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Abstract

We study the consumption based asset pricing model in a discrete time pure exchange setting with incomplete information. Incomplete information leads to a filtering problem which agents solve using the Kalman filter. We characterize the solution to the asset pricing problem in such a setting. Empirical estimation with US consumption data indicates strong statistical support for the incomplete information model versus the benchmark complete information model. We investigate the ability of the model to replicate some key stylized facts about US equity and riskfree returns.

Key phrases: asset pricing; incomplete information; Kalman filter; equity returns; riskfree returns

JEL classification: G12, G13, E43

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1. INTRODUCTION

We study a pure exchange Lucas (1978) asset pricing model in a setting with incomplete information on the stochastic dividends process. In incomplete information asset pricing models, the drift rate of the dividends process is assumed to be unobservable. Agents need to estimate this drift rate based on observed dividends in order to compute the expected future dividend payouts and hence set equilibrium asset prices. This introduces a filtering problem into asset pricing models.

Early work on incomplete information in asset pricing models used linear stochastic differential equations with Brownian motion increments to characterize the exogenous path of the dividends process. The unobservable drift rate of the dividends process is also characterized as a linear stochastic differential equation with Brownian motion increments. Dothan and Feldman (1986), Detemple (1986), Gennotte (1986), and more recently, Brennan and Xia (2001) study asset pricing / portfolio allocation problems in this setting. Linear Gaussian setting permits use of the Kalman filter to solve the filtering problem in an optimal sense. The Kalman filter is a Bayesian updating rule that permits learning about the unobservable dividend drift rate with the arrival of new information on dividends each period. Recently, David (1997) and Veronesi (2004) study asset pricing with incomplete information in a non-Gaussian setting where the unobservable dividend growth rate undergoes jumps, driven either by a Markov switching or Poisson arrival process.

All the papers discussed above on asset pricing with incomplete information formulate the problem in continuous time. In a discrete time setting, Cecchetti et al. (2000) and Brandt et al. (2000) model dividends as a random walk driven by Gaussian innovations and a drift term that follows a discrete state Markov switching process. Such a specification fails to account for autocorrelation in the dividend growth rates.

In this paper we study the asset pricing problem with incomplete information in a discrete-time continuous-state stochastic setting. We assume that the observed dividend growth rate is the sum of an unobservable persistent component and noise. The unobservable persistent component is assumed to be an autoregressive process driven by Gaussian shocks. A complete information asset pricing model is a special case. Our model allows for a simple way to numerically solve for equilibrium asset prices, and hence implied returns. The solution is a simple extension of the solution to the asset pricing problem in complete information setting studied in Burnside (1998). We characterize the solution to the asset pricing model in such a setting. We then calibrate the model to data on quarterly US per capita consumption, and study the ability of the model to replicate the unconditional moments of observed returns.

The paper is organized as follows. We describe the economic environment and the asset pricing model in section 2. We study the solution to the model in section 3. We tackle empirical issues including estimation of the model in section 4. We analyze the model implied rates of return in section 5. The last section provides some conclusions derived from the paper.

2. THE MODEL

Is this section we lay out the economic environment, including specification of exogenous stochastic processes and information structure in the asset pricing model.

2.1 Pure Exchange Economy

In a single good Lucas (1978) economy, with a representative utility-maximizing agent and a single asset that pays exogenous dividends of non-storable consumption goods, the first-order Euler condition is:

$$P_t U'(C_t) = \theta E_t U'(C_{t+1})[P_{t+1} + D_{t+1}]. \qquad (1)$$

Here, P_t is the real price of the single asset in terms of the consumption good, $U'(C)$ is the marginal utility of consumption C for the representative agent, θ is a constant subjective discount factor, D is the dividend from the single productive unit, and E_t is the mathematical expectation, conditioned on information available at time t.

Assume a constant relative risk aversion (CRRA) utility function with risk-aversion coefficient γ:

$$U(C) = (1-\gamma)^{-1} C^{(1-\gamma)}, \qquad \gamma \geq 0. \qquad (2)$$

Since consumption equals dividends in this simple model, i.e. $C = D$ every period, Equation (1) reduces to:

$$P_t D_t^{-\gamma} = E_t \theta D_{t+1}^{-\gamma}[P_{t+1} + D_{t+1}]. \qquad (3)$$

On rearranging, this yields:

$$P_t = E_t \theta \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} [P_{t+1} + D_{t+1}]. \qquad (4)$$

Let v_t denote the price-dividend ratio, i.e. $v_t = P_t / D_t$. Then, we can rewrite Equation (4) in terms of v_t as:

$$v_t = E_t \theta \left(\frac{D_{t+1}}{D_t} \right)^{1-\gamma} [v_{t+1} + 1]. \qquad (5)$$

Thus, this equation implicitly defines the solution to the asset pricing problem in this model. One specifies an exogenous stochastic process for dividends and solves for the price dividend ratio v_t.

Let $x_t = \ln(D_t / D_{t-1})$ denote the dividend growth rate. Then, we can express Equation (5) as:

$$v_t = E_t \theta \exp[(1-\gamma)x_{t+1}](v_{t+1}+1). \tag{6}$$

Defining $m_{t+1} \equiv \theta \exp[(1-\gamma)x_{t+1}]$, we can rewrite Equation (6) as:

$$v_t = E_t m_{t+1}[v_{t+1}+1]. \tag{7}$$

On forward iteration, this equation yields:

$$v_t = \sum_{i=1}^{\infty}\left(E_t \prod_{j=1}^{i} m_{t+j} \right) + \lim_{i\to\infty} E_t \prod_{j=1}^{i} m_{t+j} v_{t+i}. \tag{8}$$

One solution to the above difference equation in v_t is obtained by imposing the transversality condition:

$$\lim_{i\to\infty}\left(E_t \prod_{j=1}^{i} m_{t+j} v_{t+i} \right) = 0. \tag{9}$$

This condition rules out solutions to the asset pricing model that imply intrinsic bubbles (Froot and Obstfeld 1991). Imposing the transversality condition on Equation (8) gives:

$$v_t = \sum_{i=1}^{\infty}\left(E_t \prod_{j=1}^{i} m_{t+j} \right). \tag{10}$$

Thus, the solution to the price-dividend ratio can be found by evaluating the conditional expectations on the right hand side of Equation (10), under a specified exogenous stochastic process for the dividend growth rates.

2.2 Information Structure

We assume that dividend growth rates stochastically evolve according to the following process:

$$x_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim iid\, \mathrm{N}(0, \sigma^2) \tag{11a}$$

$$\mu_t - \bar{\mu} = \rho\left(\mu_{t-1} - \bar{\mu}\right) + \eta_t, \qquad |\rho| < 1, \qquad \eta_t \sim \mathrm{iid}\, \mathrm{N}\left(0, \sigma_\eta^2\right). \tag{11b}$$

We assume that ε_t and η_t are independent of each other contemporaneously as well as at all leads and lags.

We assume that agents in the economy have full knowledge about the structure of the economy. They know the stochastic process governing the evolution of the dividend growth rates, including the parameters of the process. They observe the dividend stream (and hence the realized dividend growth rates x_t as well). However, we assume that agents do not ever observe the persistent component μ_t (or equivalently the noise component ε_t) of the dividend growth rates.

Agents need to form conditional expectations of μ_t in order to compute the expected future dividend payouts, and hence determine equilibrium prices. Thus, agents face a filtering problem. We assume that agents form conditional expectations on μ_t based on Bayesian updating rules. Specifically, agents face a linear Gaussian filtering problem. In this case, the conditional density of μ_t is Gaussian (see, for instance, Harvey 1992, Ch.3) and, therefore, completely specified by its conditional mean and variance. These are given recursively by the classic Kalman filter.

In a benchmark full information economy, we assume that the innovation ε_t in Equation (11a) has zero variance (i.e. ε_t is trivially zero). In this case, $\mu_t = x_t$, and therefore agents actually observe μ_t. There is no filtering problem facing the agents in such an economy. This model is studied in Burnside (1998).

3. MODEL SOLUTION

We now proceed to evaluate Equation (10) for the price-dividend ratio under the assumed process for the dividend growth rates. We also study some properties of this model implied price-dividend ratio.

3.1 Solution for the P/D Ratios

Appendix A shows that v_t in Equation (10) can be reduced to:

$$v_t = \sum_{i=1}^{\infty} \theta^i . \left[E_t \exp\left\{ b_i(\mu_t - \bar{\mu}) \right\} \right] . \exp \left[\begin{array}{c} i\bar{\mu}(1-\gamma) + i\left(1-\gamma\right)^2 \dfrac{\sigma^2}{2} + \\ \left(\dfrac{1-\gamma}{1-\rho} \right)^2 \dfrac{\sigma_\eta^2}{2} \cdot \sum_{j=1}^{i} (1-\rho^j)^2 \end{array} \right] \qquad (12)$$

where $b_i = (1-\gamma)\left(\dfrac{\rho}{1-\rho} \right)(1-\rho^i)$.

As discussed in subsection 2.2, with the linear Gaussian setup that we have, the conditional density of μ_t is Gaussian, and its conditional mean and variance are given by the Kalman recursions. Using these conditional moments, the conditional expectations term $E_t \exp\{b_i(\mu_t - \bar{\mu})\}$ appearing in Equation (12) can then be evaluated using the formula for the moment generating function of Gaussian random variables.[1]

The following theorem provides conditions for the infinite series in Equation (12) to converge, and hence for the price–dividend ratio to be finite.

Theorem 1. The series in Equation (12) converges if

$$r \equiv \theta \exp\left[(1-\gamma)\bar{\mu} + (1-\gamma)^2 \frac{\sigma^2}{2} + \left(\frac{1-\gamma}{1-\rho}\right)^2 \frac{\sigma_\eta^2}{2}\right] < 1. \tag{13}$$

Proof. See Appendix B.

Finiteness of the price-dividend ratio ensures that the expected discounted utility is finite in this model (see Burnside 1998). The next theorem derives an expression for the mean of the price-dividend ratio, i.e. the unconditional expectation of v_t in Equation (12). It also provides conditions under which this mean is finite.

Theorem 2. The mean of the price dividend ratio is given by:

$$E(v_t) = \sum_{i=1}^{\infty} \theta^i \exp\left[i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \frac{b_i^2}{1-\rho^2}\frac{\sigma_\eta^2}{2} + \left(\frac{1-\gamma}{1-\rho}\right)^2 \frac{\sigma_\eta^2}{2} \cdot \sum_{j=1}^{i}(1-\rho^j)^2\right]. \tag{14}$$

It is finite if $r < 1$, where r is the constant defined in Theorem 1.

Proof. See Appendix C.

3.2 Solution under Complete Information

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[1] If $x \sim N(\mu, \sigma^2)$, then $E\{\exp(x)\} = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$.

In the complete information benchmark case, recall from subsection 2.2 that $\mu_t = x_t$, which is observed at time t. All the analyses of subsection 3.1 goes through exactly as in the incomplete information case, with some simplifications detailed below. The expression for the price-dividend ratio given in Equation (12) remains the same but with $E_t \exp\{b_i(\mu_t - \bar{\mu})\} = \exp\{b_i(x_t - \bar{\mu})\}$ and $\sigma^2 = 0$. Theorem 1 goes through as before with $\sigma^2 = 0$ imposed on r defined by Inequality (13). The mean of the price-dividend ratio given in Equation (14) remains the same but with $E_t \exp\{b_i(\mu_t - \bar{\mu})\} = \exp\{b_i(x_t - \bar{\mu})\}$ and $\sigma^2 = 0$. The condition for its finiteness given by Theorem 2 remains unchanged but with $\sigma^2 = 0$ imposed on r defined by Inequality (13).

The price-dividend ratio and its related properties in the benchmark complete information model are derived in Burnside (1998). Such a complete information model with habit formation utility as in Abel (1990) is studied in Collard et al. (2006).

4. EMPIRICAL ESTIMATION OF THE MODEL

We calibrate the asset pricing model to quarterly real per capita US consumption growth rates on non-durables and services from 1952:1 through 2004:2. Nominal seasonally adjusted per capita consumption data obtained from NIPA tables are deflated using the CPI index. Summary statistics indicate an annualized mean growth rate of 2.02 percent and a standard deviation of 1.34 percent. The first order autocorrelation coefficient is 0.18 and statistically different from 0 at the 1 percent level.

The dividend growth rates process in Equations (11) constitutes a linear Gaussian state space model. Equation (11a) is the observation equation and Equation (11b) is the state transition equation. The linear Gaussian nature of the model results in the conditional density of the state variable μ_t being Gaussian as well. The Kalman filter gives recursive formulae for obtaining the conditional mean and variance of the state variable μ_t, as well as the likelihood function.

Maximum likelihood parameter estimates of the consumption growth rate process (conditional on the first observation) in Equations (11) are reported in Table 1 (Panel A). Parameter estimates indicate a mean consumption growth rate of 0.50 percent per quarter, or 2.00 percent per annum. The autoregressive (AR) parameter ρ is estimated to be 0.74. It is statistically significantly different from 0 by the usual t-test at better than the 1 percent significance level. The signal-to-noise ratio σ_η / σ is estimated to be 0.38. Figure 1 plots the mean of the filter densities $E(\mu_t \mid x_1, x_2, ..., x_t)$, along with the observed consumption growth rates x_t.

The complete information model parameter estimates are reported in Panel B of Table 1. The AR coefficient ρ is now only 0.18. This is understandable, however, because the AR process for μ_t in Equation (11b) is now combined with the iid process for ε_t in Equation (11a), and effectively an AR model is being estimated for the resulting contaminated (with iid noise) series. Nonetheless, the AR coefficient is statistically significantly different from 0 by the usual t-test at better than the 1 percent significance level. However, the maximized log-likelihood shows a large drop. The likelihood ratio (LR) test statistic for complete information versus incomplete information model turns out to be 3.76, with a χ_1^2 p-value of 0.05. Thus, there is significant statistical support for the incomplete information model.

5. ANALYSIS OF MODEL IMPLICATIONS

In this section we discuss the implications of the theoretical model of section 2 for rates of return on risky and risk free assets, set up a simulation framework for analyses of the unconditional properties of model implied rates of returns, and report on the results obtained.

5.1 Model-Implied Rates of Return

Equilibrium gross equity returns R_t^e on assets held from period t through period t+1 are given by:

$$R_t^e = \left(\frac{P_{t+1} + D_{t+1}}{P_t}\right). \tag{15}$$

Using $v_t = P_t / D_t$ and $x_t = \ln(D_t / D_{t-1})$, this reduces to:

$$R_t^e = \left(\frac{1 + v_{t+1}}{v_t}\right)\exp[x_{t+1}]. \tag{16}$$

It is not possible to analytically evaluate the population mean of the implied equity returns, i.e. $E(R_t^e)$, in our model given the expression for v_t in Equation (12).

The price of a risk free asset P_t^f in our endowment economy guarantees one unit of the consumption good on maturity. It is given by:

$$P_t^f = \theta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\right). \tag{17}$$

With CRRA utility and $C = D$ in the model from Section 2, this reduces to:

$$P_t^f = \theta E_t\left(\frac{D_{t+1}}{D_t}\right)^{-\gamma}. \tag{18}$$

Using $x_t = \ln(D_t / D_{t-1})$, we get $P_t^f = \theta E_t[\exp(-\gamma x_{t+1})]$. Substituting for x_{t+1} using Equation (11) yields:

$$P_t^f = \theta E_t\left[\exp\left\{-\gamma\bar{\mu} - \gamma\rho(\mu_t - \bar{\mu}) - \gamma\eta_{t+1} - \gamma\varepsilon_{t+1}\right\}\right]. \tag{19}$$

Using independence of μ_t, ε_{t+1} and η_{t+1}, we can rewrite this as:

$$P_t^f = \theta \exp\left\{-\gamma\bar{\mu}\right\} E_t\left[\exp\left\{-\gamma\varepsilon_{t+1}\right\}\right] E_t\left[\exp\left\{-\gamma\rho(\mu_t - \bar{\mu})\right\}\right] E_t\left[\exp\left\{-\gamma\eta_{t+1}\right\}\right]. \tag{20}$$

We have assumed that $\varepsilon_t \sim \text{iid}\, N(0, \sigma^2)$ in Equation (11a). Therefore, using the moment generating function for the normal random variable:

$$E_t[\exp\{-\gamma\varepsilon_{t+1}\}] = \exp\left\{\frac{\gamma^2\sigma^2}{2}\right\}. \tag{21}$$

We have assumed that $\eta_t \sim \text{iid}\, N(0, \sigma_\eta^2)$ in Equation (11b). This yields:

$$E_t[\exp\{-\gamma\eta_{t+1}\}] = \exp\left\{\frac{\gamma^2\sigma_\eta^2}{2}\right\}. \tag{22}$$

Substituting Equations (21) and (22) into Equation (20) gives the price of the risk free asset:

$$P_t^f = \theta\left[\exp\left\{-\gamma\bar{\mu} + \gamma^2\sigma^2/2 + \gamma^2\sigma_\eta^2/2\right\}\right] E_t\left[\exp\left\{-\gamma\rho(\mu_t - \bar{\mu})\right\}\right]. \tag{23}$$

Gross equilibrium returns on the risk free asset R_t^f are given by:

$$R_t^f = \frac{1}{P_t^f}. \tag{24}$$

Excess returns on the risky asset over the risk free asset are given by:

$$R_t = R_t^e - R_t^f. \tag{25}$$

5.2 Simulation Setup

We undertake a simulation study in order to analyze the model implications for the endogenous rates of return. The simulations are performed in the following manner. We draw random numbers for ε_t and η_t in Equations (11) using parameter estimates reported in Table 1. The value of μ_0 is set to the unconditional mean of μ_t, equal to $\overline{\mu}$. We then use the simulated η_t series to generate a sequence $\{\mu_t, t = 1, 2, ..., T\}$ using Equation (11b) with $T = 4000$. We use this sequence and the simulated ε_t series to generate a sequence of artificial dividend growth rates $\{x_t, t = 1, 2, ..., T\}$ according to Equation (11a).

We use the simulated sequence $\{x_t\}$ and the parameter estimates from Table 1 to obtain the mean of the posterior density $E(\mu_t \mid x_1, x_2, ..., x_t)$ using the Kalman filtering equations. We use this posterior mean to evaluate the price-dividend ratios v_t in Equation (12). Calculations are done for various values for the preference parameters θ (discount factor) and γ (risk aversion coefficient) that satisfy the convergence condition $r < 1$ in Equation (13). Model-implied returns on risky and risk free assets are then generated using Equations (16), (23) and (24), and excess returns from Equation (25). In order to eliminate any effects from startup of the Kalman filter, we drop the first ten implied returns.

5.3 Analysis of Unconditional Moments

Table 2, Panel A reports unconditional moments of quarterly value-weighted excess returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period 1952:1 through 2004:2. We subtract returns on the one-month Treasury bills from nominal returns to obtain excess returns, expressed in percent per annum. Real riskfree returns are obtained by subtracting CPI inflation from the nominal T-bill returns.

Panel A indicates that quarterly excess returns have a mean of 6.83 percent per annum and a standard deviation of 16.55. Riskfree returns have a mean of 1.23 percent and a standard deviation of 1.34 percent. Explaining these set of stylized facts has proved to be a challenge in the macro-finance literature (see Mehra and Prescott 1985).

Panels B and C report the unconditional moments for returns implied by our theoretical model of section 2 using the simulation setup from subsection 5.2. Moments are reported for various values of the discount factor θ and the risk aversion coefficient γ. The maximum implied mean excess returns from our incomplete information model are only 0.07 percent and the maximum standard deviation is only 1.99 percent. On the other hand, the minimum implied mean riskfree returns from our incomplete information model is 5.26 percent but the maximum standard deviation is only 0.61 percent. Overall, it is clear from looking at both the panels that neither model does a good job of replicating the unconditional moments of excess equity or riskfree returns. This is simply a manifestation of the equity premium puzzle of Mehra and Prescott (1985).

It is clear from an examination of panels B and C that adding incomplete information to the asset pricing model raises the implied excess returns and reduces the implied riskfree returns by a small amount, about 0.01-0.02 percent per annum. It however raises the volatility of both riskfree and excess returns, but the increase is less than 0.60 percent per annum. Overall, although adding incomplete information to the standard asset pricing model moves the mean and volatility of implied returns in the right direction, the quantitative effects are too small to be of any significance in helping to resolve either the equity premium or the riskfree rate puzzles.

6. CONCLUSIONS

We study the consumption based asset pricing model of Lucas (1978) in an incomplete information setting. Although agents observe realized dividends (and hence their growth rates), they do not observe the persistent and noise components that make up the observed dividends. Estimation of the persistent component is important for evaluating conditional expectations of future dividends, used to set equilibrium asset prices. Its unobservability introduces a filtering problem that agents solve using Bayesian updating schemes. Asset pricing with complete information is a special case of our framework.

We fit the model to quarterly per capita real US consumption data. Maximum likelihood parameter estimates indicate strong support for our incomplete information model. The likelihood ratio test rejects complete information in favor of the incomplete information model. We find that although adding incomplete information to the standard asset pricing model moves the mean and volatility of implied excess and riskfree returns in the right direction, the quantitative effects are too small to be of any significance in helping to resolve either the equity premium or the riskfree rate puzzles.

APPENDIX A

Derivation of the Price-Dividend Ratio

In this appendix we derive the expression for the price dividend ratio v_t given in Equation (12). From subsection 2.1, we have $m_{t+j} \equiv \theta \exp[(1-\gamma)x_{t+j}]$. Let $\omega = 1-\gamma$. Therefore, $m_{t+j} = \theta \exp[\omega x_{t+j}]$.

$$\prod_{j=1}^{i} m_{t+j} = \prod_{j=1}^{i} \theta \exp\left[\omega x_{t+j}\right] \quad = \theta^i \exp\left(\omega \sum_{j=1}^{i} x_{t+j}\right). \tag{A1}$$

From dividend growth rate process in Equation (11a),

$$\sum_{j=1}^{i} x_{t+j} = \sum_{j=1}^{i} \mu_{t+j} + \sum_{j=1}^{i} \varepsilon_{t+j}. \tag{A2}$$

From dividend growth rate process in Equation (11b), $\mu_{t+j} - \bar{\mu} = \rho\left(\mu_{t+j-1} - \bar{\mu}\right) + \eta_{t+j}$, we have

$$\mu_{t+j} - \bar{\mu} = \rho^j \left(\mu_t - \bar{\mu}\right) + \rho^{j-1}\eta_{t+1} + \rho^{j-2}\eta_{t+2} + \ldots + \rho^2\eta_{t+j-2} + \rho\eta_{t+j-1} + \eta_{t+j}. \tag{A3}$$

Therefore,

$$\sum_{j=1}^{i} \mu_{t+j} = [\bar{\mu} + \rho(\mu_t - \bar{\mu}) + \eta_{t+1}] + [\bar{\mu} + \rho^2(\mu_t - \bar{\mu}) + \rho\eta_{t+1} + \eta_{t+2}] + \ldots$$
$$\ldots + [\bar{\mu} + \rho^i(\mu_t - \bar{\mu}) + \rho^{i-1}\eta_{t+1} + \rho^{i-2}\eta_{t+2} + \ldots. + \eta_{t+i}]$$

This can be written as:

$$\sum_{j=1}^{i} \mu_{t+j} = i\bar{\mu} + (\mu_t - \bar{\mu})\left[\frac{\rho(1-\rho^i)}{1-\rho}\right] + \frac{1}{1-\rho}\left[(1-\rho^i)\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \ldots + (1-\rho)\eta_{t+i}\right] \tag{A4}$$

Therefore, $\prod_{j=1}^{i} m_{t+j} = \theta^{i} \exp\left(\omega \sum_{j=1}^{i} x_{t+j} \right)$

$$= \theta^{i} \exp \begin{pmatrix} i\bar{\mu}\omega + \left\{ \frac{\omega\rho}{1-\rho}(1-\rho^{i}) \right\} (\mu_t - \bar{\mu}) + \\ \left(\frac{\omega}{1-\rho} \right) \left\{ (1-\rho^{i})\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \ldots + (1-\rho)\eta_{t+i} \right\} + \\ \omega \sum_{j=1}^{i} \varepsilon_{t+j} \end{pmatrix}.$$

Define $b_i = \omega\left(\frac{\rho}{1-\rho} \right)(1-\rho^{i})$. From the iid nature of ε_t and η_t, we can write:

$$\begin{aligned} E_t \prod_{j=1}^{i} m_{t+j} = \theta^{i}.\exp\left[i\bar{\mu}\omega \right].E_t \exp\left[b_i(\mu_t - \bar{\mu}) \right] \cdot \\ E_t \exp\left[\frac{\omega}{1-\rho} \left\{ (1-\rho^{i})\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \ldots + (1-\rho)\eta_{t+i} \right\} \right]. \\ E_t \exp\left[\omega \sum_{j=1}^{i} \varepsilon_{t+j} \right] \end{aligned}$$

$$\begin{aligned} E_t \prod_{j=1}^{i} m_{t+j} = \theta^{i}.\exp\left[i\bar{\mu}\omega \right].E_t \exp\left[b_i(\mu_t - \bar{\mu}) \right] \cdot \\ E_t \left[\exp\left[\left(\frac{\omega}{1-\rho} \right)(1-\rho^{i})\eta_{t+1} \right] \cdot \exp\left[\left(\frac{\omega}{1-\rho} \right)(1-\rho^{i-1})\eta_{t+2} \right] \ldots \exp\left[\left(\frac{\omega}{1-\rho} \right)(1-\rho)\eta_{t+i} \right] \right]. \\ E_t \left[\exp(\omega\varepsilon_{t+1}) \cdot \exp(\omega\varepsilon_{t+2}) \ldots \exp(\omega\varepsilon_{t+i}) \right] \end{aligned} \tag{A5}$$

Since $\varepsilon_t \sim \text{iid}\, N(0, \sigma^2)$ in Equation (11a),

$$\begin{aligned} E_t \left[\exp(\omega\varepsilon_{t+1}) \cdot \exp(\omega\varepsilon_{t+2}) \ldots \exp(\omega\varepsilon_{t+i}) \right] = \\ E_t \left\{ \exp(\omega\varepsilon_{t+1}) \right\}.E_t \left\{ \exp(\omega\varepsilon_{t+2}) \right\} \ldots E_t \left\{ \exp(\omega\varepsilon_{t+i}) \right\} \end{aligned} \tag{A6}$$

From the moment generating function of normal random variables, we have

$$E_t \left\{ \exp(\omega\varepsilon_{t+1}) \right\} = E_t \left\{ \exp(\omega\varepsilon_{t+2}) \right\} = \ldots = E_t \left\{ \exp(\omega\varepsilon_{t+i}) \right\} = \exp\left\{ \frac{1}{2}\omega^2\sigma^2 \right\}. \tag{A7}$$

Since $\eta_t \sim \text{iid}\,N(0,\sigma_\eta^2)$ in Equation (11b),

$$E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^i)\eta_{t+1}\right]\cdot\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^{i-1})\eta_{t+2}\right]...\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho)\eta_{t+i}\right]\right\}$$
$$=E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^i)\eta_{t+1}\right]\right\}E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^{i-1})\eta_{t+2}\right]\right\}...E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho)\eta_{t+i}\right]\right\} \tag{A8}$$

From the properties of normally distributed random variables, we have:

$$\frac{\omega}{1-\rho}(1-\rho^i)\eta_{t+1} \sim N\left(0,\left(\omega\frac{1-\rho^i}{1-\rho}\right)^2\sigma_\eta^2\right). \tag{A9}$$

Similarly, we have:

$$\frac{\omega}{1-\rho}(1-\rho^{i-1})\eta_{t+2} \sim N\left(0,\left(\omega\frac{1-\rho^{i-1}}{1-\rho}\right)^2\sigma_\eta^2\right) \tag{A10}$$

and so forth for all the other η's in Equation (A8).

From the moment generating function of normal random variables, we have from Equations (A9) and (A10):

$$E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^i)\eta_{t+1}\right]\right\}=\exp\left[\left(\omega\frac{1-\rho^i}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right] \tag{A11}$$

$$E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^{i-1})\eta_{t+2}\right]\right\}=\exp\left[\left(\omega\frac{1-\rho^{i-1}}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right] \tag{A12}$$

and so forth for all the other η's in Equation (A8):

$$E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho)\eta_{t+i}\right]\right\}=\exp\left[\left(\omega\frac{1-\rho}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right]. \tag{A13}$$

Substituting (A11), (A12), and (A13) into (A8), we get:

$$E_t\left\{\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^i)\eta_{t+1}\right]\cdot\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho^{i-1})\eta_{t+2}\right]...\exp\left[\left(\frac{\omega}{1-\rho}\right)(1-\rho)\eta_{t+i}\right]\right\}$$
$$=\exp\left\{\left[\left(\frac{\omega}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right]\cdot\sum_{j=1}^{i}(1-\rho^j)^2\right\} \quad \text{(A14)}$$

Substituting (A7) and (A14) into (A5) and collecting terms results in:

$$E_t\prod_{j=1}^{i}m_{t+j}=\theta^i.\left[E_t\exp\left\{b_i(\mu_t-\bar{\mu})\right\}\right].\exp\left[\begin{array}{l} i\bar{\mu}(1-\gamma)+i\left(1-\gamma\right)^2\dfrac{\sigma^2}{2}+ \\ \left\{\left[\left(\dfrac{1-\gamma}{1-\rho}\right)^2\dfrac{\sigma_\eta^2}{2}\right]\cdot\displaystyle\sum_{j=1}^{i}(1-\rho^j)^2\right\}\end{array}\right] \quad \text{(A15)}$$

recognizing that $\omega = 1-\gamma$.

Equation (10) gives:

$$v_t = \sum_{i=1}^{\infty}\left(E_t\prod_{j=1}^{i}m_{t+j}\right). \quad \text{(A16)}$$

Substituting (A15) into (A16) gives:

$$v_t=\sum_{i=1}^{\infty}\theta^i.\left[E_t\exp\left\{b_i(\mu_t-\bar{\mu})\right\}\right].\exp\left[\begin{array}{l} i\bar{\mu}(1-\gamma)+i\left(1-\gamma\right)^2\dfrac{\sigma^2}{2}+ \\ \left\{\left[\left(\dfrac{1-\gamma}{1-\rho}\right)^2\dfrac{\sigma_\eta^2}{2}\right]\cdot\displaystyle\sum_{j=1}^{i}(1-\rho^j)^2\right\}\end{array}\right] \quad \text{(A17)}$$

where, we have $b_i = \left(1-\gamma\right)\left(\dfrac{\rho}{1-\rho}\right)(1-\rho^i)$.

APPENDIX B

Proof of Theorem 1

From Equation (12),

$$v_t = \sum_{i=1}^{\infty} \theta^i . \left[E_t \exp\left\{ b_i (\mu_t - \bar{\mu}) \right\} \right] . \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i\left(1-\gamma\right)^2 \frac{\sigma^2}{2} + \\ \qquad \left\{ \left[\left(\frac{1-\gamma}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \right] \cdot \sum_{j=1}^{i} (1-\rho^j)^2 \right\} \end{array} \right] \tag{B1}$$

or, substituting $\omega = 1 - \gamma$

$$v_t = \sum_{i=1}^{\infty} \theta^i . \left[E_t \exp\left\{ b_i (\mu_t - \bar{\mu}) \right\} \right] . \exp \left[\begin{array}{l} i\bar{\mu}\omega + i\omega^2 \frac{\sigma^2}{2} + \\ \qquad \left\{ \left[\left(\frac{\omega}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \right] \cdot \sum_{j=1}^{i} (1-\rho^j)^2 \right\} \end{array} \right]. \tag{B2}$$

Let $v_t \equiv \sum_{i=1}^{\infty} z_i$. (B3)

$$\frac{z_{i+1}}{z_i} =$$

$$\frac{\theta^{i+1} E_t \exp\left\{ b_{i+1}(\mu_t - \bar{\mu}) \right\} . \exp\left[(i+1)\bar{\mu}\omega + (i+1)\frac{\omega^2\sigma^2}{2} + \left\{ \left[\left(\frac{\omega}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \right] \cdot \sum_{j=1}^{i+1} (1-\rho^j)^2 \right\} \right]}{\theta^{i} E_t \exp\left\{ b_{i}(\mu_t - \bar{\mu}) \right\} . \exp\left[i\bar{\mu}\omega + i\frac{\omega^2\sigma^2}{2} + \left\{ \left[\left(\frac{\omega}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \right] \cdot \sum_{j=1}^{i} (1-\rho^j)^2 \right\} \right]}$$

which on simplifying becomes:

$$\frac{z_{i+1}}{z_i} = \theta \frac{E_t \exp\left\{ b_{i+1}(\mu_t - \bar{\mu}) \right\}}{E_t \exp\left\{ b_{i}(\mu_t - \bar{\mu}) \right\}} \exp\left[\bar{\mu}\omega + \frac{\omega^2\sigma^2}{2} + \left\{ \left[\left(\frac{\omega}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \right] \cdot (1-\rho^{i+1})^2 \right\} \right]$$

With $|\rho|<1$ specified in Equation (11b),

$$\lim_{i\to\infty}\left|\frac{z_{i+1}}{z_i}\right| = \theta\exp\left[\bar{\mu}\omega+\frac{\omega^2\sigma^2}{2}+\left\{\left[\left(\frac{\omega}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right]\right\}\right].\lim_{i\to\infty}\frac{E_t\exp\{b_{i+1}(\mu_t-\bar{\mu})\}}{E_t\exp\{b_i(\mu_t-\bar{\mu})\}}.$$

(B4)

One can easily show that $\lim_{i\to\infty} b_{i+1} = \lim_{i\to\infty} b_i = \left(\frac{\omega}{1-\rho}\right)\rho$. Therefore, we have

$\lim_{i\to\infty}\frac{E_t\exp\{b_{i+1}(\mu_t-\bar{\mu})\}}{E_t\exp\{b_i(\mu_t-\bar{\mu})\}}=1$. Using this in (B4), we have:

$$\lim_{i\to\infty}\left|\frac{z_{i+1}}{z_i}\right| = \theta\exp\left[\bar{\mu}\omega+\frac{\omega^2\sigma^2}{2}+\left\{\left[\left(\frac{\omega}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right]\right\}\right]\equiv r \tag{B5}$$

Substituting $\omega=1-\gamma$, we get:

$$r=\theta\exp\left[\bar{\mu}(1-\gamma)+\frac{(1-\gamma)^2\sigma^2}{2}+\left\{\left[\left(\frac{1-\gamma}{1-\rho}\right)^2\frac{\sigma_\eta^2}{2}\right]\right\}\right]. \tag{B6}$$

Proof for convergence of v_t in (D1) for $r<1$ now follows from the ratio test (see, for instance, Marsden 1974, Theorem 13, p.47).

APPENDIX C

Proof of Theorem 2

Derivation of Equation (14)

From Equation (12),

$$v_t = \sum_{i=1}^{\infty} \theta^i . \left[E_t \exp\left\{ b_i(\mu_t - \bar{\mu}) \right\} \right] . \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i\left(1-\gamma\right)^2 \dfrac{\sigma^2}{2} + \\ \qquad \left\{ \left[\left(\dfrac{1-\gamma}{1-\rho} \right)^2 \dfrac{\sigma_\eta^2}{2} \right] \cdot \displaystyle\sum_{j=1}^{i} (1-\rho^j)^2 \right\} \end{array} \right]. \quad \text{(C1)}$$

Therefore, from the law of iterated expectations,

$$v_t = \sum_{i=1}^{\infty} \theta^i . \left[E \exp\left\{ b_i(\mu_t - \bar{\mu}) \right\} \right] . \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i\left(1-\gamma\right)^2 \dfrac{\sigma^2}{2} + \\ \qquad \left\{ \left[\left(\dfrac{1-\gamma}{1-\rho} \right)^2 \dfrac{\sigma_\eta^2}{2} \right] \cdot \displaystyle\sum_{j=1}^{i} (1-\rho^j)^2 \right\} \end{array} \right] \quad \text{(C2)}$$

From Equation (11b), we have $b_i(\mu_t - \bar{\mu}) \sim N\left(0, \dfrac{b_i^2 \sigma_\eta^2}{1-\rho^2}\right)$. We then have, from the moment generating function for normal random variables:

$$E[\exp\{b_i.(\mu_t - \bar{\mu})\}] = \exp\left[\frac{b_i^2 \sigma_\eta^2}{2\left(1-\rho^2\right)} \right] \quad \text{(C3)}$$

Substituting into Equation (C2) gives:

$$E(v_t) = \sum_{i=1}^{\infty} \theta^i \exp\left[i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \frac{b_i^2 \sigma_\eta^2}{2\left(1-\rho^2\right)} + \left\{ \left(\frac{1-\gamma}{1-\rho} \right)^2 \frac{\sigma_\eta^2}{2} \cdot \sum_{j=1}^{i} (1-\rho^j)^2 \right\} \right]. \quad \text{(C4)}$$

Proof of convergence of $E(v_t)$

$$\text{Let } E(v_t) \equiv \sum_{i=1}^{\infty} z_i \tag{C5}$$

Using Equation (E4), one can easily show that:

$$\lim_{i\to\infty}\left|\frac{z_{i+1}}{z_i}\right| = \theta \exp\left[\bar{\mu}\omega + \frac{\omega^2\sigma^2}{2} + \left(\frac{\omega}{1-\rho}\right)^2 \frac{\sigma_\eta^2}{2}\right] . \lim_{i\to\infty} \exp\left[\frac{\sigma_\eta^2}{2\left(1-\rho^2\right)}\left\{b_{i+1}{}^2 - b_i{}^2\right\}\right].$$

Using the definition of r in Theorem 1,

$$\lim_{i\to\infty}\left|\frac{z_{i+1}}{z_i}\right| = r. \lim_{i\to\infty} \exp\left[\frac{\sigma_\eta^2}{2\left(1-\rho^2\right)}\left\{b_{i+1}{}^2 - b_i{}^2\right\}\right] \tag{C6}$$

Following from the proof of Theorem 1 in Appendix B, it suffices to show that:

$$\lim_{i\to\infty} \exp\left\{\frac{\sigma_\eta^2}{2\left(1-\rho^2\right)}\left[b_{i+1}{}^2 - b_i{}^2\right]\right\} = 1$$

or that, $\lim_{i\to\infty}\left\{ b_{i+1}{}^2 - b_i{}^2 \right\} = 0$. With $|\rho| < 1$ specified in Equation (11b),

$b_i{}^2 = \left(\frac{\rho(1-\gamma)}{1-\rho}\right)^2 [1-\rho^i]^2$. Therefore,

$$\lim_{i\to\infty}\left\{ b_{i+1}{}^2 - b_i{}^2 \right\} = \left(\frac{\rho(1-\gamma)}{1-\rho}\right)^2 . \left[\lim_{i\to\infty}\left\{[1-\rho^{i+1}]^2 - [1-\rho^i]^2\right\}\right] = 0 .$$

REFERENCES

Abel, A., 1990. Asset prices under habit formation and catching up with the Joneses, American Economic Review Papers and Proceedings, Vol.80, No.2, 38-42.

Brandt, M.W., Q. Zeng, and L. Zhang, 2004. Equilibrium stock return dynamics under alternative rules of learning about hidden states, Journal of Economic Dynamics and Control 28, 1925-1954.

Brennan, M.J. and Y. Xia, 2001. Stock price volatility and equity premium, Journal of Monetary Economics 47, 249-283.

Burnside, C., 1998. Solving asset pricing models with Gaussian shocks, Journal of Economic Dynamics and Control 22, 329-40.

Cecchetti, S.G., Lam, P-s., Mark, N.C., 2000. Asset pricing with distorted beliefs: Are equity returns too good to be true?, American Economic Review 90, 787-805.

Collard, F., Feve, P., Ghattassi, I., 2006. Predictability and habit persistence, Journal of Economic Dynamics and Control, forthcoming.

David, A., 1997. Fluctuating confidence in stock markets: Implications for returns and volatility, Journal of Financial and Quantitative Analysis, Vol.32, No.4, 427-462.

Detemple, J.B., 1991. Further results on asset pricing with incomplete information. Journal of Economic Dynamics and Control 15, 425-453.

___________, 1986. Asset pricing in a production economy with incomplete information, Journal of Finance, Vol.41, No.3, 383-391.

Dothan, M.U. and D. Feldman, 1986. Equilibrium interest rates and multiperiod bonds in a partially observable economy, Journal of Finance, Vol.41, No.2, 369-382.

Froot, K.A., Obstfeld, M., 1991. Intrinsic bubbles: The case of stock prices, American Economic Review 81, No.5, 1189-1214.

Gennotte, G., 1986. Optimal portfolio choice under incomplete information, Journal of Finance, Vol.41, No., 733-746.

Harvey, A.C., 1992. Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, Cambridge, UK.

Lucas, Jr., R.E., 1978. Asset prices in an exchange economy, Econometrica 46, 1429-45.

Marsden, J.E., 1974. Elementary Classical Analysis, W.H. Freeman and Company, San Francisco.

Mehra, R., Prescott, E.C., 1985. The equity premium: a puzzle, Journal of Monetary Economics 15, 145-161.

Veronesi, P., 2004. Belief dependent utilities, aversion to state uncertainty, and asset prices, unpublished manuscript, GSB University of Chicago.

Table 1. Maximum Likelihood Parameter Estimates

Panel A: Incomplete Information	$\bar{\mu}$	ρ	σ_η	σ	$\log L$
	0.0050 (0.0007)	0.7396 (0.1260)	0.0022 (0.0008)	0.0058 (0.0004)	755.4003
Panel B: Complete Information	$\bar{\mu}$	ρ	σ_η		$\log L$
	0.0050 (0.0006)	0.1758 (0.0674)	0.0066 (0.0003)		753.5202

This table reports the Maximum Likelihood estimates of the model for dividend growth rates, $x_t = \mu_t + \varepsilon_t$ where $\varepsilon_t \sim \text{iid}\, N(0, \sigma^2)$ and where the unobserved persistent component μ_t follows:

$$\mu_t - \bar{\mu} = \rho\left(\mu_{t-1} - \bar{\mu}\right) + \eta_t, \qquad \text{with} \quad |\rho| < 1 \quad \text{and} \quad \eta_t \sim \text{iid}\, N(0, \sigma_\eta^2).$$

The model is calibrated to quarterly real per capita US consumption growth rates on non-durables and services from the first quarter of 1952 through the second quarter of 2004. Nominal seasonally adjusted per capita consumption data obtained from NIPA tables are deflated using the CPI index.

Panel A reports estimates for the incomplete information model given by the two equations above. Panel B reports estimates for the complete information model obtained by setting ε_t to zero (i.e. by setting $\sigma^2 = 0$).

Conditional densities of the state variable μ_t are obtained by applying the Kalman filter in panel A. Standard errors are reported below each parameter estimate.

Table 2. Unconditional Moments of Returns

Panel A: Data (1952:1 to 2004:2)			$E(R_t)$	$\sigma(R_t)$	$E(R_t^f)$	$\sigma(R_t^f)$
			6.82	16.55	1.23	1.34
Panel B: Incomplete Information	θ	γ	$E(R_t)$	$\sigma(R_t)$	$E(R_t^f)$	$\sigma(R_t^f)$
	0.98	0.60	0.07	1.99	9.39	0.24
	0.98	0.90	0.07	1.73	10.01	0.36
	0.98	1.50	0.06	1.22	11.23	0.61
	0.99	0.60	0.07	1.99	5.26	0.24
	0.99	0.90	0.07	1.72	5.86	0.36
	0.99	1.50	0.06	1.19	7.08	0.60
Panel C: Complete Information						
	0.98	0.60	0.05	1.46	9.40	0.15
	0.98	0.90	0.05	1.37	10.02	0.22
	0.98	1.50	0.05	1.21	11.25	0.36
	0.99	0.60	0.05	1.44	5.27	0.14
	0.99	0.90	0.05	1.37	5.88	0.22
	0.99	1.50	0.05	1.20	7.10	0.36

Panel A reports unconditional moments of quarterly value-weighted excess returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period going from the first quarter of 1952 through the second quarter of 2004. Excess returns are calculated over the riskfree rates proxied by the one-month Treasury bill rates. All rates are expressed in percent per annum.

Panels B and C report the unconditional moments of simulated returns obtained from the asset pricing model by feeding simulated consumption growth rates data using the estimated parameters from each of the two panels in Table 1. The statistics reported in percentage per annum are the mean $E(R)$ and standard deviation $\sigma(R)$ of excess returns R and of risk free returns R^f.

Model-implied moments are reported for a range of values for the subjective discount factor θ, and the risk-aversion coefficient γ.

Figure 1. Filter Mean $E(\mu_t \mid x_1, x_2, \ldots, x_t)$

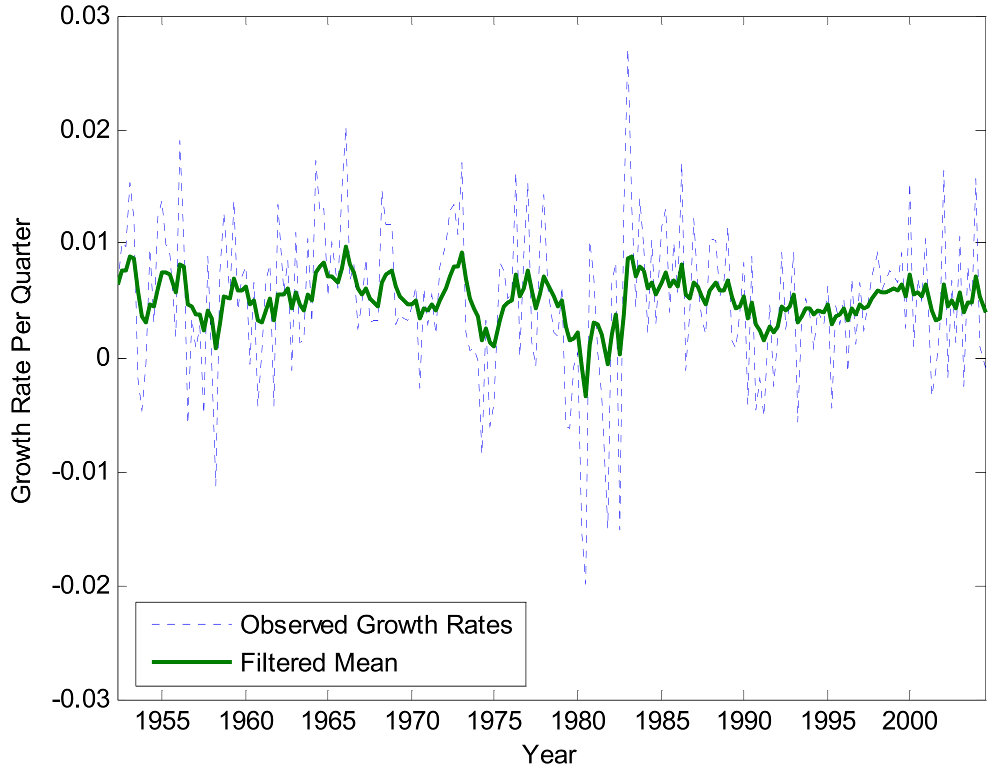

Figure 1 plots the mean of the filter densities $E(\mu_t \mid x_1, x_2, \ldots, x_t)$, along with the observed consumption growth rates x_t. The mean of the filter densities are estimated with the Kalman filter using the Maximum Likelihood parameter estimates of Panel A in Table 1.