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The last fifty years have witnessed large secular increases in educational attainment and R&D intensity. The fact that these trends have not stimulated more rapid income growth has been a persistent puzzle for growth theorists. We construct a model of endogenous economic growth in which income growth, R&D intensity, and educational attainment depend on the complexity of new technologies. An increase in complexity that makes passive learning more difficult, induces increases in R&D and education, alongside a decline in income growth. Our explanation also predicts a concurrent rise in the skill premium.

KEYWORDS: Endogenous growth, learning, R&D, educational attainment, wage inequality, technological complexity.

JEL Classifications: O40, E10.

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Figure 1 contains some US aggregate time-series that are now very familiar. Panel (a) shows a dramatic rise since 1950 in the intensity of R&D, whether measured as a proportion of the labor force or as a proportion of aggregate expenditure. Panel (b) shows a similar marked increase in educational attainment. One may quibble about the significance of these data. R&D has no doubt become more formal and, consequently, more broadly defined in the official statistics, and much of the increase in educational attainment may be a consumption good that contributes little to measured productivity growth (Klenow and Rodriguez-Clare, 1997; Dinopoulos and Thompson, 1999). Nonetheless, given the strength of the observed trends, the underlying real changes in R&D and educational attainment must be considerable. However, panels (c) and (d) show that, despite these dramatic changes in the key inputs of the knowledge production function, there has been no corresponding rise in either per capita income growth or labor productivity growth. To the contrary, long-run income growth has declined somewhat over this period.

Despite the familiarity of these data, growth theorists have made little progress explaining them. Because R&D intensity and years of schooling cannot rise forever, we must be observing transitional dynamics. Yet most growth models predict that per capita income growth will rise along a transition path characterized by rising inputs into the knowledge-creation process. In one notable exception, Jones (2002) has shown that the data are consistent with out-of-steady state predictions of a semi-endogenous growth model in which new ideas are shared across countries. In his model, per capita income growth is proportional to population growth in the steady state, but can be sustained at a constant, higher, rate when input intensity is rising. We learn much from Jones' analysis about the properties of a particular class of endogenous growth models but, because the secular increases in R&D and education are treated as exogenous data, our understanding of the evidence in Figure 1 remains incomplete.

We propose a simple explanation for Figure 1, in which rising R&D and educational attainment are *endogenous* responses to a change in the economic envi-

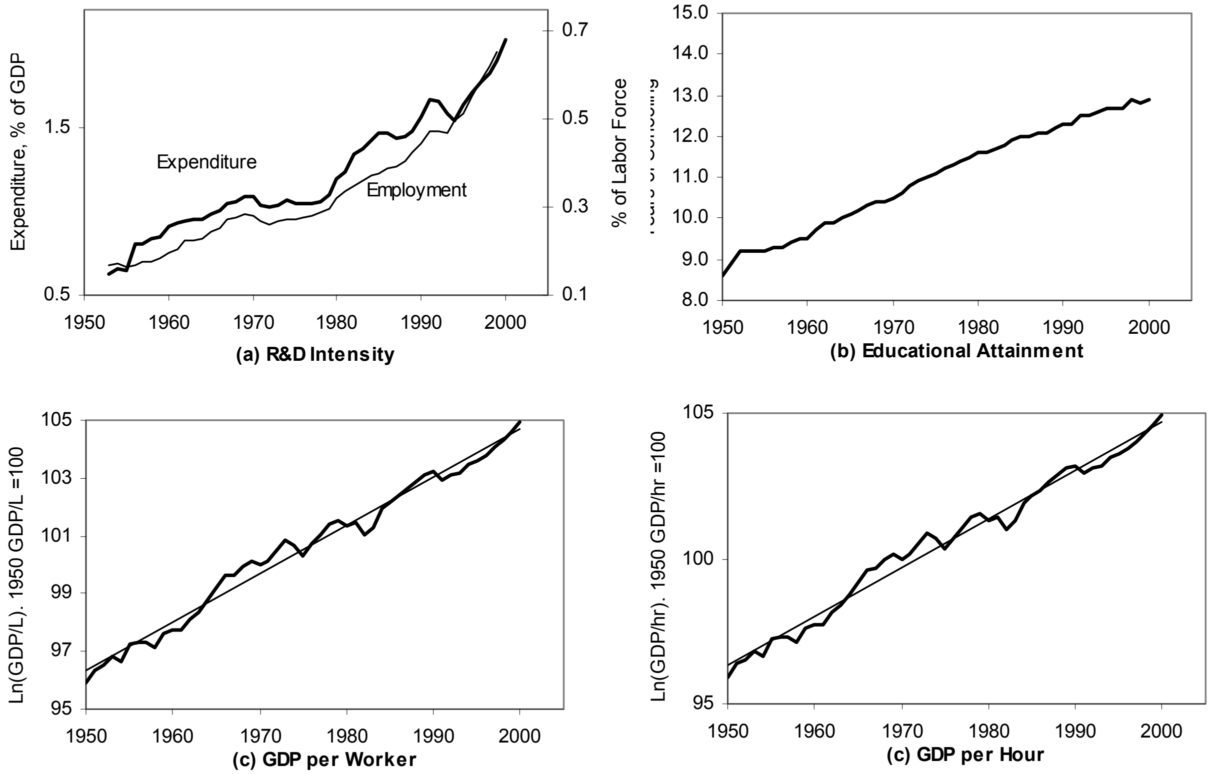

FIGURE 1. R&D, education and economic performance in the US. For sources, see the appendix.

environment, and in which the growth rate of income declines despite the rise in knowledge-creating inputs. We construct a quality ladders model that incorporates learning in the spirit of much earlier work by Young (1991, 1993), Lucas (1993), and Parente (1994). New product generations raise the quality of a product line, but firms can further raise the quality of any given generation at a rate that depends on their employment of skilled labor and the complexity of the technology they are using. Our explanation for the data is that a secular increase in complexity in the latter half of the 20th century has made passive learning more difficult. In response, firms increased their demand for skilled labor, part of which was to be engaged in applied R&D, and part in white-collar production-related employment. The increased demand for skill raised the returns to education, which in turn induced a rise in education attainment. Consequently, our explanation also is consistent with a rise in the returns to schooling and in the skill premium.

Kaboski (2001) developed an assignment model in which heterogeneous workers are assigned to a continuum of tasks that increase in complexity over time at a constant rate. More educated workers have a comparative advantage in complex tasks, and all workers seek more education as task complexity rises. Calibrating this model to the entire 20^{th} century experience, Kaboski is able to mimic some demanding empirical patterns, most notably the fall and then rise of both wage inequality and the returns to schooling over the century, at the same time that educational attainment is rising everywhere in the distribution. To accomplish this, however, Kaboski had to construct a complex model in which not only rising task complexity, but also falling fertility and rising life expectancy drive the data. We have fewer ambitions for our model, but we are able to explain Figure 1 with a simple, transparent framework.

One implication of our model is that it may generate a very different future from that implied by Jones' (2002), analysis, which contains some unpleasant arithmetic. Applying traditional growth accounting techniques, Jones concludes that rising input intensity accounts for 80 percent of post-war growth. Eventually, the secular increases in R&D intensity and educational attainment must end. When they do, income growth can be expected to decline dramatically, perhaps to no more than one-fifth of its post-war trend. What happens out of sample in our model depends of course upon the future behavior of complexity. Our model explains the data as a response to a one-time increase in complexity that slowly diffuses through the economy as firms gradually adopt new product generations that embody the new basic technology. If this is more or less what has been happening, then our model generates some pleasant arithmetic: inevitable future declines in the growth of R&D intensity and educational attainment need not presage a decline in the growth rate of income.

Our theory rests on some precise assumptions, and it is worth fixing ideas on these directly: technology has become more complex over time; learning by doing is more difficult in complex environments; and skill is more valuable in complex learning environments. None of these assumptions seems particularly contentious, but it turns out to be quite difficult to produce direct evidence for them. We do not have any easy way to measure complexity, and attempts to measure rates of passive learning have proved to be rather unreliable (Mishina, 1999; Lazonick and Brush, 1985; Sinclair, Klepper, and Cohen, 2000; Thompson, 2001). Nonetheless, there is a body of indirect evidence consistent with our assumptions, which is briefly reviewed here.

A. *Learning and complexity*

Jovanovic and Nyarko's (1995) Bayesian model of learning is perhaps the best-known study of the interaction between complexity and learning. They define complexity in terms of the number of independent tasks that must be undertaken in the production process. Their model predicts that in more complex technologies there will be more to learn, but the rate of learning is slower. Parameter estimates obtained from fitting their model to a dozen data sets are consistent with these predictions. In a series of papers (Argote, Beckman and Epple, 1990; Darr, Argote and Epple, 1995; Epple, Argote and Devadas, 1991), Argote, Epple and colleagues obtained similar results from estimating learning curves from three distinct activities – the operation of pizza franchises, an automotive assembly plant, and wartime shipbuilding. Figure 2 plots the learning curves implied by their parameter estimates. [1] If we are willing to entertain the notion that shipbuilding is a

[1] Epple and Argote assume that knowledge rises log-linearly with cumulative output and declines as a function of time. They interpret their results as evidence of organizational forgetting. Thompson (2004) has argued that forgetting may be a spurious result of assuming a learning curve in which, absent forgetting, productivity must rise without bound. In Figure 2 we simply plot the predicted productivity levels implied by the regression estimates.

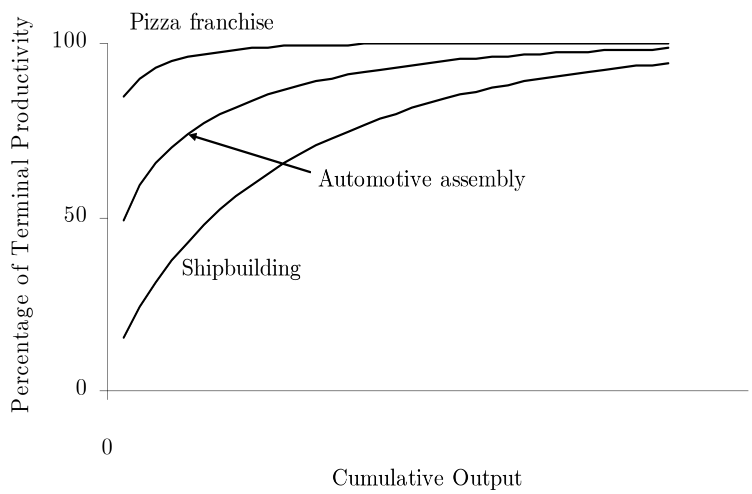

FIGURE 2. Learning curves from three industries. Curves plot the function q_t/q^*, where $q_t = K_t^{\gamma}$, $K_t = \lambda K_{t-1} + 1$. and $q^* = (1-\lambda)^{\gamma}$. Parameter estimates are: $\gamma=0.71$, $\lambda=0.93$ for shipbuilding (from Argote, Beckman and Epple, 1990, table 1, column 1); $\gamma=0.28$, $\lambda=0.92$ for automotive assembly (from Epple, Argote and Devadas, 1991, table 1, column 4); $\gamma=0.104$, $\lambda=0.80$ for pizza franchises (from Darr, Argote and Epple, 1995, table 1, column 4).

more complex task than automotive assembly, and automotive assembly is more complex than operating a pizza franchise, the learning curves yield half-lives of learning consistent with the predictions of Jovanovic and Nyarko.

Our ranking of Argote and Epple's three technologies is inevitably subjective. Unfortunately, Jovanovic and Nyarko's inferences about the relative complexity of different activities are even more problematic for our purposes, as they are obtained from the learning curves themselves. Galbraith (1990) took perhaps a more objective approach by allowing senior project engineers learning to work with new technologies to evaluate their complexity. He studied 32 instances in which high-technology companies transferred core manufacturing technology to plants located at least 100 miles from where the technology was originally in use. The senior project engineer at each recipient location was asked to rate on a five-point scale the complexity of the transferred technology relative to the recipient's existing technologies. Galbraith shows that the time it took the recipient site to reach the level of productivity at the donor site increased significantly with the complexity of the technology, even controlling for an initial loss in productivity that was higher in the more complex transfers. An increase of one on the five-point scale led to an increase in the initial productivity loss of about 16.7% and an increase in the recovery time of the lost productivity of about 15 percent.

B. *Skill and Learning*

An extensive literature on wage inequality and technology is consistent with our assumption that skilled labor has an advantage in learning more complex technologies and that, as technology became more complex in recent decades, the returns to education and unobservable skills have increased. As Figure 3 shows, there has been a marked increase in the college wage premium despite the concurrent rise in the relative supply of college graduates.

This sharp rise in the return to schooling (see also Blackburn, Bloom and Freeman, 1990; Katz and Murphy, 1992), in the premium for unobserved ability (Juhn, Murphy and Pierce, 1993; DiNardo and Pischke, 1997), and in the premium for observable indicators of cognitive ability (Murnane, Willet and Levy, 1995) are all consistent with our assertion that education and ability have become more valuable as complexity has increased. Evidence that earnings profiles are steeper for educated workers (Psacahropoulos and Layard, 1979; Knight and Sabot, 1981; Altonji and Dunn, 1995; Altonji and Pierret, 1997; Brunello and Comi, 2004; Low et al., 2004) is consistent with our assertion that educated workers are more able to learn.

If newer technologies are more complex than older technologies, our assumptions imply a positive correlation between wages and use of new technology, and this is again consistent with empirical evidence. Autor, Katz, and Krueger (1998) document an increased demand for skilled labor during the last five decades, and especially since 1970. They argue that the diffusion of computers and related

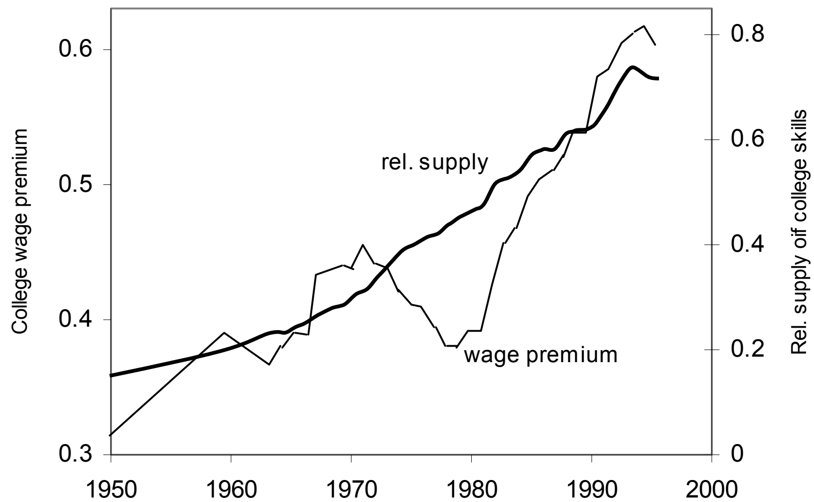

Figure 3. The college wage premium and the relative supply of college graduates. Source: Acemoglu (2002).

technologies contributed significantly to this phenomenon and show that skill upgrading occurred more rapidly in industries that are computer intensive. Berman, Bound and Griliches (1994) and Berman, Bound and Machin (1998) find large within–industry increases in the share of non-production workers in manufacturing, both in the US and in a sample of OECD countries, despite the rise in their relative wages during the 1980s and 1990s. They also show that the increase in the share of non-production workers is associated with R&D and computer investment. Allen (2001), focusing on the timeframe 1979-1989, shows that wage gaps by schooling increased the most in industries with rising R&D intensity and accelerating growth in the capital-labor ratio.[2]

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[2] Further evidence relating the wage structure to technology use can be found in Krueger (1993), Dunne and Schmitz (1995), Doms, Dunne and Troske (1997), and Thompson (2003).

Despite the wealth of wage data consistent with our assertions, we must acknowledge that the evidence is only circumstantial. Rising wage inequality is also predicted by models in which skilled individuals have an advantage simply in adopting or working with new technologies (Caselli, 1999; Galor and Moav; 2000, Lloyd-Ellis, 2002). Chari and Hopenhayn (1993) predict that workers employed on newer technologies will exhibit steeper earnings profiles, even though all workers learn at the same rate. R&D intensive industries, and industries and plants using newer technologies are likely to be more capital intensive, and capital-skill complementarity may be sufficient to explain their higher wages.

The Model

Models that combine R&D, new product generations and within-generation learning have tended to be rather complicated. As a result, they have also tended to be rather stylized. We do not depart from that "tradition" here. After laying out the model, we present our analysis in four parts. In sections A and B, we assume that skilled labor is in fixed supply. Section A characterizes the steady state, while Section B describes the dynamic responses of income and the skill premium to a one-time increase in complexity that affects each firm after it has adopted their its product generation. Sections C and D allow the supply of skill to respond endogenously to changes in demand.

A representative agent's intertemporal utility is given by

$$U = \int_0^\infty e^{-\rho t} \ln D(t) dt \ , \tag{1}$$

where

$$D(t) = \left[\int_0^1 q(i,t)^{1/\theta} x(i,t)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} , \tag{2}$$

is a quality-adjusted Dixit-Stiglitz consumption index defined over a continuum of goods of unit mass. The parameter $q(i,t)$ is an index of the quality of good i, while $x(i,t)$ denotes its quantity.

The familiar Euler equation,

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho \,, \tag{3}$$

where $E(t)$ is the agent's nominal expenditure on consumption goods, solves the consumer's intertemporal optimization problem. Nominal expenditure is the numeraire, so that $r(t)=\rho$, and instantaneous consumer demands satisfy

$$x(i,t) = \frac{q(i,t)p(i,t)^{-\theta}}{\int_0^1 q(i,t)p(i,t)^{1-\theta}\,di} \,. \tag{4}$$

Production is carried out by unskilled labor, one unit of which produces one unit of output. Let $w_u(t)$ be the wage of the unskilled. Each good is produced by a monopolistic firm i which chooses a constant markup over marginal cost, setting a price $p(i,t)=w_u(t)\theta/(\theta-1)$, and consequently facing demand

$$x(i,t) = \frac{(\theta-1)\alpha(i,t)}{\theta w_u(t)} \,, \tag{5}$$

where $\alpha(i,t) = q(i,t) / \int_0^1 q(i,t)di = q(i,t) / Q(i,t)$ is the relative quality of firm i's product. Let $(1-s(t))$ denote the supply of unskilled workers, and $G(\alpha,t)$ the distribution of relative quality. Full employment of unskilled workers requires that

$$\begin{aligned} 1-s(t) &= \frac{(\theta-1)}{\theta w_u(t)} \int_0^1 \alpha dG(\alpha,t) \\ &= \frac{(\theta-1)}{\theta w_u(t)} \,, \end{aligned} \tag{6}$$

which identifies the wage, $w_u(t)=(\theta-1)/(\theta(1-s(t)))$, product demands, $x(i,t)=\alpha(i,t)(1-s(t))$, and profits from manufacturing, $\pi(i,t)= \alpha(i,t)/\theta$.

New generations of the product arrive to each firm randomly according to an exogenous Poisson process with mean intensity μ. Let $\alpha(i,t)$ denote the relative quality of the current generation of i's product line. If the firm's next generation arrives at time t , it yields an improvement in relative quality of magnitude λ. In the absence of any other sources of quality change, it is then easy to verify that firm i's relative quality evolves according to the shot noise process

$$d\alpha(i,t) = -\alpha(i,t)g(t)dt + dq(i,t)\,, \tag{7}$$

where $g(t) = \dot{Q}(t)/Q(t)$, and $dq(i,t)$ is a Poisson process with mean intensity μ and magnitude λ. However, while manufacturing any generation of product, the firm may further enhance its relative quality by employing skilled labor. Let $s(i,t)$ denote firm i's employment of skilled labor. A fraction γ of this skilled labor is employed in formal R&D efforts while the remaining $1-\gamma$ is employed in management and related supervisory tasks. Skilled labor in either activity secures increases in relative quality by resolving quality control problems, making minor improvements in product design, and so on. We assume that if $s(i,t)$ skilled workers are employed for the interval dt, they secure an increase in relative quality of $\left(s(i,t)+\phi\right)^{\beta} dt$. Here, ϕ measures the *ease* of learning: a reduction in ϕ implies a smaller increment to relative quality for any given s, and it raises the marginal productivity of skilled labor. The evolution of firm i's relative quality therefore satisfies

$$d\alpha(i,t) = -\alpha(i,t)g(t)dt + \left(s(i,t)+\phi\right)^{\beta} dt + dq(i,t) \tag{8}$$

At each point in time, the firm must choose how much skilled labor to employ. The marginal value product of $s(i,t)$ is

$$\theta^{-1}\int_t^{\infty} \beta\left(s(i,t)+\phi\right)^{\beta-1} e^{-\rho(v-t)-\int_t^v g(y)dy} dv\,. \tag{9}$$

The immediate increment to relative quality brought about by an increase in $s(i,t)$ is $\beta\left(s(i,t)+\phi\right)^{\beta-1}$; multiplying this by θ^{-1} gives the immediate contribution to profits. The contribution decays as a result of continued growth elsewhere in the economy, and it decays in present value because of discounting. The firm chooses $s(i,t)$ at each point in time so that (9) equals the wage, $w_s(t)$ of the skilled:

$$s(i,t)=\left(\frac{\beta\int_t^{\infty}e^{-\rho(v-t)-\int_t^{v}g(y)dy}dv}{\theta w_s(t)}\right)^{\frac{1}{1-\beta}}-\phi\,. \tag{10}$$

Note that $s(i,t)$ does not depend on $\alpha(i,t)$. Hence $s(i,t)$=$s(t)$, and (10) also defines the aggregate demand for skill. For a given set of parameters, (6) and (10) therefore define wages of the skilled and unskilled.

A. *The steady state with a fixed supply of skills*

Assume for the moment that $s(t)=\overline{s}$. In the steady state, the growth rate of the economy is fixed. Let this growth rate be g. Then (10) simplifies to

$$\overline{s}=\left(\frac{\beta}{\theta(\rho+g)w_s}\right)^{\frac{1}{1-\beta}}-\phi\,. \tag{11}$$

Measuring wage inequality by the ratio of skilled to unskilled wages, from (6) and (11) we have

$$\omega=\frac{w_s}{w_u}=\frac{\beta(1-\overline{s})}{(\theta-1)(\rho+g)(\overline{s}+\phi)^{1-\beta}}\,. \tag{12}$$

which for given g is decreasing in $\overline{s}$ and ϕ. Using (8), it is easy to show that $dq(i,t)=Q(t)\left[(\overline{s}+\phi)^{\beta}\,dt+dq(i,t)\right]$. Integrating over i and dividing by $Q(t)\,dt$ yields $g=(\overline{s}+\phi)^{\beta}+\lambda\mu$, and hence

$$\omega=\frac{\beta(1-\overline{s})}{(\theta-1)\big((\rho+\lambda\mu)(\overline{s}+\phi)^{1-\beta}+(\overline{s}+\phi)\big)} \tag{13}$$

Comparing across steady states, an increase in complexity (i.e. a reduction in ϕ) is associated with a decline in the growth rate and an increase in wage equality. Because the supply of skilled labor is for the moment held fixed, a change in ϕ has no consequence for the intensity of R&D (equal to $\gamma\overline{s}$).

For any discussion of the skill premium to be interesting, we require that (13) exceed unity at $\overline{s}=0$, which requires that the parameters satisfy the restriction

$$(\theta - 1) < \frac{\beta}{(\rho + \lambda\mu)\phi^{1-\beta} + \phi}. \tag{14}$$

If this inequality is not satisfied, then unskilled labor employed in production is always more valuable than skilled labor used in advancing knowledge. In the remainder of the paper, we assume (14) holds.

Although the model is highly stylized, it generates a well-behaved steady state with a stationary cross-sectional distribution for the firm size. Because it is incidental to the main focus of the paper, this distribution is derived in the appendix.

B. *Transition dynamics with a fixed supply of skills*

Although changes in the supply of skills over time are an essential part of our story, it is easier and therefore useful to explore first the transitional adjustment of wages and economic growth to an increase in complexity when skills are held in fixed supply. Imagine at some arbitrary time 0 that a new more complex technological paradigm emerges. After this time, any new product generations adopted by firms embody a technology in which the ease of learning, ϕ, is reduced, say to ϕ'. Let $s(t;\phi)$ and $s(t;\phi')$ denote the resulting demands for skilled labor by firms that have the new and old technologies. At time t, a fraction $1 - e^{-\mu t}$ of firms are engaged with the new paradigm, while the remainder have yet to switch. Thus, aggregate growth is

$$g(t) = \left(1 - e^{-\mu t}\right)\left(\lambda\mu + \left(s(t;\phi') + \phi'\right)^{\beta}\right) + e^{-\mu t}\left(\lambda\mu + \left(s(t;\phi) + \phi\right)^{\beta}\right). \tag{15}$$

Derivation of the growth rate along the transition path is straightforward. From (10), we note that $s(t;\phi') + \phi' = s(t;\phi) + \phi$ for all t. Full employment of skilled labor further requires that $\left(1 - e^{-\mu t}\right)\left(s(t;\phi') + \phi'\right) + e^{-\mu t}\left(s(t;\phi) + \phi\right) = \overline{s}$. Combining these expressions with (15) yields

$$g(t) = \lambda\mu + \left(\overline{s} + e^{-\mu t}\phi + \left(1 - e^{-\mu t}\right)\phi'\right)^{\beta}, \tag{16}$$

which, as is often the case, is a function of an exponentially-weighted average of the old and new steady states. The growth rate declines monotonically along the transition path, and it is easy to establish that it does so at a declining rate.

We can also readily obtain an approximation to the transition path of the skill premium. Rewrite (11) for firms using technologies with complexity ϕ,

$$s(t;\phi) + \phi = \left(\frac{\beta}{\theta(\rho + g(t))w_s(t)}\right)^{\frac{1}{1-\beta}}. \tag{17}$$

Substituting the relationships between $s(t;\phi)$, $s(t;\phi')$, and $\overline{s}$, along with (6) and (16) yields

$$\omega(t) = \frac{\beta(1-\overline{s})}{(\theta-1)\big(\rho + g(t)\big)\big(g(t) - \lambda\mu\big)^{(1-\beta)/\beta}}. \tag{18}$$

where $g(t)$ is given by (16). As $g(t)$ declines monotonically, it is clear that the skill premium rises monotonically to its new steady state. Although $g(t)$ approaches its new steady state at a declining rate, the time path of $w(t)$ may either be strictly concave or S-shaped. [3]

[3] Note that (17) is not exact. The term $\big(\rho + g(t)\big)^{-1}$ is the integral of $\int_t^\infty \exp\Big(-\rho(v-t) - \int_t^v g(y)dy\Big)dv$ under the assumption that g is constant, whereas it is in fact declining monotonically. Thus, in reality, ω will somewhat exceed the level indicated by (18) along the transition path. With $g(t)$ given by (16), the integral does not exist in closed form.

C. *Endogenous supply of skills*

We now replace the representative agent with a continuum of agents with a constant death rate δ. When an agent dies, she is immediately replaced by a new agent. Upon birth, new agents can choose to pay a cost, c, to obtain education. Each agent has ability $\xi \in [0,1]$, which is a draw from the uniform distribution. If the cost is paid, and the agent chooses to train, she becomes skilled with probability ξ, and remains unskilled otherwise. Indirect utility is separable in expenditure, so the agent is concerned only to make the education choice that maximizes the discounted present value of her lifetime earnings. As an unskilled worker, this is $\int_0^\infty e^{-(\rho+\delta)t} w_u(t)dt$.

Restricting attention to the steady state with constant wages, the expected lifetime earnings of an unskilled worker is $w_u/(\rho+\delta)$. For a worker with ability ξ, expected earnings are $-c+(1-\xi)w_u/(\rho+\delta)+\xi w_s/(\rho+\delta)$ if she chooses to train. Given these payoffs, workers with ability greater than $\xi^* = c(\rho+\delta)/(w_s - w_u)$, choose to pay c, and the corresponding supply of skilled workers is $\int_{\xi^*}^1 \xi d\xi = \frac{1}{2}\left(1-(\xi^*)^2\right)$. Using (6), (11), and $g = (s+\phi)^\beta + \lambda\mu$ in the expression for ξ^*, we find that the steady-state supply of skilled labor is the smaller of the two solutions to the fixed point expression

$$s^* = \frac{1}{2}\left(1 - \frac{c^2(\rho+\delta)^2}{\left(\frac{\beta}{\theta\left((\rho+\lambda\mu)(s^*+\phi)^{1-\beta}+(s^*+\phi)\right)} - \frac{\theta-1}{\theta(1-s^*)}\right)^2}\right), \tag{19}$$

if the smallest solution is positive, and zero otherwise. The solution lies in the interval [0,½], and it is easily verified that this is decreasing in ϕ. Hence, an increase in complexity, as expected, raises the steady-state supply of skilled labor (and also R&D effort). As we have seen, g declines in response to an increase in complexity when s is fixed. This decline is now at least partially offset by a rise in s. To see what the steady-state growth rate is, we can replace s with $(g-\lambda\mu)^{1/\beta} - \phi$ in (19), to obtain

$$\tilde{g} = \phi + \frac{1}{2}\left(1 - \frac{c^2(\rho+\delta)^2}{\left(\frac{\beta}{\theta\left((\rho+\lambda\mu)\tilde{g}^{1-\beta}+\tilde{g}\right)} - \frac{\theta-1}{\theta(1-\tilde{g}+\phi)}\right)^2}\right), \tag{20}$$

where $\tilde{g} = (g - \lambda\mu)^{1/\beta}$. Equation (20) holds whenever an interior solution to (19) exists; otherwise, $\tilde{g} = \phi$. In both cases, a reduction in ϕ unambiguously reduces $\tilde{g}$ (and hence g). Hence, the induced increase in s is never sufficient to offset fully the direct growth-reducing effect of a decline in ϕ.

D. *Transition dynamics with an endogenous supply of skills*

When a technological revolution reduces ϕ, there can be no immediate response in skill supply. Existing workers have already chosen their training, and they are replaced only gradually, at the rate δdt, by new workers that have yet to choose whether to undertake training. Without an increase in skilled wages, there can be no increase in supply, so the short-term response to a reduction in ϕ, must be an increase in the skill premium as the new technological paradigm diffuses through the economy. The skilled wage increase induces a greater fraction of new workers to seek training, and so the supply of skilled workers gradually rises. Our assumption that R&D employment is proportional to s implies that R&D intensity also rises. The increased supply of skilled labor will mute the increased wage inequality, and moderate the demand for education. Because a grater fraction of the new workers is choosing education, the supply of unskilled workers must decline with time. Hence, the wages of unskilled workers rise over time. Thus, the short-term response to an increase in complexity is to overshoot the long-run equilibrium change in wage inequality. The aggregate growth rate also overshoots its new long-run equilibrium. A reduction in ϕ gradually reduces the growth rate as more and more firms are affected. Part of this decline is eventually offset by the increased skilled labor supply.

Noting that $\dot{s}(t) = \frac{1}{2}\delta\left(1-(\xi^*)^2\right) - \delta s(t)$, we can derive an approximation for the evolution of skilled labor under the assumption that new workers choose their education myopically on the basis of the current skill premium. Letting $\tilde{\phi}(t) = e^{-\mu t}\phi + (1 - e^{-\mu t})\phi'$, we can therefore write

$$\dot{s}(t) = \frac{\delta}{2}\left(1 - \frac{c^2(\rho+\delta)^2}{\left(\frac{\beta}{\theta\left((\rho+\lambda\mu)(s(t)+\tilde{\phi}(t))^{1-\beta}+(s(t)+\tilde{\phi}(t))\right)} - \frac{\theta-1}{\theta(1-s(t))}\right)^2}\right) - \delta s(t)\,. \tag{21}$$

The aggregate growth rate satisfies $g(t) = \lambda\mu + \left(s(t) + e^{-\mu t}\phi + \left(1 - e^{-\mu t}\right)\phi'\right)^{\beta}$, and wages satisfy (6) and (10). Column A of Figure 3 provides representative plots for parameter values satisfying (14).[4] The overshooting of skilled wages and growth are clearly evident. For comparison, column B plots the corresponding transition dynamics when skilled labor is in fixed supply.

Conclusions

In this paper we offer an explanation for the paradox presented by the coexistence of secular increases in R&D expenditure and educational attainment with no corresponding increases in per capita income growth. As Jones (2002) pointed out, these observations are inconsistent with most endogenous growth models. We construct a quality-ladders model in which new product generations arrive stochastically at an exogenous rate. Formal R&D and learning by doing influence the productivity of the new product. We claim that passive learning became more difficult during the latter half of the 20th century, as a result of the increased complexity of the technologies that firms have to work with. In this setting, falling per capita income growth, rising R&D expenditure and rising educational attainment are shown to be *equilibrium* responses to greater complexity.

[4] The values used in the figure are: ϕ=.1, ϕ'=.05, c=3, ρ=.03, δ=.01, β=.8, λ=.05, μ=.1, θ=3.

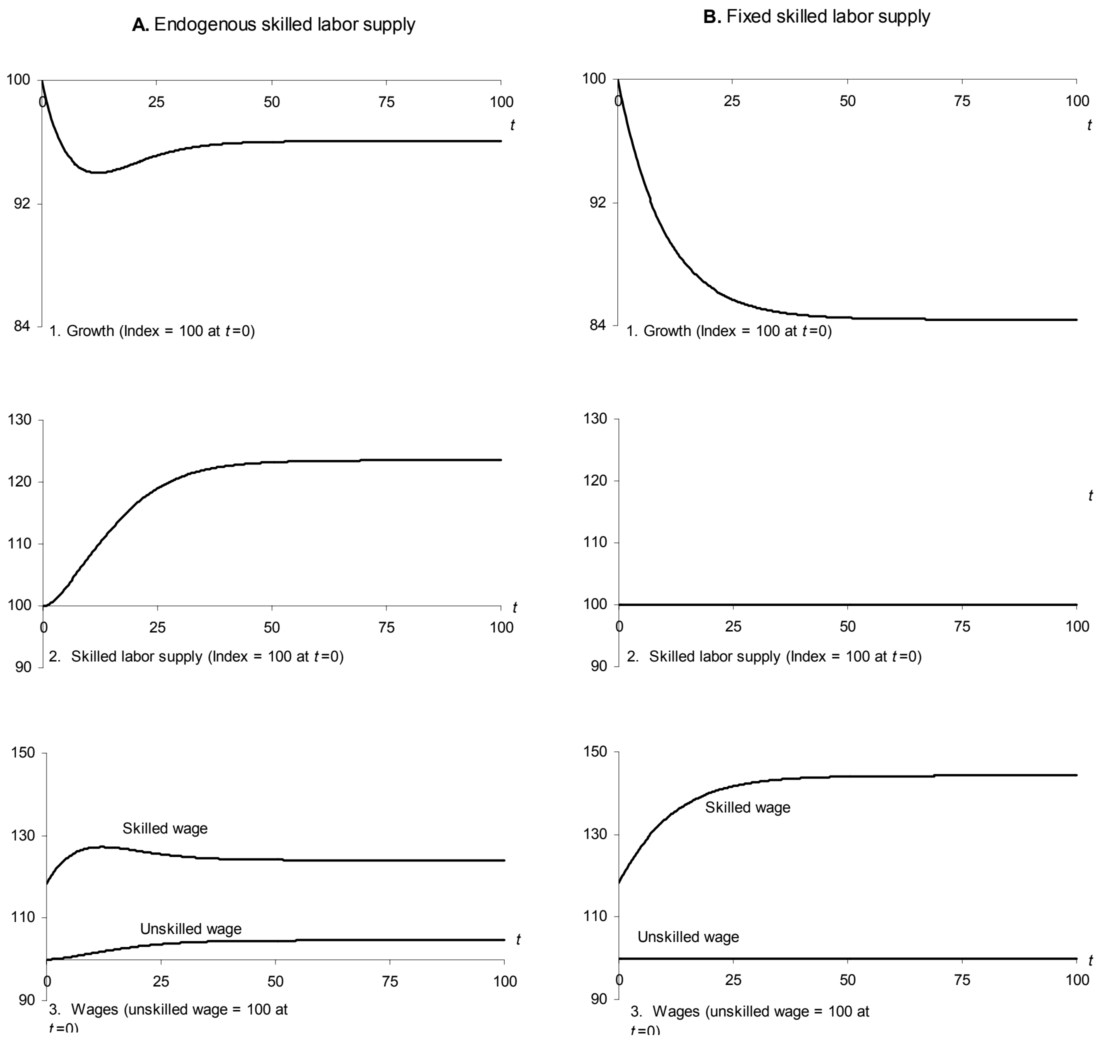

FIGURE 4. Dynamic responses to a reduction in ϕ.

Appendix

The steady-state distribution of firm size. Let $\alpha(i,0)$ denote the initial relative quality of firm i. If firm i were never to launch a new product generation, $\alpha(i,t)$ would evolve according to the differential equation

$$\dot{\alpha}(i,t) = -\alpha(i,t)g + (s+\phi)^{\beta}\ .$$

The backward solution is

$$\alpha(i,t) = \alpha(i,0)e^{-gt} + \int_0^t (s+\phi)^{\beta} e^{-g(t-s)} ds$$

$$= \alpha(i,0)e^{-gt} + \frac{(1-e^{-gt})(s+\phi)^{\beta}}{g}\ ,$$

where $g = \lambda\mu + (s+\phi)^{\beta}$. In the steady state, as $t \to \infty$, this contribution to i's relative quality therefore has mean $(s+\phi)^{\beta} / (\lambda\mu + (s+\phi)^{\beta})$ and zero variance.

In addition, however, firm i experiences Poisson jumps of intensity μ and magnitude λ, the contribution of each of which decays at the exponential rate g. Let $\tau_j(i)$ denote the arrival time of the jth product generation for firm i, and let $n(i,t)$ denote the number of new product generations that have been launched by firm i by time t. At time t, the current contribution to relative quality of a product generation of vintage $t-\tau$ is $\lambda e^{-g(t-\tau)}$. Then, including the first and all subsequent product generations yields

$$\alpha(i,t) = \alpha(i,0)e^{-gt} + \frac{(1-e^{-gt})(s+\phi)^{\beta}}{g} + \sum_{j=1}^{n(i,t)} x_j(i,t)\ ,$$

where $x_j(i,t) = \lambda e^{-g(t-\tau_j(i))}$. The $\tau_j(i)$ are i.i.d. random variables, uniformly distributed on $[0,t]$. Using the method of transformations to obtain the pdf of x, we have

$$f(x,t) = \begin{cases} (gxt)^{-1}, & \lambda e^{-gt} \le x \le \lambda \\ 0, & \text{otherwise} \end{cases}\ .$$

The characteristic function for x is

$$\phi_x(s,t) = \int_{\lambda e^{-gt}}^{\lambda} e^{\hat{i}sx} (gxt)^{-1} dx$$

$$= \int_{\lambda e^{-gt}}^{\lambda} \cos(sx)(gxt)^{-1} dx + \hat{i} \int_{\lambda e^{-gt}}^{\lambda} \sin(sx)(gxt)^{-1} dx\ ,$$

where $\hat{i} = \sqrt{-1}$. The second inequality comes from Euler's formula. Let $z(i,t)$ denote the contribution of all product generations *except* the first. As the τ_j are i.i.d., the characteristic function for $z(i,t)$ is simply the expectation of the $n(i,t)$-fold product of $\phi_x(s,t)$, where $n(i,t)$ is a Poisson r.v. with mean μt.

$$\begin{aligned}\phi_z(s,t) &= E\left[\phi_x(s,t)^{n(i,t)}\right] \\ &= \sum_{n=0}^{\infty} \frac{\phi_x(s,t)^n (\mu t)^n e^{-\mu t}}{n!} \\ &= e^{\mu t[\phi_x(s,t)-1]}.\end{aligned}$$

The last line used the series expansion $e^y = \sum_{n=0}^{\infty} y^n / n!$. The kth moment is found by differentiating $\phi_z(s,t)$ k times with respect to s, multiplying by $-\hat{i}^k$, and evaluating the resulting expression at s=0:

$$m_1(z) = E[z(i,t)] = \frac{\lambda\mu}{g}\left(1 - e^{gt}\right)$$

$$m_2(z) = E\left[z(i,t)^2\right] = \frac{\lambda^2\mu(g+2\mu) - \lambda^2\mu^2 e^{-gt} - \lambda^2\mu e^{-2gt}(g-2\mu)}{2g^2}.$$

Letting $t \to \infty$, the steady-state mean and variance of z are $\lim_{t\to\infty} E(z(i,t)) = \lambda\mu / (\lambda\mu + (s+\phi)^\beta)$ and $\lim_{t\to\infty} \mathrm{var}(z(i,t)) = \lambda^2\mu / (2(\lambda\mu + (s+\phi)^\beta))$. Finally, adding in the current contribution of the first product generation, we have

$$\lim_{t\to\infty} E(\alpha(i,t)) = \frac{(s+\phi)^\beta}{\lambda\mu + (s+\phi)^\beta} + \frac{\lambda\mu}{\lambda\mu + (s+\phi)^\beta} = 1$$

and

$$\lim_{t\to\infty} \mathrm{var}(\alpha(i,t)) = \frac{\lambda^2\mu}{2(\lambda\mu + (s+\phi)^\beta)}.$$

As neither moment depends on initial conditions, these expressions are also the moments of the steady-state cross-sectional distribution. From the relations between $\alpha(i,t)$, profits, and demands given in the main text, we conclude that the limiting distributions of relative quality, profits, and firm size are stationary, with finite variance. Having already established that $\lambda\mu + (s+\phi)^\beta$ is increasing in ϕ, it is easy to see that the variance of firm size increases complexity.

Sources of data for Figure 1. Expenditure on non federal R&D as a percentage of GDP for 1953-2002 is taken from the National Patterns of R&D resources: 2002 provided by National Science Foundation, Division of Science Resource Statistics at http://www.nsf.gov/sbe/srs/seind04/c4/fig04-05.xls .

The number of scientists and engineers engaged in R&D for the period 1950-1980 are taken from Jones (2002), and for the rest of the series (1981-1999) an estimate of the number of scientists and engineers engaged in R&D is taken from the National Patterns of R&D Resources: 2002 provided by the National Science Foundation at http://www.nsf.gov/sbe/srs/nsf03313/tables/tab8.xls. Missing data are derived from averages of adjacent years. Labor force data are taken from the Bureau of Labor Statistics. The number of scientists and engineers engaged in non federal R&D is estimated as in

Ha and Howitt (2005), being equal to overall scientists and engineers engaged in R&D multiplied by non federal R&D/ total R&D expenditure.

Average years of educational attainment in the population among persons 25 years and older are from Jones for 1950-1980. The remaining years are estimated using Jones' method. The US Census Bureau reports interval data on educational attainment at http://www.census.gov/population/socdemo/education/tabA-1.xls. In computing the average we assume that every person in a given interval had schooling equal to the interval mean. Persons that have four or more years of college are assumed to have 4 years.

GDP per worker in 1996 dollars is from the Penn World Data for the period 1950-2000 and from the Bureau of Economic Analysis (http://www.bea.gov/bea/newsrel/gdpnewsrelease.htm) for 2001-2003. To estimate Real GDP per hour we take data on Real Gross Domestic Product, 2000 Chained Dollars (1950-2003) from the Bureau of Economic Analysis, the National Income and Product Accounts Table http://www.bea.doc.gov/bea/dn/nipaweb/SelectTable.asp?Popular=Y. Employment data of civilian population of age 16 and older are from Jones for 1950-1979 and from Labor Force Statistics (Current Population Survey of the Bureau of Labor Statistics) at http://www.bls.gov/cps/cpsatabs.htm for the rest of the series. Average weekly hours of production are from the Current Population Survey (ftp://ftp.bls.gov/pub/suppl/empsit.ceseeb2.txt). Assuming an average work load of 50 weeks a year we estimate the total number of hours worked in the economy and real GDP per hour.

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