What Teaching Strategies Can I Employ to Improve Student Achievement in the Addition of Fractions with Different Denominators?

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Abstract: The purpose of this action research was to determine what instructional strategies could be used to improve student achievement in fraction addition. An eighth grade intensive math class practiced multiplication facts and hands-on applications of fractions concepts for 2 months. Pretests/posttests were used to measure improvement in computation and understanding.

Many middle school students do not recognize fractions as mathematical entities, or objects that can be referred to as real things (Sfard, 1991). Rather, they view fractions, as well as many other mathematical concepts, only as procedures that they have memorized. During class discussion, students expressed concerns with fraction addition and finding the least common denominator (LCD). As students practiced their multiplication facts, their ability to find the LCD increased but their difficulty finding equivalent fractions continued. This study researched how a combination of drill and practice, the Concrete/Semi-Concrete/Abstract Approach (CSA) for fraction concepts, and competitive games, could help eighth grade remedial students master fraction addition. The following null hypothesis was tested: Students will have no significant increase in mastering fraction addition with different denominators after 8 weeks of the combined strategies drill and practice, Concrete/Semi-Concrete/Abstract (CSA) Approach, and competitive games.

Review of Literature

Why Students Struggle with Fractions

Many middle students struggle with fractions (Suh, Moyer, & Heo, 2005). Although they learn how to add, subtract, multiply, and divide fractions for several years in school, they lack the understanding and number sense that is necessary to manipulate fractions even when presented in practical situations because they lack the understanding of fractions as actual values. The emphasis on standardized tests instruction and improper calculator use has created a considerable imbalance between comprehension and fluency (Krudwig, 2003). This lack of understanding and number sense hinders their success in other areas of mathematics. The Third International Mathematics and Science Study (Hiebert, 2003) found that U.S. teachers use most of their instructional time teaching computation procedures and little to no time on developing conceptual understanding or connecting the procedures that students are learning with the concepts that show why and how those procedures work. The data show that when students over practice procedures before they actually understand them, it becomes more difficult to make sense of them later, which sets them up for failure in future math classes (Hiebert, 2003). This occurs because as the procedure is practiced, the individual steps become more unified and are eventually stored as a single procedure (Anderson, 1983). While this compression increases speed in computation, it decreases the student’s ability to reflect on the concepts that are connected to each procedural step.

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Theoretical Considerations

Learning theory on math. “Conceptual knowledge is defined as knowledge of those facts and properties of mathematics that are recognized as being related in some way [and] is distinguished primarily by relationships between pieces of information” (Hiebert & Wearne, 1986, p. 200). Procedural knowledge has no connection to these relationships; it involves knowledge of written symbols and the set of rules that governs those symbols within a syntactic system. Working with fractions is difficult for students because they associate numerals to the syntactic system of whole numbers. The image of a fraction contains a new syntactic system within a preexisting semantic base (Hiebert & Wearne, 1986). Although the fraction“4/5” has a “4” and a “5” in it, both being greater than one, the value of “4/5” is less than one. To truly understand the meaning of this, the student cannot continue thinking of the numeral “4” as a whole number. Instead, the student must recognize the relationship between the “4” and the “5.” Students who are not aided in recognizing that different combinations of previously learned symbols can represent the same concept, such as value, continue to struggle and do not recognize the semantic link (Hiebert & Wearne, 1986).

The importance of automaticity. According to the National Council of Teachers of Mathematics (2000), knowing the basic number combinations, such as single digit multiplication and division, and having computational fluency, or automaticity, in these is essential and elemental to future success in other areas of mathematics. Without automaticity, students spend too much time and energy focusing on basic skills, such as retrieving math facts, rather than on other higher order processes involved in problem solving. Unfortunately, middle school students develop math skills without concurrent development of automaticity because they do not have to memorize basic math facts or formulas due to the availability of reference sheets and calculators for standardized tests (Krudwig, 2003).

Instructional Strategies

Drill and practice. Students need to make connections with the prior procedural, or computational, knowledge that leads to a concept as an entity. In fact, the absence of either computational or structural understanding at various stages of learning actually delays further development (Sfard, 1991). Timed practice drills provide an effective traditional means of developing automaticity when combined with untimed practice of facts for mastery.

Concrete/Semi-Concrete/Abstract (CSA) approach. The Concrete/Semi-Concrete/Abstract (CSA) approach enables students to gain understanding of concepts and fluency in computation by gradually moving through three phases. The phases move from lower level understanding to higher level understanding through the use of scaffolding activities. According to Miller, Butler, and Lee (1998), the CSA approach creates a 25-85% improvement in students’ mathematical test scores. CSA is also supported by other research findings, which show that developing visual models for fractions is a significant influential factor in building understanding for fraction computation (Suh, Moyer, & Heo, 2005).

Effects of games and competition on attitudes and learning. Some research finds that games improve or reduce the negative effects of attitude and lack of motivation, thereby increasing student performance (Druckman, 1995). Furthermore, Van Eck (2006) found that although noncompetition games do not create more positive student attitudes towards mathematics, the presence of a coach, mentor, or advisor in conjunction with competition can make learners function beyond their maximum ability. This setting can increase the positive effects of competition, such as self efficacy and positive attitude, and simultaneously decrease the stress of competition and math anxiety. Advisement and coaching was also found to lower math anxiety during non competition, as well.

Methods and Procedures

Organization

Sixteen eighth grade students in an intensive class, ages 13 through 15, attending The Charter School at Waterstone, in Homestead, Florida, participated in this study. The class was composed of students who scored a Level I or II on the Florida Comprehensive Assessment Test. Eleven (68%) of the students were Hispanic, two (12.5%) of the students were African American, and three (18.75%) of the students were White. Ten (62.5%) of the students were male, and six (32.5%) of the students were female. Only one of the participating students was an ESOL student. This student was instructed using ESOL strategies, specifically vocabulary building, extended wait time, and peer tutoring. Each student was administered a pretest on fraction addition. Students participated in 30 minute sessions of drill and practice and concept building activities for 16 days during an 8 week period. The researcher kept an anecdotal log for each meeting in order to document student success or difficulty throughout the intervention.

Timeline of Study

In the first week, all students were given a written pretest on addition of fractions. A separate pretest on multiplication facts was also administered in order to determine which multiplication facts the students were having trouble with. The automaticity aspect of the intervention focused on these fact families. Students graded their own papers and discussed areas of difficulty that they wanted to improve. In the second week, students modeled the multiplication fact families using manipulatives (concrete phase) and participated in class discussions about the connection between multiplication and addition. Students practiced giving timed responses. Incorrect answers, or answers that were too slow, were automatically corrected by their partners.

During weeks three, four, and five students drew pictorial models of several multiplication problems on white boards (semi-concrete phase). Students were allowed to choose which multiplication fact families they wanted to practice and how to practice. They were given three choices, but not limited to choosing only one: (a) write out their own practice, (b) use the printed worksheets, or (c) flash cards with a partner. Students also played the Multiplication Game, in which they competed against each other for speed. During the sixth week, students represented fractions using colored shape tiles on the projector. Students practiced drawing pictorial models of equivalent fractions and used the pictorial models to solve fraction addition problems. Students also competed to find equivalent fractions by drawing pictorial models, using white boards, and solved various fraction addition problems using the pictorial models. Students created five fraction addition problems of increasing difficulty (with their solutions) for the rest of the class to attempt to solve. Students had three resources for help in creating their own problems: the text book, their partner, or the teacher. During week seven students created story problems involving the addition of fractions with different denominators (abstract phase). In the final week all students were given a written posttest on the multiplication facts and a separate posttest on addition of fractions. Students graded their own papers and discussed areas of improvement. Students also discussed areas that still needed more work.

Statistical Analysis

All the students in the study were given the pretest and equivalent posttest on fraction addition, consisting of 12 addition problems. The tests assess the students' understanding of the necessity of equivalent fraction. Answers were considered correct as long as the students' answers were equivalent to the simplified answer provided by the answer key. The percentage scores for the fraction addition pretest and posttest were compared using a paired samples t-test. Statistical Package for the Social Sciences software was used to determine if there was any significant improvement in student achievement. All the students in the study were given the multiplication facts recall pretest and posttest. A paired samples t-test was also used to compare the pretest and posttest scores on the multiplication facts. A correlations test was also done in order to find a possible correlation between the improvement in multiplication fact recall and improved fraction addition. Because no pretests or posttests were administered for the other two strategies used in the intervention, there were no other statistical analyses done.

Results

The purpose of this study was to explore the combination of three strategies, drill and practice, CSA, and competitive games, which could be used to improve students' understanding of and achievement in the addition of fractions. The mean percentage score for the fraction addition pretest was 37.7%. Following the treatment, the posttests mean percentage score was 82% (See Figure 1). The mean difference from pretest to posttest was 44.27 with a 95% confidence interval of a 28.46-60.07 increase from pretest to posttest. The paired sample t-test shows that the difference is highly significant with a correlation of 0.51 and a p-value less than .05 (See Table 1).

Table 1
Comparison of Pretest and Posttest Mean Scores for Fraction Addition

Group	N	Mean (%)	t (2-tailed)	Significance
Pretest	16	37.7		
			5.972	0.0000006
Posttest	16	82		

Discussion

Originally, the researcher believed that the major source of student difficulty with fraction addition consisted of a lack of multiplication fact recall. Their lack of automaticity seemed to be the cause of further difficulty in finding the least common denominator when adding fractions with different denominators. However, after 2 weeks of drill and practice on multiplication facts, even though students' multiplication fact recall began to improve (See Figure 2), they continued making errors in fraction addition because their understanding of equivalent fractions did not improve automatically as a result of their increased fluency. The literature suggests that drill and practice can reaffirm skills only when the concepts from which those skills come from are mastered. Instruction focused on strategies that help make connections between procedures and meaning improves students' conceptual knowledge (Hiebert & Wearne, 1986). Therefore, the researcher's focus switched from mostly drill and practice to a combined approach in which drill and practice complemented the CSA approach through competitive activities.

Students were able to arrive at equivalent numerators and denominators, abstractly, once they realized that they could use multiplication facts rather than draw pictorial models to count units. This supports other research that shows that automaticity without conceptual understanding does not allow students to transfer knowledge to other areas of mathematics efficiently (Pesek & Kirshmer, 2002). Conversely, conceptual understanding accompanied by automaticity allows higher understanding of mathematical concepts (Sfard, 1991). The findings of this study do not support a correlation between increased multiplication fact recall and fraction addition. However, the findings suggest that increased understanding of fractions can be achieved through the combination of the three strategies used in the intervention.

Recommendations

The lack of understanding of fractions should be addressed through a combination of strategies that include drill and practice, CSA, and competitive games. The competitive games with mentoring serve as motivation for students to push themselves beyond their current ability level in fraction manipulation. Furthermore, incorporating competitive aspects into the drill and practice also makes the drill and practice activities more enjoyable for the students. The CSA approach enables students to transition from lower level understanding of fraction concepts to higher levels gradually and the increase of automaticity gained from drill and practice decreases their cognitive load, allowing for more automatic understanding and manipulation of fractions at each phase of the CSA approach. Future replication of this study would benefit from increased time for the intervention. Also, the researcher believes that the last three weeks of this study had a more powerful effect on student achievement because of the combination of the three strategies. The researcher believes that this intervention could help students master other basic math concepts, such as basic addition and subtraction with whole numbers and addition and subtraction of decimal fractions. This study is limited by its quasi-experimental design. Future research should be conducted to include a control group. Future research could also test whether this combination of strategies is beneficial in higher levels of mathematics, as well.

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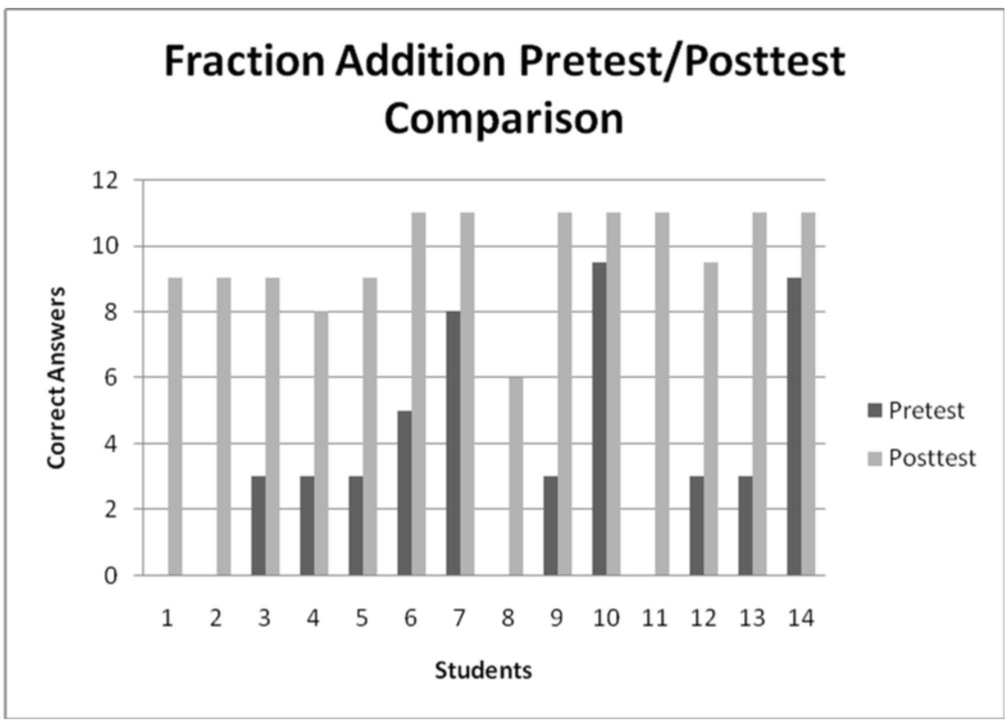

Figure 1. Fraction addition pretest/posttest comparison.

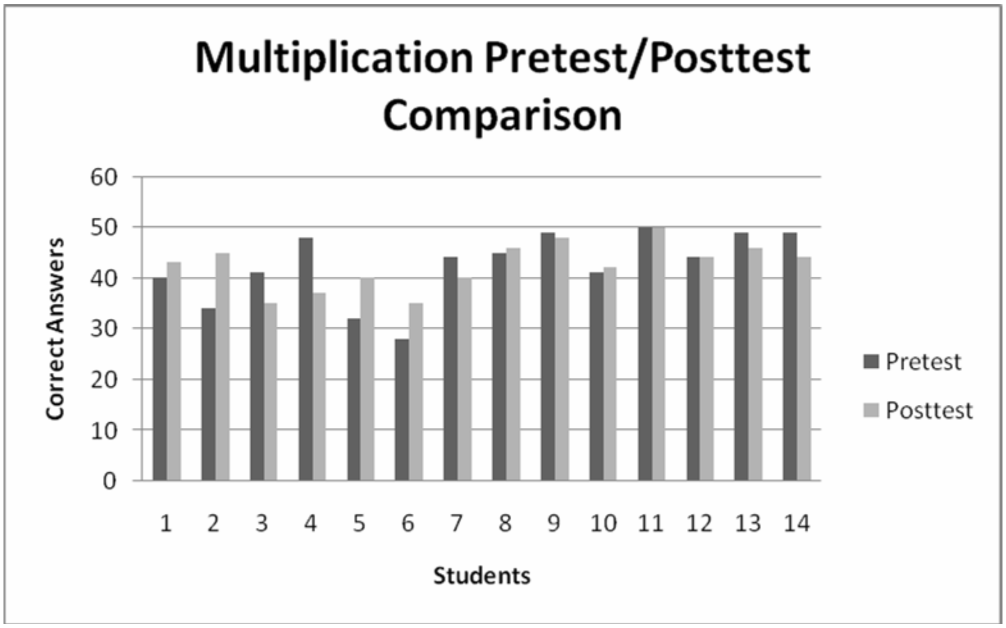

Figure 2. *Multiplication pretest/posttest comparison.*