decay function for each type of correlation. As in Sec.9.3.2, we consider the kinematics with fixed $Q^2 = 4 \text{ GeV}^2/c^2$.

Figure 31: Dependence of the Decay Function on the momentum of the registered recoil nucleon, p_{r2} , in ${}^3\text{He}(e, e'N_f N_{r2})N_{r3}$ reactions. (a), (b), (c) and (d) correspond to the types 2N-I, 2N-II, 3N-I and 3N-II kinematics respectively. Dashed lines – PWIA prediction, solid lines – IA predictions. Two pairs of curves in each figure correspond to different compositions of detected nucleons (N_f, N_{r2}).

A. Type 2N-I Correlations:

For the type 2N-I correlations, we consider first the dependence of the Decay Function on the production angle of recoil nucleon N_{r2} with respect to the direction of the virtual momentum $\mathbf{q}$. Fig. 32 shows these dependences for different values of recoil nucleon (neutron (a) and proton (b)) momentum p_{r2} for the reaction with knocked-out proton. Calculations clearly show the transition of FSI from the screening regime

at momenta $p_{r2} \leq 300$ MeV/c to the double scattering regime at $p_{r2} \geq 400$ MeV/c. This picture is clearly reminiscent the double scattering signatures of the deuteron electro-disintegration [23, 21, 22].

Figure 32: Dependence of the Decay Function on the angle of the detected recoil nucleon, θ_{r2} , with respect to $\mathbf{q}$ for different and fixed values of p_{r2} . The Decay Function is calculated for the type 2N-I SRC kinematics. (a) – $(N_f, N_{r2}) \equiv (p, n)$, (b) – $(N_f, N_{r2}) \equiv (p, p)$. Dotted lines - IA, dashed lines - IA+FSI1 (Single Rescattering only), solid line - IA+FSI. Note that for figure (b) the IA contribution at $p_{r2} = 400$ MeV/c is not shown since it is negligible due to the node in the pp momentum distribution.

Calculations also predict very different angular dependences for neutron (a) and proton (b) production in recoil kinematics. It is easy to check that the maximal FSI

effects are predicted at $\alpha_{r2} = \frac{E_{r2} - p_{r2}^z}{m} = 1$ which is analogous to the maximums of the FSI for the Spectral Functions at $\alpha_m = 1$ observed in Sec.9.3.2. Fig. 32 shows also that our best chances to extract genuine information about the type 2N-I SRC, is to concentrate on anti-parallel ($\theta_{r2} = 0^0$) and parallel ($\theta_{r2} = 180^0$) kinematics. It is worth noting that the range of θ_{r2} where FSI, are small is broader for the $\theta_{r2} = 180^0$ case.

Figure 33: Dependence of the Decay Function on the momentum of the recoil nucleon, p_{r2} calculated for the kinematics of the type 2N-I correlations (a) – $(N_f, N_{r2}) \equiv (p, n)$ and (b)– $(N_f, N_{r2}) \equiv (p, p)$. Doted lines - IA prediction, dashed and solid lines for IA+FSI1 and IA+FSI predictions respectively for parallel $\theta_m = 0^0$ kinematics. Curves with square labels correspond to IA+FSI1 and IA+FSI predictions for anti-parallel $\theta_m = 180^0$ kinematics.

Fig. 33 represents the p_{r2} momentum dependences of the Decay Function for $\theta_{r2} = 180^\circ$ and $\theta_{r2} = 0^\circ$ (marked curves) kinematics. Calculations predict qualitatively different momentum distributions for correlated pn (a) and pp (b) pairs. While for pn at both $\theta_{r2} = 180^\circ$ and $\theta_{r2} = 0^\circ$ one observes qualitatively similar momentum distributions, for pp pairs they are significantly different. This difference can be understood from Eq.(87), which defines the effective longitudinal component of missing momentum entering in the rescattering amplitude. For the type 2N-I kinematics it can be written as $p_Z = -p_{r2}\cos(\theta_{r2}) + \frac{q_0}{q}(E_m + \frac{p_{r2}^2}{4m})$. At $\theta_{r2} = 180^\circ$ ($\theta_{r2} = 0^\circ$), $p_Z > (<)p_{r2}^z$ and the FSI term is defined by the effective momentum which is larger (smaller) than measured momentum, p_{r2} . As a result, the FSI is suppressed at $\theta_{r2} = 180^\circ$ (backward) kinematics as compared to the $\theta_{r2} = 0^\circ$ (forward) kinematics. For the pp pair this difference is very dramatic due to the node in the relative momentum distribution of the pp pair.

It is worth noting that no significant triple scattering (double rescattering) contribution is observed in Figs.32 and 33. This serves as an additional indication that in the type 2N-I kinematics the reaction is defined predominantly by two-body interactions, thus representing a nearly ideal framework for studies of the 2N processes both in terms of the short range correlations and single rescattering processes.

B. Type 2N-II Correlations:

To isolate the type 2N-II correlations we identify kinematics similar to one presented in Fig. 31(b), in which missing momentum is fixed to $p_m = 100$ MeV/c in $\mathbf{q}$ direction. In Fig. 34 we consider the θ_{r2} dependence of the Decay Function for both

Figure 34: Dependence of the Decay Function on the angle of the detected recoil nucleon, θ_{r2} with respect to $\mathbf{q}$ for different and fixed values of p_{r2} . The Decay Function is calculated for the type 2N-II SRC kinematics with missing momentum fixed at $p_m = 100$ MeV/c and $\theta_m = 0^\circ$. (a) - $(N_f, N_{r2}) \equiv (n, p)$, (b) - $(N_f, N_{r2}) \equiv (p, p)$. Dotted lines - IA, dashed lines - IA+FSI1, solid line - IA+FSI.

pp (a) and pn (b) recoil pairs. Since, in these kinematics, the knocked-out nucleon is on average at a larger distance from the recoil 2N pair, one expects the lesser FSI effects due to rescattering of the knocked-out nucleon off the recoil nucleons. Calculations presented in Fig. 34 confirm this expectation. The same pattern can be seen in the momentum distribution plot in Fig. 35 which shows that the FSI is diminished practically for the whole region of recoil nucleon momenta, p_{r2} , of interest. In particular, the comparison of the type 2N-I and 2N-II correlation cases demonstrate

Figure 35: Dependence of the Decay Function on the momentum of recoil nucleon, p_{r2} calculated for the kinematics of the type 2N-II correlations with missing momentum fixed at $p_m = 100$ MeV/c and $\theta_m = 0^\circ$. (a) – $(N_f, N_{r2}) \equiv (n, p)$ and (b) – $(N_f, N_{r2}) \equiv (p, p)$. Doted lines - IA prediction, dashed and solid lines for IA+FSI and IA+FSI predictions respectively for $\theta_{r2} = 0^\circ$ kinematics. Curves with square labels corresponds to IA+FSI and IA+FSI predictions for anti-parallel $\theta_{r2} = 180^\circ$ kinematics.

that the type 2N-II makes the best case for probing the node of the relative momentum distribution in the pp pair in the ${}^3\text{He}$ wave function (see Figs.34(a) and 35(b)). However, a definitive answer on whether the node can be observed in the experiment requires a careful accounting of non-pole effects in the pair-distortion contribution. This consideration is out of scope of present dissertation. Meanwhile, we can point out, that as in the case of the type 2N-I correlations, the effects of double rescattering

are marginal which indicates again the feasibility of isolating two-body effects without complication, due to double rescattering of knocked-out nucleon off both recoil nucleons.

C. Type 3N-I Correlations:

A consideration of the angular dependence of the Decay Function for the type 3N-I correlations in Fig. 37 reveals a significant effect of FSIs for almost all angles of pair production (or θ_m). The FSI dominates especially at transverse kinematics at $\alpha_m = 1$ which reveals a significant contribution from the double rescattering processes starting at $p_m \geq 400$ MeV/c. Production of pp pairs (Fig. 37(a)) in parallel kinematics provides the best condition for probing the type 3N-I SRCs (albeit not without considerable pair distortion effects) starting at $p_m \geq 600$ MeV/c. This can be understood qualitatively. Since each rescattering transfers $k_z \approx \Delta, k_t \leq 500$ MeV/c momentum (predominantly in transverse direction; $k_t > \Delta$), and the rescattering amplitude falls exponentially with k_t^2 , it is kinematically infeasible to rescatter two nucleons above 600 MeV/c in the backward direction.

The effects discussed in the previous paragraph can be seen in more detail in the momentum distribution in Fig. 36 which confirms again that the only reasonable chance to extract the original momentum distribution in the type 3N-I correlation is to measure the coherent recoil pp pair production in the parallel kinematics $\theta_m = 0^\circ$.

Note that the significant contribution from double rescattering processes at transverse kinematics can be also understood qualitatively. To produce two coherent nu-

Figure 36: Dependence of the Decay Function on the momentum of recoil nucleon, p_{r2} , calculated for the kinematics of the type 3N-I correlations. The relation between missing and recoil momenta are the same as in Fig. 37. (a) – $(N_f, N_{r2}) \equiv (n, p)$ and (b)– $(N_f, N_{r2}) \equiv (p, p)$. Dotted lines - IA prediction, dashed and solid lines for IA+FSI1 and IA+FSI predictions respectively for $\theta_{r2} = 180^\circ$ kinematics. Curves with square labels corresponds to IA+FSI1 and IA+FSI predictions for anti-parallel $\theta_{r2} = 0^\circ$ kinematics.

cleons at large angle it is more preferable for the knocked-out nucleon to scatter consecutively off spectator nucleons. It contributes maximally in the transverse kinematics due to the eikonal nature of NN rescattering which dominates at $\alpha_m = 1$.

D. Type 3N-II Correlations:

In considering the angular dependence of the Decay Function for the type 3N-II correlations in Fig. 38, we observe overall large FSI effects except in the kinematics

Figure 37: Dependence of the Decay Function on the angle of the detected recoil nucleon, θ_{r2} , with respect to $\mathbf{q}$ for different and fixed values of p_m . The Decay Function is calculated for the type 3N-I SRC kinematics such that the recoil momentum of the detected nucleon is opposite to the missing momentum direction and defined as $p_{r2} = \frac{p_m}{2} + 50$ MeV/c. (a) - $(N_f, N_{r2}) \equiv (n, p)$, (b) - $(N_f, N_{r2}) \equiv (p, p)$. Dotted lines - IA, dashed lines - IA+FSI1, solid line - IA+FSI.

in which both recoil nucleons are produced in the backward direction compared to that of $\mathbf{q}$. In this figure this corresponds to the situation when the recoil nucleon with momentum p_{r2} is produced at $\theta_{r2} = 120^\circ$ and the fast nucleon is knocked-out in parallel kinematics ($\theta_m = 0^\circ$). This automatically puts the production angle of the second recoil nucleon at $\theta_{r3} = 120^\circ$ in the other half of the scattering plane.

One can understand the suppression of FSI in this kinematics qualitatively. It is very improbable with one single rescattering to produce two nucleons in the back-

Figure 38: Dependence of the Decay Function on the angle of the detected recoil nucleon, θ_{r2} , with respect to $\mathbf{q}$ for different and fixed values of p_{r2} . The Decay Function is calculated for the type 3N-II SRC kinematics such that $p_m = p_{r2} = p_{r3}$ and relative angles between these momentum vectors are 120° . Here, $\theta_{r2} = 120^\circ$ corresponds to $\theta_m = 0^\circ$. (a) - $(N_f, N_{r2}) \equiv (n, p)$, (b) - $(N_f, N_{r2}) \equiv (p, p)$. Dotted lines - IA, dashed lines - IA+FSI1, solid line - IA+FSI.

ward hemisphere of knocked-out energetic nucleon. One may expect that double rescattering can contribute to production of such configurations. However, as it will be discussed in the next section, it is dominant only at $\alpha_{r2} \approx \alpha_{r3} \approx 1$ which is significantly away from the considered kinematics. Note that the different angular dependence for recoil pp (a) and pn (b) pairs at momentum range $300 - 500$ MeV/c is related to the qualitative difference in the relative momentum distribution of these pairs (namely to an existing node in pp distribution). Note that recoil nucleon angles

$110 < \theta_{r2} < 130$ at $\theta_m = 0^0$ and $p_{r2} = 600$ MeV/c are kinematically forbidden since in this case $\alpha_m + \alpha_{r2} + \alpha_{r3} \geq 3$. However, the 600 MeV/c curve shows that the FSI is small at broader angular ranges starting at 80^0 to 150^0 . This may be very important for probing larger internal momenta in the type 3N-II SRCs without substantial FSI effects.

9.4.3 Conclusion on Numerical Studies of the Decay Function.

Considering the Decay Function, which is practically an unexplored quantity, we observe that within PWIA it clearly exhibits the main features of 2N and 3N correlations. Subsequent analysis of pair distortion and FSI effects revealed that the additional degrees of freedom associated with the full detection of the decay products of the reaction allows us to pinpoint unambiguously the kinematics in which the FSI or SRC plays a dominant role. Our conclusion is that the comprehensive study of the Decay Function in exclusive reactions will allow an unprecedented access to the 2N and 3N correlations in the nuclei.

10 Three Body Forces

Three-nucleon forces (3NFs) are one of the most elusive features in nuclear physics. It is true that most nuclear physics ranges from insensitive to relatively insensitive to three and more nucleon forces. The dominant ingredient in calculations of nuclear properties is the two-nucleon potential.

Three-nucleon forces nevertheless play an important role in nuclear physics because of the great precision of modern calculational methods for systems of relatively few nucleons. It has been more than 70 years since Wigner first raised the possibility that three-nucleon forces might be significant in the triton: “... one must assume a certain potential energy ... or a three-body force.” [4]. It is significant that the triton had not yet been discovered, although he predicted it would be bound by nucleon-nucleon (NN) forces alone. With all modern “second-generation” NN forces the triton is underbound by roughly $\frac{1}{2} - 1$ MeV. Two nucleon forces underestimate also the binding energy of the ${}^4\text{He}$. Unless our understanding of these NN forces is badly deficient, nuclear forces require an additional three-nucleon force. 3NFs allowed for improvements in the calculation of binding energies of nuclei $A = 3 - 9$ [56]. The 3NFs can significantly modify the present models of equations of state of nuclear matter [57], therefore, understanding 3NFs can have a significant impact on our understanding of the physics relevant to the superdense nuclear matter such as neutron stars. However, as it is mentioned in Ref. [58], an accuracy of 1% in calculations of binding energies are needed in order to systematically disentangle 3NFs forces from the overwhelming

2N interactions in few nucleon systems. This explains why we know so little about the three-nucleon forces, even though they have been introduced more than a seven decades ago.

Figure 39: Differential cross section for 65 MeV proton-deuteron scattering, showing calculations with N - N forces only (dashed lines), a full calculation with the TM 3NF (solid line), and an estimate of the effect of the 3NF alone (long-dashed line).

Another direct evidence for 3NFs is available in the Sagara discrepancy, which is shown in Fig. 39 in the differential cross section for pd scattering at 65 MeV [59]. If we ignore the effects of the Coulomb interaction in the forward direction, agreement between the experimental data and the calculation with NN forces alone (dashed line) is very good, except in the diffraction minimum where the data lie above the calculation. If one adds a 3NF the solid curve results, which nicely fills in the minimum, and agreement with the data is quite good. This is rather strong evidence for 3NFs and it exists at other energies. An estimate of the effect of the 3NF alone that is based on DWIA is shown in the long-dashed curve.

In our consideration of 3NFs, we proceed from the assumption that, just by the nature of 3N forces, they will dominate in the type 3N-II correlations. Conversely, in the case of the type 2N-I correlations, it is possible to suppress 3NFs significantly by restricting the momentum of the third spectator nucleon to be close to the zero.

Figure 40: Dependence of the Decay Function on the momentum of the recoil nucleon, p_{r2} , calculated for the kinematics of the type 3N-II correlations. The relation between missing and recoil momenta is the same as in Fig. 38. (a) – $(N_f, N_{r2}) \equiv (n, p)$ and (b) – $(N_f, N_{r2}) \equiv (p, p)$. Doted lines - IA prediction, dashed and solid lines for IA+FSI1 and IA+FSI predictions respectively for $\theta_m = 0^\circ$ ($\theta_{r2} = 120^\circ$) kinematics. Curves with square labels correspond to IA+FSI1 and IA+FSI predictions for anti-parallel $\theta_m = 180^\circ$ ($\theta_{r2} = 60^\circ$) kinematics.

Thus, our expectation is that one should observe significantly different contributions from 3NFs in 3N-II and 2N-I correlations. Furthermore, we recall our discussions

Figure 41: Three-nucleon forces in the Decay Function distribution when $(N_f, N_{r2}) \equiv (n, p)$. The relation between missing and recoil momenta is the same as in Fig. 38 (type 3N-II SRC), and the $r2$ recoil proton is detected at $\theta_{r2} = 120^\circ$ while the angle of missing momentum $\theta_m = 0^\circ$. Dotted line – PWIA, dash-dotted – IA, dashed – IA+FSI1 and solid – IA+FSI predictions. Curves with circle labels correspond to the same contributions with three-nucleon forces included in the calculation of the ^3He wave function.

of the type 2N-I and 3N-II correlations in Secs. 9.4.2(A) and 9.4.2(D) where we found that one can significantly suppress FSI effects in the type 2N-I correlations in parallel kinematics ($\theta_m = 0^\circ$) (see Fig. 33) and in the type 3N-II correlations in kinematics where we choose two recoil nucleons to be produced in the backward hemisphere with respect to $\mathbf{q}$ (see Fig. 40). Naturally, we expect that these kinematic windows are optimal for probing 3NFs. To quantify our statement, in Fig. 41 we compare the mo-

Figure 42: Three-nucleon forces in the Decay Function distribution for the type 2N-I SRC. Dependence of the Decay Function on the momentum of the recoil nucleon, p_{r2} , for the type 2N-I kinematics at $\theta_{r2} = 180^\circ$ ($\theta_m = 0^\circ$). $(N_f, N_{r2}) \equiv (n, p)$. Dotted line – PWIA, dash-dotted – IA, dashed – IA+FSI1 and solid – IA+FSI predictions. Curves with circular labels correspond to the same contributions with three-nucleon forces included in the calculation of the ${}^3\text{He}$ wave function.

momentum distributions of the Decay Function calculated in the type 3N-II kinematics in which two recoil protons are produced at 120° relative to the $\mathbf{q}$. Calculations are done for two cases: in one case we have only NN interactions while in the other case 3NFs are included according to the Tucson-Melbourne model [60]. Our calculations show a factor of two difference between calculations including only 2N forces and calculation including 2N+3NF forces. However, in contrast, the calculations in the kinematics of the type 2N-I correlations presented in Fig. 42, using the same two

models of NN interactions, predict little difference between momentum dependences of the Decay Function.

These considerations show that we can identify the domains in the kinematics of the type 3N-II correlations where FSI effects are relatively small and one has a good chance to extract the genuine information about 3NFs. This assumes one is doing simultaneous measurement also in 2N-I kinematics where the same 3NFs will give a negligible contribution.

10.1 Conclusion on Three-Nucleon Forces Studies

Our expectation is that one should observe sizably different contributions from 3NFs in 3N-II and 2N-I correlations. We found that one can significantly suppress FSI effects in the type 2N-I correlations in parallel kinematics ($\theta_m = 0^0$) (see Fig. 33) and in the type 3N-II correlations in kinematics where we choose two recoil nucleons to be produced in the backward hemisphere with respect to $\mathbf{q}$ (see Fig. 40). In Fig. 41 we compare the momentum distributions of the Decay Function calculated in the type 3N-II kinematics in which two recoil protons are produced at 120^0 relative to the $\mathbf{q}$. Calculations are done for two cases: in one case we have only NN interactions while in the other case 3NFs are included according to the Tucson-Melbourne model [60]. Our calculations show a factor of two difference between calculations including only 2N forces and calculation including 2N+3NF forces. In contrast, the calculations in the kinematics of the type 2N-I correlations presented in Fig. 42, using the same two models of NN interactions, predict little difference between momentum dependences

of the Decay Function. We thus conclude that our analysis allowed us to identify new approaches for investigating the role of the practically unknown three-nucleon forces in the ground state wave function of ^3He .

11 Comparison with Experiment

There is an emerging number of experiments dedicated to the study of exclusive electrodisintegration of ^3He [55, 61, 62]. There are also several experiments planned with the upgraded 11GeV beam energy at JLab. However there are next to none published high energy exclusive disintegration data therefore we will compare our calculations with semi-exclusive experiment, in which three-body breakup channel is isolated. Latter is closely related to the exclusive disintegration reaction studied in this dissertation.

11.1 Semiexclusive Three Body Break up of ^3He

To date, there have been only few measurements of high energy three-body breakup of ^3He in the kinematic window (See Section 2) relevant to our studies. Best suited to our kinematics are the the data of $^3\text{He}(e, e'p)pn$ experiment performed in Hall A at the Thomas Jefferson National Accelerator Facility by F. Benmokhtar *et al.* [62, 63]. These measurements were part of the E89-044 experiment *Selected Studies of the ^3He and ^4He Nuclei through Electrodisintegration at High Momentum Transfer*, [64]. This was a coincidence experiment, with the scattered electrons detected in the electron arm High Resolution Spectrometer and the knocked-out proton detected in the Hadron High Resolution Spectrometer. Note that References [62] and [63] deal specifically with the high energy three body break up of ^3He (reaction $^3\text{He}(e, e'p)pn$) whereas *two*-body breakup detected in the experiment (reaction

${}^3\text{He}(e, e'p)d$) discussed in other papers. It is also significant that the main motivations behind studies in References [62] and [63] coincide with the motivations of these dissertation, which is the studies of SRC.

The measurements in Refs. [62] and [63] were performed at fixed transferred momentum and energy, $\mathbf{q} = 1502 \text{ MeV}/c$ and $q_0 = 840 \text{ MeV}$, respectively, so $Q^2 = 1.55 \text{ GeV}^2/c^2$. The missing momentum p_m in this experiment is varied up to $1 \text{ GeV}/c$ and missing energies are probed in the continuum region, up to the pion threshold. This kinematic coverage is much more extensive than that of the two-body breakup experiments. The data were collected at two values of the beam energy, 4.8068 GeV and 1.2553 GeV . References [62] and [63] report the results of measurements in perpendicular kinematics with $x_B = \frac{Q^2}{2M_p q_0} = 0.96$ near the top of the quasifree peak ($x_B = 1$). Protons were detected at several angles relative to $\mathbf{q}$, corresponding to missing momenta $p_m \sim 0 - 1 \text{ GeV}/c$. Even though the experiment was semiexclusive i.e. only struck proton was detected, it was still possible to identify the three body break up by providing the E_m missing energy greater than the three-body breakup threshold energy.

11.2 GEA Prediction

Our analytical results for the differential cross section of exclusive ${}^3\text{He}(e, e'NN)N$ describe the situation where both knocked-out and recoil nucleons momenta have been detected. But since in above described experiment only struck proton was detected it is clear that for comparison we have to integrate by phase space of recoil system

in order to eliminate the undetected degrees of freedom. To do that, let us recall the following equation:

$$\frac{d\sigma^N}{dE_{e'}d\Omega_{e'}dp_f} = K(Q^2, \mathbf{p}_m)\sigma_{eN}S^N(E_m, \mathbf{p}_m), \quad (89)$$

we can relate differential cross section to the Spectral Function, which in turn, can be related to the Decay Function as follows:

$$\begin{aligned} \int d^3p_{r2} D^N(\mathbf{p}_m, E_m, \mathbf{p}_{r2}) &= S^N(E_m, \mathbf{p}_m); \\ \sum_N \int dE_m d^3p_m S^N(E_m, \mathbf{p}_m) &= 1. \end{aligned} \quad (90)$$

Furthermore, using the fact that integration involves all the phase space of relative momentum of recoil pn system we can use the condition of the closure which cancels out the effects due to the pair distortion. As a result only the effects associated with high energy rescattering of fast nucleon should be taken into account.

Below we present experimental data and our calculations for six-fold differential cross section of ${}^3\text{He}(e, e'p)pn$ reaction for different values of missing momenta. Dotted curves represent the PWIA prediction only, dashed curves include single rescattering and solid curves represent full calculations. As p_m increases, we can see that the broad peak in the cross section moves to higher missing energies. The arrow in the figure indicates where one would expect the peak in the cross section due to disintegration processes involving two active nucleons plus a spectator (see Eq.(81) in Sec. 9.1). The large peak in the data roughly aligned with the arrow suggests that two-nucleon disintegration processes are dominant. Our full-amplitude calculations produce rea-

sonable agreement with the data. The below results show that the developed model satisfactory describes the data already at $Q^2 = 1.5 \text{ GeV}^2/c^2$. As one expected due to the fact that kinematics of the experiment where close to perpendicular there are considerable effects due to FSI. The next five figures has been first presented in the Ref. [65].

Figure 43: Differential cross section results for the $^3\text{He}(e, e'p)pn$ reaction versus missing energy E_m at missing momentum value $p_m = 270 \text{ MeV}/c$. Solid squares from Ref. [62], dotted curve correspond to calculations with PWIA amplitude only, dashed curve = PWIA+single rescattering, solid curve = PWIA+single rescattering+double rescattering.

Figure 44: Cross section results for the $^3\text{He}(e, e'p)pn$ semiexclusive reaction for the $p_m = 340 \text{ MeV}/c$. All notations are the same as in the Fig. 43.

Figure 45: Cross section results for the $^3\text{He}(e, e'p)pn$ semiexclusive reaction for the $p_m = 440 \text{ MeV}/c$. All notations are the same as in the Fig. 43.

Figure 46: Cross section results for the $^3\text{He}(e, e'p)pn$ semiexclusive reaction for the $p_m = 620 \text{ MeV/c}$. All notations are the same as in the Fig. 43.

Figure 47: Cross section results for the $^3\text{He}(e, e'p)pn$ semiexclusive reaction for the $p_m = 800 \text{ MeV/c}$. All notations are the same as in the Fig. 43.

12 Conclusions

We have developed a theory of high-energy exclusive electrodisintegration of three-nucleon systems on the example of ${}^3\text{He}(e, e' NN)N$ reaction. Derivations done in kinematics where one nucleon in the final state can be clearly identified as a knocked-out nucleon that carries practically whole momentum of the virtual photon.

The increase of transferred energies provides a qualitative new regime in electro-nuclear reactions. The possibility to suppress the long-range phenomena in these reactions opens a completely new window in the study of microscopic properties of nuclear matter at small distances.

The theoretical approach which allows us to do consistent calculations is based on the fact that in the high energy regime new small parameters (such as $\frac{q_-}{q_+}$) emerge. We demonstrate how the existence of these parameters results to the reduction theorem which allows one to group potentially infinite number of rescattering amplitudes into a finite number covariant amplitudes.

Furthermore we identify the set of effective Feynman diagram rules which allows one to calculate these covariant amplitudes. This framework we call Generalized Eikonal Approximation (GEA). GEA is a covariant theory in which scattering amplitudes are represented as a sum of the diagrams corresponding to the n 'th order rescattering of the knocked-out nucleon with residual nucleons in the target.

Applying these effective rules to the reaction under consideration we consequently calculate PWIA scattering amplitude, pair distortion amplitude, struck nucleon sin-

gle rescattering and double rescattering amplitudes. Calculations were carried out in details and final results are presented in a form suitable for programming and numerical calculations. Manifestly covariant nature of Feynman diagrams allowed us to preserve both the relativistic dynamics and kinematics of the scattering while identifying the low momentum nuclear part of the amplitude with nonrelativistic nuclear wave function. We have obtained the analytical expression for differential cross section of $e+{}^3\text{He} \rightarrow e'+p+p+n$ reaction.

To describe the semiexclusive ${}^3\text{He}(e, e'N)NN$ reaction we use the formalism of the Decay Function which is related to the conventional Spectral Function through the integration of the phase space of the recoil nucleons.

In the numerical analyses of both Spectral and Decay Functions, we concentrate on studies of two main types of 2N and 3N correlations. For 2N correlations, we consider the ones in which the struck nucleon is initially correlated with one of the recoil nucleons while the third nucleon is spatially isolated (the type 2N-I SRC). Also for the case of 2N correlations we consider the case in which the two recoil nucleons are in 2N correlation with the struck nucleon in the mean field of the SRC pair (the type 2N-II SRC). For 3N correlations we consider the correlations in which struck nucleon with large missing momenta is correlated with the pair of coherent nucleons (the type 3N-I SRC). For the 3N correlations we also consider the case in which all three nucleons have relative momenta exceeding the characteristic mean field momentum in the nucleus and have a relative angle $\approx 120^\circ$ (the type 3N-II SRC). Particularly, in the studies of the Spectral Function, we demonstrate that it

exhibits several unique features related to the structure of 2N correlations. These are the correlation between missing momenta and missing energy and the minimum in the Spectral Function associated with the node in the relative momentum distribution of the pp pair. These results are in agreement with previous analyzes of Spectral Function (see e.g. [52]). There are, however, no clear cut signatures in the spectral function related to the 3N correlations. Within PWIA, 3N SRCs reveal only through the strength of Spectral Function at very high values of missing momenta and/or energy. Our new result in considering the Spectral Function is that the effects related to the 3N SRCs is practically washed out by the pair distortion and the FSI.

Considering the Decay Function, which is practically an unexplored quantity, we observe that within PWIA it clearly exhibits the main features of 2N and 3N correlations. Subsequent analysis of pair distortion and FSI effects revealed that the additional degrees of freedom associated with the full detection of the decay products of the reaction allows us to pinpoint unambiguously the kinematics in which the FSI or SRC plays a dominant role. Our conclusion is that the comprehensive study of the Decay Function in exclusive reactions will allow an unprecedented access to the 2N and 3N correlations in the nuclei.

We applied our theoretical model and used our code to calculate differential cross section for semiexclusive ${}^3\text{He}(e, e'p)pn$ and compared our predictions with experimental data available from Ref. [62]. While Plane Wave Impulse Approximation failed to describe experimental reality our full amplitude calculations relate to data very well even for very high values of missing momenta.

Our conclusion is that the investigation of the Decay Function opens up a completely new venue in studies of short range nuclear properties and allows us to discriminate between different orders of final state reinteractions.

References

- [1] R. Machleidt, F. Sammarruca and Y. Song, Phys. Rev. C **53**, 1483 (1996) [arXiv:nucl-th/9510023].
- [2] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C **51**, 38 (1995) [arXiv:nucl-th/9408016].
- [3] V. G. J. Stoks, R. A. M. Kompl, M. C. M. Rentmeester and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).
- [4] E. Wigner Phys. Rev. **43**, 252 (1933).
- [5] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt and B. H. J. McKellar, Nucl. Phys. A **317**, 242 (1979).
- [6] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper and R. B. Wiringa, Phys. Rev. C **56**, 1720 (1997) [arXiv:nucl-th/9705009].
- [7] L. L. Frankfurt, G. A. Miller and M. Strikman, Ann. Rev. Nucl. Part. Sci. **44**, 501 (1994) [arXiv:hep-ph/9407274].
- [8] W. Glöckle, *The Quantum Mechanical Few-Body Problem*, Springer-Verlag, 1983.
- [9] K. S. Egiyan *et al.* [CLAS Collaboration], Phys. Rev. Lett. **96**, 082501 (2006) [arXiv:nucl-ex/0508026]. K.S. Egiyan *at. al* [CLAS Collaboration], arXiv:nucl-ex/0508026.
- [10] R. J. Glauber Phys. Rev. **100**, 242 (1955)
- [11] G. R. Farrar, H. Liu, L. L. Frankfurt and M. I. Strikman, Phys. Rev. Lett. **61**, 686 (1988).
- [12] T. S. H. Lee and G. A. Miller, Phys. Rev. C **45**, 1863 (1992).
- [13] B. K. Jennings and G. A. Miller, Phys. Lett. B **318**, 7 (1993) [arXiv:hep-ph/9305317].
- [14] A. Kohama, K. Yazaki and R. Seki, Nucl. Phys. A **551**, 687 (1993).
- [15] L. L. Frankfurt, M. I. Strikman and M. B. Zhalov, Phys. Rev. C **50**, 2189 (1994).
- [16] O. Benhar, A. Fabrocini, S. Fantoni, V. R. Pandharipande and I. Sick, Phys. Rev. Lett. **69**, 881 (1992).
- [17] A. S. Rinat and M. F. Taragin, Phys. Rev. C **52**, 28 (1995) [arXiv:nucl-th/9410024].
- [18] N. N. Nikolaev, A. Szczurek, J. Speth, J. Wambach, B. G. Zakharov and V. R. Zoller, Phys. Lett. B **317**, 287 (1993).

- [19] S. Frankel, W. Frati and N. Walet, Nucl. Phys. A **580**, 595 (1994) [arXiv:nucl-th/9301011].
- [20] A. Bianconi, S. Boffi and D. E. Kharzeev, Phys. Lett. B **325**, 294 (1994) [arXiv:hep-ph/9310375].
- [21] L.L. Frankfurt, M.M. Sargsian, M.I. Strikman, Phys. Rev. **C56**, 1124 (1997)
- [22] M. M. Sargsian, Int. J. Mod. Phys. E **10**, 405 (2001).
- [23] Garrow et. al., Phys. Rev. **C66**, 044613 (2002)
- [24] C. Marchand *et al.*, Phys. Rev. Lett. **60**, 1703 (1988).
- [25] K. I. Blomqvist *et al.*, Phys. Lett. B **424**, 33 (1998). A. Bussiere *et al.*, Nucl. Phys. A **365**, 349 (1981).
- [26] D. L. Groep *et al.*, Phys. Rev. C **63**, 014005 (2001).
- [27] H. Arenhovel, W. Leidemann and E. L. Tomusiak, Phys. Rev. C **52**, 1232 (1995).
- [28] Feynman R, *Photon-Hadron Interactions*, Benjamin, Reading, Mass., 1972.
- [29] Frankfurt L L, Kondratyuk A, and Strikman M I, eprint.
- [30] Frankfurt L L, Miller G A, Sargsian M M and Strikman M I 2000 Phys. Rev. Lett. **84** 3045
- [31] P. Stoler, Phys. Rept. **226**, 103 (1993).
- [32] Y. I. Azimov, Sov.Phys. JETP, v.16, 1640 (1963); Phys.Lett., v.3, 195 (1963).
- [33] H. Cheng and T. T. Wu, *Expanding Protons: Scattering At High-Energies*, Cambridge, USA: MIT-PR. (1987).
- [34] P.D.B. Collins, *An Introduction to Regge Theory and High Energy Physics*, Cambridge University Press, Cambridge, 1977.
- [35] V. N. Gribov, JETP, **30**, 709 (1970).
- [36] L. Bertuchi, Il Nuovo Cimento A **11**, 45 (1972).
- [37] J. M. Laget, J. Phys. G **14** (1988) 1445.
- [38] G. Brown and A.D. Jackson, *The Nucleon Nucleon Interaction*, North-Holland Publishing Company, 1976.
- [39] G. D. Alkhasov, S. L. Belostotsky and A. A. Vorobev, Phys. Rept. **42**, 89 (1978).
- [40] R. A. Arndt, I. I. Strakovsky and R. L. Workman, Phys. Rev. C **62** (2000) 034005; <http://gwdac.phys.gwu.edu>

- [41] G. van der Steenhoven, Nucl. Phys. A **622**, 31C (1997).
- [42] A. Fantoni [HERMES Collaboration], DESY-HERMES-98-88 *Prepared for 18th International Conference on Physics in Collision (PIC 98), Frascati, Italy, 17-19 Jun 1998*
- [43] S. Boffi, C. Giusti, F. D. Pacati and M. Radici, *New Physics from the $(e, e'N)$ Reaction*, (1986).
- [44] M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman and L. L. Frankfurt, Phys. Rev. C **71**, 044614 (2005).
- [45] M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman and L. L. Frankfurt, Phys. Rev. C **71**, 044615 (2005).
- [46] L.L. Frankfurt and M.I. Strikman, Phys. Rep. **160**, 235 (1988).
- [47] T. De Forest, Nucl. Phys. A **392**, 232 (1983).
- [48] W. R. Gibbs and B. Loiseau, Phys. Rev. C **50**, 2742 (1994).
- [49] A. Nogga A. Kievsky, H. Kamada, W. Gloeckle, L. E. Marcucci, S. Rosati and M. Viviani, Phys. Rev. C **67**, 034004 (2003).
- [50] T.V. Abrahamyan, *Master's Thesis*, unpublished, FIU, 2003.
- [51] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C **51**, 38 (1995)
- [52] C. Ciofi degli Atti, L. P. Kaptari and D. Treleani, Phys. Rev. C **63**, 044601 (2001)
- [53] L.L. Frankfurt and M.I. Strikman, Phys. Rept. **76**, 215 (1981).
- [54] C. Ciofi degli Atti, S. Simula, L. L. Frankfurt and M. I. Strikman, Phys. Rev. C **44**, 7 (1991).
- [55] R. A. Niyazov and L. B. Weinstein [CLAS Collaboration], Phys. Rev. Lett. **92**, 052303 (2004)
- [56] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa and J. Carlson, Phys. Rev. C **64**, 014001 (2001).
- [57] H. Heiselberg and V. Pandharipande, Ann. Rev. Nucl. Part. Sci. **50**, 481 (2000).
- [58] J. L. Friar, Nucl. Phys. A **684**, 200 (2001).
- [59] H. Witala, W. Glöckle, D. Hüber, J. Golak, and H. Kamada, *Phys. Rev. Lett.* **81**, 1183 (1998).
- [60] S. A. Coon and H. K. Han, Few Body Syst. **30**, 131 (2001).

- [61] M. M. Rvachev *et al.* [The Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. **94**, 192302 (2005) [arXiv:nucl-ex/0409005].
- [62] F. Benmokhtar *et al.* [Jefferson Lab Hall A Collaboration], “Measurement of the He-3(e,e’ p)p n reaction at high missing energies and Phys. Rev. Lett. **94**, 082305 (2005) [arXiv:nucl-ex/0408015].
- [63] F. Benmokhtar, *PhD Thesis*, unpublished, Rutgers, The State University of New Jersey, 2004.
- [64] M. Epstein, A. Saha and E. Voutier (spokespersons), *Selected Studies of the ^3He and ^4He Nuclei through Electrodisintegration at High Momentum Transfer* Jefferson Lab Proposal - E-89-044.
- [65] M. Sargsian, *Invited talk at APS April Meeting*, Dallas, TX, 2006.

APPENDIX A. Three-nucleon bound systems

The system which consists of three nucleons received special attention in the nuclear theory. First of all in this case we are dealing with the next simplest bound systems after deuteron, namely ${}^3\text{H}$ and ${}^3\text{He}$ nuclei. Here is where the three-particle forces may exhibit themselves most clearly. It is therefore of evident importance to use what is known about two-nucleon forces in order to ascertain whether or not that information is adequate to account for the observed properties of $A = 3$ system. Secondly, these systems can yield new data on the two-nucleon interactions itself. While considering scattering of two nucleons, third (spectator) particle may take up or impart enough energy to the two-nucleon subsystem to preserve the energy balance. So sum of initial energies of scatterers now may be different from the sum of their final energies. This off-energy-shell scatterings are important ingredients in the dynamics of nuclei other than deuteron. In the two particle scatterings bound states cannot occur – we have as many scattering solutions as unperturbed solutions. However, if there are three or more particles participating in the process then it may well happen that two or more of them join up in the bound state. One of the most fundamental routes which was developed to handle three-particles problems in non-relativistic quantum mechanics provided by method of Faddeév equations which we are going to discuss in this chapter.

Let us consider three interacting particles, labeled 1, 2 and 3. Interaction between particles 1 and 2 we denote V_3 , between 2 and 3 — V_1 , and between 1 and 3

— V_2 . To facilitate derivation of Faddeev equations let us consider the specific case in which particle 2 scatters on particles 1 and 3. The Lippman-Schwinger equation for the transition operator T is:

$$T = V + V(E - H_0 + i\varepsilon)^{-1}T. \quad (\text{A.1})$$

Interaction V of incident particle 2 with the scatterers 1 and 3 is equal to

$$V = V_1 + V_3, \quad (\text{A.2})$$

and the unperturbed Hamiltonian H_0 therefore can be expressed as

$$H_0 = V_2 + K, \quad (\text{A.3})$$

where K is the total kinetic energy of the three particles

$$K = \frac{|\mathbf{p}_1|^2}{2m_1} + \frac{|\mathbf{p}_2|^2}{2m_2} + \frac{|\mathbf{p}_3|^2}{2m_3}. \quad (\text{A.4})$$

The very small positive adiabatic switching factor ε is introduced in order to avoid problems due to singularities which would otherwise be present when E is equal to one of the eigenvalues of H_0 and this is almost always the case for scattering problems.

Eq. (A.1) thus can be rewritten as

$$T = (V_1 + V_3) + (V_1 + V_3)G_2T, \quad (\text{A.5})$$

where

$$G_2 \equiv (E - K - V_2 + i\varepsilon)^{-1}. \quad (\text{A.6})$$

Then

$$\{1 - (V_1 + V_3)G_2\}T = (V_1 + V_3), \quad (\text{A.7})$$

and

$$T = \{1 - (V_1 + V_3)G_2\}^{-1}(V_1 + V_3). \quad (\text{A.8})$$

Let us now extract from G_2 the Green's function for the free particles G , defined as

$$G \equiv (E - K + i\varepsilon)^{-1}. \quad (\text{A.9})$$

With this definition we have

$$\begin{aligned} G_2 &= [G^{-1} - V_2]^{-1} = [G^{-1} - V_2]^{-1}G^{-1}G = \\ &= [G^{-1} - V_2]^{-1}(G^{-1} - V_2 + V_2)G = (1 + [G^{-1} - V_2]^{-1}V_2)G = \\ &= (1 + G_2V_2)G. \end{aligned} \quad (\text{A.10})$$

Solving this for G_2 we get

$$G_2 - G_2V_2G = G_2[1 - V_2G] = G, \quad (\text{A.11})$$

$$G_2 = G[1 - V_2G]^{-1}. \quad (\text{A.12})$$

In order to transform this further we first note that because $G[1 - V_2G] = G - GV_2G = [1 - GV_2]G$ we can write the following identity

$$G[1 - V_2G]^{-1} = [1 - GV_2]^{-1}G, \quad (\text{A.13})$$

so eq.(A.12) may be replaced with :

$$G_2 = [1 - GV_2]^{-1}G. \quad (\text{A.14})$$

With this we now return to eq (A.8) and obtain a new form for transition operator

T :

$$\begin{aligned}
T &= \{1 - (V_1 + V_3)[G^{-1} - V_2]^{-1}\}^{-1}(V_1 + V_3) = \\
&= \{1 - (V_1 + V_3)[1 - GV_2]^{-1}G\}^{-1}(V_1 + V_3) = \\
&= G^{-1}[1 - GV_2 - G(V_1 + V_3) + G(V_1 + V_3)] \times \\
&\times \left[1 - G \sum_{k=1}^3 V_k\right]^{-1} G(V_1 + V_3), \tag{A.15}
\end{aligned}$$

or

$$T = (V_1 + V_3) + (V_1 + V_3) \left[1 - G \sum_{k=1}^3 V_k\right]^{-1} G(V_1 + V_3). \tag{A.16}$$

Let us define new quantities T_{ij} by the means of equations

$$T_{ij} \equiv V_i \delta_{ij} + V_i \left[1 - G \sum_{l=1}^3 V_l\right]^{-1} G V_j, \quad i, j = 1, 2, 3. \tag{A.17}$$

Since we are considering particle 2 scatters on particles 1 and 3 we can represent the corresponding transition operator T as a partial sum of T_{ij} operators:

$$T = \sum_{\substack{i \neq 2 \\ j \neq 2}} T_{ij}. \tag{A.18}$$

Our goal is to obtain the set of coupled equations for operators T_{ij} and to express each one of them through others but not through itself. In order to do that we are going to transform eq.(A.17) by the following chain of identities:

$$\begin{aligned}
T_{ij} &= V_i \delta_{ij} + V_i \left\{1 - G \sum_{l=1}^3 V_l + G \sum_{k=1}^3 V_k\right\} \left[1 - G \sum_{l=1}^3 V_l\right]^{-1} G V_j = \\
&= V_i \delta_{ij} + V_i G V_j + V_i G \left(\sum_{k=1}^3 V_k\right) \left[1 - G \sum_{l=1}^3 V_l\right]^{-1} G V_j = \\
&= V_i \delta_{ij} + V_i G \sum_{k=1}^3 \left\{V_k \delta_{kj} + V_k \left[1 - G \sum_{l=1}^3 V_l\right]^{-1} G V_j\right\}, \tag{A.19}
\end{aligned}$$

but the quantity in braces is the same operator T_{ij} we defined in eq.(A.17), therefore we arrive at

$$T_{ij} = V_i \delta_{ij} + V_i G \sum_{k=1}^3 T_{kj}. \quad (\text{A.20})$$

We can solve eq.(A.20) for operator T_{ij}

$$(1 - V_i G) T_{ij} = V_i \delta_{ij} + V_i G \sum_{k \neq i} T_{kj}, \quad (\text{A.21})$$

$$T_{ij} = (1 - V_i G)^{-1} V_i \delta_{ij} + (1 - V_i G)^{-1} V_i \sum_{k=1}^3 \tilde{G}_{ik} T_{kj}, \quad (\text{A.22})$$

where we have introduced new operator $\tilde{G}_{ik}$:

$$\tilde{G}_{ik} \equiv G(1 - \delta_{ik}) = (E - K + i\varepsilon)^{-1}(1 - \delta_{ik}). \quad (\text{A.23})$$

Our final step is to make one more definition:

$$\begin{aligned} t_i &\equiv (1 - V_i G)^{-1} V_i = V_i (1 - G V_i)^{-1} = \\ &= V_i \left(G(G^{-1} - V_i) \right)^{-1} = V_i (G^{-1} - V_i)^{-1} (G^{-1} - V_i + V_i) = \\ &= V \left(1 + (G^{-1} - V_i)^{-1} V_i \right) = V_i + V_i (G^{-1} - V_i)^{-1} V_i, \end{aligned} \quad (\text{A.24})$$

or

$$t_i = V_i + V_i (E - K - V_i + i\varepsilon)^{-1} V_i. \quad (\text{A.25})$$

One can show that so defined operator t_i is the same transition operator T given by eq.(A.1) for the case of two-particle scattering at energy E by particles $j \neq i$ and $k \neq i$. Indeed, let us consider full Hamiltonian $H = H_0 + V$, where unperturbed

Hamiltonian is now just a kinetic energy K of two interacting particles and V is the coupling among them. Then eq.(A.1) might be transformed as

$$V^{-1}T = 1 + (E - (H_0 + V) + V + i\varepsilon)^{-1}T, \quad (\text{A.26})$$

$$(E - H + V + i\varepsilon)V^{-1}T = (E - H + V + i\varepsilon) + T, \quad (\text{A.27})$$

$$(E - H + i\varepsilon)V^{-1}T = (E - H + i\varepsilon) + V, \quad (\text{A.28})$$

$$T = V + V(E - H + i\varepsilon)^{-1}V, \quad (\text{A.29})$$

which is identical to eq.(A.25). Returning to eq.(A.22) and substituting new-introduced operator t_i we obtain one form of the famous Faddeév equations, namely

$$T_{ij} = t_i \delta_{ij} + \sum_{k=1}^3 t_i \tilde{G}_{ik} T_{kj}, \quad i, j = 1, 2, 3. \quad (\text{A.30})$$

As one can clearly see, this set of coupled equations is completely inhomogeneous. In other words operator T_{ij} does not appear on the right-hand side($\tilde{G}_{ik}$ gives 0 when $k = j$). Compared to initial Lippman-Schwinger equation eq.(A.1) this is significant improvement for three-particle problem. The kernels of corresponding integral equations can be shown to be compact for a broad range of potentials. The Faddeév equations are therefore an adequate starting point for a practicable solution to the problem of three interacting nucleons.

APPENDIX B. Scattering amplitude in the momentum space

Elastic scattering of two particles, just like any other two-body problem, can be treated as a scattering of one particle with the reduced mass in the field $U(\mathbf{r})$ of a fixed force center. Generally speaking, this field may have arbitrary spatial symmetry, the special case of central field we will denote as $U(r)$. Very far from interaction region the wave function of a scattered particle represents a mixture of a plane wave and a diverging spherical wave:

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) \approx e^{i\mathbf{k}\mathbf{r}\mathbf{n}\mathbf{n}'} + \frac{f(\mathbf{n}, \mathbf{n}')}{r} e^{i\mathbf{k}\mathbf{r}}, \quad (\text{B.1})$$

where scattering amplitude $f(\mathbf{n}, \mathbf{n}')$ depends on the directions of both incoming($\mathbf{n}$) and outgoing($\mathbf{n}'$) waves. Situation is much simpler in the case of centrally symmetric fields $U(r)$, where scattering amplitude only depends on the angle between $\mathbf{n}$ and $\mathbf{n}'$. For those fields we can replace $f(\mathbf{n}, \mathbf{n}') = f(\mathbf{n} \cdot \mathbf{n}') \equiv f(\theta)$. Knowledge of $f(\theta)$ is sufficient for calculation of differential cross section:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \quad (\text{B.2})$$

Solution of time independent Shrödinger equation, which describes scattering in the field $U(r)$, given by superposition of continuum wave functions corresponding to motion of particle with known energy $E = k^2/2m$ and different values l of orbital momenta:

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = \frac{1}{2k} \sum_{l=0}^{\infty} i^l (2l+1) e^{i\delta_l} R_{kl}(r) P_l(\cos \theta), \quad (\text{B.3})$$

where P_l are ordinary Legendre polynomials and δ_l are phase shifts of functions $R_{kl}(r)$.

Latter are the solutions of the radial Shrödinger equation:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_{kl}}{dr} \right) + \left[k^2 - \frac{l(l+1)}{r^2} - 2mU(r) \right] R_{kl} = 0. \quad (\text{B.4})$$

Using asymptotical form of $R_{kl}(r)$ one can express scattering amplitude through the phase shifts δ_l

$$f^{\text{on shell}}(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) (\exp(2i\delta_l) - 1) P_l(\cos \theta). \quad (\text{B.5})$$

The “on shell” tag should remind that this expansion is valid only for scattering of unbound (on-mass-shell) particles, when the corresponding field $U(r)$ is assumed to be centrally symmetric. One can’t use eq.(B.5) for bound (off shell) scatterers. Instead of central fields let us consider arbitrary $U(\mathbf{r})$ and deduce a general expression which connects the scattering amplitude $f(\mathbf{n}, \mathbf{n}')$ with the momentum space wave functions $\Psi_{\mathbf{k}}^{(+)}(\mathbf{q})$. The coordinate space Fourier-components of these functions in asymptotic limit are given by eq.(B.1).

As usually, we define direct and inverse Fourier transformations as

$$\Psi_{\mathbf{k}}^{(+)}(\mathbf{q}) = \int \psi_{\mathbf{k}}^{(+)}(\mathbf{r}) e^{-i\mathbf{q}\mathbf{r}} dV \quad (\text{B.6})$$

and

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = \int \Psi_{\mathbf{k}}^{(+)}(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}} \frac{d^3q}{(2\pi)^3}. \quad (\text{B.7})$$

Time independent Shrödinger equation in the coordinate representation given by

$$-\frac{1}{2m} \Delta \psi_{\mathbf{k}}^{(+)}(\mathbf{r}) + [U(\mathbf{r}) - E] \psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = 0. \quad (\text{B.8})$$

In the momentum space one rewrites this equation as

$$\left(\frac{q^2}{2m} - E\right) \Psi_{\mathbf{k}}^{(+)}(\mathbf{q}) + \int U(\mathbf{q} - \mathbf{q}') \Psi_{\mathbf{k}}^{(+)}(\mathbf{q}') \frac{d^3 q'}{(2\pi)^3} = 0, \quad (\text{B.9})$$

where $U(\mathbf{q})$ is a Fourier-component of $U(\mathbf{r})$:

$$U(\mathbf{q}) = \int U(\mathbf{r}) e^{-i\mathbf{q}\mathbf{r}} dV. \quad (\text{B.10})$$

Replacing last term in eq.(B.1) by function $\chi_{\mathbf{k}}(\mathbf{r})$, which is a diverging spherical wave as $r \rightarrow \infty$, one has

$$\psi_{\mathbf{k}}^{(+)}(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} + \chi_{\mathbf{k}}(\mathbf{r}). \quad (\text{B.11})$$

The corresponding Fourier-image of this equation has the form of

$$\Psi_{\mathbf{k}}^{(+)}(\mathbf{q}) = (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{k}) + \chi_{\mathbf{k}}(\mathbf{q}). \quad (\text{B.12})$$

Plugging this in eq.(B.9) and replacing $E = k^2/2m$ we get

$$\begin{aligned} & \frac{(2\pi)^3}{2m} (q^2 - k^2) \delta^3(\mathbf{q} - \mathbf{k}) + \frac{1}{2m} (q^2 - k^2) \chi_{\mathbf{k}}(\mathbf{q}) = \\ & = \int U(\mathbf{q} - \mathbf{q}') (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{k}) \frac{d^3 q'}{(2\pi)^3} + \int U(\mathbf{q} - \mathbf{q}') \chi_{\mathbf{k}}(\mathbf{q}') \frac{d^3 q'}{(2\pi)^3}. \end{aligned} \quad (\text{B.13})$$

First term on the left hand side identically vanishes and we arrive at

$$\frac{1}{2m} (k^2 - q^2) \chi_{\mathbf{k}}(\mathbf{q}) = U(\mathbf{q} - \mathbf{k}) + \int U(\mathbf{q} - \mathbf{q}') \chi_{\mathbf{k}}(\mathbf{q}') \frac{d^3 q'}{(2\pi)^3}. \quad (\text{B.14})$$

Defining a new function $F(\mathbf{k}, \mathbf{q})$ as

$$\chi_{\mathbf{k}}(\mathbf{q}) = 2m \frac{F(\mathbf{k}, \mathbf{q})}{q^2 - k^2 - i\varepsilon}, \quad (\text{B.15})$$

we immediately get from eq.(B.14)

$$F(\mathbf{k}, \mathbf{q}) = -U(\mathbf{q} - \mathbf{k}) - 2m \int \frac{U(\mathbf{q} - \mathbf{q}')F(\mathbf{k}, \mathbf{q}')}{q^2 - k^2 - i\varepsilon} \frac{d^3 q'}{(2\pi)^3}. \quad (\text{B.16})$$

Infinitesimally small term $i\varepsilon$, which was added to avoid singularities at $q = \pm k$ could be chosen to carry either “+” or “−” sign. Depending on that sign the integral in eq.(B.16) may have two different values. Indeed, according to contour integration theorem, if $z = \infty$ is at least second order 0 for analitical function $f(z)$, then

$$\int_{-\infty}^{\infty} f(x)dx = 2\pi i \sum_{\nu=1}^n \text{Res}_{a_{\nu}} f(z), \quad a_{\nu} \in +\text{Im}z. \quad (\text{B.17})$$

As we write

$$q^2 - k^2 + i\varepsilon = (q - k + i\varepsilon)(q + k - i\varepsilon), \quad (\text{B.18})$$

$$q^2 - k^2 - i\varepsilon = (q - k - i\varepsilon)(q + k + i\varepsilon), \quad (\text{B.19})$$

and check with the Fig. 17 we see that $+i\varepsilon$ determines the value of integral through the residue in $-k$ whereas $-i\varepsilon$ idicates that residue should be taken at $+k$. Choice

Figure 48: a) Poles corresponding to the choice $+i\varepsilon$, b) same for $-i\varepsilon$.

b) automatically insures that inverse Fourier transformations of $\chi_{\mathbf{k}}(\mathbf{q})$ will return

correct diverging behavior of spherical wave $\chi_{\mathbf{k}}(\mathbf{r})$ from eq.(B.11). To prove it we apply transformation of eq.(B.7) to eq.(B.15). With the definition $\mathbf{n}' = \mathbf{r}/r$ we have

$$\chi_{\mathbf{k}}(r\mathbf{n}') = 2m \int \frac{F(\mathbf{k}, \mathbf{q})}{q^2 - k^2 - i\varepsilon} e^{iqr \cos \theta} \frac{q^2 dq d\phi d(\cos \theta)}{(2\pi)^3}, \quad (\text{B.20})$$

where we have aligned z -direction with $\mathbf{n}'$.

Integration over $d\phi$ gives 2π , whereas integral over $d\mu \equiv d(\cos \theta)$ can be worked out if we slightly bend the path of integration toward the upper-half complex plane (Fig. 18) keeping boundaries fixed at -1 and 1 . Since r is arbitrary large exponent

Figure 49: Integral over $d\mu = d(\cos \theta)$.

$e^{iqr\mu}$ is highly oscillating function and the value of integral mainly defined by the regions where $\mathbf{q}/q = \pm \mathbf{n}'$. In that regions $F(\mathbf{k}, \mathbf{q}) \approx F(\mathbf{k}, \pm q\mathbf{n}')$ and can be pulled out in front of the integral. Observing that $e^{iqr\mu}$ damping very fast as it departs from the boundaries ± 1 we arrive at

$$\chi_{\mathbf{k}}(r\mathbf{n}') = -2m \frac{2\pi i}{r} \left(\int_0^\infty \frac{F(\mathbf{k}, q\mathbf{n}') e^{iqr}}{q^2 - k^2 - i\varepsilon} \frac{q dq}{(2\pi)^3} - \int_0^\infty \frac{F(\mathbf{k}, -q\mathbf{n}') e^{-iqr}}{q^2 - k^2 - i\varepsilon} \frac{q dq}{(2\pi)^3} \right). \quad (\text{B.21})$$

Substitution $q \rightarrow -q$ in the last integral yields

$$\chi_{\mathbf{k}}(r\mathbf{n}') = -\frac{im}{2\pi^2 r} \int_{-\infty}^{\infty} \frac{F(\mathbf{k}, q\mathbf{n}') e^{iqr} q dq}{q^2 - k^2 - i\varepsilon}. \quad (\text{B.22})$$

Using contour integration theorem (see eq.(B.17)) and setting $\mathbf{n} \equiv \mathbf{k}/k$ one obtains

$$\chi_{\mathbf{k}}(\mathbf{r}) = \frac{m}{2\pi} \frac{e^{ikr}}{r} F(k\mathbf{n}, k\mathbf{n}'), \quad (\text{B.23})$$

which is precisely what one would expect to get for diverging spherical wave.

From eq.(B.1) we identify

$$f(\mathbf{n}, \mathbf{n}') = \frac{m}{2\pi} F(k\mathbf{n}, k\mathbf{n}') \quad (\text{B.24})$$

and eq.(B.16) can be rewritten as

$$\chi_{\mathbf{k}}(\mathbf{q}) = 4\pi \frac{f(\mathbf{n}, \mathbf{n}')}{q^2 - k^2 - i\varepsilon}. \quad (\text{B.25})$$

With this we are able to write down the desired expression for $\Psi_{\mathbf{k}}^{(+)}(\mathbf{q})$:

$$\Psi_{\mathbf{k}}^{(+)}(\mathbf{q}) = (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{k}) + 4\pi \frac{f(\mathbf{n}, \mathbf{n}')}{q^2 - k^2 - i\varepsilon}. \quad (\text{B.26})$$

APPENDIX C. NN diffractive parameters

Figure 50: Proton-proton total cross section as a function of incoming proton momentum in the Lab. frame. Solid line is our fit to the world data.

```
function sigma_pp_tot(s)
```

C

```
pp total cross section fit in [mb] (sigma_pp)

common/par/pi,pm,pmp,pmn,tm,eb ! pm -> mass of the proton
dimension B(10)
ep = (s - 2.0*pm**2)/(2.0*pm)
p = sqrt(ep**2-pm**2)
pcm = pm*p/sqrt(s) ! in GeV
pcm = pcm/sqrt(0.389385) ! in 1/sqrt(mb)
if(p.lt.3.68) then
  call Bystricky_dat(p,0.0,0,B)!using SAID's analysis
  stot = 4.*pi*pcm * ( B(6)+B(7) )/2.
elseif ( p.le.5.87) then
  x1 = 3.68
  y1 = 41.39135
  x2 = 3.9
  y2 = 41.19135
  x3 = 5.870000
  y3 = 40.42657
  stot = cubic_spline(p,x1,x2,x3,y1,y2,y3)
elseif ( p.le.12.) then
  x1 = 5.87
  y1 = 40.42657
  x2 = 9.
  y2 = 40.19135
  x3 = 12.00000
  y3 = 40.01630
  stot = cubic_spline(p,x1,x2,x3,y1,y2,y3)
else
  stot = 48.0 + 0.522*alog(p)**2 - 4.51*alog(p)
endif
sigma_pp_tot = stot
return
end
```

Figure 51: Neutron-proton (proton-neutron) total cross section as a function of incoming neutron(proton) momentum in the Lab. frame. Solid line is our fit to the world data: squares — np , circles — pn .

```

function sigma_pn_tot(s)

C   pn total cross section fit in [mb]   (sigma_pn)

common/par/pi,pm,pmp,pmn,tm,eb ! pm -> mass of the proton
dimension B(10)
ep = (s - 2.0*pm**2)/(2.0*pm)
p = sqrt(ep**2-pm**2)
pcm = pm*p/sqrt(s) ! in GeV
pcm = pcm/sqrt(0.389385) ! in 1/sqrt(mb)
if(p.lt.1.4)then
  call Bystricky_dat(p,0.0,1,B)!using SAID's analysis
  stot = 4.*pi/pcm * ( B(6)+B(7) )/2.
elseif(p.le.3.1) then
  x1 = 1.4
  y1 = 38.42728
  x2 = 2.07800
  y2 = 41.905
  x3 = 3.1
  y3 = 43.13159
  stot = cubic_spline(p,x1,x2,x3,y1,y2,y3)
else
  stot = 47.3 + 0.513*alog(p)**2 - 4.27*alog(p)
endif
sigma_pn_tot = stot
return
end

```

Figure 52: The ratio of the real to imaginary part of the proton-proton amplitude.

```
function a_pp(s)
```

C pp Re/Im amplitudes ratio parameter (alpha_pp)

```
common/par/pi,pm,pmp,pmn,tm,eb  ! pm -> mass of the proton
dimension B(10)
ep = (s - 2.0*pm**2)/(2.0*pm)
p = sqrt(ep**2-pm**2)
a_pp = 0.0
if(p.lt.3.68) then
  call Bystricky_dat(p,0.0,0,B)!using SAID's analysis
  a_pp = ( B(1)+B(2) )/( B(6)+B(7) )
return
elseif(p.ge.3.68.and.p.lt.5.) then
  x1 = 3.68
  y1 = -0.3316863
  x2 = 4.34
  y2 = -0.352568200
  x3 = 5.0
  y3 = -0.3734501
  a_pp = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
elseif(p.ge.5..and.p.lt.101.) then
  a_pp=-0.43377 +0.13158E-01*p -0.22820E-03*p**2
&      +0.19088E-05*p**3 -0.59601E-08*p**4
return
endif
end
```

Figure 53: The ratio of the real to imaginary part of the proton-neutron amplitude.

```
function a_pn(s)
```

```
C    pn Re/Im amplitudes ratio  parameter (alpha_pn)

    common/par/pi,pm,pmp,pmn,tm,eb  ! pm -> mass of the proton
    dimension B(10)
    ep = (s - 2.0*pm**2)/(2.0*pm)
    p = sqrt(ep**2-pm**2)
    a_pn = 0.0
    if(p.lt.1.4) then
        call Bystricky_dat(p,0.0,1,B)!using SAID's analysis
        a_pn = ( B(1)+B(2))/( B(6)+B(7))
    return
    elseif(p.ge.1.4.and.p.lt.3.) then
        x1 =  1.4000
        y1 = -0.2845016
        x2 =  1.9
        y2 = -0.43915290
        x3 =  3.0
        y3 = -0.4938042
        a_pn = cubic_spline(p,x1,x2,x3,y1,y2,y3)
    return
    elseif(p.ge.3..and.p.lt.101.) then
        a_pn=-0.56207 +0.24223E-01*p -0.50362E-03*p**2
&      +0.48408E-05*p**3 -0.17331E-07*p**4
    endif
    return
end
```

Figure 54: Diffraction slope parameter for proton-proton forward scattering.

```
function bp(s)
```

C pp Slope parameter (Bpp)

```
common/par/pi,pm,pmp,pmn,tm,eb ! pm -> mass of the proton
ep = (s - 2.0*pm**2)/(2.0*pm)
p = sqrt(ep**2-pm**2)
if(p.lt.0.926)then
  x1= 0.
  y1= 0.
  x2= 0.745
  y2= 0.6
  x3= 0.926
  y3= 0.39
  bp = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
elseif(p.lt.1.2)then
  x1= 0.9300001
  y1= 0.39
  x2= 1.15
  y2= 2.0
  x3= 1.2
  y3= 3.177635
  bp = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
elseif(p.lt.1.7)then
  bp = 55.861-125.66*p+95.365*p**2-22.695*p**3
return
elseif(p.le.3.0)then
  x1 = 1.7
  y1 = 6.343315
  x2 = 2.4
  y2 = 7.5
  x3 = 3.0
  y3 = 7.903550
  bp = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
else
  bp = 8.22 + 1.10*log(p/4.)
endif
return
end
```

Figure 55: Diffraction slope parameter for proton-neutron forward scattering.

```
function bn(s)
```

C pn Slope parameter (Bpn)

```
common/par/pi,pm,pmp,pmn,tm,eb ! pm -> mass of the proton
dimension B1(10),B2(10)
ep = (s - 2.0*pm**2)/(2.0*pm)
p = sqrt(ep**2-pm**2)
if(p.lt.0.8)then
  x1= 0.
  y1= 0.
  x2= 0.545
  y2= 1.0
  x3= 0.8000000
  y3= 2.232239
  bn = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
elseif(p.lt.1.74)then
  bn = -0.41579 + 1.8333*p + 2.6484*p**2 -1.0031*p**3
return
elseif(p.le.3.0)then
  x1 = 1.740000
  y1 = 5.508093
  x2 = 2.4
  y2 = 7.202996
  x3 = 3.0
  y3 = 7.903550
  bn = cubic_spline(p,x1,x2,x3,y1,y2,y3)
return
else
  bn = 8.22 + 1.10*alog(p/4.)
endif
return
end
```

```
function cubic_spline(p,x1,x2,x3,y1,y2,y3)
```

C Approximates function with the 3-nodes cubic spline

```
dyx2 = (y3-y2)/(x3-x2)
dyx1 = (y2-y1)/(x2-x1)
gamma2 = 6.*( dyx2 - dyx1 )
yyy2 = gamma2/2./(x3-x1)
yy1 = dyx1 - yyy2*(x2-x1)/6.
yy2 = dyx2 - yyy2*(x3-x2)/3.
C1 = y1+yy1*(p-x1)+yyy2*(p-x1)**3/(x2-x1)/6.
C2 = y2+yy2*(p-x2)+yyy2*(p-x2)**2/2.
&                                -yyy2*(p-x2)**3/(x3-x2)/6.
if(p.le.x2) cubic_spline = C1
if(p.gt.x2) cubic_spline = C2
return
end
```

VITA

TIGRAN ABRAHAMYAN

May 4, 1978	Born, Yerevan, Armenia
1999	B.S. Physics Yerevan State University Yerevan, Armenia
2003	M.S. Physics Florida International University Miami, FL
2001-2003	Teaching Assistant Florida International University Miami, FL
2003	SURA/Jefferson Lab Graduate Fellow Southeastern Universities Research Association (SURA)
2003-2006	Research Assistant Florida International University Miami, FL
2007	Dissertation Year Fellow Florida International University

PUBLICATIONS AND PRESENTATIONS

T. V. Abrahamyan, M. M. Sargsian (April, 2003). *Two Nucleon Emission from ^3He at High Q^2* . Talk presented at American Physical Society's April Meeting, Philadelphia, PA.

T. V. Abrahamyan, M. M. Sargsian (May, 2004). *High Q^2 $^3\text{He}(e, e'pp)n$ Reactions*. Talk presented at American Physical Society's April Meeting, Denver, CO.

M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman, L. L. Frankfurt (2005) *Exclusive electrodisintegration of ^3He at high Q^2 . I. Generalized eikonal approximation*, Phys. Rev. C **71**, 044614.

M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman, L. L. Frankfurt (2005) *Exclusive electrodisintegration of ^3He at high Q^2 . II. Decay Function formalism*, Phys. Rev. C **71**, 044615.