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Coherent manipulation of quantum states in a coupled cavity-atom system

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Abstract

We study atomic coherence and interference in four-level atoms confined in an optical cavity and explores the interplay between cavity QED and electromagnetically induced transparency (EIT). The destructive interference can be induced in the coupled cavity-atom system with a free-space control laser tuned to the normal mode resonance and leads to suppression of the normal mode excitation. Then by adding a pump laser coupled to the four-level atoms from free space, the control-laser induced destructive interference can be reversed and the normal mode excitation is restored. When the free-space control laser is tuned to the atomic resonance and forms a Λ -type EIT configuration with the cavity-atom system, EIT is manifested as a narrow transmission peak of a weak probe laser coupled into the cavity mode. With the free-space pump laser driving the cavity-confined atoms in a four-level configuration, the narrow transmission peak of the cavity EIT can be split into two peaks and the dressed intra-cavity dark states are created analogous to the dressed states in free space. We report experimental studies of such coherently coupled cavity-atom system realized with cold Rb atoms confined in an optical cavity and discuss possible applications in quantum nonlinear optics and quantum information science.

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1. Introduction

Cavity QED has been a subject of many recent studies and has a variety of applications in quantum physics and quantum electronics [1]. The basic cavity QED system consists of a single two-level atom coupled to a single cavity mode [2]. The composite atom-cavity system exhibits a double-peaked transmission spectrum representing the two normal modes of the first excited eigenstates. When the system is resonantly coupled, the two normal modes are separated in frequency by 2g ($g = \mu \sqrt{\omega_a/2\hbar\varepsilon_0 V}$ is the atom-cavity coupling coefficient), commonly referred to as the vacuum Rabi splitting [1-2]. Observation of the two normal modes in the optical wavelength range requires a cavity-atom system with a g value greater or comparable with the decay rates of the cavity and the atomic system, which can be realized in a high finesse cavity with a small mode volume [2]. On the other hand, if N two-level atoms collectively interact with the cavity mode, the coupling coefficient becomes $G = \sqrt{Ng}$ and the vacuum Rabi splitting of the normal modes for the collectively coupled atom-cavity system becomes 2G and may then be observed in a cavity with a moderate mode volume and finesse [3-5].

Electromagnetically induced transparency (EIT) can be created in various atomic systems via coherent interactions of radiations fields and atoms [6-7]. EIT has been shown to be important for various applications in quantum optics and nonlinear optics [8-10]. Recent studies of EIT and related phenomena have been extended to coherent coupled atom-cavity systems [11-14]. It has been shown that in a coherently coupled cavity and multi-atom system, the interplay of the collective coupling of the atoms and the cavity mode, and the atomic coherence and interference manifested by EIT may lead to interesting linear and nonlinear optical phenomena [15-20].

Here we present studies of an atom-cavity system consisting of N four-level atoms confined in an optical cavity and coherently coupled from free space by two laser fields: one acts as a coupling laser and forms a Λ -type standard EIT configuration with the cavity mode; another acts as a pump laser and forms a N-type coupled atomic system with the control laser and the cavity mode. By varying the frequency detuning of the control laser, several distinct phenomena are manifested in the coupled cavity-atom system. When the control laser is tuned to the atomic resonance and the pump laser is absent, the cavity-atom system exhibit cavity EIT [11-15]. If the pump laser is turned on and tuned to the atomic resonance, the transmission peak of the cavity EIT can be split into two peaks, indicating generation of two dressed intra-cavity dark states [21-22]. On the other hand, when the control laser is tuned to the resonance of one of the two normal modes and the pump laser is off, the destructive interference is induced for the normal mode excitation, which is manifested as a narrow dip in the transmission spectrum of the probe laser coupled into the cavity mode [23-24]. When the pump laser is on and tuned to the atomic resonance, the destructive interference induced by the control laser is reversed and the light transmission of the probe laser at the resonance of the normal mode is restored. Such manipulation of the coupled cavity-atom system shows a simple way to control the quantum states of the cavity QED by the laser induced coherence and interference and may be useful in a variety of applications in nonlinear optics, and quantum information science.

2. Theoretical analysis

We consider a composite atom-cavity system that consists of a single mode cavity confining N identical four-level atoms driven by a control laser and a pump laser from free space as shown in Fig. 1. The cavity mode couples the atomic transition $|1\rangle-|3\rangle$ and $\Delta_c = \nu_c - \nu_{13}$ is the cavity-atom

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detuning. The classical control laser drives the atomic transition $|2\rangle-|3\rangle$ with Rabi frequency 2Ω , and the classical pump laser drives the atomic transition $|2\rangle-|4\rangle$ with Rabi frequency $2\Omega_s$. $\Delta = \nu - \nu_{23}$ is the control frequency detuning and $\Delta_s = \nu_s - \nu_{24}$ is the pump laser detuning. We calculate the light transmission of a weak probe laser coupled into the cavity mode as the probe frequency detuning $\Delta_p = \nu_p - \nu_{13}$ is scanned across the atomic transition frequency ν_{13} . The interaction Hamiltonian for the cavity-atom system is

$$H = -\hbar \left(\sum_{i=1}^{N} \Omega \,\hat{\sigma}_{32}^{(i)} + \sum_{i=1}^{N} \Omega_{s} \hat{\sigma}_{42}^{(i)} + \sum_{i=1}^{N} g \hat{a} \,\hat{\sigma}_{31}^{(i)}\right) + H.C. , \qquad (1)$$

where $\hat{\sigma}_{lm}^{(i)}$ (l, m=1-4) is the atomic operator for the ith atom and \hat{a} is the annihilation operator of the cavity photons. The resulting operator equations of motion for the intra-cavity light field (two-sided cavity, one input) is given by [25-26]

$$\dot{\hat{a}} = -\frac{i}{\hbar} [\hat{a}, H] - \frac{\kappa_1 + \kappa_2}{2} \hat{a} + \sqrt{\kappa_1} \hat{a}_p^{in}, \qquad (2)$$

where \hat{a}_p^m is the input probe field, and κ_1 and κ_2 are the loss rates of the cavity mirrors. The equation of the motion for the expectation value of the intra-cavity probe field is [20]

$$\dot{a} = -((\kappa_1 + \kappa_2)/2 - i\Delta_c)a + \sum_{i=1}^{N} ig\,\sigma_{31}^{(i)} + \sqrt{\kappa_1}a_p^{in}\,,\tag{3}$$

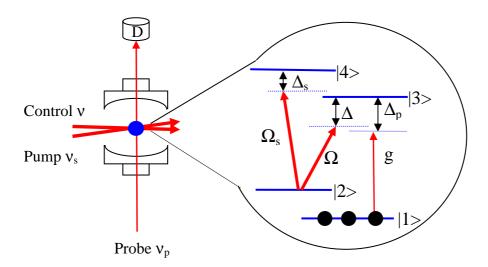


Fig.1 The schematic coupling scheme of coherently coupled four-level atoms in a cavity. A control laser drives $|2\rangle$ - $|3\rangle$ transition with Rabi frequency 2Ω and a pump laser couples $|2\rangle$ - $|4\rangle$ transition with Rabi frequency $2\Omega_s$. Δ (Δ_s) is the control (pump) detuning. The cavity mode is coupled to the atomic transition $|1\rangle$ - $|3\rangle$ with the collective coupling coefficient $\sqrt{N_g} \left(g = \mu \sqrt{\omega_a/2\hbar\varepsilon_0 V} \right)$ (Δ_c is the cavity-atom detuning). A weak probe laser is coupled into the cavity and Δ_p is its frequency detuning from the atomic transition.

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For a symmetric cavity (as in our experiment), $\kappa_1 = \kappa_2 = \kappa$. Under the condition of g<< Ω , the atomic population is concentrated in |1> and the steady-state solution of the intra-cavity probe field is given by

$$a = \frac{\sqrt{\kappa} a_p^{in}}{\kappa - i\Delta_c - i\chi},\tag{4}$$

where χ is the atomic susceptibity given by

$$\chi = \frac{ig^{2}N}{\Gamma_{3} - i\Delta_{p} + \frac{\Omega^{2}(\Gamma_{4} + \gamma_{12} - i(\Delta_{s} + \Delta_{p} - \Delta))}{(\gamma_{12} - i(\Delta_{p} - \Delta))(\Gamma_{4} + \gamma_{12} - i(\Delta_{s} + \Delta_{p} - \Delta)) + \Omega_{s}^{2}}} . (5)$$

Here Γ_3 (Γ_4) is the decay rate of the excited state $|3\rangle$ ($|4\rangle$) and γ_{12} is the decoherence rate of the ground states $|1\rangle$ and $|2\rangle$. The transmitted probe field is then given by $a_p^{out} = \sqrt{\kappa}a$.

First, we consider manipulation of cavity EIT in the four-level system. Fig. 2 plot the normalized transmission intensity $\frac{I_t}{I_{in}} = \frac{|a_p^{out}|^2}{|a_p^{in}|^2}$ of the probe laser coupled into the cavity mode

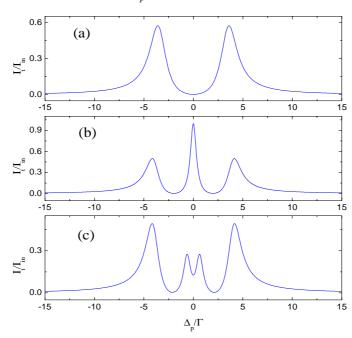


Fig. 2 The transmission intensity of the probe laser coupled into the cavity-atom system versus the probe detuning Δ_p/Γ . (a) Without the control laser ($\Omega=0$) and the pump laser ($\Omega_s=0$). (b) With the control laser at $\Delta=0$ ($\Omega=2\Gamma$), but without the pump laser ($\Omega_s=0$), (c) With both the control laser ($\Omega=2\Gamma$) and the pump laser at $\Delta_s=0$ ($\Omega_s=\Gamma$). The other parameters are $\Gamma_3=\Gamma_4=\Gamma$, $g\sqrt{N}=3.5\Gamma$, $\kappa=1.5\Gamma$, $\gamma_{12}=0.001\Gamma$, and $\Delta_c=0$.

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versus the probe frequency detuning Δ_p/Γ . Fig. 2(a) depicts the probe transmission spectrum without both the control laser and the pump laser, in which the two transmission peaks represent the two normal modes separated in frequency by the vacuum Rabi frequency $2G = 2\sqrt{Ng}$ [3-5]. When the control laser is present and tuned to the atomic resonance at Δ =0 (without the pump laser), cavity EIT is created in the cavity-atom system, and the probe transmission spectrum exhibits three peaks as shown in Fig. 2(b): the central peak at $\Delta p=0$ represents the cavity EIT, or intra-cavity dark state [11-13] and the two sideband peaks represent the normal modes of the coupled cavity-atom system [3-5], which are modified by the free-space control laser [21]. Fig. 2(c) plots the probe transmission spectrum when a resonant pump laser (Δ_s =0) and a Rabi frequency $\Omega_s = \Gamma$ is present. It shows that the central EIT peak is split into two peaks and the peak separation is approximately equal to $2\Omega_s$. This is analogous to the dressed states of a laser coupled two-level system in free space [27]. Next we consider the effect of the control-laserinduced interference on the normal mode excitation of the multi-atom cavity QED system. Fig.

3(a) plots the transmitted intensity of the probe field $\frac{I_t}{I_{in}}$ versus the probe frequency detuning

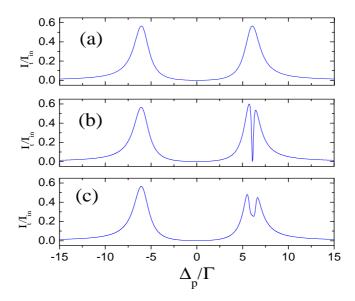


Fig. 3 The normalized transmission intensity I_t/I_{in} of the probe laser (I_{in} is the probe input intensity) through the cavity versus the probe detuning Δ_p/Γ . (a) The probe transmission spectrum without the control laser (Ω =0) and the pump laser (Ω s=0). The two transmission peaks represents the two normal modes of the cavity QED system. (b) The probe transmission spectrum with the control laser (Ω =2 Γ), but without the pump laser (Ω_s =0), (c) The probe transmission spectrum with both the control laser $(\Omega=0.5\Gamma)$ and the pump laser $(\Omega_s=0.5\Gamma)$. The relevant parameters are $\Gamma_3=\Gamma_4=\Gamma$, $g\sqrt{N} = 6\Gamma$, $\kappa = 1.5\Gamma$, $\gamma_{12}=0.001\Gamma$, and $\Delta_c=0$.

 $\Delta p/\Gamma_3$ without the control laser and the pump laser. The probe spectrum exhibits two transmission peaks representing two normal modes separated by the vacuum Rabi frequency 2G. Fig. 3(b)

doi:10.1088/1742-6596/414/1/012001

plots the probe transmission spectrum with the control laser present (Ω =0.5 Γ) and tuned to the normal mode resonance at $\Delta = \sqrt{N}g$, but without pump laser (Ω_s =0). Fig. 3(b) shows that the control laser induces the destructive interference and suppresses the normal mode excitation (the probe transmission at the normal mode resonance (Δ_p =G) is blocked) [23]. Fig. 3(c) plots the probe transmission when the pump laser is present (Ω_s =0.5 Γ) and tuned to the atomic resonance (Δ_s =0). The spectrum shows that the pump laser reverses the destructive interference and enables the excitation of the normal mode and therefore the transmission of the probe light as seen by the increased transmission at the normal mode resonance Δ_p =G.

3. Experimental results

We carried out experimental studies of the coherently coupled cavity and four-level atoms system with cold ⁸⁵Rb atoms confined in a near confocal cavity consisting of two mirrors of 5 cm curvature with a mirror separation ~ 5 cm. The empty cavity finesse is measured to be ~ 150. A detailed description of our experimental set up can be found in elsewhere [13,21] and is only briefly outlined here. Three extended-cavity diode lasers were used as the control laser that drives the ⁸⁵Rb D₁ transition F=3 to F'=3, the pump laser that couples the ⁸⁵Rb D₂ transition F=3 to F'=4, and the probe lasers that couples the ⁸⁵Rb D₁ transition F=2 to F'=3. The circularly-polarized control laser and the linearly polarized pump laser were directed to overlap the cold atoms from the open side of the cavity and propagated in the directions nearly perpendicular to the cavity axis. The probe laser was linearly polarized parallel to the propagating direction of the control laser and then after sufficient attenuation, was coupled into the cavity. The transmitted probe light was collected by a photon counter (PerkinElmer SPCM-AQR-16-FC). The probe transmission spectrum was measured as the probe laser frequency was scanned across the ⁸⁵Rb D₁ transition F=2 to F'=3.

The experiment was run sequentially with a repetition rate of 10 Hz. All lasers were turned on or off by acousto-optic modulators (AOM) according to the time sequence described below. For each period of 100 ms, ~98 ms was used for cooling and trapping of the 85 Rb atoms, during which the trapping laser and the repump laser were turned on by two AOMs while the coupling laser, the pump laser, and the probe laser were off. The time for the data collection lasted ~ 2 ms, during which the repump laser was turned off first, and then after a delay of ~0.2 ms, the trapping laser was turned off (the current to the anti-Helmholtz coils of the MOT was always kept on), and the coupling laser, the pump laser, and the probe laser were turned on. After the coupling laser, the pump laser, and the probe laser were turned on by the AOMs for 0.2 ms, the probe laser frequency was scanned across the 85 Rb D₁ F=2 \rightarrow F=3 transitions and the probe light transmitted through the cavity was then recorded versus the probe frequency detuning.

Fig. 4 plots the measured cavity transmission intensity of the probe laser I_{out}/I_{in} (I_{in} is the resonant transmission of the probe light through an empty cavity) versus the probe frequency detuning Δ_p . The empty cavity is tuned to the atomic transition frequency $\Delta_c = \nu_c - \nu_{13} = 0$ and both the control laser and the pump laser are on resonance ($\Delta \approx 0$ and $\Delta_s \approx 0$). The decay linewidth of the Rb transitions are $\Gamma_3 = 5.7$ MHz and $\Gamma_4 = 5.9$ MHz, respectively. Other parameters are $g\sqrt{N} = 20$ MHz, $\kappa = 14$ MHz, $\Omega = 12$ MHz, $\gamma_{12} = 0.01\Gamma$, and $\Delta_c = \Delta = \Delta_s = 0$. The measured spectrum was the average of 50 scans. Fig. 4(a) plots the probe transmission spectrum without both the control laser and the pump laser. The two transmission peaks represent the two normal modes

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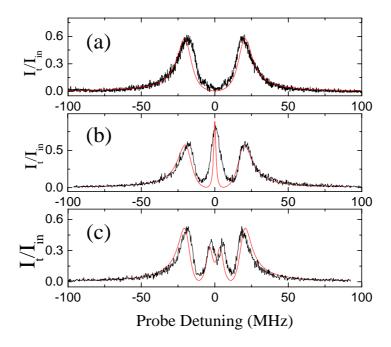


Fig. 4 The cavity transmission intensity I_t/I_{in} versus the probe detuning Δ_p . Black lines are experimental data and red lines are calculations. (a) Without both the control laser and the pump laser. (b) With the control laser ($\Omega \approx 12$ MHz) and without the pump laser. (c) With both the control laser ($\Omega \approx 10$ MHz) and the pump laser $\Omega_s \approx 8$ MHz.

separated in frequency by the vacuum Rabi frequency $(2 g\sqrt{N} \approx 40 MHz)$. Fig. 4(b) plots the probe transmission spectrum with the control laser, but without the pump laser, and exhibits the threepeaked cavity EIT spectrum: two sideband peaks located at $\Delta_p = \pm \sqrt{\Omega^2 + g^2 N}$ represent the normal modes of the coupled cavity-atom system, and a central peak at Δ_p =0 is manifested by EIT (the intra-cavity dark state) [11,13]. Fig. 4(c) plots the probe transmission spectrum with both the control laser and the pump laser, in which the cavity EIT peak is split into two peaks. We observed that the splitting is nearly equal to $2\Omega_s$ and appear at sufficiently high pump intensities $(\Omega_3 > 3 \text{ MHz})$. We are in the process of carrying out experimental studies of manipulating the excitation of the normal modes by the quantum interference in the cavity-coupled four-level atomic system. We observed destructive interference in the normal mode excitation induced by the control laser when it is tuned to the normal mode resonance (detuned from the atomic resonance by Δ =G). The experimental results are plotted in Fig. 5. Fig. 5(a) plots the probe transmission spectrum without the control laser. The measured vacuum Rabi splitting is $2G = 2g\sqrt{N} \approx 85 \text{MHz}$ (the experiment was done with the number of cold atoms about 4 times that of the experiment shown in Fig. 4). Fig. 5(b) plots the probe transmission spectrum when the control laser is present (Δ =42 MHz and Ω =12 MHz), but without the pump laser. It shows that the normal mode excitation at the resonance Δ_p =G is suppressed and a dip appears in the probe transmission. We plan to add the pump laser to form the coherently coupled four-level atom and cavity system and in subsequent experiments, expect to observe the phenomenon of reversed destructive interference and restoration of the normal mode excitation predicted in the theoretical calculation of Fig. 3(c).

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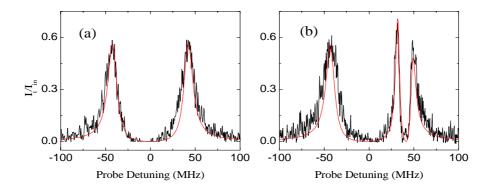


Fig. 5 The probe transmission intensity versus the probe detuning Δ_p . Black lines are experimental data and red lines are calculations. (a) Without both the control laser and the pump laser. (b) With the control laser ($\Omega \approx 12$ MHz) and without the pump laser. The parameters are $2G = 2g\sqrt{N} = 85$ $\kappa = 14$ MHz, $\Omega = 12$ MHz, $\gamma_{12} = 0.01\Gamma$, $\Delta = g\sqrt{N}$, and $\Delta_c = \Delta_s = 0$

4. Conclusion

In conclusion, we have shown that the atomic coherence and interference induced by laser fields in four-level atoms can be used to manipulate and control quantum states of the coupled cavity and atom system. Specifically, the destructive interference can be induced by a free-space control laser in the excitation of the normal mode of the cavity-atom system and can be reversed by a free-space pump laser. Under appropriate conditions, the same control laser and the pump laser can be also used to manipulate the cavity EIT and create the dressed intra-cavity dark states. The coherent coupled cavity-atom system and the interference phenomena reported may be useful to a variety of applications in nonlinear optics and quantum physics. For example, the coherently coupled cavity-atom system can be used to explore the light-control-light phenomena such as all optical switching and cross-phase modulations at ultra-low light intensities [17,24], which may be useful for quantum gates applications. The cavity-atom system can be also used to realize the broadband cavity EIT and explore possibility of applications for multi-channel and multi-color light memories [10, 28]. Furthermore, our recent study shows that the transmitted photons and reflected photons from the coherently coupled cavity-atom system are highly correlated, which may render the system useful for studies of the photon correlation and quantum entanglement for the atoms and light fields.

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