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Application of a PERT-Type System and "Crashing" in a Food Service Operation

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Abstract
A pert-type system, a combination of the program evaluation and review technique (PERT) and the critical path method (CPM), might be used by the hospitality industry to improve planning and control of complex functions. The author discusses this management science technique and how it can assist.

Keywords
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Application of a PERT-Type System and "Crashing" in a Food Service Operation

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A pert-type system, a combination of the program evaluation and review technique (PERT) and the critical path method (CPM), might be used by the hospitality industry to improve planning and control of complex functions. The author discusses this management science technique and how it can assist.

The program evaluation and review technique (PERT) and critical path method (CPM) are the two most widely known and most widely used management science techniques in use today. They can be especially helpful to the hospitality manager because these two techniques can be used to plan, schedule, and control large projects which are composed of many activities performed by various people at different work situations. For example, a catering manager could use these techniques to improve the planning and control of a complex function such as a wedding or dinner reception.

In recent years the PERT and CPM systems have been combined into what is known as a PERT-type system. This type of system uses a network diagram to graphically depict the sequential nature of the activities which comprise a particular project. A project can be defined as a group of tasks which are performed in a certain sequence to reach an objective. Each task is defined as an activity. Each activity therefore has a beginning point and an ending point. These beginning points and ending points are known as events. In the network diagram an event is represented by a circle (node) and each activity is represented by a line (branch) that extends between two events. Directional arrows are drawn on each line (branch) in order to indicate the sequence in which events must be accomplished.

Activity Identification Is First Step

The first step in applying the PERT-type system to managing a project is to specify all the tasks or activities that are required to complete the project. It is important that all activities be identified since the scheduling process hinges on the list of activities for the project. Table 1 lists the activities required to set up a banquet and serve the guests and identifies the immediate predecessors for each activity; the latter includes those activities that must be completed immediately prior to the beginning of a given activity. For example, Activity I cannot

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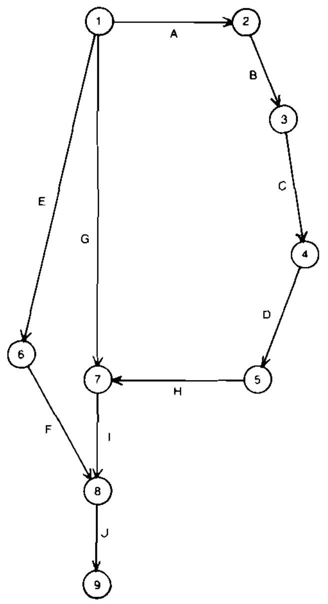
begin until Activity G is completed. Activity A can begin anytime because it does not depend on the completion of any prior activities.

Table 1
Activities Required

Activity	Description of Activity	Immediate Predecessor
A	Bring tables and chairs up from basement and arrange in hall	–
B	Pick up tablecloths from laundry and place on tables	A
C	Arrange place setting and decorations	B
D	Fill water glasses	C
E	Turn ovens on and perform equipment check	–
F	Prepare and cook main course	E
G	Prep salad and store	–
H	Seat guests	D
I	Plate salad and serve	G,H
J	Plate dinner and serve	F,I

The next step is to construct the network diagram that connects all these activities. Before constructing the network it is important to recognize two important rules, first, that each activity must only be represented by one line (branch) and, second, that no two activities can begin and end on the same two events (nodes). Figure 1 was constructed according to these two rules and is the network representation of the project described in Table 1.

Figure 1



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Time Estimates Vary for Activities

Since the PERT-type system is typically used for projects that exhibit some amount of time uncertainty the next step is to identify three time estimates for each activity. The three time estimates required are optimistic, most likely, and pessimistic. When developing the three time estimates for each activity, the judgment of the best personnel should be used. This may require the input of the manager and several employees. This will insure that the time estimates for each activity are as accurate as possible. Table 2 presents the three time estimates for each activity.

Table 2
Activity Time Estimates in Hours

Activity	Most Optimistic Time	Most Likely Time	Most Pessimistic Time
A	2.30	3.00	3.50
B	0.40	0.50	0.75
C	0.75	1.00	1.25
D	0.30	0.50	0.75
E	0.28	0.30	0.35
F	3.00	3.25	3.50
G	1.00	1.25	1.50
H	0.50	0.75	1.00
I	1.00	1.25	1.30
J	1.25	1.50	1.75

The time estimates are considered to be related in the form of a unimodal probability distribution. Because the distribution may be skewed to the right or to the left, it is necessary to make two assumptions in order to convert the most optimistic time (a), most likely time (m), and most pessimistic time (b) into estimates of the expected time and variance of the time required for each activity. The first assumption is that the activity time estimates are beta distributed which means that the expected time for each activity can be calculated by the following formula:

$$\text{Expected activity time} = 1/6\,(a + 4m + b)$$

The second assumption is that the standard deviation of the time required for each activity is equal to one-sixth of the range of reasonable possible time requirements. Therefore the standard deviation of the elapsed time required for each activity can be calculated by the following formula:

$$\text{Variance of expected activity time} = [1/6\,(b - a)] \quad 2$$

The mean activity time and activity time variance for each activity is presented in Table 3.

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Table 3
Activity Time Estimates in Hours

Activity	Most Opt. Time	Most Likely Time	Most Pess. Time	Mean Activity Time	Activity Time Variance
A	2.30	3.00	3.50	2.97	0.0400
B	0.40	0.50	0.75	0.53	0.0034
C	0.75	1.00	1.25	1.00	0.0069
D	0.30	0.50	0.75	0.51	0.0056
E	0.28	0.30	0.35	0.31	0.0001
F	3.00	3.25	3.50	3.25	0.0069
G	1.00	1.25	1.50	1.25	0.0069
H	0.50	0.75	1.00	0.75	0.0069
I	1.00	1.25	1.30	1.22	0.0025
J	1.25	1.50	1.75	1.50	0.0069

PERT Is Used to Calculate Critical Path

One of the objectives in using a PERT-type system is to calculate the longest or critical path for the network. In order to determine the critical path for a network it is necessary to determine four variables for each activity:

- EST: The earliest time an activity can start
- EFT: The earliest time an activity can finish
- LST: The latest time at which an activity can start
- LFT: The latest time at which an activity can finish

These variables are located on the network diagram according to the following format:

EST	MEAN TIME	EFT
°LST		LFT°

In order to compute these times for each activity in the network, it is necessary to make a forward pass through the network. This will allow us to calculate the EST and EFT for each activity. Once these times have been determined it will be possible to determine the LST and LFT for each activity. The rule for calculating the earliest start time for an activity leaving a node is that it is equal to the largest of the earliest finish times for all activities entering the node. The earliest finish time for an activity is simply the earliest start time plus the expected activity time for that activity. There are no activities entering node 1, so the earliest start time (EST) for activities A, G, and E is equal to zero. Since there is a single activity, activity A, leading into node 2 the early finish time (EFT) for activity A becomes the earliest start time (EST) for activity B. When calculating the EST for activity I, it is necessary to select the largest early finish time of either activity G or activity H. Since the early finish time for activity H is the highest value of the two activities, we will select that value as the EST for activity I.

The next step is to compute the latest start and finish times for each activity in the network. The rule for calculating the latest finish time for an activity that enters a node is that it is equal to the smallest of the latest start times for all activities leaving that node. The latest start time for that activity is simply the latest finish time minus the expected activity time for that activity. The latest start time (LST) for activity J in Figure 2 below is simply the latest finish time of 8.48 hours minus 1.50 hours which is that activity's expected activity time. The latest finish time (LFT) for activity F is simply the latest start time for activity J which is 6.98 hours. The LST and LFT for all activities are calculated in a similar manner. The network diagram with the four variables for each activity along with the expected time for each activity is shown in Figure 2 below.

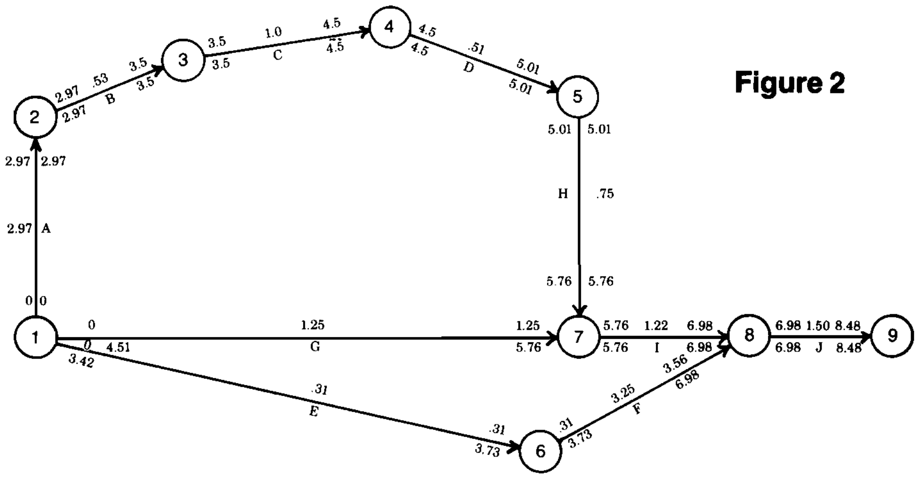

Figure 2

Now that all the times have been plotted for each activity, it is now possible to determine the minimum time required to set up the banquet and serve the guests. This is done by adding the time duration of each path through the network. Various durations of project completion are determined and the path with the longest duration is the time it will take to complete the project. In this case the project will take 8.48 hours to complete. The path associated with the longest duration is also known as the critical path because if any activities on the path are delayed, the completion of the project will also be delayed. In the example being considered, the manager should not be as concerned with the time it takes to prep the salad, turn on the ovens, or prepare and cook the main course as he/she should be concerned with activities A, B, C, D, H, I, and J. These activities lie on the critical path and could delay the dinner if they are not completed on time. The expected completion time for this event is 8.48 hours.

The critical path for the project can also be described as the path through the network where the activities have zero slack. Slackness is determined by the formula: LST - EST or LFT - EFT. Table 4 below presents the slack of each activity.

Table 4
Slack of Each Activity

Activity	Slack (LST - EST)
A	0.00
B	0.00
C	0.00
D	0.00
E	3.42
F	3.42
G	4.51
H	0.00
I	0.00
J	0.00

Probability of Completing Project Can Be Calculated

If the manager wanted to know how likely it was that the dinner would be served eight hours after the first activity began, this could be easily calculated. In order to determine the probability of completing a project by some future time it is necessary to make three assumptions. The first is that activity times are statistically independent. This means that the time it takes to complete one activity will not affect how long it takes to accomplish another activity in the project. The second is that the critical path always requires a longer total time than any other path. The third assumption is that the project completion time has a normal distribution. This assumption is based on the central limit theorem which states that the sum of several independent variables tends to have a normal distribution. The normal probability distribution can then be used to determine the probability of finishing a project by some future time. In the banquet example above, the expected completion time is simply the sum of the mean completion times for all the activities that lie on the critical path. It is computed as follows:

$$\text{Total Time} = 2.97 + .53 + 1.00 + .51 + .75 + 1.22 + 1.50 = 8.48$$

The variance of the project completion time is given by the sum of the variances of the critical path activities. It is computed as follows:

Variance of Project Completion Time = .0400 + .0034 + .0069 + .0056 + .0069 + .0025 + .0069 + .0722

Because the standard deviation is the square root of the variance, the standard deviation of the project completion time is calculated as .2687. With this information it is now possible to convert the target completion time into a standard normal random variable by subtracting the mean project completion time from it and then dividing the result by the standard deviation of the project completion time as shown below:

Prob (Completion time = 8 hours) =
P(T 8 hrs) =
P(Z Target time - Mean time/Standard deviation of completion)
= P(Z 8-8.48/.2687) = P(Z - 1.786) = .5 - .46298 = .03702

The use of a standard normal distribution table and interpolation was needed to derive the number .46298. Since the standard normal variable was a negative number, it is necessary to subtract the number extracted from the standard normal distribution table from .5. The result is .03702 or, in other words, there is a 3.7 percent probability that the dinner will be served eight hours after the set-up begins.

The probability of meeting target completion times at the completion of each event (node) can be determined by summing the expected activity times and the variances of the activity times on the longest path to each event (node) being examined. The normal distribution is then applied in the same manner as was done above. It is important to remember that there must be enough activities leading to the node being examined to justify the use of the central limit theorem. Table 5 provides the probability of meeting the schedule for each event in the network.

Crashing A Project Increases Costs

If the manager of the banquet wished to shorten the duration of the project, he/she could do so by assigning more resources to the banquet. Of course this would raise the cost of putting on the banquet. The process of shortening the duration of a project is known as "crashing." Because crashing a project will obviously raise the cost of that project, it is very beneficial to crash the project at the least additional cost. In order to determine which activity(ies) should be crashed, it is necessary to determine two time estimates and two cost estimates associated with each activity. The two time estimates needed are a normal activity time and a crash activity time. The two cost estimates in Table 6 are a normal time cost and a crash time cost for each activity of the banquet.

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Table 5
Probability of Meeting the Schedule
For Each Project Network Event

Event	Scheduled Completion Time	Mean Completion Time	Completion Time Variance	Probability of Meeting Schedule
0	0.00	0.00	0.0000	–
1	2.90	2.97	0.0400	36.3%
2	3.20	3.50	0.0434	7.5%
3	4.15	4.50	0.0503	5.9%
4	5.20	5.01	0.0559	78.8%
5	0.30	0.31	0.0001	15.9%
6	6.26	5.76	0.0628	97.7%
7	7.68	6.98	0.0653	99.7%
8	8.00	8.48	0.0722	3.7%

Table 6
Time in Hours and Cost Estimates for Project Network

Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Crash Cost/Hr
A	2.97	2.50	30.00	$45.00	$ 31.91
B	0.53	0.50	6.00	$ 7.00	$ 33.33
C	1.00	0.80	10.00	$12.00	$ 10.00
D	0.51	0.50	6.00	$ 7.00	$100.00
E	0.31	0.30	5.00	$ 5.50	$ 50.00
F	3.25	3.00	40.00	$50.00	$ 40.00
G	1.25	1.00	15.00	$16.00	$ 4.00
H	0.75	0.50	8.00	$ 9.00	$ 4.00
I	1.22	1.00	15.00	$17.00	$ 9.09
J	1.50	1.25	15.00	$17.00	$ 8.00

The crash cost per unit time is the net change in cost per unit change in time. The formula used to calculate the crash cost per hour in Table 6 is given below:

$$\text{Crash cost/Hr} = \frac{(\text{Crash cost - Normal cost})}{(\text{Normal time - Crash time})}$$

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The procedure for crashing a project at the minimum cost involves four steps:

- determine the normal time critical path and identify the critical activities
- compute the crash cost per time period for all activities in the network, using the formula provided above
- select the activity on the critical path which has the minimum crash cost per unit of time. Crash this activity to the maximum amount possible, or until the desired deadline has been achieved
- revise the network by adjusting for the time and cost assigned to the crashed activity. Then, check to see whether or not the critical path which has just been crashed is still the critical path for the network. If the original critical path is still the longest path through the network, return immediately to step 3. Otherwise, determine the new critical path, or paths, and then return to step 3[1]

The information in Table 6 was used to calculate the data presented in Table 7, which will be very useful when the manager of the banquet begins to determine the cost of shortening the time needed to begin serving the guests. In this example let us assume the manager wishes to shorten the duration of the project to 8.23, 8.00, and 7.5 hours in the most cost effective way. In order to shorten the expected project completion time to 8.23 hours the most cost effective way would be to crash activity H by the total number of hours possible (.25). This is the most cost effective way since the crash cost is only $4/hour. If the manager wished to shorten the duration of the project to eight hours, he or she would then crash activity J by .23 hours at a cost per hour of $8. It would do no good to crash activity G even though it has a lower cost because it does not lie on the critical path (see step 4 above). A summary of the project completion cost for each time frame is presented in Table 8.

Table 7
Crash Costs and Times Possible

Act	Maximum Crashing Possible (hrs)	Crash Cost Per Hour
A	0.47	$ 31.91
B	0.03	$ 33.33
C	0.20	$ 10.11
D	0.01	$100.00
E	0.01	$ 50.00
F	0.25	$ 40.00
G	0.25	$ 4.00
H	0.25	$ 4.00
I	0.22	$ 9.09
J	0.25	$ 8.00

Table 8
Summary of Time/Cost Tradeoffs

Step	Activity	Mean Project Completion Time in Hours	Project Completion Cost
0	No network crashing	8.48	$90.00
1	H crashed by .25 hrs.	8.23	$4.00 x .25 = $1.00 + $90.00 = $91.00
	J crashed by .23 hrs.	8.00	$8.00 x .23 = $1.84 + $91.00 = $92.84
3	J crashed by .02 hrs.	7.50	$8.00 x .02 = $0.16
	I crashed by .22 hrs.		$9.09 x .22 = $1.99
	C crashed by .20 hrs.		$10.00 x .20 = $2.00
	A crashed by .06 hrs.		$31.91 x .06 = $1.91
	(Total hrs. crashed = .98) $0.16 + $1.99 + $2.00 + $1.91 + $92.84 = $98.9		

The use of a PERT-type system in the hospitality field can have many applications by managers in back-of-the-house as well as front-of-the-house. The significance of PERT is that it can be used to to schedule and control individual project activities so that the project will be completed within a certain time period. It is also useful in helping the manager decide which ways are the most cost effective in shortening the length of a project.

References

[1]Shawnee K. Vickery, *Quantitative Methods: Applications to Managerial Decision Making*, (New York: John Wiley & Sons, 1987), p. 402.

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