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Energy as a substancelike quantity that flows: Theoretical considerations and pedagogical consequences

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Utilizing an energy-as-substance conceptual metaphor as a central feature of the introductory physics curriculum affords students a wealth of conceptual resources for reasoning about energy conservation, storage, and transfer. This paper first establishes the utility and function of a conceptual metaphor in developing student understanding of energy concepts. Then a curricular framework with a prominent energy-as-substance conceptual metaphor is described. The curricular framework involves both a reorganization of the content of introductory physics as well as a renewed focus. Reorganizing includes treating energy early and spiraling back to energy treatments. The refocusing includes emphasizing energy's role in modeling phenomena and attending to the tools for representing energy conservation, storage, and transfer. Implementation of the energy framework is then described in the context of a Modeling Instruction course. Finally, qualitative evidence is presented showing student use of energy conceptual resources which are promoted in the curricular implementation.

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I. INTRODUCTION

Energy and energy conservation are foundational concepts across a wide swath of different sciences, yet physics is their home. Introductory college or university physics courses formally introduce students to energy concepts that show up repeatedly in other sciences. Physics serves an important purpose by shouldering the primary responsibility for helping students to develop useful energy concepts. Embracing this responsibility to other sciences has benefits for physics as well, by making modern topics more accessible in the introductory curriculum. There is a lack of curriculum materials and instructional approaches that aid students in the development of meaningful energy concepts and that enable them to flexibly use energy to model physical situations.

The instructional treatment of energy can be greatly improved in the introductory physics curriculum. A curricular framework that begins by treating energy as a substancelike quantity that is stored and transferred affords students conceptual resources for reasoning about energy. Reorganizing and refocusing the curriculum to promote the conceptual resources this energy metaphor allows enhances the tools available to students to model physical phenomena. This restructuring and refocusing creates a coherent curricular framework with energy as a substancelike quantity that is stored and transferred as the central conceptual metaphor. In this approach energy establishes a coherent framework that scaffolds the content throughout the curriculum and thus energy analyses become prevalent.

The shift toward greater emphasis on energy is coupled with an attention to more modern topics in the introductory sequence. Topics such as cohesion, binding, phase changes, condensation, thermal expansion, elasticity, and plasticity all have macroscopic properties that can be modeled conceptually at the atomic level using energy and that provide a window into the structure of matter. Using energy to investigate microscopic phenomena such as the structure of matter is not merely an attractive theme for physics, it also integrates well across other scientific domains. The dominant shift in chemistry and biology, as well as physics, has been to study interactions at microscopic scales. This shift increases the importance placed on energy concepts, for energy analyses are more prevalent at the microscopic scale than force analyses. Increased importance of energy concepts at the forefront of science also necessitates a shift in the treatment of energy concepts, especially in introductory physics.

II. THEORETICAL CONSIDERATION

A curriculum undergirded by an energy-as-substance conceptual metaphor shifts greater attention to energy concepts. Shifting attention to energy concepts (1) helps students develop conceptual resources for reasoning with energy, (2) expands the tool set students have available to solve physics problems, and (3) promotes the coherence of the content in the introductory curriculum with an energy framework. These shifts have the added benefit of providing avenues to incorporate relevant science into the introductory curriculum by enabling students to model more complex phenomena. This paper will first describe the

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theoretical considerations of an energy-as-substance framework, then explore the pedagogical consequences of using this energy framework in physics. Finally, it will detail the implementation of this energy framework in a Modeling Instruction setting and provide qualitative evidence of student use of the energy conceptual resources available to students.

A. Conceptualizing energy as a substancelike quantity that is stored and transferred

Conceptual metaphors, grounded in embodied experience, are a powerful way to represent abstract concepts [1,2]. Lakoff and Johnson [3] claim that ‘‘all the resources used in direct, immediate understanding are pressed into service in indirect understanding via metaphor.’’ One challenge of teaching energy is developing a set of conceptual resources that effectively convey a scientific understanding of energy. In part, this derives from a lack of a definition of energy. Schroeder [4] conveys this difficulty in his text on thermodynamics: ‘‘To further clarify matters, I really should give you a precise definition of energy. Unfortunately, I can’t do this. Energy is the most fundamental dynamical concept in all of physics and for this reason, I can’t tell you what it is in terms of something more fundamental.’’ He goes on to identify fundamental qualities of energy which include conservation, storage, and transfer. It is challenging to identify conceptual resources for understanding energy, because as Feynman [5] said, ‘‘We have no idea what energy *is*.’’ Instead, we focus on promoting student understanding of *properties* of the energy concept, and this is where the conceptual metaphor of energy as a substancelike quantity that can be stored and transferred and the accompanying conceptual resources are employed.

Treating energy as a substancelike quantity uses a conceptual metaphor that compares energy to an actual physical object, as advocated by Duit [6]. Attaching the qualities of an object to energy provides a way of thinking about energy conservation, storage, and transfer. Feynman, in Ref. [5], compares energy to a child’s blocks: no matter what the child does (storing them in his toy box, giving them to a friend, losing them behind the couch) the number of blocks does not change. Establishing a substancelike conception of energy early in the first semester of introductory physics provides students with a rich set of well-established tools for reasoning about energy conservation, storage, and transfer. Empowering students with a conception of energy that is productive for them may help them overcome aversion toward energy. Students tend to not use energy or conservation of energy in analyzing physical situations. Driver and Warrington [7] found that students applied work or energy principles in less than 10% of responses to qualitative questions and that students rarely used energy principles in solving quantitative problems. Driver and Warrington concluded that application of the conservation of energy is nontrivial for students and attributed this to students not being taught to think of energy as a conserved quantity.

Swackhamer [8], building on the work of Lakoff and Johnson on conceptual metaphor [3], analyzed the energy as ‘‘stuff’’ conceptual metaphor and identified three statements derived from the metaphor which guide the development of energy concepts and an understanding of energy conservation with students:

1. As an attribute, energy is viewed as a possession that can be ‘‘stored’’ or ‘‘contained’’ in a ‘‘container,’’ namely, a physical system.
2. Energy can ‘‘flow’’ or be ‘‘transferred’’ from one container to another and so can cause changes.
3. Energy maintains its identity after being transferred.

Each of these metaphorical statements supports the development of scientifically valid conceptions of energy. Treating energy as an attribute that is stored in physical systems (which includes both objects and interactions) casts energy as a property of physical systems. Coupling energy to physical systems has two outcomes. The first is that there is no free energy. The second is that statements like ‘‘energy is lost’’ or ‘‘energy is released’’ imply that energy goes to a void, statements that contradict the idea that while energy may be transferred to a storage that is not easily recoverable, it is still transferred to some physical system.

Swackhamer’s second statement, that energy can be transferred and so can cause changes, is critical because it attaches agency to changes in energy. Energy transfers are the result of interactions between physical systems, and with each energy transfer the physical systems involved in the transfer are changed. Developing a sense of mechanism for energy transfer changes the treatment of energy from one of an accounting problem to one of modeling changes in physical systems, and modeling physical systems is the basic endeavor of practicing physicists.

The third statement, that energy maintains its identity after being transferred, is a way of reconsidering the conservation law. Two common treatments of energy, introducing energy through the work-energy theorem and describing different forms of energy, have been posited to cause problems for understanding energy conservation. First, work and energy are often conflated (even in textbooks [9]) and people often have the notion that energy takes on different forms rather than being a unitary quantity with different storage mechanisms [10]. Utilizing a substancelike conceptual metaphor for energy helps us to address each of these two common alternate conceptions of energy. Then, treating energy as a substancelike quantity emphasizes the unitary quality of energy and thus helps students avoid issues with work. One common issue with work is that work is confused with energy, which inhibits students from viewing energy as a conserved quantity [11]. Perhaps this is not surprising, as a variety of researchers criticize physicists’ treatment of work [12]. Mallinckrodt and Leff [13] identify seven distinct types of work, and Arons [14] identifies pseudowork. Utilizing a substancelike conception of energy helps students distinguish energy and the process of doing work and enhances their view of energy as conserved. Returning to Feynman's metaphor, a child's block is always a block and is never building a tower. For students, the substancelike metaphor, treating energy as blocks, separates the substance (energy or the blocks) from the process of transferring energy (doing work or building with the blocks).

In addition to alleviating confusion with work, the substancelike energy concept dictates that language of energy forms and transformation should be avoided. Falk, Herrmann, and Schmid [10] argue that conservation requires energy to be considered as a unitary quantity that may have different storages but does not take on different forms: "It is inappropriate to speak about the forms of something which itself does not change but, rather, which only changes carriers." To illustrate, they provide the example of potatoes: potatoes can be stored in trucks, bags, refrigerators, and root cellars, but changing the storage of potatoes does not change the potatoes much like gravitational energy and electric potential energy are both still energies regardless of their storage mechanism. A further point is that transfers within the system are treated the same as transfers across system boundaries. This is because the mechanisms of energy transfer from one storage to another are the same regardless if the transfer crosses system boundaries; the accounting is the only difference. The messages about energy that are developed must be attended to, and consistently utilizing a substancelike energy conception can support the development of energy conservation, storage, and transfer.

One common concern regarding the introduction of a substancelike metaphor for energy is that this may introduce wrong physics, as no such substance exists [15]. In fact, Chi [16] proposes that ontological miscategorization, such as classifying energy as stuff, may be the reason some misconceptions are particularly robust. However, Gupta *et al.* [17] counter that everyday scientific reasoning is full of examples of flexible ontologies such as those required by the use of conceptual metaphor. In short, we are able to consider energy as stuff when it is productive for understanding and reasoning and to dynamically reclassify it ontologically for other purposes. While the substance metaphor is useful it will not necessarily result in a caloric view of energy. Amin's analysis of energy concepts agrees with Gupta *et al.* and asserts that metaphor is a productive tool for developing understanding and is widely employed, not only in scientific conceptions of energy, but also in lay uses of energy [1]. Amin also conducted an analysis of energy conceptual metaphors and suggested that developing an understanding of an abstract concept may rely extensively on metaphorical projection from experiential knowledge. Metaphorical projection is particularly useful in developing an understanding of energy as it is difficult to categorize energy ontologically as it is neither stuff, nor process, nor historical event, nor, as Slotta [18] proposes, is it an "emergent process."

Treating energy as a substancelike quantity unlocks a wealth of conceptual resources for reasoning about energy conservation, storage, transfer, and agency. The conceptual resources developed in an energy-as-substance framework are continually refined throughout the introductory curriculum, and eventually the lack of a substance is addressed directly to avoid a caloric view of energy.[1] As Amin notes, the associations developed in an energy-as-substance metaphor are useful in considering the design and use of instructional representational tools ([1], p. 192). To further cultivate reasoning with energy in an energy-as-substance curricular framework, powerful tools to represent the storage and transfer of energy are utilized.

B. Supporting energy as a substancelike quantity with powerful tools for reasoning

The conceptual resources the energy-as-substance view allows are augmented with powerful tools for modeling phenomena with energy. These tools can be grouped in three categories: systemic, accounting, and functional. Systemic tools aid students in establishing the objects and interactions being considered. Accounting tools are used to represent the storage and transfer of energy within systems. Functional tools help characterize interactions by representing functional dependences of energy storage and transfer. Tools from all of these groups contribute to developing understanding of energy in conjunction with the energy-as-substance conceptual metaphor.

One goal of developing a curricular framework around energy-as-substance is to make energy a more useful approach to modeling phenomena, and the inclusion of the powerful representational tools provides multiple ways to reason about physical phenomena. Typical introductory curricula place heavy emphasis on forces, shifting toward greater emphasis on energy concepts and representations of energy storage, and transfer expands the scope of phenomena that students are adept at modeling. Developing energy concepts and representations enables students to model more complex phenomena and therefore makes more relevant science accessible in the introductory curriculum.

C. Energy-as-substance framework promotes coherent connections across the curriculum

Introducing the energy-as-substance metaphor promotes energy concepts and increases the utility of energy concepts for students as they model physical phenomena.

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[1] The lack of an energy substance is typically addressed during the second semester when students consider whether a dead battery has less mass.

Promoting the agency of energy in changing physical systems encourages students to develop understanding of when and how to use energy to model physical phenomena. This shift puts energy in a more central role within the curriculum. Several researchers have cited the benefits of the coherence of an energy-centered curriculum. They identify easy incorporation of thermodynamics [19,20], a modernization of content [21], and improved coverage of the second law of thermodynamics [22] as benefits of an energy-centered curriculum. This goal of improved modeling of phenomena emphasizes the coherent connections among the content of the introductory curriculum using energy. Incorporating energy as a central element in the introductory curriculum requires both a reorganization and a refocusing of the curriculum and pedagogy.

III. CURRICULAR AND PEDAGOGICAL CONSEQUENCES

A coherent curricular framework based on energy introduces energy concepts early in the semester, enhances the tools available for students to use energy to model phenomena, and builds students' conceptual resources for understanding energy and energy conservation. These changes reflect changes to the structure of the curriculum and focus of the pedagogy. Reorganizing the content of the introductory curriculum (1) promotes energy concepts to balance a force-centered curriculum and (2) distributes the time devoted to energy topics across the semester. This second goal is achieved by spiraling back to treat energy in parallel with force and momentum to help students develop criteria for when each approach is useful. Accompanying this reorganization, the treatment of energy is refocused. In this curricular framework the primary refocusing is to treat energy as a substancelike quantity that can be stored and transferred. Employing this conceptual metaphor encourages students to utilize the conceptual resources for reasoning using energy, energy conservation, storage, and transfer when modeling systems and interactions. Benefits of the renewed focus include (1) greater attention to energy's role in modeling physical systems and (2) awareness of the conceptual resources, representational tools, and qualitative reasoning which are afforded by an energy-as-substance framework. Either the refocusing to develop energy conceptual resources or restructuring the curriculum could conceivably stand alone. However, pairing the refocusing with a restructuring addresses the large-scale problem of students not utilizing energy resources by promoting a productive conceptual metaphor paired with tools that emphasize the utility of energy and a course organization that contributes to students seeing energy as important.

A. Content reorganization promotes energy

A reorganization of the content of introductory physics supports the renewed focus on energy in this curricular framework. Table I shows the curriculum organization of Modeling Instruction with a prominent energy framework. The reorganization involves treating energy earlier in the curriculum and spiraling back to energy repeatedly throughout the curriculum, building on the conceptual resources and refining them toward a scientifically appropriate understanding of energy.

1. Treating energy early

Reorganizing the content by introducing energy prior to forces accomplishes two things: it gives prominence to energy concepts and it requires energy to be introduced through energy conservation rather than work-kinetic energy theory (as this requires force concepts which have not yet been introduced and are posited to be counterproductive [11,23]). A common difficulty with energy is that students do not see it as a useful approach to analyzing phenomena and often confuse energy with other physics concepts. Standard introductory curriculum is organized around forces, perhaps as an historical artifact, as forces ‘‘developed’’ prior to energy. Introducing energy before forces (see Table I) establishes energy as a prominent way of modeling phenomena, promotes its importance within the discipline, and counters students' aversion to energy.

2. Spiraling back to energy treatments

Energy is particularly useful in modeling changes within a physical system. In order to emphasize this utility, the curriculum was further reorganized to treat energy in parallel with other approaches to modeling phenomena, forces, and momentum. To accomplish this, the time typically devoted to energy concepts was distributed across the semester. The distribution of time devoted to energy can be seen in Table I. The general law of energy conservation becomes a foundation for analyzing phenomena beginning in week 5 and the instruction spirals back to energy throughout the remainder of the semester[2] continually refining student understanding of energy concepts. Treating energy in parallel with other topics affords the opportunity to compare and contrast approaches to modeling phenomena. This helps students develop heuristics as to when energy is a useful approach and when forces or momentum are preferable. Explicitly supporting students' development of a sense about when one approach is preferable to another is entirely missing from standard curricula.

B. Refocusing the treatment of energy

The principal refocusing of the treatment of energy is to employ an energy-as-substance metaphor. Refocusing in

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[2] Actually, energy is a foundation throughout the remainder of the year, but in deference to brevity only the first semester is considered here.

TABLE I. A model-centered introductory mechanics curriculum with a prominent coherent energy framework. Each column represents a cycle of model development, beginning at the top with phenomenology and moving down toward greater abstraction. Instruction progresses from the left toward the right, increasing the sophistication and robustness of the models developed over time.

Time frame	Week 1	Weeks 2 and 3	Week 4	Weeks 5–8	Week 9	Week 10	Week 11	Weeks 12 and 13	Week 14
Phenomena	1D Motion	2D Motion	Ball bounce	Static coffee cup	Collision	Elastic and inelastic collisions	Return of ball bounce (compression)	Friction	Matter simulator and properties of materials
New tools	Kinematic graphs Motion maps	Motion maps Whole vectors	Energy pie charts System schema	Force diagrams		F vs t Momentum vectors	F vs d E vs d	Coupled particles (balls and springs)	
Specific laws of general models	$d = v_0 t + \frac{1}{2} a t^2$ $v = v_0 + at$	$\mathbf{d} = \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$ $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}\mathrm{t}$	$\mathrm{E}_k = \frac{1}{2} m v^2$ $E_g = mgh$	$F_g = mg$ Superposition		$\mathbf{I} = \int \mathbf{F} dt$ $\mathbf{p} = m\mathbf{v}$	$\mathbf{F} = -k\mathbf{d}$ $E = \frac{1}{2} k d^2$	$F_k = \mu_k F_N$ $F_s \leq \mu_s F_N$	$E_k = \frac{3}{2} k_B T$ $PV = NRT$
General models	1D Constant acceleration	Constant acceleration	Dynamic constant acceleration with E conservation	Dynamic constant acceleration with E conservation and force	Dynamic impulsive with E conservation and force	Dynamic nonconstant acceleration with E conservation, force and P conservation	Harmonic oscillator or elastic	Dynamic constant acceleration with E, P conservation, and force	
Theory				Newtonian	Newtonian	Newtonian	Newtonian	Newtonian	Kinetic theory
General laws			Energy conservation	Energy conservation	Energy conservation	Energy and momentum conservation	Energy and momentum conservation	Energy and momentum conservation	Energy, momentum and mass conservation

this way promotes reasoning using energy conservation, storage, and transfer when modeling physical systems and interactions.

1. Energy's role in modeling physical systems

Emphasizing energy as an approach to modeling interactions and phenomena promotes energy's footing to that of force and momentum. Typically, energy treatments are primarily focused on accounting, and do not address how energy is used by scientists. Two ways that scientists regularly utilize energy is to identify the relevant objects and interactions present in a physical system and to identify energy storages in the application of energy conservation. Scientists regularly make decisions about how they model systems, phenomena, or interactions based on energy. Decisions, such as where to identify the zero of gravitational energy within a system, which interactions to consider, or how to model an object all have a basis in energy considerations. By making the modeling of interactions and phenomena the focus, the decision to use energy, momentum, or force becomes an outcome of a choice made by the scientist rather than the focus of the instruction. Focusing the curriculum on the role that energy plays in modeling physical systems then makes these decisions explicit; often these decisions underpin the entire modeling process. [24] One example of how energy considerations motivate new models comes from the ball bounce activity (see Table I, column 4). In this activity, students drop a playground ball under a motion detector and try to answer the question, "Why doesn't it bounce back as high?" Addressing this question shifts the models from purely descriptive kinematic models to dynamic explanatory models, which is a shift best explained through energy conservation. Students must identify objects and interactions which are relevant to the situation, then must account for how energy has been stored and transferred as the result of the relevant interactions. Throughout the introductory curriculum, modeling shifts such as this are motivated by energy considerations. The emphasis on modeling interactions is further supported by the choices of tools provided to students and the continual spiraling back to energy considerations (especially the parallel treatments of energy and forces).

2. Attending to the tools for representing and reasoning about energy conservation, storage, and transfer

Standard curricula address energy as an accounting problem, where students are encouraged to check the energy before and after some interaction and then balance the books [25]. This emphasis on accounting does not help students gain a perspective on energy concepts like those of a practicing scientist—it lacks a sense of agency for energy by not attending to the interaction that causes the changes. In order to scaffold both the development of useful and canonical energy concepts and students' reasoning with energy conservation, storage, and transfer, several representational tools are introduced. These tools, which include energy pie charts, energy bar charts, system schema, and energy versus position graphs, facilitate students' reasoning using energy and build on the conceptual resources available from an energy-as-substance metaphor. Further, the curriculum reconsiders the role of tools regularly used by practicing physicists: energy versus position graphs, potential versus position graphs, and equipotential lines and surfaces.

The resulting framework for energy has been designed to highlight energy as a valuable, viable conceptual resource in modeling physical phenomena. The process of modeling phenomena inherently involves the coordinated use of representational tools. Modeling tools aid in making strong coherent connections between models. There are a number of such modeling tools embedded within this energy framework. The modeling tools fall into three primary categories: systemic, accounting, and functional tools.

Systemic tool: System schema.—The system schema explicitly identifies the physical system being studied, which is a different purpose than other tools [26,27]. For this reason, it is the lone member in the systemic class of tools. The system schema organizes the analysis of a given situation. Inherent in the statement of conservation of energy is the concept of system. Energy concepts requires careful identification of a relevant system. A system schema is a representation of the system that includes system boundaries, all relevant objects included in the system, and all relevant interactions between these objects.

Objects in a system schema are represented without any detail. Interactions between objects are represented by two-headed arrows and labeled according to the type of interaction. The system boundary is represented by a dotted line around at least one of the objects. Alone, system schema do not provide predictive power to a model, but they serve the unique purpose of explicitly identifying the system being modeled. In addition to identifying objects and interactions to be included, creating a system schema requires students to consider which objects and interactions can be excluded. This process of limiting the phenomena being modeled is a valuable skill that physicists routinely practice, but which often goes unnoticed. Figure 1 presents an example of a system schema for the following situation, "An electron, initially at rest, is accelerated across a 5 cm long region containing a constant electric field. The electric field is created by a pair of parallel oppositely charged plates and is of magnitude 30 N/C and is directed to the left."

The system schema shown in Fig. 1 clearly identifies the objects involved: the electron and the left and right plates. The interactions between objects are labeled with an e to denote electric interactions. The system boundary is defined by the dotted line enclosing all objects and interactions, identifying this as a closed system. Because no

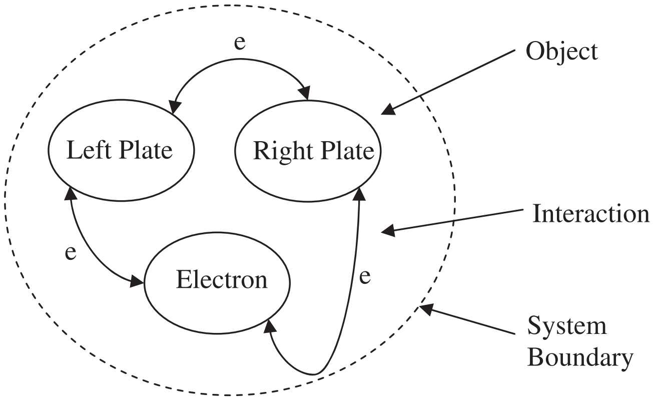

FIG. 1. Example system schema.

interactions cross the system boundary, energy remains constant within this system.

System schemas have a number of utilities. They comprise the first level of abstraction above pictorial representation by allowing for identification of objects without concern for the structure of the objects. Additionally, the system schema aids in the creation of basic energy arguments based on conservation, because it includes possible ways that energy can be stored (in objects and interactions). Local energy conservation or nonconservation is dependent on a defined system; the schema provides an outlet to make the system definition explicit. As shown above, it is easy to determine that the energy in the system must remain constant, because nothing crosses the system boundary.

Accounting tools: Energy bar charts and energy pie charts.—The second category of modeling tools in this energy framework is made up of the accounting tools. Energy pie charts, energy bar charts, and the first law of thermodynamics all represent the storage and transfer of energy. The tools used here for accounting for energy are different because students must represent the energy storages, how energy has been transferred, and the energy conservation. Accounting tools are used to keep track of energy within a system, including transfers into or out of the system. The accounting of energy resonates with energy being a unitary, conserved, substancelike quantity that is stored and transferred. Accounting for energy throughout a physical phenomenon attributes energy to the physical system that is storing the energy, ties transfers to interactions, and emphasizes the conservation of the substancelike quantity. Accounting tools not only describe energy storage and transfer, but also emphasize the unitary nature of energy and promote understanding of the concept of conservation.

(1) Energy pie charts: Energy pie charts are visual and conceptual representations of energy conservation, storage, and transfer within a system that emphasize the unitary nature of energy. Each pie represents the energy in the system and how that energy is stored. Energy transfers into the system are accompanied by an increase in the size of the pie, and, conversely, transfers out of the system decrease the size of the pie. Pies are divided according to the energy storage mechanisms being used. Practically, the divisions are not precise representations of relative amounts of energy. Deemphasizing the quantitative accounting of energy shifts the focus toward a thorough qualitative analysis of energy conservation, storage, and transfer. Energy transfers within the system are represented by changing the distribution of energy within the pies as time progresses. Figure 2 presents a set of energy pie charts based on the system schema from the previous section.

The pie charts in Fig. 2 are all the same size, representing energy conservation within the system. As the electron moves from the negatively charged plate toward the positively charged plate, the kinetic energy increases, showing that the electron is accelerating. The electric interaction energy E_{IE} of the electron-plate system decreases to zero when the electron is at the midpoint between the plates. While this example shows the utility of the energy pie charts in the representation of energy storage and transfer, it also demonstrates a limitation of the pie charts. Beyond the halfway point the electric interaction energy continues to decrease and the kinetic energy continues to increase. However, the electric interaction energy becomes negative, which is not represented well on energy pie charts. This failure provides motivation for energy bar charts, which are discussed in Sec. III B 2. In spite of the failure of pie charts to represent negative energy, they serve the essential purpose of promoting the unitary quality of energy, which is why pie charts are introduced as the first tool to represent energy conservation, storage, and transfer in the curriculum.

Pie charts exist within an intermediate level of abstraction; students can focus on energy storage and transfer in the system, but not concern themselves with the mathematics. Using energy pie charts in conjunction with system schema, students are able to make sophisticated energy arguments because energy pie charts provide a visual representation of energy conservation. Energy pie charts are also used to emphasize conceptual resources afforded by the energy-as-substance metaphor. Energy pie charts are introduced before energy bar charts in order to combat the belief that there are a number of disparate forms of energy. With pie charts, energy is a unitary quantity that is merely stored in different mechanisms rather than in different

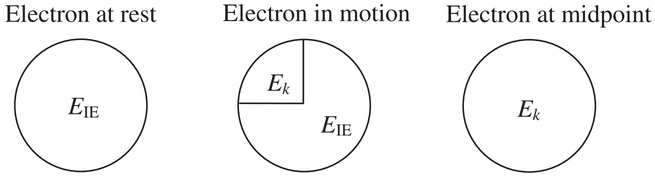

FIG. 2. Example energy pie charts (here E_{IE} is electric interaction energy).

forms. The second conception that energy pie charts combat is the notion that energy is "lost." Students are forced to account for all of the energy in the system; therefore, it is acceptable to describe energy leaving the system, but not to say that it is lost. This requires students to establish energy storage mechanisms for the energy that was previously "lost."

(2) Energy bar charts: Energy bar charts are very similar in nature to energy pie charts, but have the ability to represent negative energy [28]. Similar to pie charts, where the pie represents energy stored in the system, bar charts represent the energy in the system by the total height of the bars. The different bars represent different energy storage mechanisms. In addition to the ability to represent negative energy, the bar charts are different in that they are more suited to quasinumeric calculations. Using the previous electron in a constant field example, the bar charts in Fig. 3 demonstrate the similarities and differences between bar charts and pie charts.

Energy bar charts are introduced later than energy pie charts for a number of reasons. First, because energy bar charts separate energy storage into different bars, bar charts may support the idea that energy exists in a number of different forms. Introducing pie charts first emphasizes the unitary nature of energy. After students have developed their sense about the unitary quality of energy, they can change representations to use bar charts when they need to include negative energy in their models. Then bar charts are introduced as a more general tool for representing energy.

Additionally, bar charts lend themselves to a more quantitative analysis. Students are encouraged to first develop qualitative analyses of situations before modeling the phenomenon mathematically. Bar charts are more quantitative, and accordingly pie charts are used first to promote the use of the representational tools to develop models of phenomena. Subsequently, students are eventually introduced to bar charts because bar charts are a more general tool.

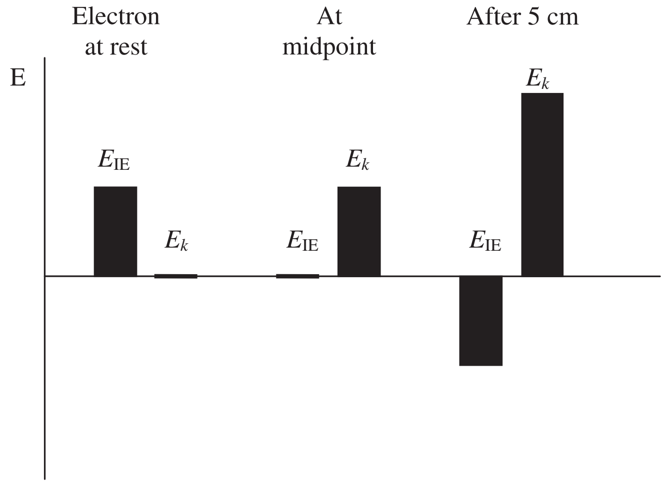

FIG. 3. Example of energy bar charts.

Functional tools: Energy versus position graphs, potential versus position graphs, and equipotential surfaces.—A third category of modeling tools, functional tools, exists within this energy framework. Functional tools include energy versus position graphs, potential versus position graphs, and equipotential surfaces. These tools are all utilized in standard physics curricula and in the daily practice of physics. Although they are not unique to this energy framework, the focus on energy-as-substance and the organization of the curriculum increases the emphasis on these representational tools. This approach highlights the role of energy in modeling phenomena. Because the content has been reorganized to treat energy and forces in parallel, tools that represent the functional dependence of energy on the interaction are particularly important. Further, because the tools are coupled to the interactions, they also can be coordinated with other approaches to modeling the interactions, such as forces and fields. Coordinating these representations with other approaches helps to increase the coherent connections among topics in the curriculum. Because these tools are familiar, only a brief description of their utility is provided.

(1) Energy versus position graphs: Energy versus position graphs represent the functional dependence of energy in an interaction. These graphs are among the most widely used representational tools; both physicists and chemists use them to explain a variety of phenomena. Among the phenomena that can be explained based on interaction energy versus distance graphs are binding, cohesion, compressibility, conductivity, frictional energy transfers, phase changes, physical bonding, and thermal expansion. Further, the relationship between force and energy is represented effectively with interaction energy graphs. The force between two particles can be found by the negative slope of the interaction energy graph. By exploiting this relationship, equations describing the energy stored in a spring, internal energy due to friction, and gravitational and electric interaction energies can all be developed analytically. Relating force to energy is an important component of this energy framework. It is through these types of relationships that coherence among topics in the curriculum is developed.

Energy versus position graphs also play an important role in the modeling of physical systems. For example, students are initially taught that gravitational interaction energy equals mass × strength of the gravitational field × height, $E_{\mathrm{IG}} = mgh$. However, this model is only valid near Earth's surface. When the mgh model is extended to a height of infinity, the model breaks down. This model breakdown provides the motivation for universal gravitation. Energy versus position graphs allow students to consider the ranges of validity for the two models. Near Earth the graph of gravitational interaction energy appears linear and has a slope of $-mg$, which is of course the attractive force between Earth and the object. However, that only holds true when the energy can be linearized. As height increases, the graph becomes less linear and the model of mgh may no longer be useful. This interpretation is difficult without the use of energy versus position graphs.

(2) Potential versus position graphs: Potential versus position graphs are used in essentially the same manner as energy versus position graphs. The primary difference is just a matter of interpretation: energy versus position graphs represent the energy in an interaction and potential versus position graphs represent the possibility for energy in an interaction. Instead of interpreting the interaction energy graph to find the force between particles, potential graphs can be interpreted to find the field produced by the particle by taking the negative derivative. Potential graphs close the loop by relating field to force, force to interaction energy, interaction energy to potential, and interaction energy back to field.

(3) Equipotential surfaces: Equipotential surfaces are the final tool used extensively in this energy framework. Equipotentials are two-dimensional spatial representations of potential. In many respects, they are identical to potential graphs, but they are not confined to one dimension. Equipotentials can be used to reason about forces and fields. Again, because they relate forces to energy, they are useful within this energy framework. In practice, equipotential surfaces are used frequently, in weather maps and geographic relief maps, which can connect students' physics knowledge to other real world applications.

There are a number of other representational tools including kinematic graphs, motion maps, force diagrams, momentum vectors, field lines, and field vectors. All of these are used in conjunction with this curricular framework, and all are, at some point, related to energy. The tools that have been presented are tools that are used to represent or reason conceptually about energy conservation, storage, and transfer. The description of these tools should be sufficient to understand the use of modeling tools in the subsequent discussion of the implementation of this curricular framework in a Modeling Instruction class.

IV. IMPLEMENTATION OF THE ENERGY FRAMEWORK IN MODELING INSTRUCTION

The energy framework presented here has been implemented primarily in Modeling Instruction courses. This is a natural outcome, as the framework was developed to use Modeling Instruction in university physics. The Modeling Instruction context in which this curriculum was developed has profound impacts on the energy framework. Modeling Instruction was created as an instructional approach to approximate the activities of a working physicist [29,30]. One challenge with traditional introductory physics is that the content is not organized around models but around discrete topics. This contributes to students developing a fragmented view of the content of introductory physics, where they see it more as a collection of topics, problems, and equations than as a unified body of knowledge [31,32]. Modeling Instruction addresses this problem by organizing the content and instruction around a small number of basic models and the process of generalizing the characteristics of the general models from a set of situation-specific models. Within Modeling Instruction, models primarily serve an organizing role, while the instruction focuses on the processes of modeling. Many groups refer to these processes as model-based reasoning (tools, application, and reasoning), which can be summarized with the overriding question "How are you going to model this situation?" Because of the central role that energy plays in science, promoting the role of energy enhances the conceptual resources that students have available to them when modeling phenomena.

The epistemological foundations of Modeling Instruction make implementation of the energy framework a complementary extension of Modeling Instruction. From a modeling epistemology, models are seen as the basic epistemological structures of a scientific paradigm [33]. Halloun provides a useful depiction of the place of models in the knowledge structure of a paradigm, reproduced in Fig. 4. Utilizing an analogy from Halloun, the position of models in the knowledge structure of a scientific paradigm is between theory and concept, much as the position of dog is between animal and retriever. Halloun argues that concepts are too overspecified to comprise the content of a course and that general laws of a theory (such as Newton's laws) are too divorced from any specific physical situation to comprise the content of a course. Thus models are the most cognitively basic category and therefore should form the content of a course in physics.

A course organized around a small number of models and the process of generating, using, and reasoning with models has a greater coherence of content. Local coherence of models (i.e., within a family of models such as constant force models) comes from the coordination of multiple representations corresponding to a set of physical phenomena which can be modeled by the family of models. In contrast, traditional curriculum organization is driven by a mixture of general laws, general laws of a theory, and specific concepts. However, models are only coherent within a family of models, so in the adaptation of

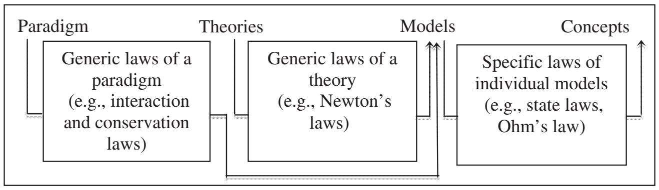

FIG. 4. Schematic of conceptual organization in a scientific paradigm as proposed by Halloun, reprinted with permission from I. A. Halloun, *Modeling Theory in Science Education* (Kluwer, Dordrecht, 2004), p. 23.

Modeling Instruction to university physics, this energy framework sought ways to increase the coherence between models and to emphasize the centrality of models to physics knowledge and to highlight the relationship with fundamental laws of the paradigm. Table I shows a model-centered curriculum organization with an included energy framework. Each column in Table I represents a cycle of model development within the curriculum. Each column has a specific phenomenon in the top row, which is modeled through the use of representational tools and used to generate a more general family of models. At the base of each row are the general laws of the paradigm of introductory physics, which constrain the development of the family of models and provide ways to analyze new phenomena. Two features of the energy framework are enacted in Modeling Instruction. Because energy conservation is a general law, it forms a basis across topics in the curriculum beginning in about the week 4 of the semester. Further, energy is used as a way of building conceptual connections between cycles of model development, as can be seen in the columns in Table I.

From this perspective on the organization of the knowledge structure and curriculum organization, energy conservation is a generic law of the paradigm of introductory physics and is applicable to all models developed throughout introductory physics. Because of the broad applicability of energy within the paradigm of physics, curricula can feature energy as a coherent organizing topic throughout the curriculum. (Note that a similar argument could be made for momentum, and Chabay and Sherwood [34] have developed an outstanding textbook which does just this.) In short, this energy framework developed in the context of Modeling Instruction because it supports students building models of physical phenomena and also provides a means to link models across the curriculum.

V. EVIDENCE OF STUDENT USE OF ENERGY CONCEPTUAL RESOURCES

Implementing the energy-as-substance curricular framework in Modeling Instruction has provided opportunities to evaluate the claim that students are afforded energy conceptual resources. In support of this claim, I draw on qualitative evidence of students in a large class discussion utilizing the conceptual resources. The evidence provided herein is not intended to allow for causal conclusions, but instead is intended to bolster the theoretical arguments made above and to illustrate the affordances the energy-as-substance framework provides students.

A. Qualitative data collection and analysis

The evidence collected comes from a Modeling Instruction class at Florida International University. The author was the instructor of the course and the energy-as-substance curricular framework was implemented. The Modeling Instruction course was calculus-based university physics which fulfilled requirements for a variety of majors primarily including engineering, biology, and physics. There were 30 students enrolled, including 63% female and 37% male, the students came from a variety of ethnic backgrounds (70% Hispanic, 13% Black, 13% Asian, and 3% White). The class was conducted in an integrated lab and lecture, studio format. In the Modeling course, the content was developed primarily through inquiry-based labs, conceptual reasoning activities, and cooperative group problem solving. Students typically sit at tables in groups of three and work on conceptual reasoning and group problem solving activities students. For each activity, each group of students prepares one small portable whiteboard. The entire class then meets in a larger circle where they then share and discuss their whiteboards. These whiteboard discussions comprise roughly 1/3 of the class time.

During fall 2010, every course meeting of the Modeling Instruction class at FIU was recorded by two videographers. Each videographer would choose one group of 3 students each day and place a wireless microphone on the table of the group. The videographers would then follow each group through small group discussions and the large group consensus building whiteboard discussions. From this data corpus, episodes were identified and transcribed. Episodes were identified as a result of the researcher's experience with the energy curriculum and chosen as exemplars of reasoning patterns employed by students in an energy-as-substance framework. While the particular conceptual reasoning in these episodes is unique, the tone, the student participation, and the frequent use of energy as a conceptual tool are common. Two episodes from the Fall 2010 class were identified to exemplify students utilizing the conceptual resources afforded by the energy-as-substance conceptual metaphor and curriculum.

1. Students resolving force of motion with energy

In the first episode, students have prepared whiteboards with system schema, force diagrams, motion maps, and kinematic graphs in response to the following prompt from the instructor: ‘‘Model the following two situations—A person throws their keys into the air. Part a) while the person is throwing the keys upward, and part b) while the keys are on their way upward.’’ Students have had 15 minutes to prepare whiteboards, and have come together to present their whiteboards in a large circle. The transcript begins after one group has explained part (a) where the force diagram includes a force from the hand. In this episode, a student named Sergio presents his group's whiteboard, which includes a force diagram with a downward force of gravity and an upward (but smaller) force of the hand on the keys. Sergio describes his belief that the keys still have some force from the hand after leaving it. The professor then leads a discussion which guides students toward the Newtonian model and, in the course of the discussion, students use energy as a way of making sense of Sergio's notion of impetus.

1 *Sergio:* In the first one we said that the force of the
2 hand is larger than the force of gravity so that the hand
3 and the keys are moving up and then once the keys
4 leave the hand they still have that force that was
5 imparted upon them by the hand but now that force
6 is smaller than gravity's so that it's starting to slow
7 down as it is going up and then eventually comes
8 back down.
9 *Female Student 1:* I don't agree, sorry.
10 *Male Student 1:* Huh?
11 *Sergio:* It's ok.
12 *Dr. Brewe:* Why?
13 *Female Student 1:* 'Cause now the keys are not in the
14 hand as much it is already in the air, so it is going to
15 be the opposite of (inaudible).
16 *Sergio:* But doesn't it have that force imparted on it
17 by the hand? That's why it was able to go up in the air.
18 *Male Student 1:* Yeah but it doesn't. . .
19 *Female Student 1:* Yeah but it is hanging out more in
20 the. . .
21 *Female Student 2:* You have it stated after the keys
22 leave the hand, so the hand isn't really touching the
23 key anymore.
24 *Male Student 2:* There is no contact.
25 *Female Student 2:* Yeah there is no contact force.
26 *Sergio:* I'm not saying that it is a contact force, I'm
27 saying it is a force. . .
28 *Many Students at once*: (Inaudible)
29 *Dr. Brewe:* So. . .stop, stop, stop. (Points to small
30 personal whiteboard he is holding while walking to
31 the center of the circle) What are they saying?
32 (Pointing to system schema on whiteboard he is
33 holding)
34 *Female Student 3:* It is just the key and the earth.
35 *Dr. Brewe:* It is just the key and the earth. Is the
36 hand involved, is the hand interacting with the keys
37 anymore?
38 *Several Students:* No.
39 *Dr. Brewe:* No. Was it before?
40 *Several Students:* Yes.
41 *Dr. Brewe:* Yes! So you've got the right idea
42 (pointing to the system schema for part a on
43 Sergio's group's whiteboard) but here (pointing to
44 the force diagram for part b on the whiteboard) is
45 the hand interacting with the key anymore?
46 *Several Students:* No.
47 *Dr. Brewe:* No. Alright. If the hand isn't interacting
48 with the key anymore, then what?
49 *Female Student 3:* It's just. . .
50 *Dr. Brewe:* It can't be exerting a force on it, can it?
51 Now if you think about it, if I'm gonna toss my keys
52 up (tosses keys up and tries to pull them back once
53 they are out of his hand), I can't pull them back to
54 me. I can't push them back away. I can't. (Tosses
55 keys again) Once I am no longer interacting with
56 them. . . once I've let them go, that bird's out of the
57 nest baby!
58 *Many Students* (Laugh heartily)
59 *Male Student 4:* Are you a blackbelt?
60 *Dr. Brewe:* No, so this is the idea, I mean, if you're
61 no longer interacting with the keys, what's, can you
62 be, can you be exerting a force on them?
63 *Several Students:* No.
64 *Dr. Brewe:* No. And this is consistent with our rules
65 for the system schema. So what's a good force
66 diagram for the keys after they leave the hand?
67 *Female Student 2:* Just the force of gravity.
68 *Dr. Brewe:* It has to just be the force of gravity. But
69 wait that doesn't make any sense, how are they going
70 up?
71 *Male Student 4:* Because they have an initial
72 velocity.
73 *Female Student:* (simultaneous with Male Student
74 4). . .energy
75 *Dr. Brewe:* What did you do?
76 *Male Student 5:* You threw them up and gave them
77 the energy of the hand
78 *Dr. Brewe:* You transferred energy to them, you gave
79 them kinetic energy. So then as they continue
80 upward, you have given them that kinetic energy,
81 so what's happening?
82 *Female Student 4:* It is transferring energy.
83 *Dr. Brewe:* It is transferring energy as they go up. . .
84 transferring it to. . ..
85 *Female Student 4:* Gravitational.
86 *Dr. Brewe:* Gravitational as they go up. Alright. So
87 the only interaction that is happening is that
88 gravitational interaction and that gravitational
89 interaction is what is causing the energy to
90 transfer from kinetic to gravitational.

In this episode, Sergio begins by presenting his belief that the hand exerts a force on the keys even after they have left the hand (lines 1–8, 16–17), which is consistent with a commonly held belief that objects in motion have an impetus force [35]. Several students disagree and begin to discuss with Sergio (lines 9–15, 18). In order to bring the class toward consensus, Dr. Brewe focuses the attention on the interpretation of the system schema representational tool (lines 29–62), which helps the students to identify that there is no longer an interaction between the hand and the keys and that therefore there is no force from the hand on the keys. In lines 68–70, Dr. Brewe returns to the common belief that the hand exerts a force of motion on the keys. When asked how the keys were moving upward after leaving the hands, one student responds with energy (lines 73–74). In lines 76–77, another student elaborates and explains that energy is transferred to the keys from the hand. This discussion continues (lines 78–90) to a tentative resolution where the explanation for the keys being allowed to move upward is the result of a student reasoning about energy as a substancelike quantity (the hand gave energy to the keys) and using an energy explanation rather than a force explanation.

This episode exemplifies several features of a course implementing an energy-as-substance curricular framework. First, energy preceded forces in the curriculum; therefore, when students were struggling with a commonsense notion of impetus force, they had energy conceptual resources which allowed them to make sense of their existing understanding and to help fit that with their formal understanding. Second, the students' use of energy (admittedly scaffolded by the professor) is consistent with a substance metaphor. The hand gives energy to the keys (lines 76–77) and the energy is transferred to gravitational (lines 82–85); both of these uses of energy are consistent with an energy-as-substance conceptual metaphor. Finally, the students successfully switch from reasoning with forces to reasoning with energy (lines 67–85), illustrating the flexibility of their use of energy concepts and the willingness to value energy-based arguments in solving problems.

2. Students using energy rather than forces when solving a problem

The second episode comes from a whiteboard discussion during which students are discussing the following problem, which was chosen to give students practice decomposing vectors. The episode follows a lab on Newton's second law, and the expectation of the professor was that students would use forces and kinematics to solve the problem (see Fig. 5).

The transcript begins after four groups of three students have already presented their whiteboards, all having analyzed the situation with forces, and all arriving at the same acceleration of $a = 2.59\ \mathrm{m/s^2}$. Each of the four groups used a slightly different approach (rotated reference frame, nonrotated reference frame, and two different variations on using kinematics) to arrive at an answer. The pedagogical purpose of this whiteboard session is to illustrate that multiple pathways exist to solve a simple problem. In this episode, Brenda presents her group whiteboard which, rather than using a force-based approach, uses energy to solve the problem. The professor, seeing this solution, asks Brenda to describe her group's approach to solving the problem as a way of highlighting different approaches.

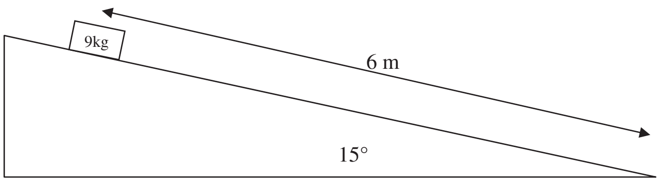

FIG. 5. Diagram of problem posed to Brenda's group.

1 *Dr. Brewe:* You guys took an entirely different
2 approach, so I would like you to explain what you've
3 done.
4 *Brenda:* Um we were kind of confused how to start,
5 and, I, I don't like vectors. They always give me
6 problems, so we kind of went a different way. Um,
7 we found that, yes we started off thinking that, oh
8 we can find the components of this angle of the
9 F_c [the contact force] but then we were kind of,
10 we had too many variables so we kind of left that.
11 So we said, 'What do we know?'. And we know
12 that the hypotenuse of the ramp or whatever is
13 6 meters so we tried to find the sides, right?
14 So given that the height of the ramp is 1.55 we
15 know. . .we can use what we know. (Pointing to
16 energy pie charts on her whiteboard showing all
17 energy stored in gravitational interaction at the
18 beginning.) We said E_g is equal to mgh, right,
19 so since the mass is 9 and the accel. . ..gravitational
20 strength is 10 and the height of the ramp is 1.55,
21 we can find E_g, so we are also saying that E_g
22 is equal to E_k at the end of the ramp.
23 *Girl 2:* Sorry, where did you get the 1.55?
24 *Brenda:* We found, using this angle.
25 *Girl 2:* Um, but. . .ok.
26 *Brenda:* And then so since $\mathrm{E_g}$ is equal to $\mathrm{E_k}$ we
27 found the velocity, the final velocity and the final
28 velocity is 5.57. So if you look at a velocity graph,
29 we are assuming the initial velocity is zero so
30 given that the we can find the time using one half
31 base times height from the velocity graph so and
32 we know that the acceleration is $\mathrm{v_f - v_o}$ over
33 time, so you can find your acceleration using and
34 we got 2.59.

In this episode, Brenda begins by acknowledging that she prefers not to decompose vectors (lines 5–10) and felt that there were too many variables to solve the problem. Instead, her group determined the height of the ramp, allowing them to solve for the gravitational energy (lines 11–19). Subsequently, she used the initial gravitational energy to solve for the final kinetic energy (lines 21–22) and then used kinematics (graphically) to solve for the final velocity and the acceleration (lines 26–34).

This second episode further illustrates that students find the conceptual tools, such as the energy pie charts, useful for reasoning. Brenda's group used the pie charts to support her explanation of their approach to solving the problem (lines 15–18). Brenda's group has not approached the problem as anticipated by the professor, and has avoided the intended activity of vector decomposition. Yet Brenda's group's solution to the problem illustrates that when energy concepts are emphasized as a viable approach to solving problems, students will utilize these methods. Further, this episode provides an example of students who, when facing a difficulty solving a problem one way, find another viable pathway (lines 5–18). Using multiple approaches and working forward to solve a problem is one feature of expertise in solving problems [36]. It should be noted that the professor highlighted this whiteboard, which thereby provided students the opportunity to compare the solution using energy to a force-based solution, which is consistent with the goals of the course content reorganization.

Together these episodes exemplify students using the conceptual resources afforded them by an energy-as-substance curricular framework. Students utilize energy resources flexibly in differing contexts such as solving problems or addressing commonly held beliefs. In both episodes presented, these conceptual resources would not be available to students without reorganizing the course content to emphasize energy concepts.

VI. SUMMARY

Treating energy with a conceptual metaphor of a substancelike quantity that can be stored and transferred provides students and instructors conceptual resources that contribute to the development of useful energy conceptions. However, simply including this metaphor for energy is not sufficient to promote energy as a viable way of modeling physical systems. The curriculum needs to be reorganized and refocused on energy as a central, coherent theme. This can and has been accomplished by incorporating multiple representational tools that support the energy framework by enhancing students' capacity to model physical phenomena. Examples of student reasoning using energy to address common conceptions and to solve problems captures the power of the reasoning tools available with implementation of the energy-as-substance framework

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