7-22-2008

Essays on Exchange Rate Economics

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ESSAYS ON EXCHANGE RATE ECONOMICS

A dissertation submitted in partial fulfillment of the

requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Yan Shu

2008

To: Dean Kenneth G. Furton
    College of Arts and Sciences

This dissertation, written by Yan Shu, and entitled Essays on Exchange Rate Economics, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

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Date of Defense: July 22, 2008

The dissertation of Yan Shu is approved.

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Florida International University, 2008

DEDICATION

I dedicate this dissertation to my parents, sister, other family members, and friends. The completion of this work would have not been possible without their continuous support, love and encouragement.

ACKNOWLEDGMENTS

I would like to particularly thank my advisor and committee chair Dr. Prasad Bidarkota. I want to thank him sincerely for his insightful advice and guidance, and for his boundless generosity and patience toward me.

I would also like to thank my committee members for their time, support and patience during the course of writing this dissertation. I deeply appreciate Dr. Cem Karayalcin who is always willing to give me his continuous encouragement and thoughtful comments. I am very grateful to Dr. Mihaela Pintea and Dr. Brice Dupoyet for taking their time to read drafts of this manuscript and for valuable advice.

I want to extend my gratitude to other professors in the Department of Economics who benefited me greatly through the courses I took with them: Dr. John Boyd, Dr. Panagis Liossatos, Dr. Peter Thompson, Dr. Jesse Bull and Dr. Richard Chisik. They are very nice and helpful and make my five years of graduate work totally enjoyable.

Finally, I want to acknowledge the financial support of the FIU University Graduate School in the form of the Dissertation Year Fellowship.

ABSTRACT OF THE DISSERTATION

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Professor Prasad Bidarkota, Major Professor

Exchange rate economics has achieved substantial development in the past few decades. Despite extensive research, a large number of unresolved problems remain in the exchange rate debate. This dissertation studied three puzzling issues aiming to improve our understanding of exchange rate behavior. Chapter Two used advanced econometric techniques to model and forecast exchange rate dynamics. Chapter Three and Chapter Four studied issues related to exchange rates using the theory of New Open Economy Macroeconomics.

Chapter Two empirically examined the short-run forecastability of nominal exchange rates. It analyzed important empirical regularities in daily exchange rates. Through a series of hypothesis tests, a best-fitting fractionally integrated GARCH model with skewed student-t error distribution was identified. The forecasting performance of the model was compared with that of a random walk model. Results supported the contention that nominal exchange rates seem to be unpredictable over the short run in the sense that the best-fitting model cannot beat the random walk model in forecasting exchange rate movements.

Chapter Three assessed the ability of dynamic general-equilibrium sticky-price monetary models to generate volatile foreign exchange risk premia. It developed a tractable two-country model where agents face a cash-in-advance constraint and set prices to the local market; the exogenous money supply process exhibits time-varying volatility. The model yielded approximate closed form solutions for risk premia and real exchange rates. Numerical results provided quantitative evidence that volatile risk premia can endogenously arise in a new open economy macroeconomic model. Thus, the model had potential to rationalize the Uncovered Interest Parity Puzzle.

Chapter Four sought to resolve the consumption-real exchange rate anomaly, which refers to the inability of most international macro models to generate negative cross-correlations between real exchange rates and relative consumption across two countries as observed in the data. While maintaining the assumption of complete asset markets, this chapter introduced endogenously segmented asset markets into a dynamic sticky-price monetary model. Simulation results showed that such a model could replicate the stylized fact that real exchange rates tend to move in an opposite direction with respect to relative consumption.

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CHAPTER 1

INTRODUCTION

The past few decades have seen a substantial development on exchange rate economics. Given the strong interest in the exchange rate among academics, policy-makers and practitioners, it is not surprising that exchange rate economics is one of the most heavily studied areas in International Finance. Since the advent of generalized floating exchange rates in 1973, lots of theoretical, applied economists and econometricians have made enormous effort to find what determined the level, or the change, of a floating exchange rate and understand its relationship to other economic fundamentals. Nevertheless, exchange rate economics remains an extremely challenging area in the sense that, despite this extensive research, a large number of unresolved issues remain in the exchange rate debate. In this dissertation, I investigate the following three research problems and address them in three chapters respectively.

The first problem concerns the short-run forecastablity of nominal exchange rates. In exchange rate economics, a robust finding by Meese and Rogoff (1983), and the extensive literature that followed over next two decades, is that although existing macro-structural models can explain some aspects of exchange rate dynamics, they perform no better than the random walk model in forecasting at short horizons. In Chapter 2 of this dissertation, "On the short-run forecastability of exchange rates", I attempt to answer the question about whether the exchange rate is inherently predictable from the perspective of time series analyses. I collect daily spot exchange rates for the Japanese Yen and the British Pound against the US Dollar over the period spanning January 1996 through

December 2005. I identify statistical regularities in these data. Among them, an important feature is long memory in exchange rate volatility. Neglect of this feature may render the misspecification on the conditional mean of exchange rates. Then I propose a very general econometric model that can statistically describe complex behavior in exchange rates. This is an Artificial Neural Network (ANN) model, with Generalized Autoregressive Conditional Heteroskedasticity (GARCH) for capturing time-varying volatility. After testing various versions of the model through rigorous hypothesis tests, I pin down a best-fitting model that features Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) with the Skewed Student's t innovations. I compare the forecasting performance of this best-fitting model with that of the random walk model. Modeling and forecasting results indicate that daily exchange rates exhibit non-linearity and long memory in volatility but linearity in the conditional means. They seem to be non-forecastable over the short run in the sense that even the best-fitting model cannot beat the random walk model in forecasting based on normal evaluating criteria.

The contention that exchange rates may not be predictable at least in the short run has a direct implication on the monetary economics studying interest rates and exchange rates. The standard macro monetary models link both nominal interest rate differentials and expected nominal exchange rate changes to the conditional means of two variables, the household's marginal utility growth differential and the inflation differential across countries, through log-linearizing the Euler equations. If the exchange rate is not predictable, expected nominal exchange rate changes behave as innovations. On the other hand, nominal interest rate differentials do not follow random walks as observed in the

data. Thus, the equation of interest rate differentials should contain other terms than the conditional means of the log of two variables above[1]. The natural candidates are their conditional variances and/or covariances omitted during the log-linearization of the models.

Also being aware of the Uncovered Interest Parity (UIP) puzzle in International Finance, I explore the second problem about the relationship between interest rate differentials and exchange rates, which is the subject of Chapter 3 of this dissertation, "Foreign Exchange Risk Premium in a New Open Economy Macroeconomic Model". According to the standard UIP condition, the expected changes in exchange rates should equal the interest rate differentials between the domestic and foreign country. Therefore, a simple regression of exchange rate changes on interest rate differentials should produce a regression coefficient of one. However, empirical work, beginning with Hansen & Hodrick (1980) and Fama (1984), consistently produces a regression coefficient that is not only smaller than one but very often negative. Considering that the exchange rate may not be predictable in the short run, one may naturally propose an explanation to rationalize this puzzle: there exists a risk premium, expressed as the conditional variances and/or covariances of relevant economic variables, in the foreign exchange rate market that drives a wedge between interest rate differentials and expected exchange rate changes.

On the other hand, recent literature on New Open Economy Macroeconomics (NOEM) developed by Obstfeld and Rogoff (1995) shows considerable promise in understanding exchange rate behavior. I borrow their setup but modify the standard

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[1] See Alvarez, Atkeson and Kehoe (2007) for detailed analysis.

model by including a cash-in-advance constraint and an exogenous monetary growth process with time-varying volatility. The purpose of the study is to examine whether such a model can generate volatile risk premium to rationalize the UIP puzzle. First, I derive equilibrium equations of the model. Second, I log-linearize these equations around the steady state of the economy. In doing so, I end up with variance-covariance terms of the system variables determining the foreign exchange risk premium. Third, I calibrate the model and simulate the dynamics of the implied risk premium and examine the second moment properties of interest. Simulation results provide quantitative evidence that might explain the UIP puzzle. In addition, the analysis also shows that the near-random walk behavior of exchange rates can arise endogenously in a New Keynesian monetary model.

Similar to other studies on the dynamics of real exchange rates, such as Chari, Kehoe and McGrattan (2002, hereafter CKM), Chapter 3 generates a correlation coefficient of unity between the real exchange rate and relative consumption across countries. However, empirical evidence suggests that the correlation between two variables is small and often negative. CKM labeled the discrepancy between the model and the data as the consumption-real exchange rate anomaly. Attempting to account for this anomaly is the subject of Chapter 4 of this dissertation, "Resolving Consumption-Real Exchange Rate Anomaly with Sticky Prices and Endogenously Segmented Markets". To study this third problem, I introduce endogenously segmented asset markets by assuming that households need to pay a fixed cost to exchange bonds and money in international financial markets. By modifying the behavior of households and keeping other economic activities same as those in Chapter 3, I relate real exchange rates to the

consumption of active households who participate asset markets by paying the fixed cost. Similarly, I derive equilibrium equations of this new model and log-linearize them around the steady state of the economy. Then I calibrate and simulate the model. Both impulse response functions and other numerical results show that such a model can replicate the stylized fact that real exchange rates tend to move in opposite direction with respect to relative consumption.

Therefore, in all three studies, I attempt to understand the dynamics of either nominal or real exchange rates, aiming to shed light on three unresolved problems mentioned above in exchange rate economics.

CHAPTER 2

ON THE SHORT-RUN FORECASTABILITY OF EXCHANGE RATES

2.1 Introduction

Understanding and forecasting exchange rate movements are clearly important to a wide range of decision problems. For example, at the microeconomic level individual investors consider to purchase/sell foreign currency denominated assets. And up to a macro level, central banks make monetary policies based on (implicit or explicit) inflation targeting.

Unfortunately researchers have found modeling and forecasting exchange rates extremely difficult. More than twenty years ago, Meese and Rogoff (1983) empirically analyzed several important macro-structural models based on monetary and asset theories of exchange rate determination. They found that none of these models could outperform the naïve random walk model in terms of out-of-sample forecast accuracy at the short horizons. Somanath (1986) and Boothe and Glassman (1987) also confirmed this finding for a number of key exchange rates. Consistent with the result from exchange rate model surveys (such as Mussa, 1990, Frankel and Rose, 1995), these discoveries lead to the consensus that the traditional macro-fundamental models are unsatisfactory, especially in the short run.

With the availability of high-frequency financial data, such as hourly, minutely or even real-time data, a growing body of literature focuses on the market microstructure to address the shortcoming of the macro approach (see Evans and Lyons 2003, 2004). This type of micro-structural modeling encapsulates issues relating to information

asymmetries and heterogeneity of market participants. Currently, it is still in the first stage of its development as a promising theoretical model for exchange rates.

Another strand of the literature has developed with the help of advanced time series techniques and reported strong evidence showing the existence of non-linearities in exchange rate movements. Many researchers have pursued nonlinear modeling of exchange rates, but with little success. For example, Engel and Hamilton (1990) and Engel (1994) showed that the Markov Switching model in general does not generate superior forecasts to the Random Walk model (RW) with/without drift. By using locally weighted regression, a nearest-neighbor nonparametric technique, Diebold and Nason (1990) reported that their model was unable to provide a lower root mean square prediction error than the RW with weekly data for ten exchange rates. On the other hand, some studies did claim to have beaten the random walk model. But in the light of the subsequent literature, however, these forecasting results turn out to be fragile in the sense that it is generally hard to replicate the superior forecasting performance for alternative periods and/or alternative currencies.

Contrary to these pessimistic findings, there seems to have some success in the application of Artificial Neural Network (ANN) model. Weigend *et al* (1991) observed that forecasts generated from the neural network were better than chance according to the out-of-sample correlation coefficients. Kuan and Liu (1995) used backpropagation and recurrent ANNs to investigate the out-of-sample forecasting ability on five exchange rates. They found that for the Japanese yen and British pound, ANNs exhibited significant forecasting improvement (relative to the random walk model); but for the remaining three currencies, the Canadian dollar, the Deutsche mark, and the Swiss franc,

ANNs had inferior performance. In a more recent study Chen and Leung (2004) applied an Error Correction Neural Network model to predict exchange rates and good forecasting results were obtained with their model. While accepting ANN as one of powerful forecasting tools, it is important to point out that these successes, more or less, were stemming from the specific forecasting evaluation criteria: for example, Weigend's out-of-sample correlation coefficients, Kuan and Liu's processing on the accuracy tests. Chen and Lung compared the performance of their model with that of the single-stage neural network model rather than with the random walk model.

From above, one may ask a natural question about the reason why the exchange rate is so difficult to forecast: Is it because we have not gotten the right model yet? Or is it because the exchange rate is actually non-forecastable? To answer this question, I take a closer look at the predictability of exchange rate returns by trying to describe time series data on the exchange rate with a 'proper' statistical model at first.

As is well known, the ability to forecast the behavior of a given system hinges on two types of knowledge. One is the law underlying a given phenomenon. The other relies on the discovery of strong empirical regularities in observations of the system. In current case, the theoretical model for exchange rate dynamics is either unsatisfactory or premature. Consequently, we need to obtain the second type of knowledge by focusing on the time series model to describe exchange rates. In addition, this method is justified by other reasons. First, with some degree of market efficiency, one may expect that most information is included in recent returns. Thus, it is natural to take (functions of) past returns as explanatory variables for current returns and volatility. Second, as previous studies have shown, other explanatory variables, such as dividend yields, term structure

variables and macroeconomic variables, have been found mainly useful for predicting returns at longer horizons ranging from one year to several decades. Finally, early in 1982, Wallis had argued that structural model based forecasts may have larger mean square errors than time series forecasts.

I put due emphasis on appropriate specification of both first and second order conditional moments in the hope that final inferences concerning predictability are free from any possible consequences of misspecification of the underlying model. This is the major difference between my work and that of Brooks (1997) where he tested the forecastability over a bunch of existing time series models without the consideration of model relevance to the data. I then evaluate the forecasting performance of the model chosen in this study *vis-à-vis* that of the random walk model by comparing the mean error, the mean absolute error and the root mean square error, etc. Although there are evidences that these conventional criteria may "mask the superiority" of non-linear models (Clements and Smith 2001), judgments based on such forecasting accuracy indices are of the greatest economic value in practice. Results show that daily exchange rates exhibit non-linearity and long memory in volatility but linearity in the conditional means. They seem to be non-forecastable in the short run in the sense that even the best-fitting model cannot beat the random walk model in forecasting.

The remainder of the chapter is set out as follows. In Section 2.2, I describe daily exchange rates and analyze their empirical regularities in details. I set up and estimate the most general empirical model for exchange rates in Section 2.3. I also conduct various interested hypothesis tests to determine the most relevant model describing the sample behavior. In Section 2.4, I present the forecasting results using the chosen model and

evaluate its performance against that of the benchmark model. Finally, Section 2.5 concludes the chapter with some discussion on future research.

2.2 Data and Empirical Characteristics

The data used in the modeling and forecasting exercise are daily nominal exchange rates for the Japanese yen (JPY) and the British pound (GBP) relative to the US Dollar (USD) over the period from January 2, 1996 to December 30, 2005. I conduct estimations over the first eight years of the sample. And the period from December 31, 2003 to December 30, 2005 is reserved for the forecasting exercise. After omitting weekend and other holiday non-trading periods, as detailed in Andersen, Bollerslev, Diebold, and Labys (1999), I am left with a total of 2,515 complete days. All the data are obtained from the Board of Governors of the Federal Reserve System's release. They are the noon buying rates in New York City for cable transfers payable in foreign currencies with the unit as foreign currency per US dollar. By using noon rates, I avoid the problem of intraday periodicity, for instance, open hour and close hour effects of the market.

The selection of JPY and GBP is due to the following considerations. First, these two currencies are major rivals of the US Dollar. Certainly, Euro is a potentially good choice but it occurred after January 1999. For the sake of comparison, I do not consider Euro here. Second, I want to study the exchange rate behavior in the economies with different degree of stability. The sample period includes several important economic events. In 1997-1998, Southeast Asia experienced the financial crisis. Japanese economy was heavily influenced by this crisis and its own collapse of asset bubbles. To fight with deflation, Bank of Japan applied quantitative easing monetary policy in March 2001 and ended it in March 2006. Plus, the Chinese Yuan revaluation in July 2005 gives another

shock to the Japanese Yen. The appreciation of the Chinese Yuan helped lift the yen against most rivals. By contrast, there is no big policy change in British economy during the sample period. Since October 1992 Bank of England (BOE) has always applied inflation targeting policy. Only in January 2004, BOE decreased the inflation target rate from 2.5% to 2%.

To avoid problems arising from non-stationary observed in the exchange rate data, I compute the difference between natural logarithms of the original exchange rate series.[2] Let S_t denote the exchange rate at time t, the return is defined as $y_t = 100 * \log(S_t / S_{t-1})$. Both the raw series and the return series are graphed in Figure 2.1. From this figure, we can clearly see the raw series is non-stationary and the return series exhibits bouts of intense volatility followed by periods of tranquility.

In the following I identify various empirical characteristics of the foreign exchange rate by analyzing the data in hand carefully. This will then pre-determine the relevant model for the exchange rate.

<u>1) Non-normal, fat-tailed distribution</u>

The descriptive statistics of daily returns are presented in Table 2.1. It is evident that the mean return is quite small while the range of the return is relatively large for both series. The estimated skewness of JPY is big and negative and that of GBP is smaller and positive. Both *p*-values imply asymmetric distribution. Their kurtoses are significantly higher than that of a normal distribution where this value is three. Along with the Jarque-

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[2] The data were tested for the presence of unit root nonstationarity using the Augmented Dickey Fuller test (The results are not reported here to save the space). The level data were found in both cases to be strongly $I(1)$, but there was no evidence of nonstationarity in the return series.

Bera test, these statistics show that the return series are characterized by fat tailed distributions, as is usually the case in such financial data. The kernel plots (Figure 2.2) of the unconditional distribution confirm that the return is unimodal with higher peak and fatter tail than Gaussian distribution.

2) Serial unautocorrelation, non-linear dependence and volatility clustering

I also report in Table 2.1 the Ljung-Box Q test for the first five lags and the result leads me to accept serial unautocorrelation (this result is not sensitive to the choice of lag order), which is consistent with other studies (e.g. Hsieh 1989). For visual purpose, Figure 2.3.a shows this point from return series' correlograms. Goldfeld-Quandt test rejects homoskedasticity for both returns.

The plot of autocorrelation and partial autocorrelation of squared returns (Figure 2.3.b) visualizes somehow non-linear dependence. McLeod and Li's Test[3], reported in Table 2.1, detects non-linearity in the return series. Engle's Lagrange Multiplier (LM) test also finds significant ARCH effect in JPY and 95% significant ARCH in GBP. The ARCH effect is an econometric name of volatility clustering, which means that large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes. Moreover, Bollerslev, Engel and Nelson (1994) illustrated that volatility clustering or heteroskedasticity gives rise to thick tails or leptokurtosis.

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[3] The McLeod and Li test (McLeod and Li, 1983) can be used as a portmanteau test of non-linearity with the null hypothesis that the series is independently and identically distributed (i.i.d.).

3) Leverage effects

      I compute LM statistics to identify the leverage effect that is common in financial data. Financial data is frequently found possessing the leverage effect to the extent that the magnitude of the response of asset prices to shocks depends on whether the shock is negative or positive. Through LM test I find leverage effects in GBP returns at 10% significance level and stronger effects in JPY returns. This result contrasts with Cont's finding (2001) that Gain/loss asymmetry is not observed in exchange rates. The difference could be due to the existence of big skewness. Actually, Meddahi and Renault (2000) argued that the leverage effect and conditional skewness are essentially different manifestation of same phenomenon. I will reconsider this effect in the text later.

4) Long memory in volatility

      From both Figure 2.3.a and Figure 2.3.b, we can see autocorrelation functions of the JPY squared returns decay very slow compared with those of the JPY returns; and there is no discernable difference on that for the GBP series----both returns and squared returns. This invokes me to check the existence of long memory. The phenomenon of long memory has been known since the time that ancient Egyptian hydrologists studied the flows and inflows of the river Nile. The idea is very simple and states that the effects of an event (shock) persist over a long period of time. The so-called long memory property is usually defined in relation to the autocorrelations of the process by requiring that the dependence between distant observations be significantly different from zero. Technically, a long memory process is characterized by a fractional degree of integration

that is less than one but greater than zero. Here I apply KPSS [4] to rudimentarily test for long memory in both conditional mean and conditional volatility. Results are displayed in Table 2.1 and KPSS statistics accept the null of short memory in the returns at the 5% level of significance. (Critical value of KPSS = 0.463). The KPSS statistic in the squared returns, however, clearly rejects the null for JPY and lightly rejects it for GBP.

5) Days-of-the-Week effects

Since I analyze daily data in this study, it is natural to check the presence of the Days-of-the-Week effects. These effects may result from significant differences in the volume of information relevant to the trading on particular days, causing consistently different patterns in the mean and variance movements. I split the whole sample into five sub-samples by weekdays and compute the first four moments of each sample. Results are showed in Table 2.2. I conduct the F test (not reported in Table 2.2 explicitly) and find no significant difference among these moments across weekdays. These are consistent with the findings of Yamori and Kurihara (2004) that the Days-of-the-Week effect disappears for almost all currencies in the 1990s and later.

In summary, the preceding analysis indicates that the empirical distribution of returns in the foreign exchange market is non-normal with very thick tails. The leptokurtosis reflects the fact that the large returns occur more often than what is predicted by the normal distribution. The empirical distribution confirms the presence of a time-varying variance or volatility clustering. A more significant result is the

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[4] The KPSS test statistic was originally developed by Kwiatkowski *et al* (1992) to test an $I(0)$ null hypothesis against an $I(1)$ alternative. It was subsequently extended by Lee and Schmidt (1996) to test an $I(0)$ null against a stationary $I(d)$ process.

asymmetric distribution of the returns. Big negative skewness of JPY, for example, implies that positive shocks (i.e. shocks that lead to a JPY depreciation) are more likely than negative shocks. The response of the market may depend on the sign of the shock (i.e. the leverage effect). No Days-of-the-Week effect is observed in the mean return and volatility.

Furthermore, I also notice that the impact of some economic events I mentioned earlier on exchange rates is represented as more volatility in JPY than in GBP. This is not surprising because during the sample period, Japanese government and monetary authority intervened its economy more often in attempt to lead recovery from the economy recession following the overwhelming Southeast Asian financial crisis. The higher degree of leptokurtosis reflects uncertainty of government policies and other economic fundamentals. Indeed, a highly concentrated market is likely to exhibit more volatility. Thus, it seems to be promising to capture these economic effects by properly modeling the volatility.

2.3 Exchange Rates Modeling

2.3.1 Overview of the Models

This sub-section presents an overview of the models to be used, given the statistical properties of the exchange rate returns found above. Generally, time-varying heteroskedasticity is modeled by the linear GARCH (p, q) model of Bollerslev (1986) i.e.

$$y_t = b_0 + \varepsilon_t, \qquad \varepsilon_t \mid \Omega_{t-1} \sim D(0, h_t)$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}$$

(2.1)

where b_0 is a constant explained as the conditional mean of the series $\{y_t\}$; innovations conditioned on the information set Ω_{t-1} at time $(t-1)$ follow some type of distribution to be specified in practice; ω, α_i and β_j are constant and non-negative parameters in the conditional variance equation. This specification allows the conditional variance to be dependent on the past information, which would induce variability over time. More specifically, the conditional variance is explained by past shocks and past variances. The specification search tells that GARCH (1,1) model is good enough to capture the ARCH effect in the data. Hereafter, I will focus on the GARCH-type models where both p and q equal one.

I employ the Skewed Student's t as the innovation's conditional distribution with zero mean and variance h_t, i.e. $\varepsilon_t \mid \Omega_{t-1} \sim skewt(0,\ h_t,\ nu,\ \lambda)$ where nu is the degree of freedom and λ is the skewed parameter. Its density function is as follows:

$$f(x\mid nu,\lambda)=\begin{cases} bc\left[1+\dfrac{1}{nu\text{-}2}\left(\dfrac{bx+a}{1-\lambda}\right)^2\right]^{-(nu+1)/2}, & if\ x<-\dfrac{a}{b} \\[4mm] bc\left[1+\dfrac{1}{nu\text{-}2}\left(\dfrac{bx+a}{1+\lambda}\right)^2\right]^{-(nu+1)/2}, & if\ x\geq-\dfrac{a}{b} \end{cases} \tag{2.2}$$

where, $2 < nu < \infty$, $-1 < \lambda < 1$, and a, b, c are constants specified as

$$a\ =\ 4\lambda c(nu\text{-}2)/(nu\text{-}1)$$

$$b^2\ =\ 1+3\lambda^2\text{-}a^2$$

$$c\ =\ \Gamma((nu+1)/2)/(\sqrt{\pi(nu\text{-}2)}\Gamma(nu/2))$$

The GARCH-process with the skewed student density is a useful extension since it takes account of skewness, leptokurtosis and the influence of outliers if any.

To account for the possible leverage effect, I use GJR-GARCH model to capture different responses of the returns to negative or positive shocks. This model was introduced by Glosten, Jogannathan, and Rankle (1993) and the conditional variance specification is of the form:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (2.3)$$

where d_{t-1} is a dummy variable that is equal to 1 if $\varepsilon_{t-1} < 0$ and zero otherwise. It is this extra term that allows the leverage effect, as the impact of ε_{t-1} on h_t depends on whether the shock is negative or positive. In the event that a negative shock is realized, the impact on the volatility will be $(\alpha_1 + \alpha_2)$ and α_1 when the shock is positive.

In any stationary GARCH model, memory decays exponentially fast. For example, if $\{\varepsilon_t\}$ are GARCH (1,1), the $\{\varepsilon_t^2\}$ have autocorrelations $\rho_k = (\alpha_1 + \beta_1)^k$. Specifically, if $\alpha_1 = .1$, $\beta_1 = .7$ and $k = 20$, we would get $\rho_{20} = .012$. This seems an unrealistically fast decay. On the other hand, for any integrated GARCH, that is IGARCH, where $\alpha_1 + \beta_1 = 1$, $\rho_k = 1$ for all k, there is no decay at all. This seems unrealistically slow. What we need, then, is a richer class of models allowing intermediate degrees of volatility persistence. Baillie, Bollerslev and Mikkelsen (1996) introduced the fractionally integrated GARCH model (FIGARCH) to account for long memory in the conditional variance. Chung (1999) slightly modified the original model to solve the problems in both estimation and interpretation of the resulting estimates. According to his suggestion, a FIGARCH (1, d, 1) model can be written as:

$$h_t = (1 - \beta_1 L)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d(\varepsilon_t^2 - h_0) + \beta_1 h_{t-1} \qquad (2.4)$$

here L is the lag operator and h_0 is a constant with unclear definition (see Chung 1999). d is the fractional degree of integration of ε_t^2. β_1 and ψ_1 are constant parameters and subjected to a set of conditions given in Chung (1999) for the conditional variance to be strictly positive. Applying this model to our case, we can get FI-GJR-GARCH model with the form of:

$$h_t = (1 - \beta_1 L)(1 + \delta d_t)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d [(1 + \delta d_t)\varepsilon_t^2 - h_0] + \beta_1 h_{t-1} \qquad (2.5)$$

The detailed deduction is shown in Appendix A.

It is noteworthy that the modified FIGARCH model will not necessarily collapse into the GARCH model when the fraction power d is equal to zero. Chung (1999) gave one reason to it in his paper. When d equals zero, the resulted coefficient of squared error terms, i.e. $(\psi_1 - \beta_1)$, is not equivalent to that in the GARCH model.

So far, I have discussed model specifications on the conditional variance. Now I turn to the model for the conditional mean. Based upon the above analysis of data properties, we need to identify whether the linear or non-linear model will be used to describe the conditional mean. Here I employ the Artificial Neural Network (ANN) model to detect any possible non-linearity in the conditional mean. ANN is used in a large variety of modeling and forecasting problems. The major reason for its increasing popularity is that this model has been shown to be able to approximate almost any nonlinear function arbitrarily close (Hornik, Stinchcombe and White 1989). But one exception exists. Franses and van Dijk (2000) proved that ANN cannot capture GARCH-type non-linearity. So I will combine ANN with GARCH-class model to consider non-linearity.

For any neural network, we have the input layer, the black-box-like hidden layer and the output layer (see Kuan and Liu 1995, among any others). To build up an ANN model, we need to identify inputs, the number of hidden layers, the activation function that conditions neurons' links in each hidden layer and outputs as well. The most commonly used activation function is the logistic function taking the form of $f(x) = \dfrac{1}{1+e^{-x}}$. In financial practice, it is sufficient to consider one input, which is the last period exchange rate return in current case, and one hidden layer. Here the output is current exchange rate return.

With all these considerations, I set up the most general model, called ANN (1,1)-FIGJRGARCH (1,d, 1)-SKEWT, as follows:

$$y_t = \mu_1 + \phi_1 y_{t-1} + \gamma * \frac{1}{1+\exp(-\mu_2 - \phi_2 y_{t-1})} + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda) \tag{2.6}$$

$$h_t = (1-\beta_1 L)(1+\delta d_t)\varepsilon_t^2 - (1-\psi_1 L)(1-L)^d [(1+\delta d_t)\varepsilon_t^2 - h_0] + \beta_1 h_{t-1}$$

where: $d_t = \begin{cases} 1, & if\ \varepsilon_t < 0 \\ 0, & otherwise \end{cases}$.

Henceforth, I will refer to this model as Model 1.

2.3.2 Estimation Results

The most straightforward estimation method for the complicated Model 1 is the Approximate Maximum Likelihood Estimation (AMLE), which is also called the Conditional Sum of Squares (CSS) estimation. The specific estimation issues around this method are addressed in Appendix A. According to the results presented in Table 2.3, the estimates of the fractional degree of integration are in line with the findings of Tse

(1998), Teyssière (1998) or Beine, Laurent and Lecourt (2002): d equals 0.22, and 0.37 respectively for the JPY and the GBP. The t tests show that d, nu, and λ are all statistically significant at 5% level. These illustrate the relevance of the Skewed Student's t distribution and the presence of long memory for both JPY and GBP returns.

2.3.3 Hypothesis Tests

In this sub-section, I attempt to pin down the best-fitting model for exchange rate series through the following sets of hypothesis tests.

<u>1) Test for Linearity in the conditional mean</u>

I test Model 1 against the null hypothesis $\gamma = 0$ for this purpose. Under the null, Model 1 reduces to AR (1)-FIGJRARCH (1, d, 1)-SKEWT model, called Model 2, which can be written as

$$y_t = \mu_1 + \phi_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda) \qquad (2.7)$$
$$h_t = (1 - \beta_1 L)(1 + \delta d_t)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d [(1 + \delta d_t)\varepsilon_t^2 - h_0] + \beta_1 h_{t-1}$$

where: $d_t = \begin{cases} 1, & if\ \varepsilon_t < 0 \\ 0, & otherwise \end{cases}$.

The Maximum Likelihood (ML) estimates of Model 2 are reported in Table 2.4. Most estimates, even the log-likelihood value, are generally similar to those of Model 1. The Likelihood Ratio (LR) test has an approximate χ^2 distribution under the null. The test result is presented in the second row of both Table 2.10.a and 2.10.b, respectively for JPY/USD and GBP/USD. For both returns I accept the null hypothesis that there is the linearity in the condition mean. That means our GARCH-type model may have already

captured all underlying non-linearity in the exchange rate series. Thus, I accept Model 2 as the relevant model for the moment.

2) Test for Leverage effect

I perform this test based on the value of δ, the coefficient of dummy variable d_t. The null model is obtained by setting $\delta = 0$ in Model 2. This reduces to Model 3 which is AR (1)-FIGARCH (1, d, 1)-SKEWT:

$$y_t = \mu_1 + \phi_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda) \tag{2.8}$$
$$h_t = (1 - \beta_1 L)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d (\varepsilon_t^2 - h_0) + \beta_1 h_{t-1}$$

Table 2.5 displays the ML estimates of Model 3. And the third row of Table 2.10.a presents the LR test and so does Table 2.10.b. These results lead to accept the null again, i.e. no leverage effect. As I talked in Section 2.2, the asymmetric response to negative and positive shocks may have been captured by the Skewed Student's t distribution. Now I accept Model 3 as a more relevant model than Model 2.

3) Test for Long memory in the conditional variance

In the absence of long memory, the null model is the AR (1)-GARCH (1,1)-SKEWT, or Model 4, which can be written as

$$y_t = \mu_1 + \phi_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda) \tag{2.9}$$
$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

The ML estimates of this model are shown in Table 2.6. We can see from there the estimated sum of slope coefficients in the conditional variance equation, i.e. $\alpha_1 + \beta_1$, is very close to 1 for both returns, indicating that the volatility process is highly

persistent. In particular, the estimate of β_1 in the GARCH model is very high and falls considerably when changing from Model 2 to Model 3, which are consistent with the findings of Baillie, Bollerlev and Mikkelsen (1996). They claimed that, in the presence of long memory, there is an upward bias in the GARCH estimates due to the fact that the GARCH model does not take into account the long memory component of the volatility process. Furthermore, since FIGARCH and GARCH model do not belong to a family of nested models as mentioned previously, we cannot use the conventional LR test to discriminate them. Here I employ Wright (1998)'s nonparametric rank test (c.f. Beine and Laurent, 2003). This test can be used as a misspecification test suitable for GARCH and FIGARCH models and is more powerful when residuals are highly non-normal, which is particularly relevant in current application. Specifically, for a fixed k, the test statistic $S(k)$ is given by:

$$S(k) = T \sum_{i=1}^{k} \rho(S_{1t}, S_{1t-i})^2 \tag{2.10}$$

where $\rho(.,.)$ denotes the sample autocorrelation function and S_{1t} is given by:

$$S_{1t} = \left[r(z_t^2) - \frac{T+1}{2} \right] \bigg/ \sqrt{\frac{(T-1)(T+1)}{12}} \tag{2.11}$$

with $r(z_t)$ being the rank of z_t among $z_1, z_2, ..., z_T$, which are the standardized residuals of the estimated model. These statistics follow a $\chi^2(k)$ distribution under the null hypothesis of correct specification in the conditional variance. The test results for both returns are shown in the fourth row of both Table 2.10.a and 2.10.b respectively. These results validate the FIGARCH model specification to some extent. Hence I still accept Model 3.

4) Test for GARCH-like effect

If there is no GARCH-like effect in exchange rate dynamics, then the null hypothesis will be $\psi_1 = \beta_1 = d = 0$ in Model 3. Thus, Model 3 becomes the simple AR(1)-SKEWT:

$$y_t = \mu_1 + \phi_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda)$$

(2.12)

I call it Model 5 and its ML estimates are presented in Table 2.7. The maximized log-likelihood value shows a big drop. The LR tests, reported in the fifth row of both Table 2.10.a and 2.10.b, easily reject the null. This leaves me Model 3 again.

5) Test for Normality

We know the Skewed Student's t distribution becomes normal distribution when $\lambda = 0$ and $nu = \infty$. This yields Model 6, AR(1)-FIGARCH(1,d,1)-NORMAL, which can be written as

$$y_t = \mu_1 + \phi_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim N(0,\ h_t)$$
$$h_t = (1 - \beta_1 L)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d (\varepsilon_t^2 - h_0) + \beta_1 h_{t-1}$$

(2.13)

The ML estimates and the LR tests are reported in Table 2.8 and the sixth row of Table 2.10.a and 2.10.b respectively. Likewise it is easy to reject the null hypothesis of normality, thereby maintaining the relevance of Model 3.

6) Test for Autoregression in the conditional mean

The last but not the least interesting hypothesis test is to check whether there are autoregressive terms in the conditional mean at all. In this case, the null model is

obtained by setting $\phi_1 = 0$ in Model 3. This reduced to Model 7, FIGARCH(1,d,1)-SKEWT, with the formula of

$$y_t = \mu_1 + \varepsilon_t$$
$$\varepsilon_t \mid \Omega_{t-1} \sim skewt\ (0,\ h_t,\ nu,\ \lambda) \tag{2.14}$$
$$h_t = (1 - \beta_1 L)\varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d(\varepsilon_t^2 - h_0) + \beta_1 h_{t-1}$$

Model 7's ML estimates are listed in Table 2.9. The maximized log-likelihood value has no significant change. Actually the LR test results in the seventh row of Table 2.10.a and 2.10.b lead me to accept the null hypothesis, thereby embracing Model 7.

2.3.4 Summary and Inference

From the above estimation and test exercises I select the FIGARCH-SKEWT model (Model 7) as the final model. As a whole, this model matches the dynamics of daily exchange rate returns better than other models and explains the data's empirical regularities such as non-normality, long memory in volatility, and volatility clustering, etc. Furthermore, the fact that the model is fit for both JPY and GBP shows the volatile macro policy shocks to JPY may be identified as the outliers in the overall exchange rate dynamics and can be captured by appropriately modeling its volatility.

Results also provide some useful evidence for the Martingale Hypothesis of exchange rates, although I do not strictly test it here. A martingale means that the realizations of a stochastic process are uncorrelated but not necessarily independent. Specifically, this process has a constant Conditionally Expected Return (CER). In current case, that is $E(y_{t+1} \mid \Omega_t) = \mu$, where μ is a constant. A martingale is not a random walk process although the latter has a constant CER, too. In addition, the random walk model

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assumes that there is no autocorrelation among variances, i.e. that y_t is statistically independent of past observations $y_{t-1}, y_{t-2}, \ldots$. Clearly, it is not the case for exchange rates that are of one sort or another non-linear dependence.

2.4 Exchange Rate Forecasting

Now I turn to evaluate the forecasting performance of the empirical model chosen in the previous section. Following the convention in the literature, I use the Random Walk with Drift model (RWD) as my benchmark model. The out-of-sample forecasts of exchange rates are constructed on the basis of estimated models. In particular, I calculated forecasts over the period from December 31, 2003 to December 30, 2005. The forecast horizons, n, are chosen to be one-day, one-week, two-week and one-month ahead, or 1, 5, 10, 20-step-ahead. Let the actual value of the series at time t and an n-step-ahead forecast of that value made at time t be written as y_{t+n} and $f_{t,n}$ respectively. I define $f_{t,n} = E(y_{t+n} | \Omega_t)$ which means that the n-step-ahead forecast of the series made at time t is the expected value of the series n periods in the future given all information available at time t.

I employ traditional measures of forecasting accuracy such as the Mean Error (ME), the Mean absolute Error (MAE) and the Root Mean Square Error (RMSE). These forecast error statistics are defined as:

$$ME = \frac{1}{T}\sum_{t=1}^{T}(y_{t+n} - f_{t,n})$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|y_{t+n} - f_{t,n}|$$

(2.15)

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_{t+n} - f_{t,n})^2}$$

In order to test whether the forecast from two competing models are equally accurate, I apply the Diebold and Mariano (1995) (DM) test. This statistic is designed as follows: let us assume that a pair of models produces the n-step-ahead forecast errors $\left\{\hat{\varepsilon}^{(1)}_{t+n|t}, \hat{\varepsilon}^{(2)}_{t+n|t}\right\}$ and that the quality of the forecasts is measured by a specified loss function $g(\hat{\varepsilon}_{t+n|t})$ of the forecast errors. We can define the loss differential between the two competing forecasts as $l_t = g(\hat{\varepsilon}^{(1)}_{t+n|t}) - g(\hat{\varepsilon}^{(2)}_{t+n|t})$. The test is then based on the following large sample statistic:

$$DM = \frac{\bar{l}}{T^{-1}\cdot 2\pi\cdot\hat{h}_{\bar{l}}(0)} \sim N(0,1)$$

(2.16)

where $\bar{l}$ is the sample average of l_t, and $2\pi\cdot\hat{h}_{\bar{l}}(0)$ is the spectral density at frequency zero which is estimated in the usual way as two-sided weighted sum of available autocorrelations (see Newey and West, 1987). I use Andrews (1991) approximation rule to set the truncation lags and define the loss functions as $l_t = (\hat{\varepsilon}^{(1)}_{t+n|t})^2 - (\hat{\varepsilon}^{(2)}_{t+n|t})^2$ for the MSE test.

Table 2.11.a and 2.11.b report the results on the forecast performance of two models: FIGARCH-SKEWT and RWD for JPY and GBP. According to the above three forecasting error statistics, the FIGARCH-SKEWT model does not have significant

improvement over the RWD model in the short-term forecasting. And the DM test confirms this point. This result shows that although the FIGARCH-SKEWT model can describe exchange rate dynamics very well in terms of capturing all relevant empirical regularities in the data, this advantage does not provide the help in the forecasting of exchange rate returns in the context.

2.5 Conclusions

In this study, I first analyze daily exchange rates of the Japanese Yen and British Pound against the US Dollar over the period from Year 1996 to Year 2003. Based on the empirical characteristics found in the data, I propose the ANN-FIGJRGARCH model with the Skewed Student's t distribution as the general model that is applicable to describe the behavior of exchange rates in the sample. Through parameter estimations and hypothesis tests in Section 2.3, I find that the fractionally integrated GARCH model with the Skewed Student's t distribution captures all important empirical regularities: non-normality, non-linearity, long memory in volatility and volatility clustering. The acceptance of this model could be interpreted as useful evidence for the Martingale hypothesis of daily exchange rates, thereby the foreign exchange market efficiency, although it is not a strict test for this topic. Finally I apply this best-fitting model to the out-of-sample forecasting of the exchange rate and compare its performance with that of the naïve random walk model in Section 2.4. I get the "negative" result----the FIGARCH-SKEWT model cannot beat the random walk model in terms of forecasting fit according to the traditional evaluation criteria. These modeling and forecasting results support the contention that exchange rates may not have the short-run forecastability while they do not follow exact random walks.

As mentioned in the introduction of this chapter, the microstructure model of the exchange rates has received more and more attention from both academic and practical researchers. The out-of-sample forecasting from this type of model, or some combined forecasting, is worthy of further study.

Table 2.1: Summary Statistics and Preliminary Tests

	Japanese Yen	British Pound
Mean	0.002	-0.007
Standard Deviation	0.729	0.476
Maximum	3.240	2.528
Minimum	-5.630	-2.005
Skewness ($p-$ value for skewness $= 0$)	-0.664 (0.00)	0.094 (0.043)
Kurtosis ($p-$ value for Kurtosis $= 3$)	7.73 (0.00)	4.368 (0.00)
Jarque-Bera Test ($p-$ value for normality)	2024.613 (0.00)	157.602 (0.00)
Ljung-Box Q(5) ($p-$ value for unautocorrelation)	6.985 (0.222)	8.046 (0.154)
Goldfeld-Quandt Test ($p-$ value for homoskedasticity)	1.883 (0.00)	1.170 (0.006)
LM Test for ARCH ($p-$ value for no ARCH effect)	168.556 (0.00)	4.058 (0.04)
LM Test for Leverage ($p-$ value for no Leverage effect)	125.911 (0.00)	6.389 (0.094)
KPSS Test in Returns (Critical Value for short memory)	0.161 (0.463)	0.209 (0.463)
KPSS Test in Squared Returns (Critical Value for short memory)	1.634 (0.463)	0.474 (0.463)
McLeod and Li's Test ($p-$ value for linearity)	148.633 (0.00)	4.036 (0.044)

Table 2.2: Days of the Week Summary

JPY returns

	ALL DAYS	MON	TUE	WED	THU	FRI
Mean	0.002	-0.048	-0.018	-0.052	0.061	0.060
Standard Deviation	0.729	0.701	0.759	0.720	0.707	0.748
Skewness	-0.664	-0.301	-1.605	-0.782	-0.533	0.025
Kurtosis	7.734	6.686	13.543	7.062	4.901	4.788

GBP returns

	ALL DAYS	MON	TUE	WED	THU	FRI
Mean	-0.007	0.002	-0.011	-0.053	0.007	0.023
Standard Deviation	0.476	0.474	0.457	0.479	0.499	0.470
Skewness	0.049	0.5849	0.030	-0.188	-0.249	0.171
Kurtosis	4.368	5.637	3.775	3.815	4.123	4.407

Table 2.3: Parameter Estimates for ANN-FIGJRGARCH-SKEWT model

Parameters	JPY	GBP
μ_1	-0.009 (2.130)	-0.035 (6.563)
ϕ_1	-0.016 (0.045)	0.022 (0.148)
γ	0.051 (4.210)	0.048 (13.278)
μ_2	0.010 (1.689)	-0.011 (4.970)
ϕ_2	0.010 (3.436)	0.010 (9.293)
h_0	0.325 (0.102)	0.127 (0.042)
δ	0.180 (0.242)	0.211 (0.146)
ψ_1	0.271 (0.137)	0.441 (0.064)
β_1	0.459 (0.175)	0.754 (0.049)
d	0.220 (0.080)	0.378 (0.090)
nu	5.784 (0.731)	6.480 (0.917)
λ	-0.057 (0.032)	-0.016 (0.031)
Log Likelihood	-2011.632	-1280.040

Note: All estimates are rounded off to the third decimal place. Hessian-based standard errors for parameter estimates are listed in parentheses.

Table 2.4: Parameter Estimates for AR-FIGJRGARCH-SKEWT model

Parameters	JPY	GBP
μ_1	0.0167 (0.015)	-0.011 (0.010)
ϕ_1	-0.016 (0.022)	0.022 (0.023)
h_0	0.326 (0.107)	0.126 (0.042)
δ	0.178 (0.262)	0.213 (0.147)
ψ_1	0.273 (0.125)	0.442 (0.064)
β_1	0.462 (0.163)	0.753 (0.049)
d	0.222 (0.081)	0.377 (0.091)
nu	5.783 (0.741)	6.479 (0.908)
λ	-0.057 (0.032)	-0.016 (0.031)
Log Likelihood	-2011.633	-1280.040

Note: See the note under Table 2.3.

Table 2.5: Parameter Estimates for AR-FIGARCH-SKEWT model

Parameters	JPY	GBP
μ_1	0.017 (0.014)	-0.010 (0.010)
ϕ_1	-0.016 (0.021)	0.022 (0.023)
h_0	0.404 (0.079)	0.174 (0.046)
ψ_1	0.283 (0.101)	0.417 (0.058)
β_1	0.515 (0.116)	0.765 (0.045)
d	0.264 (0.054)	0.417 (0.078)
nu	5.976 (0.718)	7.045 (0.912)
λ	-0.059 (0.031)	-0.016 (0.030)
Log Likelihood	-2012.103	-1281.513

Note: See the note under Table 2.3.

Table 2.6: Parameter Estimates for AR-GARCH-SKEWT model

Parameters	JPY	GBP
μ_1	0.015 (0.014)	-0.012 (0.010)
ϕ_1	-0.016 (0.021)	0.025 (0.022)
ω	0.005 (0.002)	0.002 (0.001)
α_1	0.031 (0.008)	0.038 (0.009)
β_1	0.959 (0.010)	0.955 (0.012)
nu	5.83 (0.754)	6.251 (0.884)
λ	-0.062 (0.031)	-0.019 (0.031)
Log Likelihood	-2011.929	-1284.158

Note: See the note under Table 2.3.

Table 2.7: Parameter Estimates for AR-SKEWT model

Parameters	JPY	GBP
μ_1	0.010 (0.016)	-0.007 (0.011)
ϕ_1	-0.020 (0.021)	0.022 (0.021)
σ	0.734 (0.024)	0.481 (0.012)
nu	4.247 (0.416)	5.911 (0.824)
λ	-0.053 (0.030)	-0.008 (0.30)
Log Likelihood	-2084.917	-1318.321

Note: See the note under Table 2.3.

Table 2.8: Parameter Estimates for AR-FIGARCH-NORMAL model

Parameters	JPY	GBP
μ_1	0.015 (0.015)	-0.010 (0.010)
ϕ_1	0.014 (0.024)	0.038 (0.024)
h_0	0.442 (0.063)	0.221 (0.017)
α_1	0.282 (0.125)	0.052 (0.026)
β_1	0.465 (0.130)	0.140 (0.00)
d	0.254 (0.043)	0.140 (0.03)
Log Likelihood	-2075.187	-1337.469

Note: See the note under Table 2.3.

Table 2.9: Parameter Estimates for FIGARCH-SKEWT model

Parameters	JPY	GBP
μ_1	0.016 (0.014)	-0.011 (0.010)
h_0	0.405 (0.080)	0.174 (0.046)
ψ_1	0.285 (0.101)	0.418 (0.058)
β_1	0.517 (0.116)	0.768 (0.044)
d	0.266 (0.054)	0.420 (0.079)
nu	6.009 (0.726)	7.010 (0.901)
λ	-0.059 (0.031)	-0.015 (0.030)
Log Likelihood	-2012.381	-1281.973

Note: See the note under Table 2.3.

Table 2.10.a: Hypotheses Test (JPY/USD)

	Model 1	Model 2	Model 3	Model 4
LR ($\gamma = 0$) against Model 2	0.001 (0.97)	--	--	--
LR ($\delta = 0$) against Model 3	--	0.94 (0.33)	--	--
Wright's test (Correct specification)	--	--	8.468 (0.132)	13.210 (0.021)
LR (no GARCH) against Model 5	--	--	145.628 (0.00)	--
LR (normal) against Model 6	--	--	126.168 (0.00)	--
LR (no AR) against Model 7	--	--	0.558 (0.46)	--

Notes: 1. The table presents Likelihood Ratio (LR) statistics and their associated $p-$values in parentheses for all tests between nested models. LR ($\gamma = 0$) is a test for linear conditional means. LR ($\delta = 0$) tests for the absence of asymmetric effects remained. LR (no GARCH) is a test for the homoskedasticity. And LR (normal) tests for normal distributions. Last, LR (no AR) tests whether there exist autoregressive terms in the conditional mean or not.

2. Wright's test for a correct specification in the conditional variance is conducted by setting $k = 5$ for Model 3 and Model 4, respectively.

3. Model 1 is the ANN-FIGJRGARCH-SKEWT model; Model 2 is the AR-FIGJRGARCH-SKEWT model; Model 3 is the AR-FIGARCH-SKEWT model; Model 4 is the AR-GARCH-SKEWT model; Model 5 is the AR-SKEWT model; Model 6 is the AR-FIGARCH-NORMAL model; and Model 7 is the FIGARCH-SKEWT model. The symbol "--" means "not-applicable".

Table 2.10.b: Hypotheses Test (GBP/USD)

	Model 1	Model 2	Model 3	Model 4
LR ($\gamma = 0$) against Model 2	0.00 (0.99)	--	--	--
LR ($\delta = 0$) against Model 3	--	2.945 (0.09)	--	--
Wright's test (Correct specification)	--	--	7.372 (0.194)	10.481 (0.063) *
LR (no GARCH) against Model 5	--	--	73.616 (0.00)	--
LR (normal) against Model 6	--	--	111.912 (0.00)	--
LR (no AR) against Model 7	--	--	0.919 (0.34)	--

Note: See notes under Table 2.10.a. The symbol "*" indicates a rejection at 10% level.

Table 2.11.a: Forecast Performance (JPY/USD)

		FIGARCH-SKEWT	RWD
ME	1	-0.319	-1.783
	5	-0.482	-1.945
	10	-0.516	-1.980
	20	-0.611	-2.075
MAE	1	45.264	45.311
	5	45.436	45.489
	10	45.723	45.779
	20	45.923	45.986
RMSE	1	60.478	60.504
	5	60.609	60.639
	10	60.881	60.911
	20	61.188	61.220
DM	1	0.39 (0.70)	
	5	0.46 (0.64)	
	10	0.45 (0.65)	
	20	0.41 (0.68)	

Notes: 1. The table presents the 1-, 5-, 10-, and 20-step ahead forecasts from both FIGARCH-SKEWT and RWD model.

2. DM tests for equal forecasting accuracy between these two models and their associated $p-$values are reported in parentheses.

Table 2.11.b: Forecast Performance (GBP/USD)

		FIGARCH-SKEWT	RWD
ME	1	-1.488	-1.885
	5	-1.978	-2.375
	10	-1.843	-2.239
	20	-1.878	-2.275
MAE	1	45.955	45.956
	5	45.727	45.729
	10	45.660	45.661
	20	44.975	44.979
RMSE	1	58.801	58.813
	5	58.609	58.624
	10	58.557	58.571
	20	57.741	57.756
DM	1	0.64 (0.52)	
	5	0.91 (0.36)	
	10	0.71 (0.48)	
	20	1.36 (0.17)	

Note: See the note under Table 2.11.a.

Figure 2.1: Sequence Plot

Japanese Yen per Dollar

Japanese Yen Log-return

British Pound per Dollar

British Pound Log-return
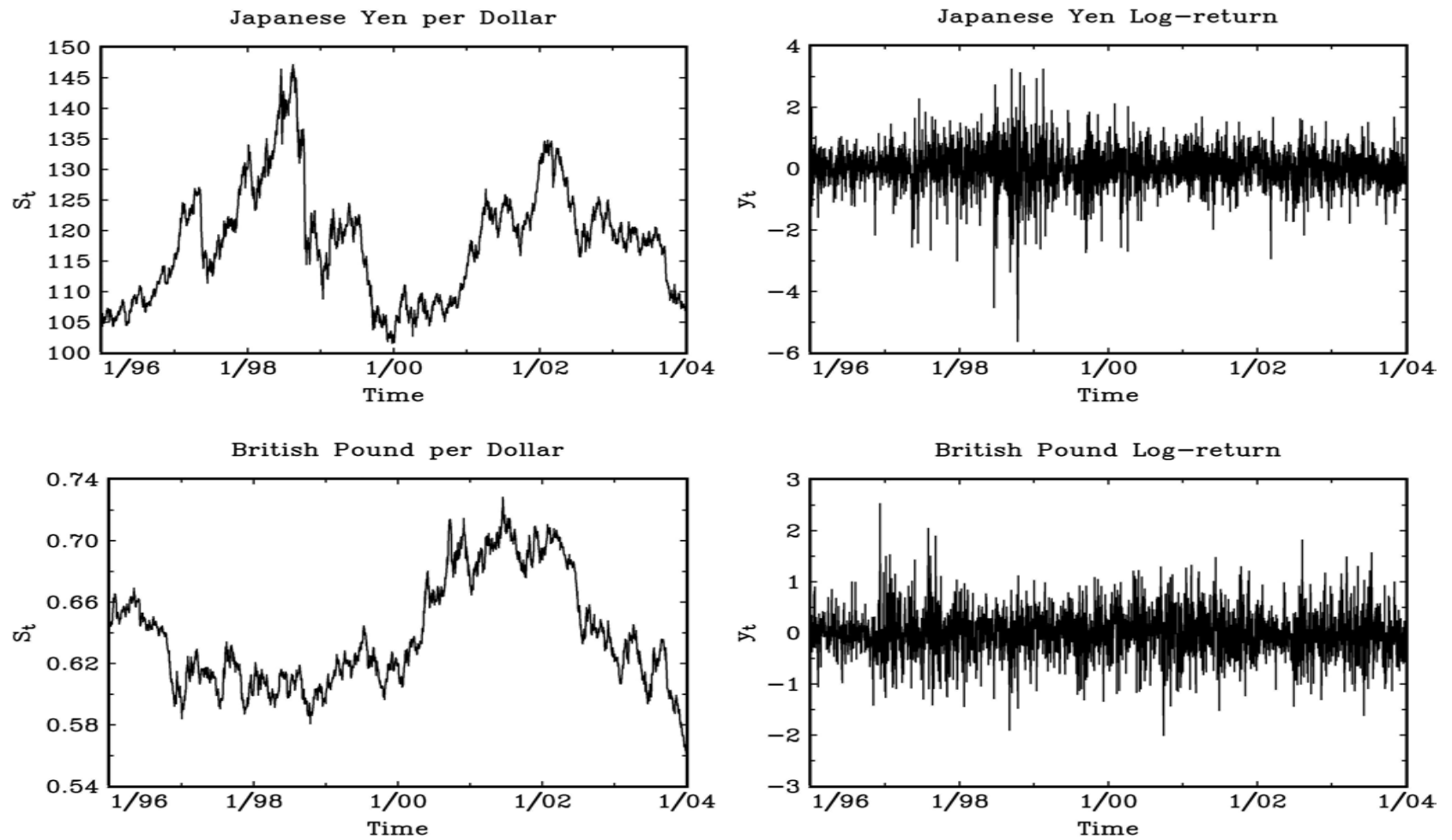
Figure 2.2: Kernel Density Plot

Japanese Yen
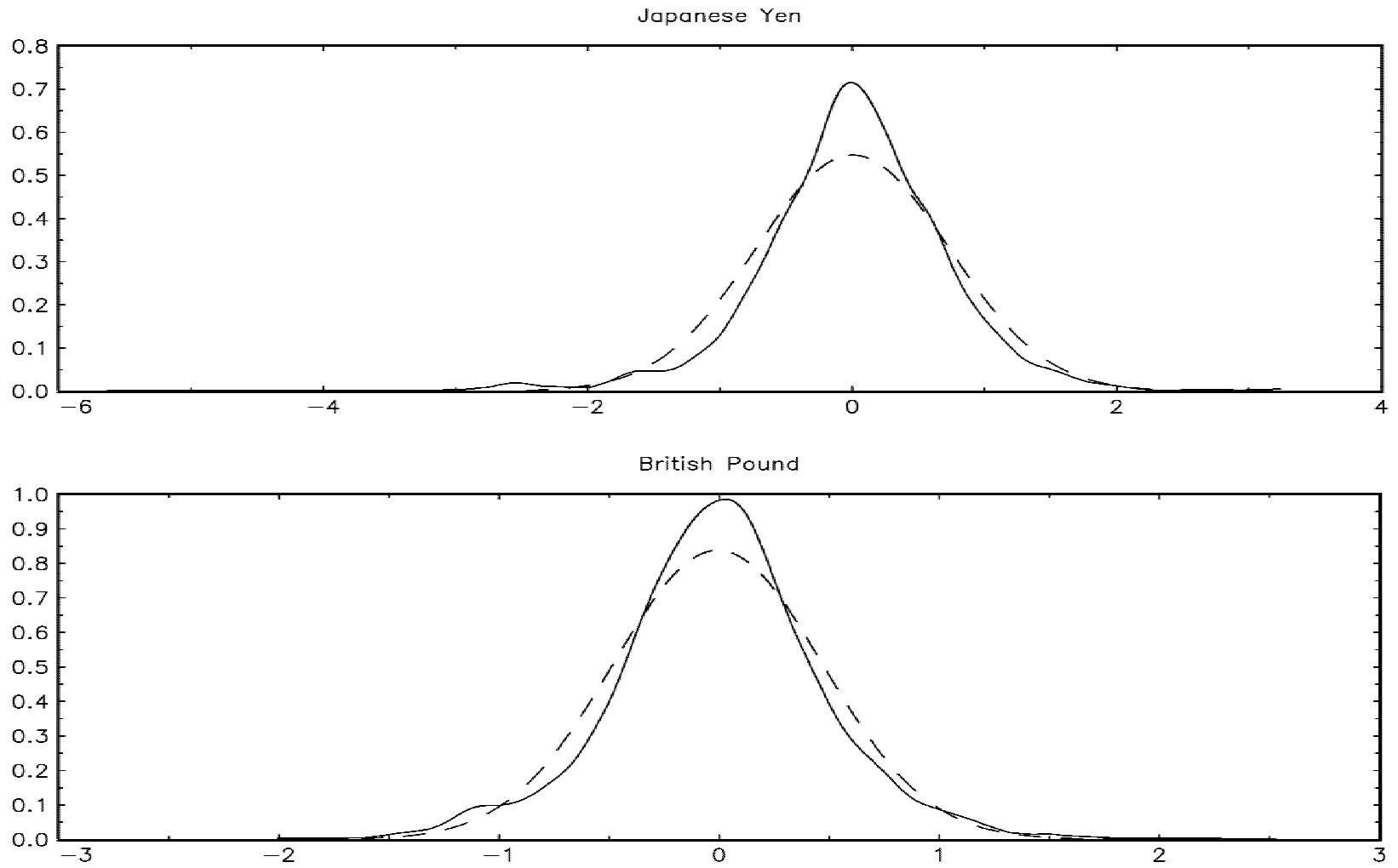
British Pound

Figure 2.3.a: Autocorrelation and Partial Autocorrelation of Returns

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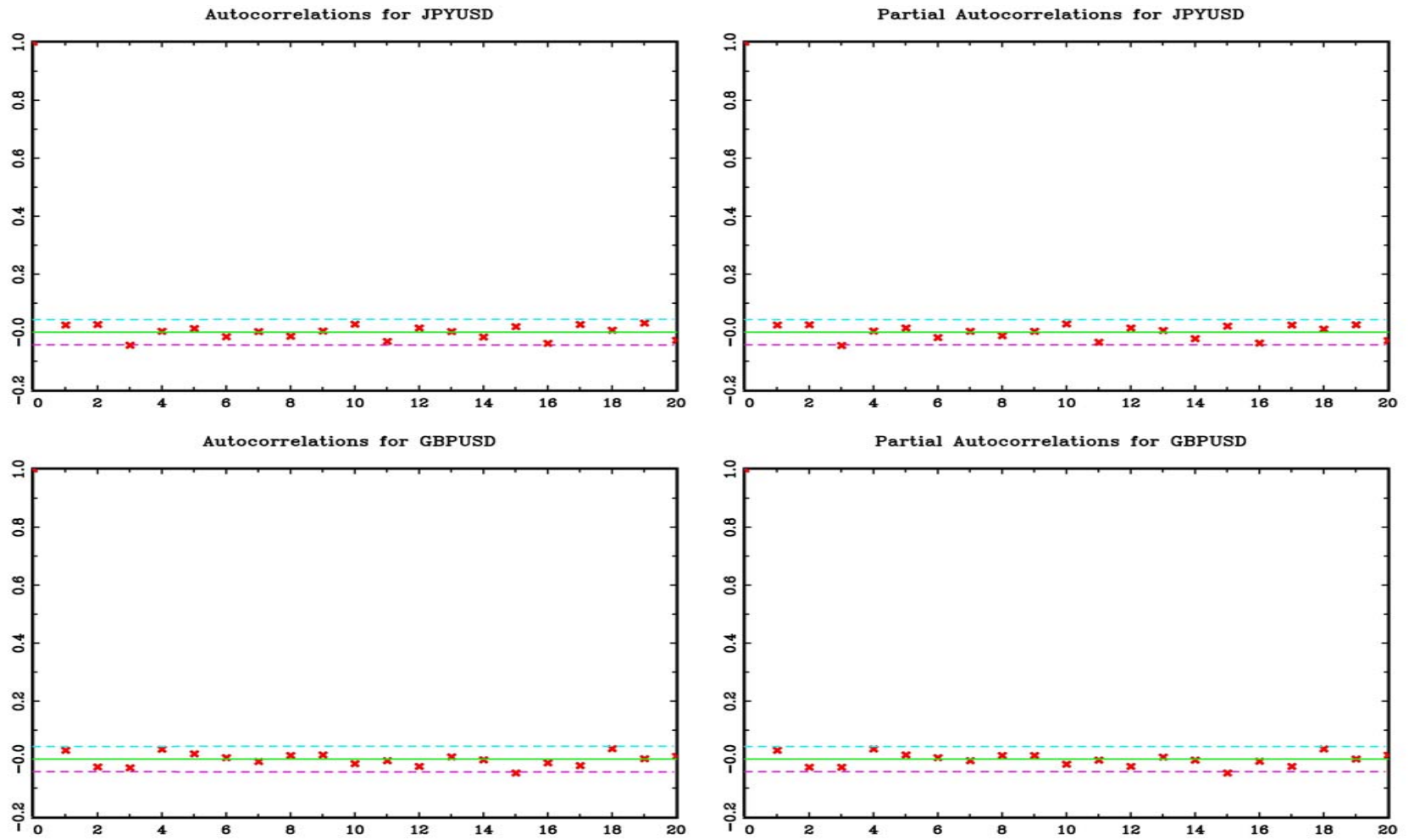
Figure 2.3.b: Autocorrelation and Partial Autocorrelation of Squared Returns

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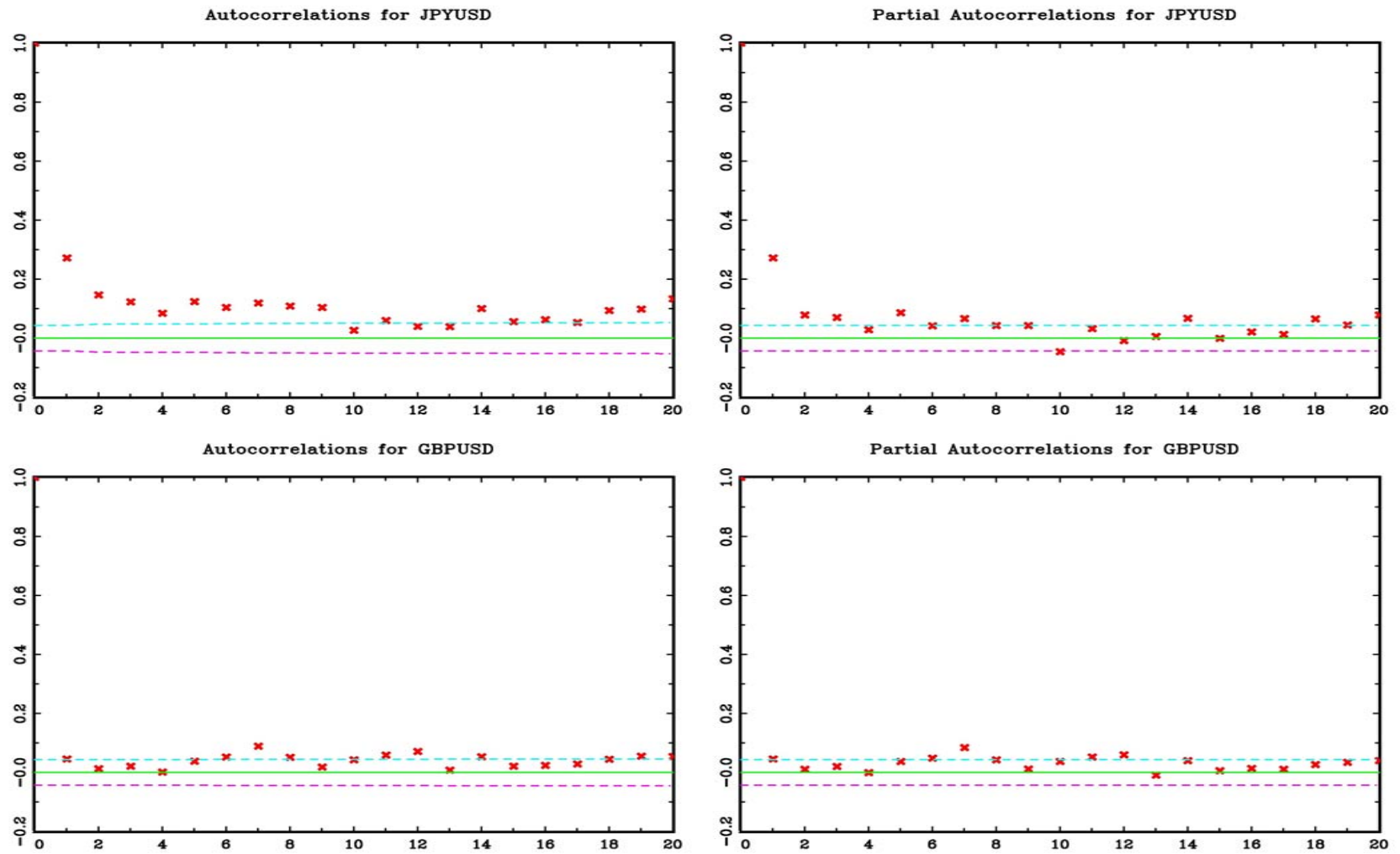
CHAPTER 3

FOREIGN EXCHANGE RISK PREMIUM IN A NEW OPEN ECONOMY MACROECONOMIC MODEL

3.1 Introduction

In International Finance, an important theoretical building block is the Uncovered Interest rate Parity (UIP) condition, which is a no-arbitrage profit condition for financial assets. A typical investor can either hold domestic risk-free nominal bonds, receiving interest rate i_t, or invest abroad, converting his currency by the exchange rate S_t, receiving the foreign interest rate i_t^*, and then converting back to domestic currency by the future exchange rate expected at time t, $E_t S_{t+1}$ [5]. No-arbitrage profit implies that returns from these two investment strategies must be equalized. When interest rates are low, the following log approximations are often used for the standard UIP condition:

$$E_t(s_{t+1} - s_t) = (i_t - i_t^*) \tag{3.1}$$

where lower-case s_t is the natural log of the nominal exchange rate S_t.

When rational expectations are assumed, a simple linear regression of exchange rate variations on interest rate differentials should yield a slope coefficient of unity and an intercept of zero. More formally, empirical UIP regressions take the form of

$$s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + \vartheta_{t+1} \tag{3.2}$$

where ϑ_{t+1} is assumed to be standard Gaussian. Thus, UIP implies $\beta_0 = 0$ and $\beta_1 = 1$. However, empirical work finds significantly negative slope coefficients from these

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[5] $E_t(\cdot)$ is the expectation operator conditional on the information available at time t.

regressions[6]. The international finance literature refers to negative UIP slope as the UIP puzzle or the forward premium anomaly.

Logically, the puzzle must reflect the failure of one or both legs of the joint hypotheses of rational expectations and risk neutrality. Rational expectations ensure that expectations of future variables, including the exchange rate, incorporate all information available at the time the expectations are formed. Thus, the difference between the *ex ante* expected future exchange rate and the ex post realized future spot rate is just a white noise error term. Risk neutrality implies that a typical investor is indifferent between the alternative investment strategies described above. In other words, he does not demand additional compensation for the investment on the foreign exchange market. Therefore, there exist two possible theoretical explanations for the puzzle: risk premia and/or expectation errors. Much of burgeoning literature focuses on these two explanations. I only review some of the more recent developments here.[7]

Bacchetta and van Wincoop (2005) who obtain a negative UIP slope coefficient attribute this to rational inattention in the sense that investors face information collecting and processing costs and therefore optimally choose not to frequently update information and revise their investment decisions. In a related paper, Chakraborty and Evans (2008) replace rational expectations by perpetual learning and find a negative relation that becomes stronger when the fundamentals are near random walk. Their simulations show that perpetual learning may explain the puzzle.

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[6] Excellent reviews of this literature can be found in Hodrick (1987), Baillie and McMahon (1989), Froot and Thaler (1990), and Engel (1996).

[7] See Sarno (2005) for a recent survey on this.

In this study, I follow the alternative path by maintaining the assumption of rational expectations and presenting a risk premium explanation for the UIP puzzle. The hypothesis of rational expectation is the building block of modern macroeconomics. Maintaining the assumption of rational expectations would strengthen and simplify our analysis. I also believe, like others in the literature, that the risk premium is the most natural and appealing explanation to the violation of UIP. Understanding the risk premium is important because it is a crucial determinant of the equilibrium level of exchange rates. Furthermore, it appears that understanding the risk associated with the behavior of exchange rates is fundamental to designing optimal policies.

With the incorporation of a risk premium, expected changes in exchange rates are equal to interest rate differentials up to a time-varying risk premium (rp_t):

$$E_t(s_{t+1} - s_t) = (i_t - i_t^*) + rp_t \tag{3.3}$$

With the maintained assumption of rational expectations, realized exchange rate variations equal expected changes plus a forecast error η_{t+1}, which is orthogonal to all the information available at time t, i.e. $s_{t+1} = E_t(s_{t+1}) + \eta_{t+1}$. Thus the UIP slope coefficient β_1 in Equation (3.2) would be then equal to:

$$\beta_1 = \frac{\text{cov}\left(s_{t+1} - s_t, \ i_t - i_t^*\right)}{\text{var}(i_t - i_t^*)} = \frac{\text{cov}\left(E_t(s_{t+1} - s_t) + \eta_{t+1}, \ E_t(s_{t+1} - s_t) - rp_t\right)}{\text{var}\left(E_t(s_{t+1} - s_t) - rp_t\right)}$$

$$= \frac{\text{var}\left(E_t(s_{t+1} - s_t)\right) - \text{cov}\left(E_t(s_{t+1} - s_t), \ rp_t\right)}{\text{var}\left(E_t(s_{t+1} - s_t)\right) + \text{var}(rp_t) - 2\,\text{cov}\left(E_t(s_{t+1} - s_t), \ rp_t\right)} \tag{3.4}$$

Two necessary conditions to obtain a negative β_1 can be derived from the above equation (3.4), like in Fama (1984):

$$\text{cov}\left(E_t(s_{t+1} - s_t), \, rp_t\right) > 0$$
$$\text{var}(rp_t) > \text{var}\left(E_t(s_{t+1} - s_t)\right)$$

(3.5)

Any rational expectations economic model that accounts for the UIP puzzle should generate these two volatility relations.

One strand of the literature is based on the dynamic, two-country, general equilibrium model of Lucas (1982) with flexible prices in an endowment economy. Verdelhan (2006) studies the UIP puzzle using the Campbell and Cochrane (1999) preferences with habit formation in a non-monetary economy with trading costs. Moore and Roche (2007) generate the negative slope in the UIP regression by extending the two-country monetary model to include a consumption externality with habit persistence. In addition, they claim that the model can simultaneously solve the Meese-Rogoff forecasting puzzle and the exchange rate disconnect puzzle. Habit formulation has the important implication that countercyclical risk premia arise endogenously as risk aversion increases in recessions. But in a production economy, habit formation in the utility function is not enough to generate a reasonable risk premium[8]. Ljungqvist and Uhlig (2000) have pointed out that there are problems in expanding the Campbell and Cochrane (1999) habits to a production economy.

My study contributes to this line of research by considering the effects of nominal price rigidities on the foreign exchange risk premium in a production economy. With price rigidities, the real economy becomes subject to nominal (e.g. monetary) shocks. I

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[8] See Rouwenhorst (1995) for the equity-premium study with production economies.

set up a model along the lines of the standard New Open Economy Macro literature, which has become the workhorse model in international macroeconomics, and study its implications for asset pricing. In particular, one purpose of this work is to examine whether general equilibrium sticky-price monetary models can generate volatile enough foreign exchange risk premia that satisfy the two volatility relations described above.

This study is specifically motivated by two papers. Engel (1999) and Obstfeld and Rogoff (2003) analytically demonstrate how foreign exchange risk premia can arise endogenously in sticky-price models with synchronized price setting. Engel (1999) notes that a cash-in-advance formulation for money demands holds "the greatest promise for generating large risk premiums". I extend their analyses to consider a production economy with a cash-in-advance constraint, monopolistic competition, and local currency pricing with a Calvo-type staggered price setting mechanism. Monetary policy follows an exogenous process with the growth rate of money supply subject to shocks with time-varying volatility. To render the model economy stationary, I normalize nominal variables with the price index and express the standard UIP condition in real terms. I then log-linearize the equilibrium equations of the model around the steady state of the economy without ignoring the second moments. Thus, the current values of relevant variables depend not only on their expected future values but also on their conditional variances and covariances with other variables. As a result, I am able to analytically derive closed form solutions to the model and an expression for the foreign exchange risk premium. I then calibrate the deep model parameter values and simulate resulting model using closed form solutions and the risk premium expression. Finally, I examine impulse

responses of selected variables to monetary shocks and the second moment properties of interest.

As in here, Moon (2007) studies the forward premium anomaly in a sticky-price model and finds that the model can generate volatile risk premium that satisfies the volatility relations. He uses the model developed in Chari, Kehoe, and McGrattan (2002) where money balances are modeled as part of the utility function. My study differs in one aspect that I include a cash-in-advance constraint. Feenstra (1986) shows that the cash-in-advance framework is a special case of money-in-the-utility function approach where the real balance component of utility takes the Leontief form. It is encouraging that results in this study only require the simplest possible monetary specification. My study also differs from Moon (2007) in that I seek to an explicit expression for the foreign exchange risk premium in a New Open Economy Macroeconomics framework. Towards this end, I analytically characterize the equilibrium conditions of the model and derive its closed form solutions. Moon (2007) instead focuses only on numerical analysis and simulation of the model.

The results can be summarized as follows. I show that the near-random walk behavior of exchange rates can be derived endogenously in a general equilibrium sticky-price monetary model. This result is consistent with that in Moon (2007). My closed form solutions for the dynamics of real exchange rates and of the risk premium facilitate both impulse response analysis and numerical analysis. The impulse response functions of the model to a positive money supply shock display that realized real exchange rate changes and real interest rate differentials move in the different direction along the path of their mean reverting. And the quantitative results show that the model produces the requisite

second-moment properties of the risk premium and the expected exchange rate changes, and thus can potentially explain the UIP puzzle.

Thus, the contribution of this study can be summarized as follows. First, a dynamic general equilibrium sticky price model is developed that generates highly volatile foreign exchange risk premium able to explain the UIP puzzle. Second, it provides theoretical foundations for empirical findings on GARCH-in-Mean effects in real exchange rates. Third, it derives closed form solutions involving second moments in the equilibrium equations of a dynamic stochastic general equilibrium model.

This chapter is structured as follows. Section 3.2 introduces the model. Section 3.3 provides the solution method, solves the model analytically, derives closed from solutions for the foreign exchange risk premium and real exchange rates, and calibrates the model as well. Impulse response analysis and other numerical results are presented in Section 3.4. Section 3.5 concludes.

3.2 The Model

The world consists of two countries: the home country (H) and the foreign country (the "rest of the world", F). Each is characterized by (i) a representative infinitely lived household, (ii) a representative final-goods producer, (iii) a continuum of intermediate-goods producers indexed by $i \in [0, 1]$, and (iv) a government. Tradable intermediate goods composites are used to produce the final goods in both countries. The final goods are used exclusively for consumption and are not tradable between the two countries. If not mentioned otherwise, the following applies to both countries. Foreign variables are denoted by an asterisk, and where necessary also by an F subscript.

3.2.1 The Household

A representative household decides about its labor supply L_t and consumption of the final goods C_t to maximize its expected whole life utility, which is assumed to be separable between two arguments:

$$\max_{C_t, L_t} E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, L_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\rho} - 1}{1-\rho} - \theta L_t \right) \tag{3.6}$$

where β is the subjective discount factor, and ρ denotes the constant coefficient of risk aversion. θ is a preference parameter associated with labor supply. Here I assume it is constant. The linear form of disutility from labor is used to capture fluctuations in the labor market and can be justified by the indivisible labor assumption as in Hansen (1985).

The household faces two constraints: a cash-in-advance constraint and a budget constraint. I assume that households need cash to purchase consumption goods. The Cash-In-Advance (CIA) constraint then dictates that in every period t

$$P_t C_t \leq M_t \tag{3.7}$$

where P_t is the price level and M_t is the quantity of currency at time t in the Home country. The constraint implies a unit consumption elasticity and a zero interest elasticity of money demand. While this is a somewhat unappealing feature of the CIA approach, it can be justified with the very low empirical estimates of the interest sensitivity of money demand (see Sriram, 2001).

The timing of the cash-in-advance constraint follows Carlstrom and Fuerst (2001): At the beginning of time t, household enters asset markets where it acquires cash for its projected consumption and engages in bond trading. Home households can hold

three types of nominal assets: non-interest bearing home money M; state contingent

home bonds $B_H(\xi^t)$; and state contingent foreign bonds $B_F(\xi^t)$. Home (Foreign) bonds

are issued in the home (foreign) country and pay off one unit of home (foreign) currency

after holding for one period if state ξ^t occurs and zero otherwise. The state

$\xi^t = (\xi_1, ..., \xi_t)$ consists of the history of aggregate events through period t, where ξ_t

denotes the aggregate event in period t. I denote as $f(\xi_t)$ the density of the probability

distribution over such histories. The aggregate event ξ_t itself consists of (μ_t, μ_t^*) since

the only uncertainty in this economy is money growth shocks in two countries, where μ_t

is the growth rate of money stocks in home country in period t and similarly μ_t^* is the

growth rate of foreign money. Financial markets are assumed to be complete.

The household's optimization problem is also restricted by the following asset

market constraint:

$$
\begin{aligned}
& M_t + \int J(\xi^{t+1}, \xi^t) B_{Ht+1}(\xi^{t+1}) d\xi^{t+1} + S_t \int J^*(\xi^{t+1}, \xi^t) B_{Ft+1}(\xi^{t+1}) d\xi^{t+1} \\
& \leq W_t L_t + D_t + M_{t-1} + B_{Ht}(\xi^t) + S_t B_{Ft}(\xi^t) + T_t
\end{aligned}
\tag{3.8}
$$

The left-hand side of Equation (3.8) comprises the household's accumulation of

money and nominal bonds. $J(\xi^{t+1}, \xi^t)$ (respectively, $J^*(\xi^{t+1}, \xi^t)$) denotes the price at

time t of the home (foreign)-currency denominated bonds conditional upon state ξ^t

occurring in period t. The right-hand side describes the household's income from labor

effort with wage rate (W), profits or dividends from firms (D), cash that has not been

spent in the previous period, maturing bonds, and lump-sum government transfers (T). If

the rate of return on bonds is positive, the cash-in-advance constraint (3.7) binds in every

period in equilibrium and agents will only hold the amount of money that is necessary to purchase their consumption. Hence, they do not hold money between periods and M_{t-1} in Equation (3.8) is zero. See Helpman (1981) for an early treatment of this issue. As in Obstfeld and Rogoff (2003), there is no capital accumulation in the model.

From the first-order conditions for the household's problem, first, we can derive a risk-free nominal bond price in domestic currency:

$$\int J(\xi^{t+1}, \xi^t) d\xi^{t+1} = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+1}} \right] \tag{3.9}$$

where $E_t[\cdot]$ denotes the expected value of variables dated $\tau \geq t$ conditional on the current state, ξ^t. Specifically, for a given variable x, $E_t x_\tau(\xi^\tau) = \int x_\tau(\xi^\tau) f(\xi^\tau) d\xi^\tau$.

Let $(1+i_t)$ be the gross risk-free interest rate of home country. Using this, Equation (3.9) can be rewritten in the familiar form:

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \cdot \frac{P_t}{P_{t+1}} \cdot (1+i_t) \right] = 1 \tag{3.10}$$

Second, I obtain the intra-temporal substitution condition between labor supply and consumption:

$$\theta = C_t^{-\rho} \frac{W_t}{P_t} \tag{3.11}$$

Third, I derive home household's optimal foreign-currency denominated bond holdings:

$$S_t \int J^*(\xi^{t+1}, \xi^t) d\xi^{t+1} = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \cdot \frac{P_t}{P_{t+1}} \cdot S_{t+1} \right] \tag{3.12}$$

Let $1+i_t^*$ denote the gross risk-free interest rate of foreign country. Equation (3.12) can be rewritten as follows:

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\cdot\frac{P_t}{P_{t+1}}\cdot\frac{S_{t+1}}{S_t}\cdot(1+i_t^*)\right]=1 \tag{3.13}$$

The foreign country household has analogous first-order conditions.

Equations (3.10) and (3.13) together imply the uncovered interest rate parity condition:

$$\frac{S_t(1+i_t)}{1+i_t^*}=\frac{E_t\left[\left(\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right)\cdot S_{t+1}\right]}{E_t\left(\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right)} \tag{3.14}$$

Combining Equation (3.12) with the foreign household counterpart of Equation (3.9), we can obtain:

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\cdot\frac{P_t}{P_{t+1}}\cdot\frac{S_{t+1}}{S_t}\right]=E_t\left[\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{P_t^*}{P_{t+1}^*}\right] \tag{3.15}$$

If asset markets are complete, as is the case here, Equation (3.15) holds in each state ξ^{t+1}. Thus, the household optimization problem also produces the following perfect risk-sharing condition in complete asset markets[9]:

$$S_t\left(\frac{C_t^{-\rho}}{P_t}\right)=\kappa\frac{C_t^{*-\rho}}{P_t^*} \tag{3.16}$$

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[9] Recent work by Brandt, Cochrane and Santa-Clara (2006) suggests that international risk sharing is very high in the real world.

where κ is a constant that depends on initial conditions (see CKM, 2002, and Gali and Monacelli, 2005). I assume that the initial state of the economy lies in a symmetric equilibrium and thus normalize κ to 1.

3.2.2 The Final-goods Producer

Final-goods producers are perfectly competitive. They use intermediate goods composites from both countries ($\underline{Y}_H$ and $\underline{Y}_F$, respectively) to produce a single country-specific perishable commodity (Y or Y^*) using the following technology:

$$Y_t = \frac{\underline{Y}_{Ht}^{\psi}\, \underline{Y}_{Ft}^{1-\psi}}{\psi^{\psi}(1-\psi)^{1-\psi}} \tag{3.17}$$

where ψ is the weight or share of the home intermediate goods composite required for final-good production. It also can be treated as an openness index. Foreign final-goods producers use the same technology to produce Y^* by using $\underline{Y}_F^*$ and $\underline{Y}_H^*$ as inputs.

The final-goods producer takes input prices as given and solves the following problem:

$$\max_{\{\underline{Y}_{Ht},\underline{Y}_{Ft}\}}\; P_t Y_t - P_{Ht}\underline{Y}_{Ht} - P_{Ft}\underline{Y}_{Ft} \tag{3.18}$$

subject to (3.17), where P_{Ht} and P_{Ft} are home prices of home and foreign intermediate goods, respectively. Here it is assumed that exports are invoiced in the currency of the importing country. This assumption, often called local currency pricing, was introduced by Betts and Devereux (1996, 2000) into Obstfeld and Rogoff's (1995) model to characterize the pricing-to-market behavior by monopolistic firms (intermediate-goods producers in this study).

The solution to the above problem yields input demands:

$$Y_{Ht} = \psi \frac{P_t Y_t}{P_{Ht}}$$

$$Y_{Ft} = (1-\psi) \frac{P_t Y_t}{P_{Ft}}$$

(3.19)

The zero-profit condition implies that the price of the final goods is given by

$$P_t = P_{Ht}^{\psi} P_{Ft}^{1-\psi}$$

(3.20)

The problem faced by the foreign final-goods producer can be described in an analogous manner.

3.2.3 The Intermediate-goods Producer

The Home (Foreign) intermediate-goods composite used by final-goods producers is made from a continuum of differentiated intermediate goods indexed by i $(j) \in [0, 1]$ described by the following equation:

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}(i)^{\frac{\upsilon-1}{\upsilon}} di \right]^{\frac{\upsilon}{\upsilon-1}} \qquad Y_{Ft} = \left[\int_0^1 Y_{Ft}(j)^{\frac{\upsilon-1}{\upsilon}} dj \right]^{\frac{\upsilon}{\upsilon-1}}$$

(3.21)

where $\upsilon > 1$ is the elasticity of substitution between different intermediate goods.

Let $P_{Ht}(i)$ (respectively, $P_{Ft}(j)$) be the price of Home (Foreign) intermediate goods i (j) in the Home market. From (3.21), it is easy to find the demand for individual intermediate goods:

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\upsilon} Y_{Ht}$$

(3.22)

$$Y_{Ft}(j) = \left(\frac{P_{Ft}(j)}{P_{Ft}} \right)^{-\upsilon} Y_{Ft}$$

(3.23)

Thus P_{Ht} and P_{Ft} are defined as follows:

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\upsilon} di \right]^{\frac{1}{1-\upsilon}} \qquad P_{Ft} = \left[\int_0^1 P_{Ft}(j)^{1-\upsilon} dj \right]^{\frac{1}{1-\upsilon}} \qquad (3.24)$$

The representative firm, i, in the home country produces its differentiated goods using the following technology:

$$\underline{Y}_{Ht}(i) + \underline{Y}_{Ht}^*(i) = AL_t(i) \qquad (3.25)$$

where $\underline{Y}_{Ht}^*(i)$ is foreign demand for home intermediate goods, $L_t(i)$ is labor input used in the production of intermediate goods i, and A is a technology parameter. I assume here A is constant.

With the wage rate W_t taken as given, the representative producer solves a cost minimization problem in order to choose labor demand. This yields the marginal cost

$$MC_t(i) = MC_t = \frac{W_t}{A} \qquad (3.26)$$

This marginal cost is identical for all intermediate goods firms.

Intermediate-goods producers are monopolistically competitive. Firm i sets different nominal prices, $P_{Ht}(i)$ and $P_{Ht}^*(i)$, taking as given the aggregate demand and the price level in each country. Typically, such pricing-to-market behavior gives rise to violation of the law of one price among traded goods, and ultimately to a departure from purchasing power parity. Empirically, Knetter (1989, 1993) provide strong evidence in favor of pricing-to-market.

Nominal prices are assumed to be sticky. Price stickiness is modeled as in Calvo (1983). That is, an individual firm has a probability $1-\phi$ of re-setting its price at any

time t. I assume that otherwise it will just charge a price equal to last period's price, adjusted for the long-run inflation rate (π). Let $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ denote the optimal prices set by a typical firm in period t in the home and foreign countries, respectively. It is not necessary to index $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ by individual firm because all firms that change their prices at a given time choose the same new price. The probability that $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ last at least until period τ, for $\tau \geq t$, is $\phi^{\tau-t}$. Therefore, when an individual firm re-sets its price, it does so by solving the following problem:

$$\max_{\{\tilde{P}_{Ht}, \tilde{P}_{Ht}^*\}} E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} \phi^{\tau-t} \{[\pi^{\tau-t}\tilde{P}_{Ht} - MC_\tau] \underline{Y}_{H\tau}(i) + [S_\tau \pi^{\tau-t}\tilde{P}_{Ht}^* - MC_\tau] \underline{Y}_{H\tau}^*(i)\}$$

$$s.t.$$

$$\underline{Y}_{H\tau}(i) = \left(\frac{\pi^{\tau-t}\tilde{P}_{Ht}}{P_{H\tau}}\right)^{-\upsilon} \underline{Y}_{H\tau} \tag{3.27}$$

$$\underline{Y}_{H\tau}^*(i) = \left(\frac{\pi^{\tau-t}\tilde{P}_{Ht}^*}{P_{H\tau}^*}\right)^{-\upsilon} \underline{Y}_{H\tau}^*$$

where $\rho_{t,\tau}$ is the pricing kernel between period t and τ. I assume that all firms are owned by the home representative household. Let $\rho'_{t,\tau}$ be the discounted marginal rate of substitution between period t and period τ consumption; thus,

$$\rho_{t,\tau} \equiv \frac{\beta^{\tau-t} U_{C\tau}}{U_{Ct}} * \frac{P_t}{P_\tau} = \beta^{\tau-t} \left(\frac{C_\tau}{C_t}\right)^{-\rho} * \frac{P_t}{P_\tau} \equiv \rho'_{t,\tau} * \frac{P_t}{P_\tau}.$$

First-order conditions give the optimal prices:

$$\tilde{P}_{Ht} = \frac{\upsilon}{\upsilon-1} \cdot \frac{E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} (\phi\pi^{-\upsilon})^{\tau-t} MC_\tau P_{H\tau}^{\upsilon} \underline{Y}_{H\tau}}{E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} (\phi\pi^{1-\upsilon})^{\tau-t} P_{H\tau}^{\upsilon} \underline{Y}_{H\tau}} \tag{3.28}$$

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$$\tilde{P}_{Ht}^* = \frac{\upsilon}{\upsilon-1} \cdot \frac{E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} (\phi\pi^{-\upsilon})^{\tau-t} MC_\tau P_{H\tau}^{*\ \upsilon} Y_{H\tau}^*}{E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} (\phi\pi^{1-\upsilon})^{\tau-t} S_\tau P_{H\tau}^{*\ \upsilon} Y_{H\tau}^*} \tag{3.29}$$

Assuming that price changes are independent across firms, the law of large numbers implies that only a fraction $1-\phi$ of firms charge up-to-date optimal prices at any time t. A fraction $\phi^{t-\tau}(1-\phi)$ of firms charge outdated prices for $\tau \le t$. That is, prices are not synchronized across firms. Some firms set a new price at time τ in the past and would not have changed it as of time t. It follows that P_{Ht} and P_{Ht}^* can be written, respectively, as:

$$P_{Ht} = \left[(1-\phi)\tilde{P}_{Ht}^{1-\upsilon} + \phi(\pi P_{Ht-1})^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}$$

$$P_{Ht}^* = \left[(1-\phi)\tilde{P}_{Ht}^{*\ 1-\upsilon} + \phi(\pi P_{Ht-1}^*)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}} \tag{3.30}$$

From the production function (3.25), we can easily get the labor demand for intermediate goods by firm i:

$$L_t(i) = \frac{1}{A} \left[\underline{Y}_{Ht}(i) + \underline{Y}_{Ht}^*(i) \right] \tag{3.31}$$

Substituting Equation (3.22) and the foreign counterpart of Equation (3.23) into the above equation, and aggregating over firms (i), we can get the aggregate demand for labor:

$$L_t = \int_0^1 L_t(i)di = \int_0^1 \frac{1}{A} [\underline{Y}_{Ht}(i) + \underline{Y}^*_{Ht}(i)]di$$

$$= \frac{1}{A}\left[\left(\frac{P'_{Ht}}{P_{Ht}}\right)^{-\upsilon} \underline{Y}_{Ht} + \left(\frac{P'^*_{Ht}}{P^*_{Ht}}\right)^{-\upsilon} \underline{Y}^*_{Ht}\right]$$

(3.32)

where $P'_{Ht} = \left[\int_0^1 P_{Ht}(i)^{-\upsilon} di\right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{P}_{Ht}^{-\upsilon} + \phi(\pi P'_{Ht-1})^{-\upsilon}\right]^{-\frac{1}{\upsilon}}$

$$P'^*_{Ht} = \left[\int_0^1 P^*_{Ht}(i)^{-\upsilon} di\right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{P}_{Ht}^{*-\upsilon} + \phi(\pi P'^*_{Ht-1})^{-\upsilon}\right]^{-\frac{1}{\upsilon}}$$

3.2.4 The Government

In both countries, the government represents the fiscal and monetary authority. For simplicity, I assume there is no government spending or investment. Each period, the government makes lump-sum transfers to households. Transfers are financed by printing additional money. Thus, the government budget constraint in the home country is

$$T_t = M_t - M_{t-1}$$

(3.33)

Money is exogenously supplied according to the following growth rule:

$$M_t = \mu_t * M_{t-1}$$

(3.34)

where μ_t denotes stochastic home money growth rate. Home money growth rate is assumed to be log-normal and exhibit the time-varying volatility, which can be described by the following AR (1)-GARCH (1, 1) model:

$$\ln \mu_{t+1} = (1-\sigma)\ln(\mu) + \sigma \ln \mu_t + \varepsilon_{t+1} \qquad 0 < \sigma < 1$$

(3.35)

$$\varepsilon_{t+1} = \sqrt{h_{t+1}} v_{t+1} \qquad v_{t+1} \sim N(0,1)$$

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \alpha_2 h_t$$

where μ is the steady-state gross rate of money growth. ε_{t+1} is the stochastic disturbance term to the home money growth rate with zero conditional mean and h_{t+1} conditional variance. This process is justified by empirical analysis of U.S. money supply as shown in the context later. The Foreign country has analogous dynamics for its monetary growth. I assume that the money supply process for each country evolves independently of the other.

3.2.5 The Market Clearing Conditions

The goods market clearing condition is $Y_t = C_t$. The money market clearing condition is already embedded in the CIA condition where money demand for purchasing goods equals the money stock in the economy. The foreign country has analogous market clearing conditions. International bond markets clear by $B_{Ht} + B_{Ht}^* + B_{Ft} + B_{Ft}^* = 0$.

The competitive general equilibrium in this model is attained when households, final-goods producer, and intermediate-goods producers simultaneously solve their optimal problems subject to the market clearing conditions above.

3.3 Model Solution

3.3.1 Solution Method

Since the model is non-linear and does not yield closed-form solutions for general paths of the variables of interest, I consider a log-linear approximation around a non-stochastic zero growth steady state. To make local approximation techniques valid, we need to consider the stationarity problems of the model economy. First, the assumption of complete international asset markets *per se* induces stationarity in the equilibrium dynamics of net foreign assets (see Schmitt-Grohe and Uribe, 2003). Second, I allow for

positive money growth, thereby a positive long-run inflation rate in our model. Therefore, we need to normalize all nominal variables to render them stationary. Without a growth element in technology, all real variables are stationary. So I transform all nominal variables into their real counterparts through dividing them by their relevant find goods price indexes. In Appendix B, I list all the resulting equilibrium equations after this normalization and also the solution to the steady state of our resulting stationary system. Appendix C lists equilibrium equations log-linearized around the steady state. Below I use lower-case letters to denote real variables corresponding to their nominal counterparts. I also use the notation of the circumflex to denote the log-deviation of a variable from its steady-state value (say, $\hat{\alpha}_t = \log \alpha_t - \log \alpha$).

3.3.2 Solving Analytically

From the log-linearized equilibrium equations as listed in Appendix C, we can obtain a bivariate system which fully describes the dynamics of the relative real money balance and the relative inflation across Home and Foreign:

$$\hat{m}_t - \hat{m}_t^* = (\hat{m}_{t-1} - \hat{m}_{t-1}^*) - (\hat{\pi}_t - \hat{\pi}_t^*) + (\hat{\mu}_t - \hat{\mu}_t^*)$$

$$E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) = \left[\frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} \right](\hat{\pi}_t - \hat{\pi}_t^*) - \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}(\hat{m}_{t-1} - \hat{m}_{t-1}^*) \qquad (3.36)$$

$$- \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}(\hat{\mu}_t - \hat{\mu}_t^*) - \frac{1-\phi}{\beta\phi} N_t$$

where $\pi_t (\pi_t^*)$ denotes the home (foreign) gross inflation rate; and

$$N_t \equiv \frac{1}{2}\beta\phi\left[\mathrm{var}_t(\hat{\pi}_{t+1}) - \mathrm{var}_t(\hat{\pi}^*_{t+1})\right]$$

$$+\frac{1}{2}\rho^2\beta\phi(1-\beta\phi)(1-\beta\phi-2\psi)\left[\mathrm{var}_t(\hat{m}_{t+1}) - \mathrm{var}_t(\hat{m}^*_{t+1})\right]$$

$$+\rho\beta\phi(1-\beta\phi-\psi)[\mathrm{cov}_t(\hat{m}_{t+1},\hat{\pi}_{t+1}) - \mathrm{cov}_t(\hat{m}^*_{t+1},\hat{\pi}^*_{t+1})]$$

This system can be written as:

$$E_t X_{t+1} = G_t + A_0 X_t + B_0 \eta_t \tag{3.37}$$

where $X_t = (\hat{m}_{t-1} - \hat{m}^*_{t-1}, \ \hat{\pi}_t - \hat{\pi}^*_t)'$, $\eta_t = \hat{\mu}_t - \hat{\mu}^*_t$ and G_t, A_0 and B_0 are as follows:

$$G_t = \begin{pmatrix} 0 \\ -\dfrac{1-\phi}{\beta\phi}N_t \end{pmatrix}$$

$$A_0 = \begin{bmatrix} 1 & -1 \\ -\dfrac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} & \dfrac{1}{\beta}+\dfrac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} \end{bmatrix} \tag{3.38}$$

and

$$B_0 = \begin{pmatrix} 1 \\ -\dfrac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} \end{pmatrix}$$

As shown in Blanchard and Kahn (1980), this system has a unique saddle-path stable solution if and only if the number of eigenvalues of the matrix A_0 outside the unit circle is equal to the number of non-predetermined variables. Of two variables in this system, $(\hat{m}_{t-1} - \hat{m}^*_{t-1})$ is predetermined and $(\hat{\pi}_t - \hat{\pi}^*_t)$ is forward-looking[10]. Hence, the

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[10] It means that "the non-predetermined variables depend on the past only through its effect on the current predetermined variables" (see Blanchard and Kahn 1980).

matrix A_0 must have one stable and one unstable characteristic root in order to have a unique saddle-path stable solution to the system.

Proposition One eigenvalue of Matrix A_0 is less than 1 and the other is greater than 1

Proof.

$$\det(A_0 - \lambda I) = (1 - \lambda)\left(\frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} - \lambda\right) - \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}$$

$$= \lambda^2 - \left(1 + \frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}\right)\lambda + \frac{1}{\beta}$$

Suppose λ_1 and λ_2 are two roots of the characteristic equation $\det(A_0 - \lambda I) = 0$, then

$$(1 - \lambda_1)(1 - \lambda_2) = 1 + \lambda_1\lambda_2 - (\lambda_1 + \lambda_2)$$

$$= -\frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} < 0$$

Therefore, one eigenvalue is less than one and the other is greater than one. ■

The matrix A_0 has one eigenvalue within the unit circle and the other one outside the unit circle. Thus, a unique stable solution always exists for any sensible values of all behavior parameters (ρ, β, ϕ) in the system. This unique stable solution can be found using the method of undetermined coefficients, by guessing[11]

$$\hat{\pi}_t - \hat{\pi}_t^* = A_1(\hat{m}_{t-1} - \hat{m}_{t-1}^*) + A_2\eta_t + A_3(h_t - h_t^*) + A_4(\varepsilon_t^2 - \varepsilon_t^{*2}) \tag{3.39}$$

This implies:

$$\hat{m}_t - \hat{m}_t^* = (1 - A_1)(\hat{m}_{t-1} - \hat{m}_{t-1}^*) + (1 - A_2)\eta_t - A_3(h_t - h_t^*) - A_4(\varepsilon_t^2 - \varepsilon_t^{*2}) \tag{3.40}$$

and

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[11] See Appendix D for forming this guess.

$$E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) = A_1(\hat{m}_t - \hat{m}_t^*) + A_2\sigma\eta_t + (A_3 + A_4)[\alpha_1(\varepsilon_t^2 - \varepsilon_t^{*2}) + \alpha_2(h_t - h_t^*)] \quad (3.41)$$

Plugging these relations into System (3.36), we can get four equations to determine the values for four coefficients in the solution form:

$$1'\; A_1^2 + \left[\frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} - 1\right]A_1 - \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi} = 0$$

$$2'\; \left[A_1 - \sigma + \frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}\right]A_2 = A_1 + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}$$

$$\quad (3.42)$$

$$3'\; \left[A_1 - \alpha_1 - \alpha_2 + \frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}\right]A_3$$

$$= \frac{1}{2}\alpha_2(1-\phi)\left[A_2^2 + (1-\beta\phi)(1-\beta\phi-2\psi)\rho^2(1-A_2)^2 + 2(1-\beta\phi-\psi)\rho A_2(1-A_2)\right]$$

$$4'\; A_4 = A_3\alpha_1/\alpha_2$$

These four equations are quite complicated and my method will focus on calibrating deep parameters of the model to compute values for the above coefficients, thereby obtaining the closed form solution to this theoretical model.

3.3.3 Implications for Foreign Exchange Risk Premium

I express the UIP condition in Equation (3.14) in real terms[12] and then log-linearize around the steady state. I obtain the following equation:

$$E_t(\hat{q}_{t+1} - \hat{q}_t) = (1-\beta)(\hat{r}_t - \hat{r}_t^*) - \frac{1}{2}\mathrm{var}_t(\hat{q}_{t+1}) + \rho\,\mathrm{cov}_t(\hat{c}_{t+1}, \hat{q}_{t+1}) \quad (3.43)$$

where q_t denotes the real exchange rate and $r_t(r_t^*)$ denotes the home (foreign) real interest rate.

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[12] See the fourth equation under B.2 in Appendix B.

When expressed in real terms and log-linearized around the steady state, the standard UIP condition in Equation (3.1) takes the form

$$E_t(\hat{q}_{t+1} - \hat{q}_t) = (1 - \beta)(\hat{r}_t - \hat{r}_t^*) \tag{3.44}$$

Comparing Equations (3.43) and (3.44), we can define the foreign exchange risk premium (rp_t) as the deviation from the standard UIP condition:

$$rp_t \equiv -\frac{1}{2}\text{var}_t(\hat{q}_{t+1}) + \rho\,\text{cov}_t(\hat{c}_{t+1}, \hat{q}_{t+1}) \tag{3.45}$$

The log-linearized forms of the normalized CIA constraint and the risk-sharing condition[13] jointly imply:

$$\hat{q}_t = \rho(\hat{m}_t - \hat{m}_t^*) \tag{3.46}$$

where $m_t(m_t^*)$ denotes the home(foreign) real money balances.

Combining Home normalized CIA constraint, Equation (3.46) can be rewritten as

$$rp_t = \frac{1}{2}\rho^2\left[\text{var}_t(\hat{m}_{t+1}) - \text{var}_t(\hat{m}_{t+1}^*)\right] \tag{3.47}$$

Equation (D.3) in Appendix D provides an expression for the relative conditional variance term in the above Equation (3.47). I thus obtain the following explicit expression for foreign exchange risk premium:

$$rp_t = \frac{1}{2}\rho^2(1 - A_2)^2[\alpha_1(\varepsilon_t^2 - \varepsilon_t^{*2}) + \alpha_2(h_t - h_t^*)] \tag{3.48}$$

where $h_t(h_t^*)$ is the conditional variance of home (foreign) monetary shocks given in Equation (3.35).

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[13] See Equations (C.1) and (C.4) in Appendix C.

3.3.4 Implications for Dynamics of Real Exchange Rates

From Equation (3.46), we know that the dynamics of real exchange rates are the same as those of real money differentials up to a coefficient of risk aversion. The latter evolves according to Equation (3.40) in equilibrium.

Updating Equation (3.40) one period forward and subtracting from the original equation, we can get:

$$(\hat{m}_{t+1} - \hat{m}_{t+1}^*) - (\hat{m}_t - \hat{m}_t^*) = -A_1(\hat{m}_t - \hat{m}_t^*) + (1-A_2)\sigma\eta_t + (1-A_2)(\varepsilon_{t+1} - \varepsilon_{t+1}^*)$$
$$- A_3\alpha_2(h_t - h_t^*) - A_3\alpha_1(\varepsilon_t^2 - \varepsilon_t^{*2}) - A_4(\varepsilon_{t+1}^2 - \varepsilon_{t+1}^{*2}) \tag{3.49}$$

Substituting the solution to Equation (3.40) into Equation (3.49), we can obtain the following expression for changes in real money differentials:

$$(\hat{m}_{t+1} - \hat{m}_{t+1}^*) - (\hat{m}_t - \hat{m}_t^*) = (1-A_2)(\varepsilon_{t+1} - \varepsilon_{t+1}^*) + (A_1A_4 - A_3\alpha_1)(\varepsilon_t^2 - \varepsilon_t^{*2})$$
$$- A_4(\varepsilon_{t+1}^2 - \varepsilon_{t+1}^{*2}) - A_4\sum_{i=2}^{\infty}(1-A_1)^i(\varepsilon_{t-i}^2 - \varepsilon_{t-i}^{*2})$$
$$+ (1-A_2)(\sigma - A_1)\eta_t - A_1(1-A_2)\sum_{i=2}^{\infty}(1-A_1)^i\eta_{t-i} \tag{3.50}$$
$$+ A_3(A_1 - \alpha_2)(h_t - h_t^*) + A_1A_3\sum_{i=2}^{\infty}(1-A_1)^i(h_{t-i} - h_{t-i}^*)$$

A noteworthy feature of Equation (3.50) is that the $(\varepsilon_{t+1} - \varepsilon_{t+1}^*)$ term dominates the right-hand side, implying that real money differentials, thereby real exchange rates, follow near-random walks.[14]

3.3.5 Calibration

To calibrate the model, I take one quarter as time unit. The analytical solutions to the system (3.36) involve seven relevant parameters in the model: $\rho, \beta, \phi, \psi, \sigma, \alpha_1, \alpha_2$.

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[14] This result is confirmed by our baseline numerical exercise where
$A_1 = 0.603$; $A_2 = 0.793$; $A_3 = -0.0055$; $and\ A_4 = -0.019$.

The simulation exercise requires another two parameters, μ and ω in the money growth process. When applicable I use parameter values that are standard in the literature (Bergin, 2004; Christiano, Eichenbaum and Evans, 2005; and Collard and Dellas, 2006). I estimate all parameters of the AR-GARCH model of monetary growth by matching U.S. quarterly data. The parameter values are presented in Table 3.1.

1) Preference

The subjective discount factor β is equal to 0.99. This implies to attain a 4% annual real interest rate in the steady state. The coefficient of risk aversion ρ is set to 7. A value below 10 for this coefficient is considered acceptable according to finance literature. For the sensitivity analysis I also use $\rho = 9$.

2) Technology

The openness index ψ is set to 0.75 to match the fact that the ratio of imports to GDP is 15% in the U.S. For the price stickiness parameter, I set ϕ at 0.7. With this calibration, intermediate-goods firms reset their prices around a year on average. In the sensitivity analysis I also consider $\phi = 0.5$.

3) Shock process

As reported in Table 3.2, quarterly M1 growth rates in the U.S. exhibit a strong ARCH effect. This supports our specification of a time-varying volatility process for money growth rates. Parameter values in Equation (3.35) are estimated from the seasonally adjusted quarterly US data of M1 for the period 1973:1 to 2006:2, obtained from International Financial Statistics. The long-run value of money growth rates (μ) is

0.013; autocorrelation of growth rates (σ) is 0.61; and three other coefficients in the GARCH model, $\omega, \alpha_1, \alpha_2$, are 0.00, 0.19 and 0.57, respectively.

3.4 Results

Below I evaluate the effects of combining pricing-to-market, the cash-in-advance constraint, and monetary growth with time-varying volatility on the foreign exchange risk premium. First, I look at the empirical evidence about the UIP puzzle. Second, I describe the dynamics of the model by analyzing impulse responses of selected variables to a positive money supply shock. Last, I simulate the model to obtain second moments of interest.

3.4.1 Estimation of the UIP Slope and Risk Premium in the Data

In the context, the empirical UIP test takes the form of a regression of real exchange rate changes on real interest rate differentials between home and foreign country:

$$q_{t+1} - q_t = b_0 + b_1(r_t - r_t^*) + \vartheta_{t+1} \tag{3.51}$$

The estimation exercise from the G7 countries' data[15] confirms the empirical finding of negative coefficient b_1 (-1.83). I measure fitted values from the regression (3.51) less real exchange rates differentials by the foreign exchange risk premium. Similarly, this negative UIP slope implies the following volatility relations:

$$\text{cov}\left(E_t(q_{t+1} - q_t), \ rp_t\right) > 0$$
$$\text{var}(rp_t) > \text{var}\left(E_t(q_{t+1} - q_t)\right) \tag{3.52}$$

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[15] See Appendix E for details.

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This study asks whether or not our theoretical economy model can generate these two relations.

3.4.2 Model Implications for Moments of Interest

I derive second moment statistics of interested variables by simulating the model using the calibration in Table 3.1. I generate artificial data series for real exchange rates, the expected real exchange rate changes, the real money differentials and the foreign exchange risk premium by simulating the model 1000 times with a sample length of 140 periods each, which is comparable to the time interval from the period 1973:1 to 2006:2. The results for the moments of interest of relevant variables are presented in Table 3.3. The second column shows their statistical properties in the data. The third column and on report the numerical results of theoretical models. The statistics reported in the table are averages of sample moments across 1000 simulations.

The main findings from the baseline parameterization of the model are: (a) The variance of the risk premium is greater than that of the expected changes in real exchange rates. The standard deviation of the risk premium relative to that of money stocks is 3.67 while that of the expected changes in real exchange rates is 2.29. (b) The covariance of the risk premium with the expected change in real exchange rates is positive. The cross correlation between these two quantities is 0.22. (c) The baseline model generates volatile enough real exchange rates which match the data (2.55 vs. 2.23). (d) The volatility of risk premia is still less than that in the data (3.67 vs. 4.56). (e) The cross correlation between the real exchange rate and relative consumption is 1, which is a common feature of the frictionless asset pricing in most macro models. (f) The model implied UIP slope is negative although its absolute value is less than that in the data.

The results from the sensitivity analysis are reported in the remaining columns of Table 3.3. The sensitivity analyses show that: (a) Changes in the coefficient of risk aversion do not affect most quantitative results reported above. But we can see a significant increase in the volatility of risk premia when we increase the value of risk aversion. Specifically, increasing this coefficient to 9 results in the volatility of the risk premium 4.73 times that of money stocks, which matches the data better. (b) Decreasing the price stickiness to some extent ($\phi = 0.5$) does not affect the relative ordering of the two volatilities: the risk premium is still more volatile than the expected real depreciation.

Therefore, numerical results show that the theoretical model can replicate negative UIP slope as observed in the data.

3.4.3 Model Dynamics

Figure 3.1 plots impulse responses of the model to one unit of positive money supply shock in the home country at time $t = 1$. The model generates a rise in real exchange rates ($\hat{q}_t$) and relative inflation ($\hat{\pi}_t - \hat{\pi}_t^*$) after a money injection, which is consistent with the empirical evidence documented in Christiano, Eichenbaum and Evans (2005), among others. The first graph displays the dynamics of money growth shocks. As the shock dies out, real exchange rates continue to increase and then decrease towards its steady state. It is because in the model, the impact of money supply shocks on real exchange rates, or relative real money balances, mainly depends on three factors: one is the shock itself which positively influences $\hat{m}_t - \hat{m}_t^*$ in a direct way; the second one is the changing conditional variance which also positively affects $\hat{m}_t - \hat{m}_t^*$; and the third one is its mean-reverting power. When the first two factors outweigh the last one in initial

periods after money injection, relative real money balances still increase over these periods. The dynamics of realized changes in real exchange rates is displayed in the middle-left graph. The model generates similar dynamics for risk premia. Relative real interest rates decrease on impact reflecting the liquid effect and then increase over some period. Finally, relative real interest rates falls toward to its equilibrium value. Therefore, impulse response functions show that there is situation in which realized changes in real exchange rates and relative real interest rates move in the opposite direction.

3.5 Conclusions

This study seeks to provide risk premium-based explanation for the UIP puzzle in a sticky-price New Keynesian monetary model. The model is characterized by cash-in-advance constraints, pricing-to-market, and an exogenous monetary growth process with time-varying volatility. I log-linearize the equilibrium equations of the model around the steady state taking explicitly into account the second moments of variables. The setup makes it possible to derive a closed form expression for the model-implied foreign exchange risk premium. I then calibrate the model, simulate the dynamics of the implied risk premium and examine the second moment properties of interest. Simulation results show that our model can generate volatile enough risk premia in the sense of satisfying two requisite volatility relations thereby potentially yielding an explanation for the UIP puzzle. In addition, my analysis also shows that the near-random walk behavior of exchange rates can arise endogenously in a New Keynesian monetary model.

My focus on deriving a closed form solution to the risk premium forces us here to limit the analysis to a very simple and stylized setup. Numerical analyses of more sophisticated models that include incomplete asset markets, investment and adjustment

costs in investment, more realistic monetary policy rules (e.g. Taylor rule) and other features are natural next step to better understand the foreign exchange risk premium in New Keynesian models.

On the other hand, the finding of perfect correlation between real exchange rates and relative consumption is in contrast with empirical evidences. CKM labeled this discrepancy as the consumption-real exchange rate anomaly. I investigate this issue in the next chapter.

Table 3.1: Parameterization

Parameter	Calibrated Value	Description
		Preference
β	0.99	Discount factor
ρ	7 (9)	Coefficient of risk aversion
		Technology
ψ	0.75	Openness index
ϕ	0.7 (0.5)	Probability of resetting prices
		Shock process
μ	0.0128	Long-run money growth rate
σ	0.609	Persistence of monetary shock
ω	0.00	
α_1	0.19	Coefficients in the GARCH model
α_2	0.565	

Note: 1. Preference and technology parameters are selected from standard practice.

2. Shock process is estimated from the US M1 data.

3. Numbers in parentheses are used in sensitivity analyses.

Table 3.2: Diagnostic Tests on the Quarterly Growth Rates of M1 in the US

Mean	0.013
Standard Deviation	0.014
Skewness (p – value for skewness = 0)	0.199 (0.173)
Kurtosis (p – value for Kurtosis = 3)	2.814 (0.670)
Jarque-Bera Test (p – value for normality)	1.081 (0.583)
Ljung-Box Q(4) (p – value for unautocorrelation)	107.629 (0.000)
Goldfeld-Quandt Test (p – value for homoskedasticity)	2.147 (0.001)
LM Test for ARCH(12) (p – value for no ARCH effect)	27.469 (0.007)

Table 3.3: Second Moments of Real Exchange Rates and Risk Premium

	Data	Baseline	$\rho = 9$	$\phi = 0.5$
Standard deviation relative to M1				
q_t	2.23	2.55	2.89	2.05
$E_t(q_{t+1} - q_t)$	--	2.29	3.12	2.31
rp_t	4.56	3.67	4.73	2.89
Cross-correlation				
$\left(E_t(q_{t+1} - q_t),\ rp_t\right)$	--	0.22	0.16	0.18
$\left(q_t,\ (c_t - c_t^*)\right)$	-0.37	1.00	1.00	1.00
UIP slope				
b_1	-1.83	-1.20	-1.4	-0.92

Notes: 1. The statistics under the header of *Data* are computed from G7 countries data, which are logged and HP filtered with quarterly frequency over the period from 1973:1 to 2006:2.

2. The statistics in the remaining columns are based on 1000 simulations of the model. *Baseline* denotes the model with relatively high price stickiness and low risk aversion, where $\phi = 0.7$ and $\rho = 7$.

3. The standard deviations of real exchange rates and risk premium are divided by the standard deviation of M1. The symbol "--" means "non-applicable".

Figure 3.1: Impulse Response Functions to a Positive Money Supply Shock
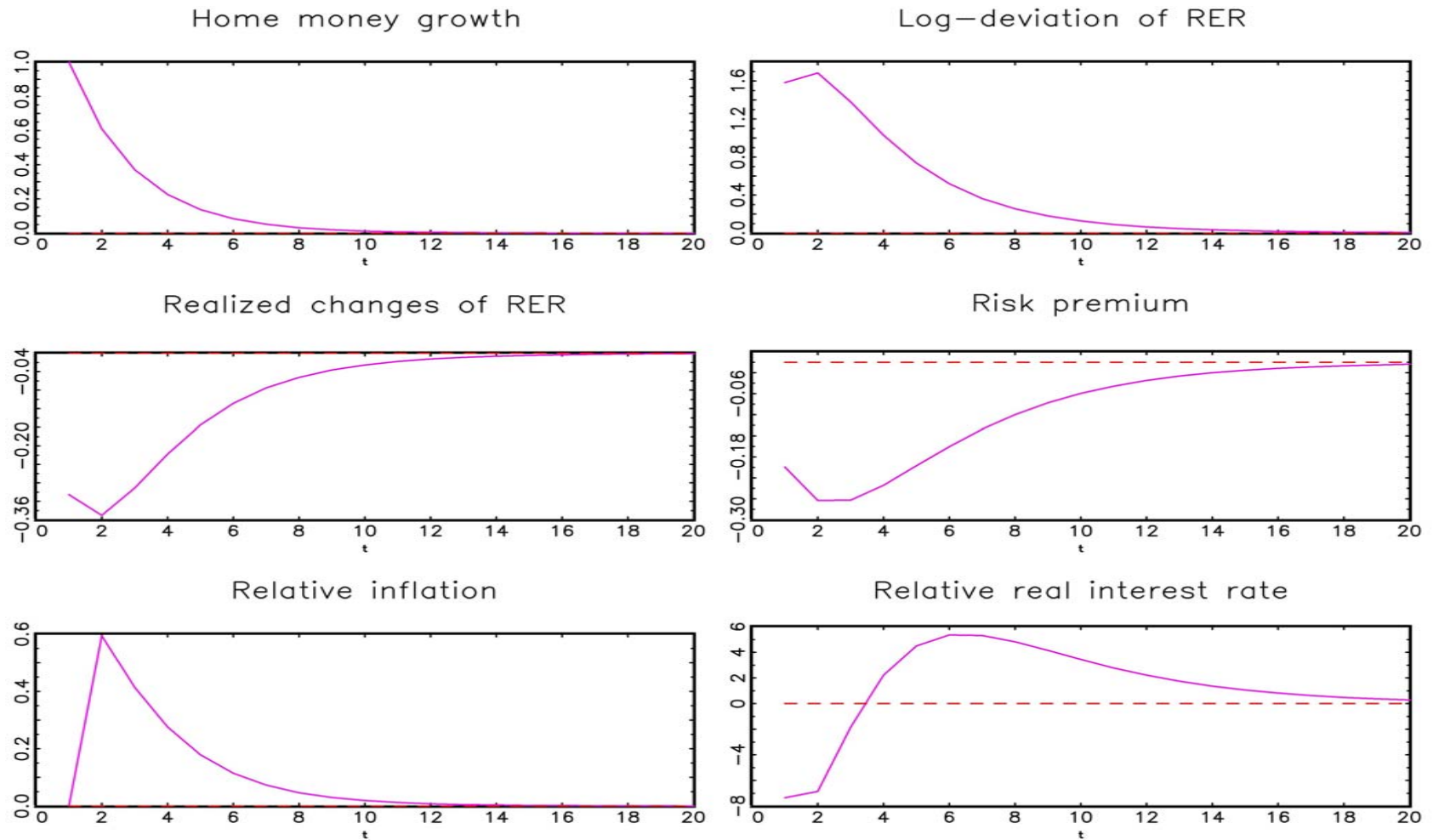
CHAPTER 4

RESOLVING CONSUMPTION-REAL EXCHANGE RATE ANOMALY WITH STICKY PRICES AND ENDOGENOUSLY SEGMENTED MARKETS

4.1 Introduction

Most international macro models predict that, under the assumption of perfect financial markets, the correlation between real exchange rates and relative consumption level across countries is close to unity. This model's feature is in sharp contrast with empirical evidence[16], which suggests that the correlation between these two variables is small and often even negative. Chari, Kehoe and McGrattan (2002, hereafter CKM) labeled the discrepancy between the model and the data as the consumption-real exchange rate anomaly.

This anomaly will occur in any model with frictionless asset markets and homothetic preferences separable in consumption and leisure because in such a model the real exchange rate is tightly linked to the marginal utilities of consumption of domestic and foreign households. Frictionless asset markets imply a high, if not perfect, risk sharing. Therefore, the theoretical solution to this anomaly lies in introducing frictions into asset markets to generate a stochastic wedge between the real exchange rate and the ratio of marginal utilities of household consumption.

Recent theoretical papers assume an incomplete asset market structure as a necessary condition for explaining the observed empirical evidence. In CKM domestic and foreign agents are only allowed to trade in a non-state contingent nominal bond. But

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[16] See Backus and Smith (1993), Corsetti, Dedola and Leduc (2004).

the correlation between real exchange rates and relative consumption in their model is still perfect as in the complete market case. They conclude by saying that the most widely used form of asset market incompleteness does not eliminate the anomaly.

On the other hand, studies by Corsetti, Dedola and Leduc (2004), Benigno and Thoenissen (2005) and Selaive and Tuesta (2003, 2006) introduce other frictions along with asset market incompleteness in an attempt to replicate the stylized fact. Corsetti, Dedola and Leduc (2004) highlight the role of distributive services and show that a low price elasticity of demand for import goods can hinder risk sharing and it might contribute to the anomaly. Benigno and Thoenissen (2005) introduce non-tradable goods to allow for the possibility that the real exchange rate and relative consumption move in opposite directions when following a productivity shock to the domestic traded goods sector. Selaive and Tuesta (2003, 2006) consider a richer model in which prices are sticky. The 2003 paper introduces a cost of bond holding and shows the importance of financial frictions in breaking the link between the real exchange rate and relative consumption. The 2006 paper attributes a key role for non-tradable good along with productivity shocks in explaining the anomaly.

In this work I contribute to the current literature by maintaining the assumption of complete asset markets but introducing fixed costs to for trading in bonds and money in international financial markets into a sticky-price dynamic general equilibrium model, which is mostly similar to the setup in Chapter Three but the behavior of households.

My study is motivated by two papers. Brandt, Cochrane and Santa-Clara (2006) calculate an index of international risk sharing and shows that risk sharing is very high in the real world, which is contrary to the standard findings based on household

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consumption data. This leads me to think of such situation where asset market is complete and risk sharing condition holds. But the ratio of marginal utilities of consumption in the risk sharing condition is not same as that of aggregate consumption. Alvarez, Atkeson and Kehoe (2007) provide an idea about how to find the right marginal utilities. In their paper agents must pay a fixed cost to transfer money between the goods market and the asset market. The real exchange rate is equal to the ratio of marginal utilities of active household consumption, not aggregate consumption. Since their model is a flexible price model where nominal monetary shocks have no real effects in aggregate, the correlation between the real exchange rate and relative aggregate consumption is always zero. I incorporate their idea of endogenously segmented asset markets to a sticky-price dynamic general equilibrium monetary model and try to examine whether such a model can generate a negative correlation between the real exchange rate and relative consumption level across two countries.

My study results suggest that the combination of sticky prices and endogenously segmented asset markets is a promising avenue for resolving the consumption-real exchange rate anomaly. Indeed, the calibrated correlation between these two variables in my model is close to that in the data for a wide range of plausible parameters values. Impulse response functions show that when a positive monetary shock in the domestic country occurs, real exchange rates increase on impact and gradually decrease over time returning to their equilibrium value. On the other hand, relative aggregate consumption across the two countries decreases due to the relative inflation distortion on inactive household consumption in the model economy. Relative aggregate consumption continues to decrease when such distortion is enhanced and then increase over time

towards its steady state. In addition, my model generates high volatility of real exchange rates that matches the data but less persistence than observed, which seems to be a common characteristic of pure sticky-price models.

The remainder of the chapter is structured as follows. Section 4.2 presents the basic structure of the model and markets clearing conditions. Section 4.3 provides the solution method and calibrates the model. The results are discussed in Section 4.4. Section 4.5 concludes.

4.2 The Model

The model in this chapter is similar to that in Chapter 3, which belongs to the line of New Open Economy Macroeconomic models. The difference between two models mainly lies in the household behavior. Here households are not completely homogenous as in the last model. Instead, households face different fixed costs when they need to transfer between cash and bonds in the asset markets. Thus, below I place emphasize on the description of the household problem and briefly introduce the behavior of other agents in the economy.

The world still consists of two countries: the home and the foreign. Each is characterized by (i) a continuum of infinitely lived households of measure 1, (ii) a representative final-goods producer, (iii) a continuum of intermediate-goods producers indexed by $i \in [0, 1]$, and (iv) a government. Trade in this economy occurs in both asset markets and goods markets. In the asset markets, households trade the local currency and home and foreign bonds. Each household must pay a real fixed cost γ for each transfer of cash to or from bonds. The government introduces currency via open market operations. In the goods market, internationally traded intermediate goods composites are used to

produce the final good in both countries. The final good is used exclusively for consumption and is not tradable between the two countries. Households buy the local good subject to a cash-in-advance constraint. The only source of uncertainty in this economy is shocks to money growth in the two countries. If not mentioned otherwise, the following applies to both countries.

4.2.1 The Household

Households are heterogeneous on fixed cost γ and homogeneous otherwise in both countries. This fixed cost, which is in units of the local goods, is constant over time for any specific household, but it varies across households in both countries according to a uniform distribution $G(\gamma)$ with density $g(\gamma)$ on $[0, \gamma_{max}]$. Thus, households are indexed by their fixed cost γ.

In period 0, there is no trade in goods markets. All households are identical. In the asset market, home households have M_0 units of home money, B_{H0} units of home government bonds and B_{F0} units of foreign government bonds, which are claims on B_{H0} home currencies and B_{F0} foreign currencies in that period, respectively.

The timing within each period $t \geq 1$ for a home household of any type is illustrated in Figure 4.1 (similar to that in Alvarez, Atkeson and Kehoe, 2007). The household enters period t with cash $(W_{-1}L_{-1} + D_{-1})$ obtained from labor and ownership income in period $t-1$, which is only available at the beginning of next session and can only be used to buy goods in the following period. Here W is the wage rate; L is the labor supply; and D denotes profits or dividends from firms which are owned by households.

The household also enters the period with state-contingent home and foreign bonds, $B_H(\xi^t)$ and $B_F(\xi^t)$. Home (Foreign) bonds are issued in the home (foreign) country and pay off one unit of home (foreign) currency after holding for one period if state ξ^t occurs and zero otherwise. The state $\xi^t = (\xi_1,...,\xi_t)$ consists of the history of aggregate events through period t, where ξ_t denotes the aggregate event in period t. I denote as $f(\xi_t)$ the density of the probability distribution over such histories. The aggregate event ξ_t itself consists of (μ_t, μ_t^*) since the only uncertainty in this economy is money growth shocks in the two countries, where $\mu_t (\mu_t^*)$ is the growth rate of money stock in home (foreign) country in period t.

Given the price level P, the household takes the starting cash with real value $(n = (W_{-1}L_{-1} + D_{-1})/P)$ and then splits into a worker and a consumer. The worker supplies labor and property rights in order to receive the income $(WL + D)$ that will be delivered at the beginning of next period. The consumer chooses whether or not to pay the fixed cost to transfer an amount of cash Px with real value x to or from bonds in the asset market. This fixed cost is paid in cash obtained in the asset market. Therefore, starting bonds (B_H and B_F) are either reinvested in a complete asset market to purchase new bonds (B_H' and B_F') at the price of J and J^* or, if the fixed cost is paid, traded with cash. The asset market constraint is:

$$B_H + S * B_F = \int J * B_H' + S * \int J^* * B_F' + P(x + \gamma) \text{ if fixed cost is paid;}$$

$$\text{and } B_H + S * B_F = \int J * B_H' + S * \int J^* * B_F' \text{ otherwise,}$$

where S denotes the nominal exchange rate which is the home price of a unit of foreign currency. And then the consumer buys goods subject to the cash-in-advance constraint. The worker rejoins the consumer at the end of the period. Likewise, I call those that pay the fixed cost and transfer cash as active households and those that do not as inactive households. In this sense the asset market is segmented.

More formally, I consider now the problem of household of type γ in the home country. Let $Z(\xi^t, \gamma)$ denote an indicator variable that is equal to one if there is a transfer in the asset market and zero if not. In period t, given the price level $P(\xi^t)$, wage rate $W(\xi^t)$ and dividends $D(\xi^t)$, the household decides about its labor supply $L(\xi^t, \gamma)$, consumption of the final good $C(\xi^t, \gamma)$ and $Z(\xi^t, \gamma)$ to maximize its expected whole life utility, which is assumed to be separable between consumption and labor:

$$\max \sum_{t=1}^{\infty} \beta^t \int_{\xi^t} U(C(\xi^t, \gamma),\ L(\xi^t, \gamma)) f(\xi^t) d\xi^t$$

$$= \sum_{t=1}^{\infty} \beta^t \int_{\xi^t} \left[V(C(\xi^t, \gamma)) - \theta L(\xi^t, \gamma) \right] f(\xi^t) d\xi^t$$

(4.1)

where β is the subjective discount factor, and $V(C(\xi^t, \gamma))$ denotes the sub-utility function of consumption. θ is a preference parameter associated with labor supply.

The household faces one transition law (4.2) and two constraints: the asset market constraint (4.3) and the cash-in-advance constraint (4.4).

$$n(\xi^{t+1}, \gamma) = \frac{W(\xi^t) L(\xi^t, \gamma) + D(\xi^t)}{P(\xi^{t+1})}$$

(4.2)

$$P(\xi^t)\left[x(\xi^t,\gamma)+\gamma\right]Z(\xi^t,\gamma)+\int_{\xi_{t+1}}J(\xi^t,\xi_{t+1})B_H(\xi^{t+1},\gamma)d\xi^{t+1}$$

$$+S(\xi^t)*\int_{\xi_{t+1}}J^*(\xi^t,\xi_{t+1})B_F(\xi^{t+1},\gamma)d\xi^{t+1}\le B_H(\xi^t,\gamma)+S(\xi^t)*B_F(\xi^t,\gamma) \tag{4.3}$$

$$C(\xi^t,\gamma)\le n(\xi^t,\gamma)+x(\xi^t,\gamma)Z(\xi^t,\gamma) \tag{4.4}$$

If the rate of return on bonds is positive, the cash-in-advance constraint (4.4) binds in every period in equilibrium and agents will only hold the amount of money that is necessary to purchase their consumption. In this situation, a household's decision on whether to pay the fixed cost to transfer in period t affects only its current consumption and bonds holdings and does not impact the real balances it holds in the following period. In addition to this sequence of constraints, I also bound real bond holdings by some large constants.

Let $C_A(\xi^t,\gamma)$ and $C_I(\xi^t,\gamma)$ denote the consumptions of an active and an inactive household for a given ξ^t and γ, respectively. According to the definition of the indicator variable Z, we can easily derive from Equation (4.4) that $C_I(\xi^t,\gamma)=n(\xi^t,\gamma)$.

The household's problem turns into two decision-making problems as it splits into the worker and the consumer. The worker chooses labor supply with the knowledge that the household will consume the real value of the amount of cash obtained from producers if he is inactive next period. The first-order condition for the worker's problem[17] gives the inter-temporal substitution condition between labor supply and consumption:

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[17] The worker's problem is as follows:

$$\max \sum_{t=0}^{\infty}\beta^t\int_{\xi^t}\left[V(C_I(\xi^t,\gamma))-\theta L(\xi^t,\gamma)\right]f(\xi^t)d\xi^t$$

s.t. $P(\xi^{t+1})n(\xi^{t+1},\gamma)=W(\xi^t)L(\xi^t,\gamma)+D(\xi^t)$; $C_I(\xi^{t+1},\gamma)=n(\xi^{t+1},\gamma)$.

$$\beta * \frac{W(\xi^t)}{P(\xi^t)} * \int_{\xi_{t+1}} \left[V'(C_I(\xi^{t+1}, \gamma)) / \pi(\xi^{t+1}) \right] f(\xi_{t+1}) d\xi_{t+1} = \theta \qquad (4.5)$$

where $\pi(\xi^{t+1}) = P(\xi^{t+1}) / P(\xi^t)$ denotes the gross inflation rate and $V'(\cdot)$ is the marginal utility of consumption. This equation implies that the consumption of inactive household in the following period is independent of γ.

Given the initial real money balance $n(\xi^t, \gamma)$, the consumer decides $Z(\xi^t, \gamma)$ and consumption pattern by maximizing his sub-utility from consumption:

$$\max \sum_{t=1}^{\infty} \beta^t \int_{\xi^t} V(C(\xi^t, \gamma)) f(\xi^t) d\xi^t$$

subject to Equation (4.3) and Equation (4.4).

I first solve for the consumption pattern given the consumer's choice of Z. When $Z(\xi^t, \gamma)$ takes the value of zero, the consumer will only consume $C_I(\xi^t, \gamma) = C_I(\xi^t)$ which is independent of γ as we discussed above. When $Z(\xi^t, \gamma)$ is equal to one, the first-order condition of the consumer's problem implies the relationship between his consumption pattern and asset prices:

$$J(\xi^t, \xi_{t+1}) = \beta * \frac{V'(C_A(\xi^{t+1}, \gamma))}{V'(C_A(\xi^t, \gamma))} * \frac{P(\xi^t)}{P(\xi^{t+1})} \qquad (4.6)$$

$$J^*(\xi^t, \xi_{t+1}) = \beta * \frac{V'(C_A(\xi^{t+1}, \gamma))}{V'(C_A(\xi^t, \gamma))} * \frac{P(\xi^t)}{P(\xi^{t+1})} * \frac{S(\xi^{t+1})}{S(\xi^t)} \qquad (4.7)$$

The foreign country consumer has analogous first-order conditions.

Let $Q(\xi^t) \equiv \dfrac{S(\xi^t)*P^*(\xi^t)}{P(\xi^t)}$ denote the real exchange rate in period t. Combining

with the foreign analogous condition of Equation (4.6), I obtain the risk-sharing condition

in complete asset markets:

$$\frac{Q(\xi^{t+1})}{Q(\xi^t)} = \frac{V'(C_A^*(\xi^{t+1},\gamma))/V'(C_A(\xi^{t+1},\gamma))}{V'(C_A^*(\xi^t,\gamma))/V'(C_A(\xi^t,\gamma))} \tag{4.8}$$

Equation (4.8) can be written as follows:

$$Q(\xi^t) = \kappa \cdot \frac{V'(C_A^*(\xi^t,\gamma))}{V'(C_A(\xi^t,\gamma))} \tag{4.9}$$

where κ is a constant that depends on initial conditions. Again I assume that the initial

state of the economy lies in a symmetric equilibrium and thus normalize κ to 1.

Next I turn to determine the optimal choice of $Z(\xi^t,\gamma)$. I suppose that there exists

a social planner choosing $Z(\xi^t,\gamma)$ and $C(\xi^t,\gamma)$ to solve the following static planning

problem:

$$\max \int_\gamma \int_{\xi^t} V(C(\xi^t,\gamma))f(\xi^t)g(\gamma)d\xi^t d\gamma$$

subject to two constraints,

$$\int_\gamma \left[C(\xi^t,\gamma) + \gamma * Z(\xi^t,\gamma) \right] g(\gamma)d\gamma = Y(\xi^t) \tag{4.10}$$

$$C(\xi^t,\gamma) = C_A(\xi^t,\gamma)*Z(\xi^t,\gamma) + C_I(\xi^t)*(1-Z(\xi^t,\gamma)) \tag{4.11}$$

where $Y(\xi^t)$ denotes total final goods in the home country. Equation (4.10) captures the

resource constraint on the consumption and cash transfer where each transfer consumes

γ units of the home good. Equation (4.11) defines the aggregate consumption. Here the

planning weight for households of type γ is the fraction of households of such type.

The first-order condition for an active household's consumption gives

$$V'(C_A(\xi^t,\gamma))f(\xi^t) = \lambda(\xi^t) \tag{4.12}$$

where $\lambda(\xi^t)$ is the Lagrange multiplier on the resource constraint. This condition clearly implies that all active households choose the same consumption level, which is independent of γ. We denote this consumption as $C_A(\xi^t)$.

For the planning problem, the increment to the Lagrange of setting $Z(\xi^t,\gamma) = 1$ is

$$V(C_A(\xi^t))f(\xi^t)g(\gamma) - \lambda(\xi^t)\left[C_A(\xi^t) + \gamma - C_I(\xi^t)\right] \tag{4.13}$$

which is the direct utility gain minus the cost of cash transfer. The increment to the Lagrangian of setting $Z(\xi^t,\gamma) = 0$ is

$$V(C_I(\xi^t))f(\xi^t)g(\gamma) \tag{4.14}$$

which is only the direct utility gain from consumption without cash transfer. Subtracting (4.14) from (4.13) and using (4.12) to substitute the Lagrange multiplier gives a cutoff rule to guide the choice of $Z(\xi^t,\gamma)$. More formally, let

$$H = \left[V(C_A(\xi^t)) - V(C_I(\xi^t))\right] - V'(C_A(\xi^t))\left[C_A(\xi^t) + \gamma - C_I(\xi^t)\right]$$

Given $C_A(\xi^t)$, there exists a cutoff level of fixed costs to allow H equal to zero. I denote the cutoff level as $\overline{\gamma}(\xi^t)$ which is relevant to current aggregate events. Specifically, I have

$$\left[V(C_A(\xi^t)) - V(C_I(\xi^t))\right] - V'(C_A(\xi^t))\left[C_A(\xi^t) + \overline{\gamma}(\xi^t) - C_I(\xi^t)\right] = 0 \tag{4.15}$$

Thus, the household of type γ pays the fixed cost and consumes $C_A(\xi^t)$, that is $Z(\xi^t,\gamma) = 1$, when $\gamma \le \overline{\gamma}(\xi^t)$ and thereby H is greater than zero. Otherwise, it does not

pay the fixed cost and simply consumes $C_I(\xi^t)$, that is $Z(\xi^t, \gamma) = 0$. The law of large numbers implies that a fraction $G(\bar{\gamma}(\xi^t))$ of households consume $C_A(\xi^t)$ at any time t with the state ξ^t. The rest of households consume $C_I(\xi^t)$. The asset market is endogenously segmented. Apparently, only active households in the asset market absorb extra money introduced by the government via open market operations.

And the resource constraint (4.10) reduces to

$$C_A(\xi^t)G(\bar{\gamma}(\xi^t)) + C_I(\xi^t)\left[1 - G(\bar{\gamma}(\xi^t))\right] + \int_0^{\bar{\gamma}(\xi^t)} \gamma g(\gamma) d\gamma = Y(\xi^t) \qquad (4.16)$$

The social planner derive $C_A(\xi^t)$ and $\bar{\gamma}(\xi^t)$ as the solutions to (4.15) and (4.16).

There are analogous household's problems in the foreign country. In what follows, I introduce some notations for simplicity. $E_t[\cdot]$ denotes the expected value of variables dated beyond t conditional on the current state, ξ^t. Specifically, for a given variable α, $E_t\alpha(\xi^{t+1}) = \int_{\xi_{t+1}} \alpha(\xi^t, \xi_{t+1}) f(\xi_{t+1}) d\xi_{t+1}$. I also abbreviate $\alpha(\xi^t)$ as α_t for a given variable α.

4.2.2 The Final-goods Producer

The behavior of final-goods producers is the same as that in Chapter 3. They are perfectly competitive and use intermediate-goods composites from both countries ($\underline{Y}_H$ and $\underline{Y}_F$, respectively) to produce a single country-specific perishable commodity (Y or Y^*) using the following technology:

$$Y_t = \frac{\underline{Y}_{Ht}^{\psi} \underline{Y}_{Ft}^{1-\psi}}{\psi^{\psi}(1-\psi)^{1-\psi}} \qquad (4.17)$$

where ψ is the weight or share of the home intermediate-goods composite required for final-goods production. Foreign final-goods producers use the same technology to produce Y^* by using $\underline{Y}_F^*$ and $\underline{Y}_H^*$ as inputs.

The final-goods producer takes input prices as given and solves the following problem:

$$\max_{\{\underline{Y}_{Ht},\underline{Y}_{Ft}\}} P_t Y_t - P_{Ht}\,\underline{Y}_{Ht} - P_{Ft}\,\underline{Y}_{Ft} \tag{4.18}$$

subject to (4.17), where P_{Ht} and P_{Ft} are home prices of the home and foreign intermediate goods, respectively. Here it is assumed that exports are invoiced in the currency of the importing country.

4.2.3 The Intermediate-goods Producer

Similarly, the home (foreign) intermediate-goods composite used by final-goods producers is made from a continuum of differentiated intermediate goods indexed by i $(j) \in [0, 1]$ described by the following equation:

$$\underline{Y}_{Ht} = \left[\int_0^1 \underline{Y}_{Ht}(i)^{\frac{\upsilon-1}{\upsilon}} \, di \right]^{\frac{\upsilon}{\upsilon-1}} \qquad \underline{Y}_{Ft} = \left[\int_0^1 \underline{Y}_{Ft}(j)^{\frac{\upsilon-1}{\upsilon}} \, dj \right]^{\frac{\upsilon}{\upsilon-1}} \tag{4.19}$$

where $\upsilon > 1$ is the elasticity of substitution between different intermediate goods.

Let $P_{Ht}(i)$ (respectively, $P_{Ft}(j)$) be the price of Home (Foreign) intermediate good i (j) in the Home market. From (4.19), it is easy to find the demand for individual intermediate goods:

$$\underline{Y}_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\upsilon} \underline{Y}_{Ht} \tag{4.20}$$

$$Y_{Ft}(j) = \left(\frac{P_{Ft}(j)}{P_{Ft}}\right)^{-\upsilon} Y_{Ft} \tag{4.21}$$

Thus P_{Ht} and P_{Ft} are defined as follows:

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\upsilon} di\right]^{\frac{1}{1-\upsilon}} \qquad P_{Ft} = \left[\int_0^1 P_{Ft}(j)^{1-\upsilon} dj\right]^{\frac{1}{1-\upsilon}} \tag{4.22}$$

The representative firm, i, in the home country produces its differentiated goods using the following technology:

$$Y_{Ht}(i) + Y_{Ht}^*(i) = AL_t(i) \tag{4.23}$$

where $Y_{Ht}^*(i)$ is the foreign demand for home intermediate goods, $L_t(i)$ is the labor input used in the production of intermediate good i, and A is a technology parameter.

With the wage rate W_t taken as given, the representative producer solves a cost minimization problem in order to choose labor demand. This yields the marginal cost

$$MC_t(i) = MC_t = \frac{W_t}{A} \tag{4.24}$$

This marginal cost is identical for all intermediate-goods firms.

Intermediate-goods producers are monopolistically competitive. Firm i sets different nominal prices, $P_{Ht}(i)$ and $P_{Ht}^*(i)$, taking as given the aggregate demand and the price level in each country. Typically, such pricing-to-market behavior gives rise to violation of the law of one price among traded goods, and ultimately to a departure from purchasing power parity.

Nominal prices are assumed to be sticky. Price stickiness is modeled as in Calvo (1983). That is, an individual firm has a probability $1-\phi$ of re-setting its price at any

time t. I assume that otherwise it will just charge a price equal to last period's price, adjusted for the long-run inflation rate (π). Let $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ denote the optimal prices set by a typical firm in period t in the home and foreign countries, respectively. It is not necessary to index $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ by individual firm because all firms that change their prices at a given time choose the same new price. The probability that $\tilde{P}_{Ht}$ and $\tilde{P}_{Ht}^*$ last at least until period τ, for $\tau \geq t$, is $\phi^{\tau-t}$. Therefore, when an individual firm re-sets its price, it does so by solving the following problem:

$$\max_{\{\tilde{P}_{Ht},\tilde{P}_{Ht}^*\}} E_t \sum_{\tau=t}^{\infty} \rho_{t,\tau} \phi^{\tau-t} \{[\pi^{\tau-t}\tilde{P}_{Ht} - MC_{\tau}]\underline{Y}_{H\tau}(i) + [S_{\tau}\pi^{\tau-t}\tilde{P}_{Ht}^* - MC_{\tau}]\underline{Y}_{H\tau}^*(i)\}$$

$s.t.$

$$\underline{Y}_{H\tau}(i) = \left(\frac{\pi^{\tau-t}\tilde{P}_{Ht}}{P_{H\tau}}\right)^{-\upsilon} \underline{Y}_{H\tau} \tag{4.25}$$

$$\underline{Y}_{H\tau}^*(i) = \left(\frac{\pi^{\tau-t}\tilde{P}_{Ht}^*}{P_{H\tau}^*}\right)^{-\upsilon} \underline{Y}_{H\tau}^*$$

where $\rho_{t,\tau}$ is the pricing kernel between period t and τ. I assume that all firms are owned by the home household, and thus according to the household's problem in Section 4.1, the asset pricing kernel depends on the behavior of active households. Formally,

$$\rho_{t,\tau} \equiv \beta^{\tau-t} \frac{V'(C_A(\xi^{\tau},\gamma))}{V'(C_A(\xi^t,\gamma))} * \frac{P(\xi^t)}{P(\xi^{\tau})}.$$

Assuming that price changes are independent across firms, the law of large numbers implies that only a fraction $1-\phi$ of firms charge up-to-date optimal prices at any time t. A fraction $\phi^{t-\tau}(1-\phi)$ of firms charge outdated prices for $\tau \leq t$. That is, prices are not synchronized across firms. Some firms set a new price at time τ in the past and

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would not have changed it as of time t. It follows that P_{Ht} and P_{Ht}^* can be written, respectively, as:

$$P_{Ht} = \left[(1-\phi)\tilde{P}_{Ht}^{\,1-\upsilon} + \phi(\pi P_{Ht-1})^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}} \tag{4.26}$$

$$P_{Ht}^* = \left[(1-\phi)\tilde{P}_{Ht}^{*\,1-\upsilon} + \phi(\pi P_{Ht-1}^*)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}} \tag{4.27}$$

From the production function (4.23), we can easily get the labor demand for intermediate goods by firm i:

$$L_t(i) = \frac{1}{A} \, [\underline{Y}_{Ht}(i) + \underline{Y}_{Ht}^*(i)] \tag{4.28}$$

Substituting Equation (4.20) and the foreign counterpart of Equation (4.21) into the above equation, and aggregating over firms (i), I can get the aggregate demand for labor:

$$L_t = \int_0^1 L_t(i)di = \int_0^1 \frac{1}{A} \, [\underline{Y}_{Ht}(i) + \underline{Y}_{Ht}^*(i)]di$$

$$= \frac{1}{A} \left[\left(\frac{P'_{Ht}}{P_{Ht}}\right)^{-\upsilon} \underline{Y}_{Ht} + \left(\frac{P'^*_{Ht}}{P_{Ht}^*}\right)^{-\upsilon} \underline{Y}_{Ht}^* \right]$$

$$\text{where} \quad P'_{Ht} = \left[\int_0^1 P_{Ht}(i)^{-\upsilon} di \right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{P}_{Ht}^{\,-\upsilon} + \phi(\pi P'_{Ht-1})^{-\upsilon} \right]^{-\frac{1}{\upsilon}}$$

$$P'^*_{Ht} = \left[\int_0^1 P_{Ht}^*(i)^{-\upsilon} di \right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{P}_{Ht}^{*\,-\upsilon} + \phi(\pi P'^*_{Ht-1})^{-\upsilon} \right]^{-\frac{1}{\upsilon}}$$

(4.29)

4.2.4 The Government

In both countries, the government represents the fiscal and monetary authority. For simplicity, I assume there is no government spending or taxation. Each period, the

government introduces money via open market operations. Thus, the government budget constraint in the home country at period $t \geq 1$ is

$$M_t - M_{t-1} = (B_{Ht} + B_{Ht}^*) - E_t(J_{t+1}*(B_{Ht+1} + B_{Ht+1}^*)) \tag{4.30}$$

with M_0 given, and at period $t = 0$, the constraint is $B_H + B_H^* = E_0(J_1*(B_{H1} + B_{H1}^*))$.

Money is exogenously supplied according to the following growth rule at $t \geq 1$:

$$M_t = \mu_t * M_{t-1} \tag{4.31}$$

4.2.5 The Market Clearing Conditions

The goods market clearing condition is the resource constraint (4.16) in Section 4.2.1. The money market clearing condition is

$$\int_{\gamma} \left[n_t + (x_t(\gamma) + \gamma)Z_t(\gamma) \right] g(\gamma) d\gamma = \frac{M_t}{P_t}$$

Or: $\qquad C_{At}G(\overline{\gamma}_t) + C_{It}\left[1 - G(\overline{\gamma}_t)\right] + \int_0^{\overline{\gamma}_t} \gamma g(\gamma) d\gamma = \frac{M_t}{P_t} \qquad$ (4.32)

Last, bond markets clear:

$$\int_{\gamma} \left[B_{Ht}(\gamma) + B_{Ht}^*(\gamma) \right] g(\gamma) d\gamma = B_{Ht} + B_{Ht}^* \tag{4.33}$$

The foreign country has analogous market clearing conditions. International bond markets clear by $B_{Ht} + B_{Ht}^* + B_{Ft} + B_{Ft}^* = 0$.

The competitive general equilibrium in this model is attained when households, final-goods producer, and intermediate-goods producers simultaneously solve their optimal problems subject to the market clearing conditions above.

4.3 Solution Technique and Calibration

4.3.1 Solution Technique

Like in Chapter 3, I consider a log-linear approximation around a non-stochastic zero growth steady state. Following the same approach, I normalize all nominal variables to render them stationary. I transform the nominal variables into their real counterparts through dividing them by their relevant final goods price indexes. In Appendix F, I list the solution to the steady state of our resulting stationary system and also all equilibrium equations log-linearized around this steady state. I use lower-case letters to denote real variables corresponding to their nominal counterparts. The circumflex denotes the log-deviation of a variable from its steady-state value ($\hat{\alpha}_t = \log \alpha_t - \log \alpha$). Worthy of notice, unlike the study in Chapter 3, here I neglect conditional variance/covariance terms when I log-linearize equilibrium equations. This is because conditional variances/covariances have little impact on the current level of economic variables as shown in the last chapter. These terms would not affect the relationship between real exchange rates and relative consumption in this study.

The log-linearization yields a system of linear difference equations which can be expressed as a dynamic system of the following form:

$$A_0 K_t + A_1 E_t (K_{t+1}) + A_2 \eta_t = 0 \tag{4.34}$$

where A_0, A_1 and A_2 are coefficient matrices whose cells are non-linear functions of model parameters. K_t is ordered so that the non-predetermined variables appear first and the predetermined variables appear last. η_t denotes relative monetary growth rates. Given

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the parameters of the model, which we calibrate in the next sub-section, I solve this system using Blanchard and Kahn (1980) solution algorithm.

4.3.2 Calibration

To calibrate the model, I take one quarter as the time unit. The calibration of model parameters follows standard practice in the literature where applicable. The parameter values are presented in Table 4.1.

1) Preference

I choose the following functional form to capture the sub-utility from consumption: $V(C_t) = \ln(C_t)$ which means the coefficient of risk aversion of households is constant and takes the value of 1. In choosing the parameters of total utility function, I set the subjective discount factor β equal to 0.99 and the labor preference parameter θ equal to 1. The maximum fixed cost $\gamma_{\max}$ is 0.1 when households transfer cash. $\gamma_{\max}$ is 0.06 in the sensitivity analysis for a different degree of market segmentation.

2) Technology

The openness index ψ is set to 0.75 to match the fact that the ratio of imports to GDP is 15% in the U.S. Monopolistically competitive intermediate-goods producers deliver a 20% profit margin, implying the elasticity of substitution between different intermediate goods υ equal to 6. For the price stickiness parameter, I set ϕ at 0.7. With this calibration, intermediate-goods firms reset their prices every three and half quarters on average. In the sensitivity analysis I also consider $\phi = 0.5$. The labor productivity is equal to 1.

3) Shock process

Home money growth rate is assumed to be log-normal and can be described by the following autoregressive model:

$$\ln(\mu_{t+1}) = (1-\sigma)\ln(\mu) + \sigma\ln(\mu_t) + \varepsilon_{t+1}$$

$$0 < \sigma < 1, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$$

(4.35)

where μ is the steady-state rate of money growth. ε_{t+1} is the stochastic disturbance term to the home money growth rate following normal distribution with mean zero and variance σ_ε^2. The Foreign country has analogous dynamics for its money growth. I assume that the monetary shock process for each country evolves independently of the other. As discussed before, the time-varying conditional variance of the money supply shock would have a negligible impact, if not nothing, on the relationship between real exchange rates and relative consumption. Therefore, I neglect more complicated modeling of the money growth process.

I estimate Equation (4.35) using U.S. quarterly data of M1 for the period 1973:1 to 2006:2. The long-run value of money growth rates (μ) is 1.0124; autocorrelation of growth rates (σ) is 0.538; and the standard deviation of disturbances (σ_ε) is 0.0113.

4.4 Results

4.4.1 Model Implications for Moments of Interest

I derive second moment statistics of interested variables by simulating the model using the calibration in Table 4.1. I mainly generate artificial data series for real exchange rates, money stock and relative aggregate consumption by simulating the model 1000 times with a sample length of 140 periods each, which is comparable to the time interval

from the period 1973:1 to 2006:2. Table 4.2 presents a selection of second moments from the data and compares them with moments generated by the simulation of the model economies.

The actual data are obtained from Datastream, International Financial Statistics, and OECD Main Economic Indicators Database. I choose the United States as the home country, and an aggregate of the remaining G7 is used for the foreign country. All data are seasonally adjusted quarterly series and logged as well as Hodrick-Prescott filtered.

The results for the moments of interest of relevant variables in the data are presented in the first column of Table 4.2. The second column of Table 4.2 reports these moments generated from the model with baseline calibration. The statistics are averages of sample moments across 1000 simulations.

The last row of Table 4.2 presents cross-correlations between real exchange rates and relative aggregate consumption. The result shows that our model with the baseline calibration can generate a negative correlation between these two variables and match the data (-.31 vs. -.37). In addition, my model generates volatile real exchange rates, as suggested by the data. The standard deviation of real exchange rates is 2.27 times the standard deviation of money stock, which is close to 2.23 in the data. The consumption is relatively smooth whose standard deviation is close to that in the data (.73 vs. .76). As CKM (2002), my pure sticky price model generates less persistence in real exchange rates and consumption than data. Introducing endogenously segmented asset markets does not help improve the model to generate enough persistence in these variables.

I perform sensitivity analyses by decreasing ϕ and γ_{max} in the model respectively in attempt to shed a light on how the extent of price stickiness and the degree of asset

markets segmentation would affect our results. By numerical exercises on the cutoff level of fixed costs around the steady state, I have the conclusion that decreasing γ_{max} will increase the fraction of active households, which causes a decrease on the degree of market segmentation defined as the fraction of inactive households. When there is no market segmentation in the economy, the fraction of inactive households is zero.

The results of the sensitivity analysis are reported in the remaining columns of Table 4.2. The analyses show that with a specifically lower price-stickiness, both real exchange rates and consumption have less volatility and persistence as expected. Consequently, there is a higher negative cross-correlation between them (-.66). The theory behind a higher correlation is that when ϕ decreases, firms can more timely adjust the price in response to a monetary shock to avoid its impact on real outputs. Thus, the dynamics of current consumption of inactive households mainly depends on last period real exchange rates and current relative inflation rates in our model, where they have an opposite influence on C_I. Recalling that current real exchange rates are equal to the ratio of marginal utilities of current active household consumption in both countries, we can expect to see a higher negative interdependence between aggregate consumption and real exchange rates when both volatility and persistence of real exchange rates decrease.

When the degree of asset market segmentation falls or, in other words, more households participate in the asset markets, there are not many changes in the volatility and persistence of variables compared with those in the baseline model. The absolute value of correlation coefficient between real exchange rates and relative consumption decreases along with the fraction of inactive households (-.08). This result is intuitive

because the fewer inactive households, the smaller the distorting effect of inflations on aggregate consumption. So is the impact of relative inflation on relative consumption. The correlation coefficient of our interest increases (its absolute value decreases) till one where there is no market segmentation.

4.4.2 Model Dynamics

In this sub-section, I analyze the dynamics of the model combining sticky prices and endogenously segmented asset markets in response to money supply shocks in the home country.

Figure 4.2 plots impulse responses of the model to one unit of positive money supply shock at time $t = 1$. The model generates a rise in real exchange rates ($\hat{q}_t$), relative inflation ($\hat{\pi}_t - \hat{\pi}_t^*$) and relative real money balances ($\hat{m}_t - \hat{m}_t^*$) after a money injection, which is consistent with the empirical evidence documented in Christiano, Eichenbaum and Evans (2005), among others. The first graph displays the dynamics of shocks. As the shock dies out, real exchange rates decrease and relative inflation falls. In the model, the impact of money supply shocks on relative real money balances mainly depends on two effects: one is the shock's direct effect which positively influences $\hat{m}_t - \hat{m}_t^*$ and the other is the indirect impact from relative inflation which negatively affects $\hat{m}_t - \hat{m}_t^*$. When the latter outweighs the former in initial periods after money injection, relative real money balances still increase over these periods. And then real money declines towards its steady-state value when shock dies at a higher speed than relative inflation falls.

The remaining four graphs in Figure 4.2 display the household's response on consumption pattern to a monetary shock. When money supply increases, relative

consumption of inactive households $(\hat{c}_{it} - \hat{c}_{it}^*)$ decreases because of higher inflation. Inactive households consume goods with the real cash balance they initially hold. Inflation is distorting that is consistent with the analysis in Alvarez, Atkeson and Kehoe (2007). My model also has the same property that the cutoff level of fixed costs $(\bar{\gamma}_t)$ to transfer cash and bonds in the home country increases following a positive money shock. More households become active since the cost of not involving with the asset market increases when inflation increases. My model is different from theirs in the responses of active households consumption. Figure 4.2 shows that the consumption of home active households $(\hat{c}_{at})$ increases on impact and continues to increase and then declines towards its steady state. In Alvarez, Atkeson and Kehoe (2007), $\hat{c}_{at}$ increases first and then decrease monotonically as shocks die out. I attribute this difference to the property that real money balances still increase over some initial periods after shock in our sticky-price model. Only active households absorb the injected money at current period to increase their consumption. When real money balance continues to increase, they keep increasing consumption. Finally, the impact of a money supply shock on aggregate consumption is displayed in the last graph, where relative aggregate consumption $(\hat{c}_t - \hat{c}_t^*)$ decreases immediately and continues decreasing and then increases towards its steady state. Therefore, I visually show that there exists a negative relationship between real change rates and relative aggregate consumption.

4.5 Conclusions

This study seeks to resolve the consumption-real exchange rate anomaly labeled by CKM (2002), which refers to a discrepancy between most international macro models

and data. Most international macro models with frictionless asset markets predict that cross-correlation between real exchange rates and relative consumption is close to unity but this correlation is often negative in the data. Current literatures on this issue mainly focus on introducing incomplete asset markets to break the tight link between these two variables of interest. In this study, I show that maintaining the assumption of complete asset markets, a dynamic sticky-price monetary model augmented with endogenously segmented asset markets can generate negative cross-correlations close to those observed in the data.

In such a model, market segmentation renders the real exchange rate equal to the ratio of marginal utilities of consumption of active households, who participate in asset markets by paying a fixed cost to transfer assets between cash and bonds. Real exchange rates depreciate (increase) in response to a positive money supply shock in the domestic country. The presence of price-stickiness generates real effects of monetary shocks and substantially distorts the effect of inflation on the consumption of inactive households. The result is a decrease in relative aggregate consumption, thereby a negative correlation between real exchange rates and relative consumption.

Table 4.1: Parameterization

Parameter	Calibrated Value	Description
		Preference
β	0.99	Discount factor
θ	1	Labor preference
γ_{max}	0.1 (0.06)	Maximum fixed cost of cash transfer
		Technology
ψ	0.75	Openness index
υ	6	Elasticity of substitution across intermediate goods
ϕ	0.7 (0.5)	Probability of resetting prices
A	1	Labor productivity
		Shock process
μ	1.0123	Long-run money growth rate
σ	0.538	Persistence of monetary shock
σ_{ε}	0.0113	Standard deviation of disturbances

Note: Numbers in parentheses are used in sensitivity analyses.

Table 4.2: Second Moments of Real Exchange Rates and Consumption

Statistics	Data	Baseline	Low price stickiness $(\phi = 0.5)$	Low market segmentation $(\gamma_{max} = 0.06)$
Standard deviation relative to M1				
Real exchange rates	2.23	2.27	1.39	2.20
Consumption	0.76	0.73	0.61	0.72
Autocorrelation				
Real exchange rates	0.85	0.71	0.60	0.74
Consumption	0.91	0.78	0.49	0.85
Cross-correlation				
Between real exchange rates and relative consumption	-0.37	-0.31	-0.66	-0.09

Notes: 1. The statistics under the header of *Data* are computed from G7 countries data, which are logged and HP filtered with quarterly frequency over the period from 1973:1 to 2006:2.

2. The statistics in the remaining columns are based on 1000 simulations of the model. *Baseline* denotes the model with relatively high price stickiness and high asset market segmentation, where $\phi = 0.7$ and $\gamma_{max} = 0.1$.

3. The standard deviations of real exchange rates and consumption are divided by the standard deviation of M1.

Figure 4.1: Timing in the Two Markets

Asset Market

| Starting Bonds B_H, B_F | Rate of money growth μ observed | *Asset Market Constraint* Bonds: $$B_H + S^* B_F = \int J^* B_H' + S^* \int J^* {}^* B_F' + P(x + \gamma)$$ if active. $$B_H + S^* B_F = \int J^* B_H' + S^* \int J^* {}^* B_F'$$ if inactive. | Ending Bonds $B_H{}'$, $B_F{}'$ |

If transfer x, pay fixed cost $P\gamma$

Cash-in-Advance Constraint
Consumption:
$c = n + x$ if active.
$c = n$ if inactive.

Goods Market

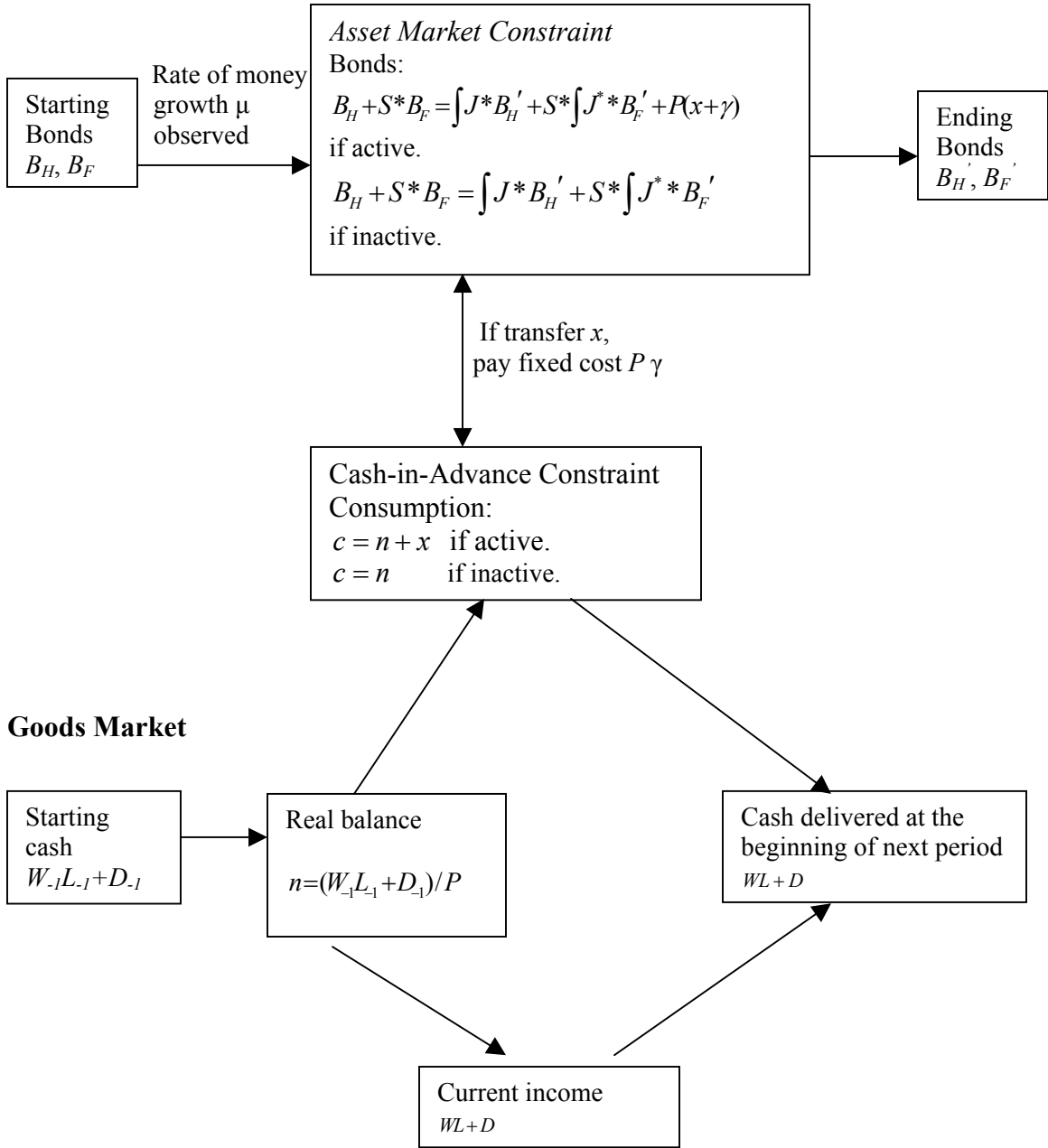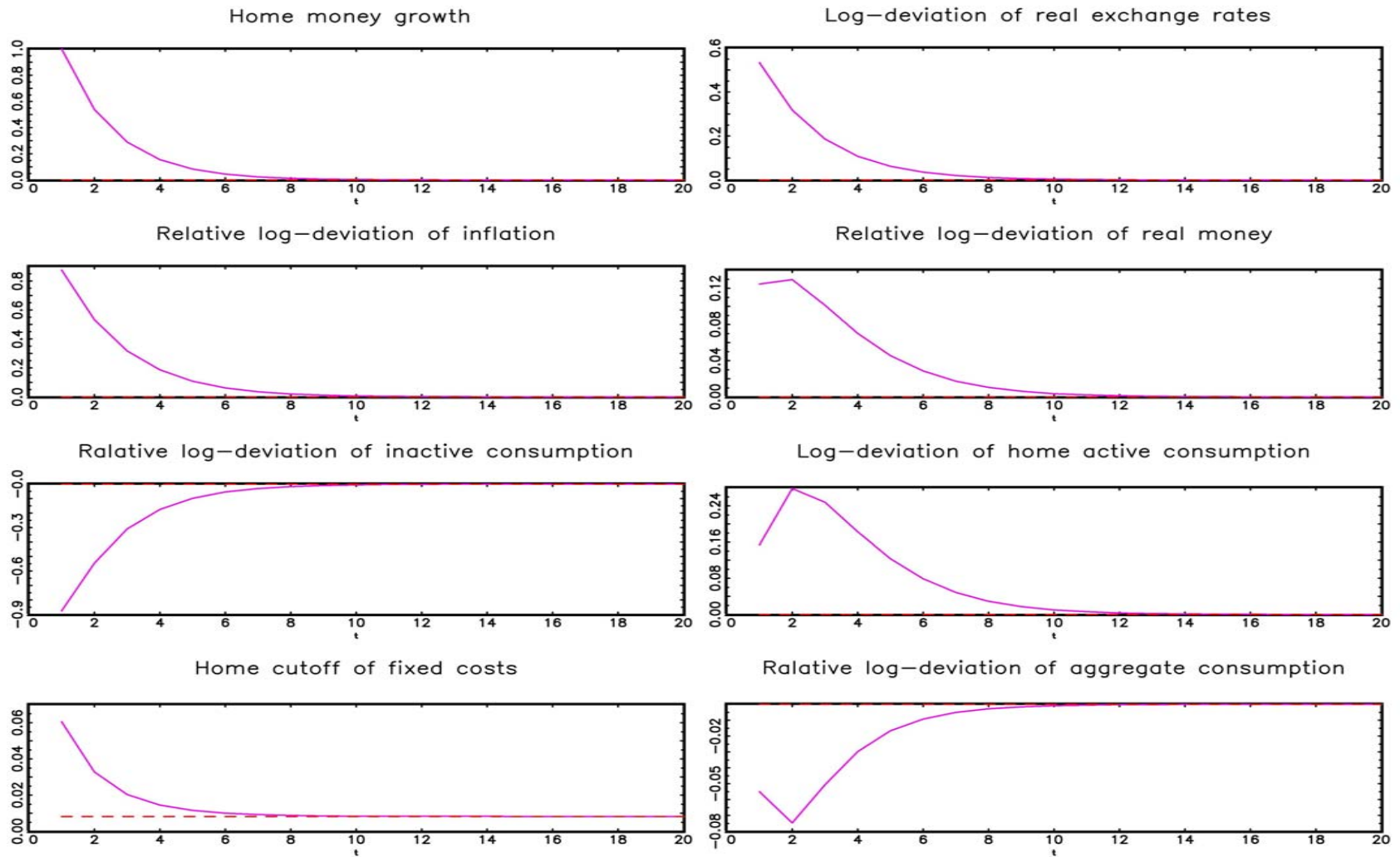| Starting cash $W_{-1}L_{-1} + D_{-1}$ | Real balance $n = (W_{-1}L_{-1} + D_{-1})/P$ | Cash delivered at the beginning of next period $WL + D$ |

Current income
$WL + D$

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Figure 4.2: Impulse Response Functions to a Positive Money Supply Shock

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APPENDIX A

Deduction and Estimation of FIGJRGARCH Model

The conditional variance in the general GJR-GARCH model takes the form of

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{A.1}$$

From this equation I get

$$
\begin{aligned}
(1 - \beta_1 L) h_t &= \omega + \alpha_1 (1 + \frac{\alpha_2}{\alpha_1} d_{t-1}) \varepsilon_{t-1}^2 \\
&\equiv \omega + \alpha_1 (1 + \delta d_{t-1}) \varepsilon_{t-1}^2 \\
&= \omega + \alpha_1 L (1 + \delta d_t) \varepsilon_t^2
\end{aligned}
\tag{A.2}
$$

Define $g(\varepsilon_t) = (1 + \delta d_t) \varepsilon_t^2$ and $v_t = g(\varepsilon_t) - h_t$. Equation (A.2) becomes:

$$(1 - \beta_1 L - \alpha_1 L) g(\varepsilon_t) = \omega + (1 - \beta_1 L) v_t \tag{A.3}$$

Following the modification on the original GARCH model by Chung (1999), I rewrite Equation (A.3) as:

$$(1 - \beta_1 L - \alpha_1 L)[g(\varepsilon_t) - h_0] = (1 - \beta_1 L) v_t \tag{A.4}$$

Then, the lag polynomial $(1 - \beta_1 L - \alpha_1 L)$ can be factorized as $(1 - \psi_1 L)(1 - L)^d$. So I have

$$(1 - \psi_1 L)(1 - L)^d [g(\varepsilon_t) - h_0] = (1 - \beta_1 L) v_t \tag{A.5}$$

Plugging the formula of $g(\varepsilon_t)$ and v_t into Equation (A.5), finally I get

$$h_t = (1 - \beta_1 L)(1 + \delta d_t) \varepsilon_t^2 - (1 - \psi_1 L)(1 - L)^d [(1 + \delta d_t) \varepsilon_t^2 - h_0] + \beta_1 h_{t-1} \tag{A.6}$$

The most straightforward estimation method for this model is the Approximate Maximum Likelihood Estimation (AMLE), which is also called the Conditional Sum of

Squares (CSS) estimation. The CSS method was originally proposed in the context of ARFIMA models by Hosking (1984). A key step is to compute the fractional differencing operator $(1-L)^d$ defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \tag{A.7}$$

with $\Gamma(.)$ being the gamma function. Here I expand it with the binomial expansion (A.7) and truncates the infinite series at the first available observation.[18]

In addition, in order to make the conditional variance in Equation (A.6) non-negative, I restrict $h_0 \geq 0$, $0 < \delta < 1$, and $0 < \psi_1 < \beta_1 < d < 1$ by following Chung (1999).

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[18] Thanks Dr. Bidarkota for valuable guidance on this GAUSS programming. The specific estimation procedures also can refer to Bidarkota and Kiani (2004).

APPENDIX B

EQUILIBRIUM EQUATIONS AND STEADY STATE

In this appendix, I normalize all nominal variables, list the resulting equilibrium equations, and finally derive the steady state of the model.

B.1 Defining Real Variables

I make most nominal variables stationary by dividing them by the relevant final-goods price index. Let

$$m_t = \frac{M_t}{P_t}; \qquad\qquad m_t^* = \frac{M_t^*}{P_t^*}$$

$$\omega_t = \frac{W_t}{P_t}; \qquad\qquad \omega_t^* = \frac{W_t^*}{P_t^*}$$

$$b_h(\xi^{t+1}) = \frac{B_H(\xi^{t+1})}{P_t}; \qquad\qquad b_f^*(\xi^{t+1}) = \frac{B_F^*(\xi^{t+1})}{P_t^*}$$

$$b_f(\xi^{t+1}) = \frac{B_F(\xi^{t+1})}{P_t^*}; \qquad\qquad b_h^*(\xi^{t+1}) = \frac{B_H^*(\xi^{t+1})}{P_t}$$

$$p_{ht} = \frac{P_{Ht}}{P_t}; \qquad\qquad p_{ft}^* = \frac{P_{Ft}^*}{P_t^*}$$

$$p_{ft} = \frac{P_{Ft}}{P_t}; \qquad\qquad p_{ht}^* = \frac{P_{Ht}^*}{P_t^*}$$

$$\tilde{p}_{ht} = \frac{\tilde{P}_{Ht}}{P_t}; \qquad\qquad \tilde{p}_{ft}^* = \frac{\tilde{P}_{Ft}^*}{P_t^*}$$

$$\tilde{p}_{ft} = \frac{\tilde{P}_{Ft}}{P_t}; \qquad\qquad \tilde{p}_{ht}^* = \frac{\tilde{P}_{Ht}^*}{P_t^*} \qquad\qquad \text{(B.1)}$$

For nominal interest rates, nominal exchange rates and nominal price index itself,

I respectively define the gross real interest rate $1 + r_t = (1 + i_t)\dfrac{P_t}{P_{t+1}}$; the real exchange rate

$Q_t = \dfrac{S_t P_t^*}{P_t}$ and the gross inflation rate $\pi_t = \dfrac{P_t}{P_{t-1}}$.

B.2 Stationary Equilibrium Equations

Take the home country for example. Equilibrium equations can be expressed in

real terms as follows:

1) CIA constraint: $C_t = m_t$

2) Labor supply function: $\theta = C_t^{-\rho} \omega_t$

3) Consumption Euler equation: $E_t \left[\beta \left(\dfrac{C_{t+1}}{C_t} \right)^{-\rho} (1 + r_t) \right] = 1$

4) UIP condition in real terms: $\dfrac{Q_t (1 + r_t)}{1 + r_t^*} = \dfrac{E_t \left[C_{t+1}^{-\rho} \cdot Q_{t+1} \right]}{E_t (C_{t+1}^{-\rho})}$

5) Risk-sharing condition: $Q_t = \kappa \left(\dfrac{C_t^*}{C_t} \right)^{-\rho}$

6) Home intermediate-goods demand function: $\underline{Y}_{Ht} = \psi \dfrac{Y_t}{p_{ht}}$

7) Foreign intermediate-goods demand function: $\underline{Y}_{Ft} = (1 - \psi) \dfrac{Y_t}{p_{ft}}$

8) Intermediate-goods' prices relation: $p_{ht}^{\psi} p_{ft}^{1-\psi} = 1$

9) Marginal cost of intermediate-goods producer: $mc_t = \dfrac{\omega_t}{A}$

10) Optimal pricing conditions:

$$\tilde{p}_{ht} = \frac{\upsilon}{\upsilon-1} \cdot \frac{E_t \sum_{\tau=t}^{\infty} \rho'_{t,\tau} (\phi\pi^{-\upsilon})^{\tau-t} mc_\tau p_{h\tau}{}^{\upsilon} \underline{Y}_{H\tau} (\prod_{k=t+1}^{\tau} \pi_k)^{\upsilon}}{E_t \sum_{\tau=t}^{\infty} \rho'_{t,\tau} (\phi\pi^{1-\upsilon})^{\tau-t} p_{h\tau}{}^{\upsilon} \underline{Y}_{H\tau} (\prod_{k=t+1}^{\tau} \pi_k)^{\upsilon-1}}$$

$$\tilde{p}_{ht}^* = \frac{\upsilon}{\upsilon-1} \cdot \frac{E_t \sum_{\tau=t}^{\infty} \rho'_{t,\tau} (\phi\pi^{-\upsilon})^{\tau-t} mc_\tau p_{h\tau}^{*}{}^{\upsilon} \underline{Y}_{H\tau}^* (\prod_{k=t+1}^{\tau} \pi_k^*)^{\upsilon}}{E_t \sum_{\tau=t}^{\infty} \rho'_{t,\tau} (\phi\pi^{1-\upsilon})^{\tau-t} Q_\tau p_{h\tau}^{*}{}^{\upsilon} \underline{Y}_{H\tau}^* (\prod_{k=t+1}^{\tau} \pi_k^*)^{\upsilon-1}}$$

11) Intermediate-goods price index:

$$p_{ht} = \left[(1-\phi)\tilde{p}_{ht}{}^{1-\upsilon} + \phi\left(\frac{\pi p_{ht-1}}{\pi_t} \right)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}$$

$$p_{ht}^* = \left[(1-\phi)\tilde{p}_{ht}^{*}{}^{1-\upsilon} + \phi\left(\frac{\pi p_{ht-1}^*}{\pi_t^*} \right)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}$$

12) Labor demand function:

$$L_t = \int_0^1 L_t(i)di = \int_0^1 \frac{1}{A} [\underline{Y}_{Ht}(i) + \underline{Y}_{Ht}^*(i)]di$$

$$= \frac{1}{A}\left[\left(\frac{p'_{ht}}{p_{ht}} \right)^{-\upsilon} \underline{Y}_{Ht} + \left(\frac{p'^{*}_{ht}}{p_{ht}^*} \right)^{-\upsilon} \underline{Y}_{Ht}^* \right]$$

where $\quad p'_{ht} = \left[\int_0^1 p_{ht}(i)^{-\upsilon}di \right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{p}_{ht}{}^{-\upsilon} + \phi\left(\frac{\pi p'_{ht-1}}{\pi_t} \right)^{-\upsilon} \right]^{-\frac{1}{\upsilon}}$

$$p'^{*}_{ht} = \left[\int_0^1 p_{ht}^*(i)^{-\upsilon}di \right]^{-\frac{1}{\upsilon}} = \left[(1-\phi)\tilde{p}_{ht}^{*}{}^{-\upsilon} + \phi\left(\frac{\pi p'^{*}_{ht-1}}{\pi_t^*} \right)^{-\upsilon} \right]^{-\frac{1}{\upsilon}}$$

13) Money supply: $m_t = \mu_t m_{t-1}/\pi_t$

B.3 Steady State

Here I present the steady state of the model. I derive the analytical solution for the zero growth steady state of the two economies in the absence of monetary shocks. Under symmetry, households from both countries hold zero assets in the steady state: $b_h = b_f = b_f^* = b_h^* = 0$. Here, I use variables without time script to denote steady state values. I impose symmetry to find the steady state values of the remaining variables in the model. The symmetric property of the solution is verified by using GAUSS to solve the steady state numerically.

$$r = r^* = \frac{1-\beta}{\beta}$$

$$\pi = \mu$$

$$\omega = \omega^* = \left(\frac{\upsilon - 1}{\upsilon} \cdot A \right)$$

$$C = C^* = Y = Y^* = m = m^* = \left(\frac{\omega}{\theta} \right)^{\frac{1}{\rho}}$$

$$p_h = \tilde{p}_h = p_f^* = \tilde{p}_f^* = 1$$

$$p_h^* = \tilde{p}_h^* = p_f = \tilde{p}_f = 1$$

$$Q = 1$$

$$\underline{Y}_H = \underline{Y}_F^* = \psi C$$

$$\underline{Y}_F = \underline{Y}_H^* = (1-\psi)C$$

$$L = L^* = \frac{C}{A}$$

(B.2)

APPENDIX C

LOG-LINEARIZED EQUILIBRIUM EQUATIONS

In this appendix, I list all log-linearized equilibrium equations. Non-linear structural equations of the model economies are log-linearly approximated around the steady state of the economies. The whole system is written in the following fifteen variables[19]:
$$\hat{c}_t - \hat{c}_t^*, \hat{m}_t - \hat{m}_t^*, \hat{\omega}_t - \hat{\omega}_t^*, m\hat{c}_t - m\hat{c}_t^*, \hat{r}_t - \hat{r}_t^*, \hat{\bar{p}}_{ft} - \hat{\bar{p}}_{ft}^*, \hat{\bar{p}}_{ft} - \hat{\bar{p}}_{ht}^*,$$
$$\hat{p}_{ht} - \hat{p}_{ft}^*, \hat{p}_{ft} - \hat{p}_{ht}^*, \underline{\hat{y}}_{ht} - \underline{\hat{y}}_{ft}^*, \underline{\hat{y}}_{ft} - \underline{\hat{y}}_{ht}^*, \hat{y}_t - \hat{y}_t^*, \hat{\ell}_t - \hat{\ell}_t^*, \hat{\pi}_t - \hat{\pi}_t^*, \hat{q}_t$$
These equations are used in linearized form, mainly expressed as differences between the home country variables and foreign country counterparts[20]. Here, the circumflex of a variable denotes the log-deviation from its steady-state value ($\hat{\alpha}_t = \log \alpha_t - \log \alpha$).

Listed below are fifteen linear conditions that describe the dynamics of these variables. The dynamics of the inflation rate, thereby that of real money balances, is the key to understanding the behavior of real exchange rates and the foreign exchange risk premium in the model. It is not feasible to solve directly for the dynamics of each country's inflation rate individually in the model. Therefore, I solve for the dynamics of the other useful linear combinations of these variables------the sum of the home and foreign inflation rates ($\hat{\pi}_t + \hat{\pi}_t^*$). To accomplish this, I define two more sequences:

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[19] The household problem implies two pairs of redundant equations about the holding of home and foreign bonds. It means that households adjust their bonds holding in an arbitrary fixed proportion and the way of holding bonds does not affect the dynamics of other variables in the model. Therefore, I do not list bonds variables explicitly here.

[20] Following common practice in both theoretical and empirical macroeconomics literature (see Bergin 2004), I assume that Home and Foreign economies have the same deep behavioral parameters.

$(\hat{\tilde{p}}_{ht} + \hat{\tilde{p}}_{ft}^{*})$ and $(\hat{\tilde{p}}_{ft} + \hat{\tilde{p}}_{ht}^{*})$. With the assumption of log-normal distributed money supply

growth rate, all variables of interest are also following the log-normal distribution in the

model economy.

$$\hat{c}_t - \hat{c}_t^{*} = \hat{m}_t - \hat{m}_t^{*} \tag{C.1}$$

$$\hat{\omega}_t - \hat{\omega}_t^{*} = \rho(\hat{c}_t - \hat{c}_t^{*}) \tag{C.2}$$

$$(1-\beta)(\hat{r}_t - \hat{r}_t^{*}) = \rho E_t(\hat{c}_{t+1} - \hat{c}_{t+1}^{*}) - \rho(\hat{c}_t - \hat{c}_t^{*}) - \frac{1}{2}\rho^2[\text{var}_t(\hat{c}_{t+1}) - \text{var}_t(\hat{c}_{t+1}^{*})] \tag{C.3}$$

$$\hat{q}_t = \rho(\hat{c}_t - \hat{c}_t^{*}) \tag{C.4}$$

$$\hat{y}_t - \hat{y}_t^{*} = \hat{c}_t - \hat{c}_t^{*} \tag{C.5}$$

$$\underline{\hat{y}}_{ht} - \underline{\hat{y}}_{ft}^{*} = (\hat{y}_t - \hat{y}^{*}) - (\hat{p}_{ht} - \hat{p}_{ft}^{*}) \tag{C.6}$$

$$\underline{\hat{y}}_{ft} - \underline{\hat{y}}_{ht}^{*} = (\hat{y}_t - \hat{y}_t^{*}) - (\hat{p}_{ft} - \hat{p}_{ht}^{*}) \tag{C.7}$$

$$\psi(\hat{p}_{ht} - \hat{p}_{ft}^{*}) + (1-\psi)(\hat{p}_{ft} - \hat{p}_{ht}^{*}) = 0 \tag{C.8}$$

$$m\hat{c}_t - m\hat{c}_t^{*} = \hat{\omega}_t - \hat{\omega}_t^{*} \tag{C.9}$$

$$\hat{\tilde{p}}_{ht} - \hat{\tilde{p}}_{ft}^{*} = \beta\phi E_t(\hat{\tilde{p}}_{ht+1} - \hat{\tilde{p}}_{ft+1}^{*}) + (1-\beta\phi)(m\hat{c}_t - m\hat{c}_t^{*}) + \beta\phi E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{*})$$
$$+ \frac{1}{2}\beta\phi\{\text{var}_t[(1-\beta\phi)m\hat{c}_{t+1} + \hat{\pi}_{t+1}] - \text{var}_t[(1-\beta\phi)m\hat{c}_{t+1}^{*} + \hat{\pi}_{t+1}^{*}]\}$$
$$- \beta\phi\rho\{\text{cov}_t[\hat{c}_{t+1}, (1-\beta\phi)m\hat{c}_{t+1} + \hat{\pi}_{t+1}] - \text{cov}_t[\hat{c}_{t+1}^{*}, (1-\beta\phi)m\hat{c}_{t+1}^{*} + \hat{\pi}_{t+1}^{*}]\}$$
$$\tag{C.10}$$

$$\hat{\tilde{p}}_{ht} + \hat{\tilde{p}}_{ft}^{*} = \beta\phi E_t(\hat{\tilde{p}}_{ht+1} + \hat{\tilde{p}}_{ft+1}^{*}) + (1-\beta\phi)(m\hat{c}_t + m\hat{c}_t^{*}) + \beta\phi E_t(\hat{\pi}_{t+1} + \hat{\pi}_{t+1}^{*})$$
$$+ \frac{1}{2}\beta\phi\{\text{var}_t[(1-\beta\phi)m\hat{c}_{t+1} + \hat{\pi}_{t+1}] + \text{var}_t[(1-\beta\phi)m\hat{c}_{t+1}^{*} + \hat{\pi}_{t+1}^{*}]\}$$
$$- \beta\phi\rho\{\text{cov}_t[\hat{c}_{t+1}, (1-\beta\phi)m\hat{c}_{t+1} + \hat{\pi}_{t+1}] + \text{cov}_t[\hat{c}_{t+1}^{*}, (1-\beta\phi)m\hat{c}_{t+1}^{*} + \hat{\pi}_{t+1}^{*}]\}$$

$$\hat{\bar{p}}_{ft} - \hat{\bar{p}}_{ht}^* = \beta\phi E_t(\hat{\bar{p}}_{ft+1} - \hat{\bar{p}}_{ht+1}^*) + (1-\beta\phi)(m\hat{c}_t^* - m\hat{c}_t + 2\hat{q}_t) + \beta\phi E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*)$$

$$+ \frac{1}{2}\beta\phi\left\{\operatorname{var}_t[(1-\beta\phi)(m\hat{c}_{t+1}^* + \hat{q}_{t+1}) + \hat{\pi}_{t+1}] - \operatorname{var}_t[(1-\beta\phi)(m\hat{c}_{t+1} - \hat{q}_{t+1}) + \hat{\pi}_{t+1}^*]\right\}$$

$$- \beta\phi\rho\left\{\operatorname{cov}_t[\hat{c}_{t+1},(1-\beta\phi)(m\hat{c}_{t+1}^* + \hat{q}_{t+1}) + \hat{\pi}_{t+1}] - \operatorname{cov}_t[\hat{c}_{t+1}^*,(1-\beta\phi)(m\hat{c}_{t+1} - \hat{q}_{t+1}) + \hat{\pi}_{t+1}^*]\right\}$$

$$\hat{\bar{p}}_{ft} + \hat{\bar{p}}_{ht}^* = \beta\phi E_t(\hat{\bar{p}}_{ft+1} + \hat{\bar{p}}_{ht+1}^*) + (1-\beta\phi)(m\hat{c}_t^* + m\hat{c}_t) + \beta\phi E_t(\hat{\pi}_{t+1} + \hat{\pi}_{t+1}^*)$$

$$+ \frac{1}{2}\beta\phi\left\{\operatorname{var}_t[(1-\beta\phi)(m\hat{c}_{t+1}^* + \hat{q}_{t+1}) + \hat{\pi}_{t+1}] + \operatorname{var}_t[(1-\beta\phi)(m\hat{c}_{t+1} - \hat{q}_{t+1}) + \hat{\pi}_{t+1}^*]\right\}$$

$$- \beta\phi\rho\left\{\operatorname{cov}_t[\hat{c}_{t+1},(1-\beta\phi)(m\hat{c}_{t+1}^* + \hat{q}_{t+1}) + \hat{\pi}_{t+1}] + \operatorname{cov}_t[\hat{c}_{t+1}^*,(1-\beta\phi)(m\hat{c}_{t+1} - \hat{q}_{t+1}) + \hat{\pi}_{t+1}^*]\right\}$$

$$- 2\beta\phi\operatorname{var}_t[(1-\beta\phi)\hat{q}_{t+1}]$$

$$\tag{C.11}$$

$$\hat{p}_{ht} = (1-\phi)\hat{\bar{p}}_{ht} + \phi(\hat{p}_{ht-1} - \hat{\pi}_t)$$

$$\hat{p}_{ht} - \hat{p}_{ft}^* = (1-\phi)(\hat{\bar{p}}_{ht} - \hat{\bar{p}}_{ft}^*) + \phi(\hat{p}_{ht-1} - \hat{p}_{ft-1}^*) - \phi(\hat{\pi}_t - \hat{\pi}_t^*)$$

$$\tag{C.12}$$

$$\hat{p}_{ht}^* = (1-\phi)\hat{\bar{p}}_{ht}^* + \phi(\hat{p}_{ht-1}^* - \hat{\pi}_t^*)$$

$$\hat{p}_{ft} - \hat{p}_{ht}^* = (1-\phi)(\hat{\bar{p}}_{ft} - \hat{\bar{p}}_{ht}^*) + \phi(\hat{p}_{ft-1} - \hat{p}_{ht-1}^*) - \phi(\hat{\pi}_t - \hat{\pi}_t^*)$$

$$\tag{C.13}$$

$$\hat{\ell}_t = \psi\underline{\hat{y}}_{ht} + (1-\psi)\underline{\hat{y}}_{ht}^*$$

$$\hat{\ell}_t - \hat{\ell}_t^* = \psi(\underline{\hat{y}}_{ht} - \underline{\hat{y}}_{ft}^*) - (1-\psi)(\underline{\hat{y}}_{ft} - \underline{\hat{y}}_{ht}^*)$$

$$\tag{C.14}$$

$$\hat{m}_t - \hat{m}_t^* = (\hat{m}_{t-1} - \hat{m}_{t-1}^*) - (\hat{\pi}_t - \hat{\pi}_t^*) + (\hat{\mu}_t - \hat{\mu}_t^*) \tag{C.15}$$

APPENDIX D

EXPRESSIONS OF CONDITIONAL VARIANCE/COVARIANCE

To help make an appropriate guess for the solution to System (3.36) in the text, we need to know first what N_t looks like. In particular, I ask here how to express the relative conditional variance of gross inflation rates or real money balances and relative conditional covariance of these two variables. It is the key to deriving an explicit expression for the foreign exchange risk premium, as we can see from Equation (3.47). To do so, I simultaneously solve for the dynamics of additional two variables: the world real money balances $(\hat{m}_t + \hat{m}_t^*)$ and the sum of national inflation rates $(\hat{\pi}_t + \hat{\pi}_t^*)$. Following similar techniques used in the text, I obtain the following equations in these two variables:

$$\hat{m}_t + \hat{m}_t^* = (\hat{m}_{t-1} + \hat{m}_{t-1}^*) - (\hat{\pi}_t + \hat{\pi}_t^*) + (\hat{\mu}_t + \hat{\mu}_t^*)$$

$$E_t(\hat{\pi}_{t+1} + \hat{\pi}_{t+1}^*) = \left[\frac{1}{\beta} + \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}\right](\hat{\pi}_t + \hat{\pi}_t^*) - \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}(\hat{m}_{t-1} + \hat{m}_{t-1}^*) \qquad \text{(D.1)}$$
$$- \frac{\rho(1-\phi)(1-\beta\phi)}{\beta\phi}(\hat{\mu}_t + \hat{\mu}_t^*) - \frac{1-\phi}{\beta\phi} N_t'$$

where N_t' contains conditional variance and covariance terms which are similar to those in N_t and is derived from log-linearized equilibrium equations in Appendix C. Considering together both System (3.36) in the text and System (D.1) here, we can infer that Home inflation rate does not respond contemporaneously to Foreign monetary shock, and vise versa. Further, Home inflation responds to Home shock in the same magnitude as that Foreign inflation responds to Foreign shock. Therefore, our model economy

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produces the following relation in, say, the relative conditional variance of national inflation rates:

$$\text{var}_t(\hat{\pi}_{t+1}) - \text{var}_t(\hat{\pi}_{t+1}^*) = \lambda^2[\text{var}_t(\hat{\mu}_{t+1}) - \text{var}_t(\hat{\mu}_{t+1}^*)] \tag{D.2}$$

where λ is the response coefficient of inflation rates to domestic monetary shocks which is a function of deep parameters. This is why I guess the coefficient A_2 in front of η_t in the solution form (3.39).

In the same way, we can express the relative conditional variance of real money balances as:

$$\text{var}_t(\hat{m}_{t+1}) - \text{var}_t(\hat{m}_{t+1}^*) = (1 - A_2)^2[\text{var}_t(\hat{\mu}_{t+1}) - \text{var}_t(\hat{\mu}_{t+1}^*)] \tag{D.3}$$

and the relative conditional covariance between real money balances and national inflation rates as:

$$\text{cov}_t(\hat{m}_{t+1}, \hat{\pi}_{t+1}) - \text{cov}_t(\hat{m}_{t+1}^*, \hat{\pi}_{t+1}^*) = A_2(1 - A_2)[\text{var}_t(\hat{\mu}_{t+1}) - \text{var}_t(\hat{\mu}_{t+1}^*)] \tag{D.4}$$

From the AR-GARCH model of monetary growth (Equation (3.35)), I obtain

$$\begin{aligned}\text{var}_t(\hat{\mu}_{t+1}) - \text{var}_t(\hat{\mu}_{t+1}^*) &= \text{var}_t(\varepsilon_{t+1}) - \text{var}_t(\varepsilon_{t+1}^*)\\&= E_t(h_{t+1} - h_{t+1}^*) = \alpha_1(\varepsilon_t^2 - \varepsilon_t^{*2}) + \alpha_2(h_t - h_t^*)\end{aligned} \tag{D.5}$$

Finally, we can express the foreign exchange risk premium as:

$$rp_t = \frac{1}{2}\rho^2(1 - A_2)^2[\alpha_1(\varepsilon_t^2 - \varepsilon_t^{*2}) + \alpha_2(h_t - h_t^*)] \tag{D.6}$$

which is Equation (3.48) in the text.

APPENDIX E

DETAILS OF DATA

Data for the US is used for the home country, and an aggregate of the remaining G7 is used for the foreign country. Five series are needed in our context to perform the UIP test, and to compute the cross correlation between real exchange rate changes and real money differential changes. They are the money supply, interest rates, price levels, exchange rates, and output. Money supply is measured as either M0 or M1 or M2, the interest rate as either the Treasury bill rate or money market rate or call money rate, the price level as the CPI, the exchange rate for each country as the bilateral rate with the US dollar, the output as national GDP. Output data is needed mainly for obtaining time-varying weights to compute Foreign aggregate variables. Specifically, aggregate variables of the remaining G7 countries are computed as a geometric weighted average, where the weights are based on each country's share of total real GDP. All data are seasonally adjusted quarterly series for the period 1973:1 to 2006:2, obtained from Datastream, International Financial Statistics, and OECD Main Economic Indicators Database.

E.1 Original Data Series

1) M1 money supply, billions of dollars, U.S.

2) M1 money supply, billions of yens, Japan

3) M0 money supply, billions of British pounds, U.K.

4) M1 money supply, billions of Canadian dollars, Canada

5) M1 money supply, billions of French francs, France

6) M1 money supply, billions of Deutsche marks, Germany

7) M2 money supply, billions of Italian liras, Italy

8) 3-month Treasury Bill Rate, US

9) Call money rate, Japan

10) 3-month Treasury Bill Rate, UK

11) 3-month Treasury Bill Rate, Canada

12) 3-month Treasury Bill Rate, France

13) Call money rate, Germany

14) Money market rate, Italy

15) Consumer price index, US

16) Consumer price index, Japan

17) Consumer price index, UK

18) Consumer price index, Canada

19) Consumer price index, France

20) Consumer price index, Germany

21) Consumer price index, Italy

22) Nominal exchange rate, Japan – US

23) Nominal exchange rate, US – UK

24) Nominal exchange rate, Canada – US

25) Nominal exchange rate, France – US

26) Nominal exchange rate, Germany – US

27) Nominal exchange rate, Italy – US

28) Gross Domestic Product, US

29) Gross Domestic Product, Japan

30) Gross Domestic Product, UK

31) Gross Domestic Product, Canada

32) Gross Domestic Product, France

33) Gross Domestic Product, Germany

34) Gross Domestic Product, Italy

E.2 Constructed Data Series

1) Real GDP Share, Japan

2) Real GDP Share, UK

3) Real GDP Share, Canada

4) Real GDP Share, France

5) Real GDP Share, Germany

6) Real GDP Share, Italy

7) Aggregate M1, Japan, UK, Canada, France, Germany, Italy

8) Aggregate CPI, Japan, UK, Canada, France, Germany, Italy

9) Aggregate interest rate, Japan, UK, Canada, France, Germany, Italy

10) Aggregate exchange rate, Japan, UK, Canada, France, Germany, Italy

11) Log CPI differential, US – Japan, UK, Canada, France, Germany, Italy

12) Change in log real exchange rate, US – Japan, UK, Canada, France, Germany, Italy

13) Real interest rate differential, US – Japan, UK, Canada, France, Germany, Italy

14) Change in log real money differential, US – Japan, UK, Canada, France, Germany, Italy

APPENDIX F

STEADY STATE AND LOG-LINEARIZED EQUILIBRIUM EQUATIONS

In this appendix, I list the steady state solution to the normalized model and all equilibrium equations when non-linear structural equations are log-linearly approximated around their steady states. In what follows, all lower-case letters denote real variables corresponding to their nominal counterparts in the model. The circumflex on a variable denotes log-deviation from its steady-state value ($\hat{\alpha}_t = \log \alpha_t - \log \alpha$).

F.1 Steady State

When applicable I derive the analytical solution for a zero growth steady state of the two-country economies in the absence of monetary shocks. I use variables without time script to denote steady state values. Steady state value for consumption of active households (C_A, C_A^*) and cutoff levels of the fixed cost ($\overline{\gamma}, \overline{\gamma}^*$) are solved numerically using GAUSS. I impose symmetry to find the steady state of the model economy. The symmetric property of the solution is verified when using GAUSS to solve the steady state numerically.

$$\pi = \mu$$

$$\omega = \omega^* = \left(\frac{\upsilon - 1}{\upsilon} \cdot A \right)$$

$$C_I = C_I^* = \frac{\beta}{\theta \pi} \cdot \omega$$

$$Y = Y^* = m = m^* = C_I \cdot \pi$$

$$p_h = \tilde{p}_h = p_f^* = \tilde{p}_f^* = 1$$

$$p_h^* = \tilde{p}_h^* = p_f = \tilde{p}_f = 1$$

$$Q = 1$$

$$\underline{Y}_H = \underline{Y}_F^* = \psi Y$$

$$\underline{Y}_F = \underline{Y}_H^* = (1-\psi)Y$$

$$L = L^* = \frac{Y}{A}$$

$$d = d^* = (1-\frac{\omega}{A}) \cdot Y$$

F.2 Log-linear Equilibrium Equations

I list here log-linearized equilibrium equations system of interest, mainly expressed as differences between the home country variables and foreign country counterparts. I call it *difference system*. Since it is not feasible to solve directly and analytically for the dynamics of each country's fundamentals, I need to simultaneously solve a system consisting of the summation of home variables and foreign variables, which I call *summation system*. When taking both countries as a whole, the exchange rate will drop off from the whole system. Therefore those equations containing the exchange rate in *difference system* are different from those in *summation system*. I only list such equations for *summation system*. The remainder has the same functional form in both systems.

$$\hat{\omega}_t - \hat{\omega}_t^* = E_t(\hat{c}_{It+1} - \hat{c}_{It+1}^*) + E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) \tag{F.1}$$

$$E_t(\hat{c}_{It+1} - \hat{c}_{It+1}^*) + E_t(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) = (2\psi - 1)*(\hat{y}_t - \hat{y}_t^*) + 2(1-\psi)*\hat{q}_t$$

$$E_t(\hat{c}_{It+1} + \hat{c}_{It+1}^*) + E_t(\hat{\pi}_{t+1} + \hat{\pi}_{t+1}^*) = (\hat{y}_t + \hat{y}_t^*) \tag{F.2}$$

$$g_1*(\hat{c}_{At} - \hat{c}_{At}^*) + g_2*(\hat{c}_{It} - \hat{c}_{It}^*) = Y*(\hat{y}_t - \hat{y}_t^*)$$

$$\text{where:} \quad g_1 = 1/\gamma_{max}*(C_A - C_I + \overline{\gamma}) + 1/\gamma_{max}*\overline{\gamma}C_A \tag{F.3}$$

$$g_2 = 1/\gamma_{max}*(C_A - C_I + \overline{\gamma})*(C_I - C_A) + (1 - 1/\gamma_{max}*\overline{\gamma})C_I$$

$$\hat{q}_t = (\hat{c}_{At} - \hat{c}^*_{At}) \tag{F.4}$$

$$\hat{y}_t - \hat{y}^*_t = \hat{m}_t - \hat{m}^*_t \tag{F.5}$$

$$\underline{\hat{y}}_{ht} - \underline{\hat{y}}^*_{ft} = (\hat{y}_t - \hat{y}^*_t) - (\hat{p}_{ht} - \hat{p}^*_{ft}) \tag{F.6}$$

$$\underline{\hat{y}}_{ft} - \underline{\hat{y}}^*_{ht} = (\hat{y}_t - \hat{y}^*_t) - (\hat{p}_{ft} - \hat{p}^*_{ht}) \tag{F.7}$$

$$\psi(\hat{p}_{ht} - \hat{p}^*_{ft}) + (1-\psi)(\hat{p}_{ft} - \hat{p}^*_{ht}) = 0 \tag{F.8}$$

$$m\hat{c}_t - m\hat{c}^*_t = \hat{\omega}_t - \hat{\omega}^*_t \tag{F.9}$$

$$\tilde{\hat{p}}_{ht} - \tilde{\hat{p}}^*_{ft} = \beta\phi E_t(\tilde{\hat{p}}_{ht+1} - \tilde{\hat{p}}^*_{ft+1}) + (1-\beta\phi)(m\hat{c}_t - m\hat{c}^*_t) + \beta\phi E_t(\hat{\pi}_{t+1} - \hat{\pi}^*_{t+1}) \tag{F.10}$$

$$\tilde{\hat{p}}_{ft} - \tilde{\hat{p}}^*_{ht} = \beta\phi E_t(\tilde{\hat{p}}_{ft+1} - \tilde{\hat{p}}^*_{ht+1}) + (1-\beta\phi)(m\hat{c}^*_t - m\hat{c}_t + 2\hat{q}_t) + \beta\phi E_t(\hat{\pi}_{t+1} - \hat{\pi}^*_{t+1})$$

$$\tilde{\hat{p}}_{ft} + \tilde{\hat{p}}^*_{ht} = \beta\phi E_t(\tilde{\hat{p}}_{ft+1} + \tilde{\hat{p}}^*_{ht+1}) + (1-\beta\phi)(m\hat{c}^*_t + m\hat{c}_t) + \beta\phi E_t(\hat{\pi}_{t+1} + \hat{\pi}^*_{t+1}) \tag{F.11}$$

$$\hat{p}_{ht} - \hat{p}^*_{ft} = (1-\phi)(\tilde{\hat{p}}_{ht} - \tilde{\hat{p}}^*_{ft}) + \phi(\hat{p}_{ht-1} - \hat{p}^*_{ft-1}) - \phi(\hat{\pi}_t - \hat{\pi}^*_t) \tag{F.12}$$

$$\hat{p}_{ft} - \hat{p}^*_{ht} = (1-\phi)(\tilde{\hat{p}}_{ft} - \tilde{\hat{p}}^*_{ht}) + \phi(\hat{p}_{ft-1} - \hat{p}^*_{ht-1}) - \phi(\hat{\pi}_t - \hat{\pi}^*_t) \tag{F.13}$$

$$\hat{m}_t - \hat{m}^*_t = (\hat{m}_{t-1} - \hat{m}^*_{t-1}) - (\hat{\pi}_t - \hat{\pi}^*_t) + (\hat{\mu}_t - \hat{\mu}^*_t) \tag{F.14}$$

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