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Draft Abstract/Summary

The study of L^2(D) spaces, or the spaces of square-integrable functions with domain D, play a fundamental role in many areas of mathematics, physics and engineering. One particular aspect of it that is of interest is the study of Riesz basis of exponential type, which are of the form {e^(2pi*i*a_n*x))} where n is a natural number and a_n belongs to R^d, as the domain D changes. The aim of this project is to prove the existence as well as to construct an explicit exponential basis when D is a triangle in R^2. Past results have proven the existence of exponential bases for a finite union of intervals as well for multi-rectangles; however, no explicit constructions for these bases have been found. As such, the first step is to find an explicit exponential basis for a finite union of intervals which would then provide an explicit exponential basis for multi-rectangles. Then a sequence of multi-rectangles that approximate a triangle must be found, thereby providing the corresponding sequence of exponential basis for them. The next step would be to find the limit of that sequence of exponential bases (if it exists) and finally, show that the limit is an exponential basis for the approximated triangle. Such a result would of great theoretical importance because it could be used to find exponential bases where D is a more “complex” polygon. Furthermore, it would allow for the approximation of functions that have such domains through an infinite sum of exponential basis. This could be of great use in practical areas such as Fourier analysis and sampling theory.