2014

Light controlling light in a coupled cavity-atom system

Bichen Zhou
Department of Physics, Florida International University, bzou@fiu.edu

Zheng Tan
Department of Physics, Florida International University; Chinese Academy of Sciences, ztan@fiu.edu

Yifu Zhu
Department of Physics, Florida International University, yifuzhu@fiu.edu

Light controlling light in a coupled cavity-atom system

Bichen Zou[1], Zheng Tan[1-2], and Yifu Zhu[1]

[1] *Department Physics, Florida International University, Miami, FL, USA*
[2] *Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan, China*

Abstract. We present a theoretical model of light controlling light in a multi-atom cavity QED system consisting of three-level atoms confined in a cavity and interacting with a control laser from free space. A signal laser is coupled into the cavity and provides two output light channels: the transmitted signal light through the cavity and the reflected signal light from the cavity. We show that with the cavity electromagnetically induced transparency manifested by the free-space control light, the amplitude and the phase of the intra-cavity signal light can be manipulated by the free-space control laser. We analyze the cavity transmitted signal light and the cavity reflected signal light, and show that the two output channels of the signal light are complementary and the cavity-atom system is a versatile system for studies of all-optical switching and cross-phase modulation at low control intensities.

1. Introduction

Cavity quantum electrodynamics (cavity QED) studies the coherent interactions of two-level atoms and photons in a cavity and has found a variety of applications in quantum physics and quantum electronics [1-4]. In recent years, studies of cavity QED have been extended to the interactions of the cavity mode and multi-level atoms. There were earlier studies of optical bistability in three-level atoms confined in an optical cavity [5]. Recent studies of atom-cavity interactions have been directed to a composite system of an optical cavity and coherently prepared multi-level atoms, in which the atomic coherence and interference such as electromagnetically induced transparency (EIT) [6] can be used to manipulate quantum states of the coupled cavity and atom system and selectively enhance the optical nonlinearities [7-11]. It has been shown that in a coherently coupled cavity and multi-atom system, the interplay of the collective coupling of the atoms and the cavity mode, and the atomic coherence and interference manifested by EIT may lead to interesting linear and nonlinear optical phenomena [12-21].

Here we present a theoretical study of an atom-cavity system consisting of N three-level atoms confined in an optical cavity and coherently coupled from free space by a control laser. The system forms a Λ-type standard EIT configuration with the cavity mode. We show that the free-space control laser induces EIT for the intra-cavity signal field, which can be used to control both the amplitude and phase of the cavity-reflected signal light and the cavity-transmitted signal light. Under appropriate conditions, all-optical switching and large cross-phase modulation (XPM) for the reflected signal field and transmitted signal field can be obtained with a weak control field. The cavity-atom system provides an example of resonant nonlinear optics at low light intensities and can be used to explore fundamental studies of light controlling light phenomena.

2. Theoretical analysis

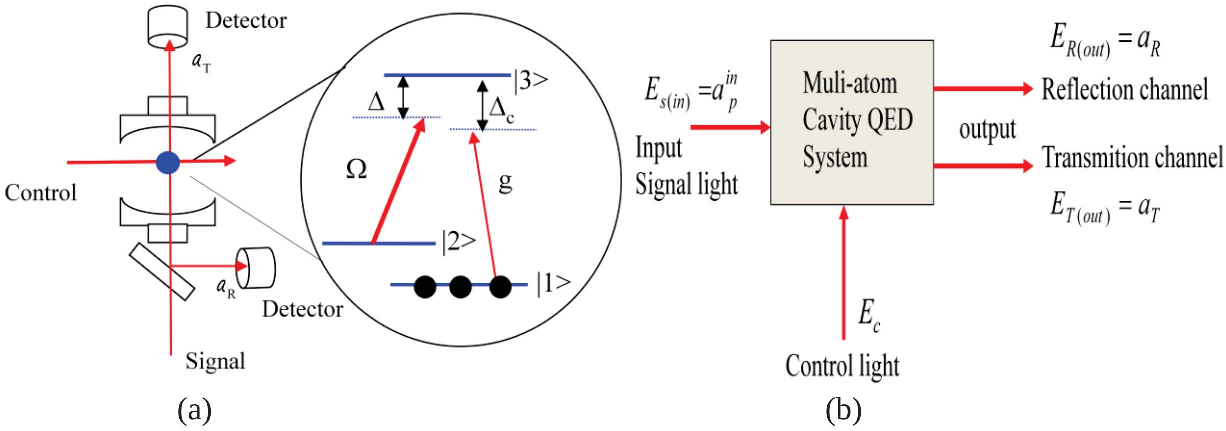

Fig.1 (a) Schematic coupling scheme of coherently coupled three-level atoms in a cavity. A control laser drives |2> - |3> transition with Rabi frequency 2Ω. Δ is the control detuning. A cavity is coupled to the atomic transition |1> - |3> with the collective coupling coefficient $\sqrt{N}g$ (Δ_c is the cavity-atom detuning). A signal laser is coupled into the cavity and Δ_p is its frequency detuning from the atomic transition. The cavity transmitted signal light and the cavity reflected signal light are collected by two detectors. (b) Schematic diagram of the light input and output channels. The cavity-atom system provides two output signal channels: the reflection channel and the transmission channel.

Fig. 1 shows the composite atom-cavity system that consists of a single mode cavity containing N Λ-type three-level atoms interacting with a control laser from free space. The cavity mode couples the atomic transition |1>-|3> and the classical control laser drives the atomic transition |2>-|3> with Rabi frequency 2Ω. $\Delta = \nu - \nu_{23}$ is the control frequency detuning and $\Delta_c = \nu_c - \nu_{13}$ is the cavity-atom detuning. A signal laser is coupled into the cavity mode and its frequency is detuned from the atomic transition |1>-|3> by $\Delta_p = \nu_p - \nu_{13}$. The interaction Hamiltonian for the coupled cavity-atom system is

$$H = -\hbar\left(\sum_{i=1}^{N} \Omega \hat{\sigma}_{32}^{(i)} + \sum_{i=1}^{N} g\hat{a}\hat{\sigma}_{31}^{(i)}\right) + H.C. \ , \qquad (1)$$

where $\hat{\sigma}_{lm}^{(i)}$ (l, m=1-4) is the atomic operator for the ith atom, $g = \mu_{13}\sqrt{\omega_c / 2\hbar\varepsilon_0 V}$ is the cavity-atom coupling coefficient, and $\hat{a}$ is the annihilation operator of the cavity photons. The resulting equation of motion for the expectation value the intra-cavity light field (two-sided cavity, one input) is given by [22] is

$$\dot{a} = -((\kappa_1 + \kappa_2)/2 - i\Delta_c)a + \sum_{i=1}^{N} ig\sigma_{31}^{(i)} + \sqrt{\kappa_1/\tau}\,a_p^{in} \, , \qquad (2)$$

where a_p^{in} is the input signal field, $\kappa_i = \frac{T_i}{\tau}$ (i=1-2) is the loss rate of the cavity field of the mirror i (Ti is the mirror transmission and τ is the photon round trip time inside the cavity). For simplicity, we consider a symmetric cavity such that $\kappa_1 = \kappa_2 = \kappa/2$. With g<<Ω, the atomic population is concentrated in |1> and the steady-state solution of the intra-cavity signal field is given by

$$a = \frac{\sqrt{\kappa/\tau}\, a_p^{in}}{\kappa - i\Delta_c - i\chi}, \qquad (3)$$

where χ is the atomic susceptibility given by $\chi = \dfrac{ig^2 N}{\Gamma_3 - i\Delta_p + \dfrac{\Omega^2}{\gamma_{12} - i(\Delta_p - \Delta)}}$. Here Γ_3 is the spontaneous decay rate of the excited state |3> and γ_{12} is the decoherence rate between the ground states |1> and |2>. The transmitted signal field is then given by $a_T = \sqrt{\kappa\tau}\, a$ and the reflected signal field from the cavity is $a_R = \sqrt{\kappa\tau}\, a - a_p^{in}$. We are interested in the regime of parameters near cavity EIT in which the laser fields are near or at resonance with the atomic transitions, and under the conditions of low light intensities in which the intra-cavity field is very weak and the control field is well below the saturation level. We show that a weak control light can induce large nonlinearities in the cavity-atom system, which then may be used to control the amplitude and phase of the cavity transmitted signal field and the cavity reflected signal field [23]. The primary control parameters of the light-control-light system are the frequency and intensity of the control laser that are characterized by the control detuning Δ and control Rabi frequency Ω respectively.

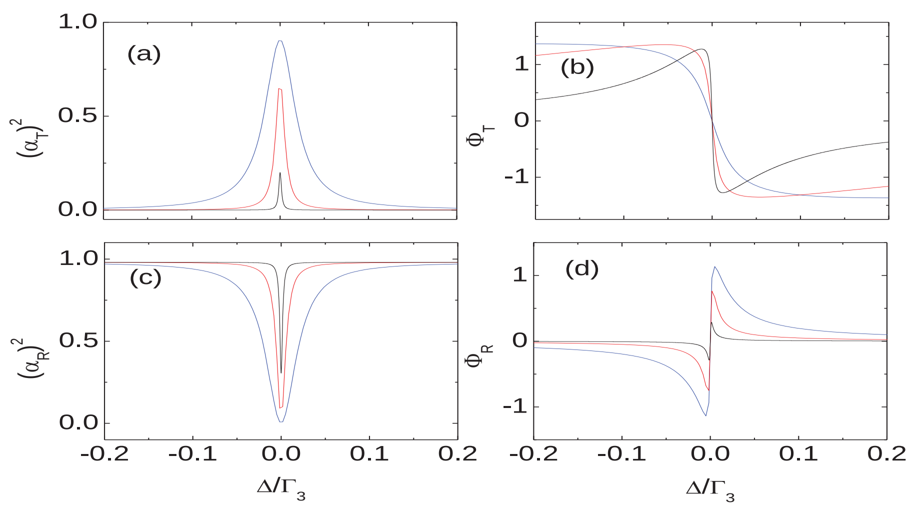

Fig. 2 (a) The intensity ratio α_T^2 and the phase shift Φ_T of the cavity transmitted field versus the control frequency detuning Δ/Γ_3. (c) The intensity ratio α_R^2 and (d) the phase shift Φ_R of the cavity reflected field versus the control frequency detuning Δ/Γ_3. The parameters used in the calculations are $g\sqrt{N} = 10\Gamma$, $\kappa = 2\Gamma_3$, γ_{12}=0.001Γ, and $\Delta_c = \Delta_p = 0$. The control Rabi frequency Ω=0.2Γ,0.5Γ, and Γ for the black lines, red lines, and blues lines respectively.

Fig. 2(a) and 2(b) plot the intensity ratio of the cavity transmitted field, $\frac{I_T}{I_{in}} = \alpha_T^2$ and the phase shift Φ_R versus the control frequency detuning Δ/Γ_3 respectively. Fig. 2(c) and 2(d) plot the intensity ratio of the cavity reflected field, $\frac{I_R}{I_{in}} = \alpha_R^2$ and the phase shift Φ_T versus Δ/Γ_3 respectively. The relevant parameters are $g\sqrt{N} = 10\Gamma_3$, $\kappa = 2\Gamma_3$, γ_{12}=0.001Γ_3, and $\Delta_c = \Delta_p = 0$. The transmission spectrum in Fig. 2(a) and 2(b) exhibit the standard cavity EIT spectral peak at Δ=0 with a peak linewidth that is substantially smaller than the decay width Γ_3 and increases with the increasing Ω of the control laser. The phase shift of the transmitted field varies rapidly across the resonance at Δ=0 and has a line shape of the anomalous dispersion. Concomitantly, the reflected field from the cavity exhibits a dip with the linewidth matching the peak linewidth of the transmitted field and its phase shift has a lineshape of the normal dispersion with a steep slope across Δ=0. The calculations of Fig. 2 are done with the signal laser tuned to the resonance Δp=0 and show that the transmitted field and the reflected field can be controlled by varying the frequency of the control laser. For example, Fig. 2(a) and 2(c) show that with a resonant control laser (Δ=0), the transmitted light field can be turned on or off by turning on or off of the control light, and at the same time and in contrast, the reflected light field is turned off or on. Thus the coupled cavity-atom system effectively performs all-optical switching with reciprocal functions for the transmission and reflection with a weak control laser. When the control laser is tuned slightly away from the resonance ($\Delta \neq 0$ but $\Delta \ll \Gamma_3$), large phase shifts are induced on the transmitted field and the reflected field (Fig. 2(b) and 2(d)), which effectively performs the cross-phase modulation (XPM) on the transmitted/reflected field with a weak control light near the atomic resonance.

2-1 All-optical switching

Fig. 2 shows that a weak control laser can be used to control the amplitude and phase of both the transmitted and reflected light fields from the cavity. In particular, when Δ=0 and $\Delta_c = \Delta_p = 0$, cavity EIT is established, and the coupled cavity-atom system can perform the all-optical switching function with two complementary output channels. Under the resonance condition (Δ=0 and $\Delta_c = \Delta_p = 0$), the reflected signal intensity is

$$I_R = \frac{g^4 N^2 \gamma_{21}^2 I_{in}}{(\kappa(\Omega^2 + \Gamma_3\gamma_{21}) + g^2 N\gamma_{21})^2}, \tag{4}$$

and the transmitted signal intensity is

$$I_T = \frac{\kappa^2 (\Omega^2 + \Gamma_3\gamma_{21})^2 I_{in}}{(\kappa(\Omega^2 + \Gamma_3\gamma_{21}) + g^2 N\gamma_{21})^2}. \tag{5}$$

The performance of an all-optical switch can be characterized by the switching efficiency defined as $\eta = \frac{I(|1>) - I(|0>)}{I_{in}}$, where I(|1>) is the signal output intensity when the switch is closed and I(|0>) is the signal output intensity (leakage) when the switch is open (I(|1>) > I(|0>)), and I_{in} is the input signal intensity. Eq. (4) indicates that $I_R(\Omega=0) > I_R(\Omega\neq0)$ so for the all-optical switching operating on the reflection output channel, we designate $I_R(\Omega\neq0)$ =I(|0>) for the open state |0> of the switch and $I_R(\Omega=0)$=I(|1>) for the closed state |1> of the switch, then the switching efficiency for the reflection output channel is

$$\eta_R == \frac{g^4 N^2}{(\kappa\Gamma_3 + g^2 N)^2} - \frac{g^4 N^2 \gamma_{21}^2}{(\kappa(\Omega^2 + \Gamma_3\gamma_{21}) + g^2 N\gamma_{21})^2} . \qquad (6)$$

Eq. (5) indicates that $I_T(\Omega\neq 0) > I_T(\Omega=0)$. Therefore, for the all-optical switching with the transmission output channel, we designate $I_T(\Omega=0)=I_T(|0>)$ for the open state |0> of the switch and $I_R(\Omega\neq 0)=I_T(|1>)$ for the closed state |1> of the switch. Then, the switching efficiency for the transmission output channel is

$$\eta_T = \frac{\kappa^2(\Omega^2 + \Gamma_3\gamma_{21})^2}{(\kappa(\Omega^2 + \Gamma_3\gamma_{21}) + g^2 N\gamma_{21})^2} - \frac{\kappa^2 \Gamma_3{}^2}{(\kappa\Gamma_3 + g^2 N)^2} . \qquad (7)$$

Eq.(6) and Eq.(7) show that under appropriate conditions, the switching efficiency for the reflection channel as well as the transmission can be near unity. In order to provide a detailed picture for the study of all-optical switching in the coherently coupled cavity-atom system, we present numerical calculations in Figures 3-6 and show that practical parameters can be identified, in which the all-optical switching of the two output channels with the switching efficiency near unity can be obtained.

The switching efficiency η_R and η_T depend on the collective coupling coefficient $g\sqrt{N}$, the control laser Rabi frequency 2Ω, the cavity decay rate κ, and the deocherence rate γ_{21}. In order to clarity the performance characteristics of the coupled cavity-atom system, we plot in Fig. 3 to

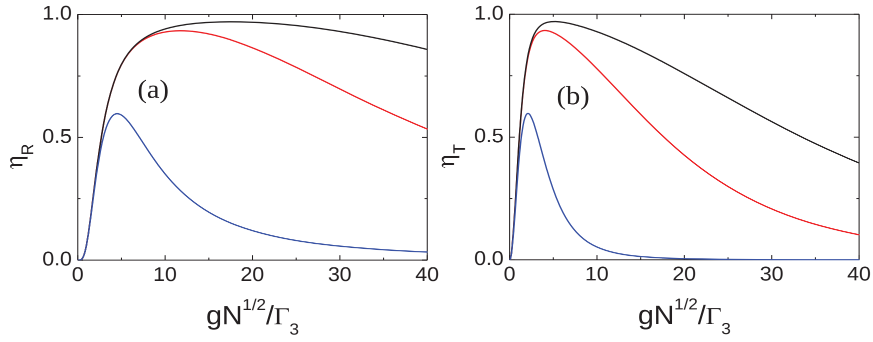

Fig. 3 (a) Switching efficiency of the reflected signal light η_R and (b) switching efficiency of the transmitted signal light η_T versus $g\sqrt{N}/\Gamma_3$ with $\gamma_{12}=0.0001\Gamma_3$ (black lines), $\gamma_{12}=0.001\Gamma_3$ (red lines) and $\gamma_{12}=0.01\Gamma_3$ (blues lines), respectively. Other parameters are $\Delta p=\Delta= g\sqrt{N}$. $\Omega=0.3\Gamma_3$, and $\kappa=3\Gamma_3$.

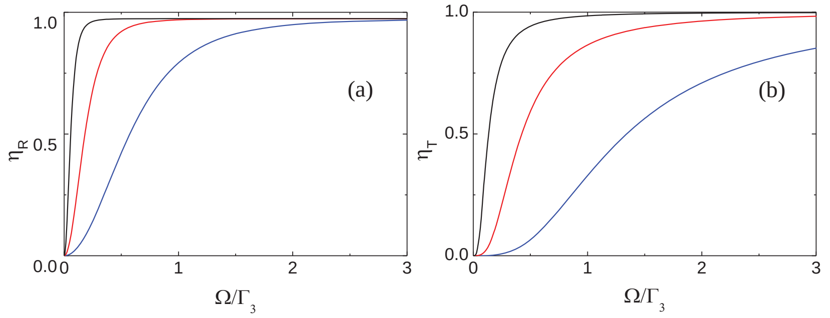

Fig. 4 (a) Switching efficiency of the reflected signal light η_R and (b) switching efficiency of the transmitted signal light η_T versus Ω/Γ_3 with γ_{12}=0.0001Γ_3 (black lines), γ_{12}=0.001Γ_3 (red lines) and γ_{12}=0.01Γ_3 (blues lines), respectively. Other parameters are $\Delta p=\Delta= g\sqrt{N}=20\Gamma_3$, $\Omega=0.3\Gamma_3$ and $\kappa=3\Gamma_3$,

Fig. 6 separately the switching efficiency η_R and η_T under the resonance conditions Δ=0 and $\Delta_c = \Delta_p = 0$, versus these parameters with practical values obtainable experimentally [9]. Fig. 3

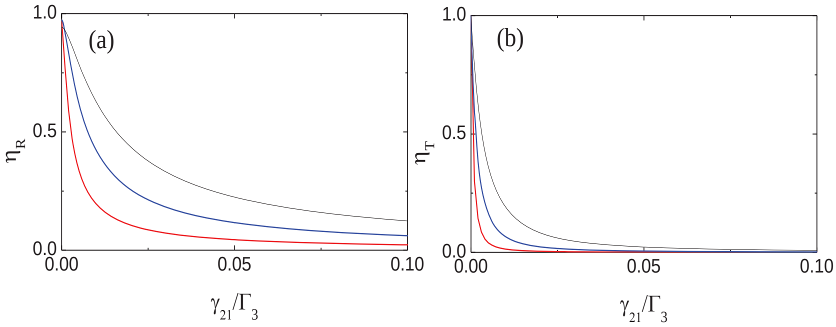

Fig. 5 (a) Switching efficiency of the reflected signal light η_R and (b) switching efficiency of the transmitted signal light η_T versus the decoherence rate γ_{21}/Γ_3 with $g\sqrt{N}=20\Gamma_3$ (red lines), $g\sqrt{N}=15\Gamma_3$ (black lines) and $g\sqrt{N}=10\Gamma_3$ (blues lines), respectively. Other parameters are $\Omega=0.5\Gamma_3$ and $\kappa=3\Gamma_3$.

shows that η_R and η_T increases initially with the collectively coupling coefficient $g\sqrt{N}$ and are maximized at moderate $g\sqrt{N}$ value. Fig. 4 shows that the all-optical switching of the cavity-atom system can be done with a weak control laser. The switching efficiency increases rapidly with the increasing control field, but saturates as Ω values near the decay rate Γ_3. Fig. 5 shows that η_R and η_T decreases monotonically with increasing γ_{21}, which indicates that the all- optical switching is a coherent process and the switching efficiency (particularly η_T) depends sensitively

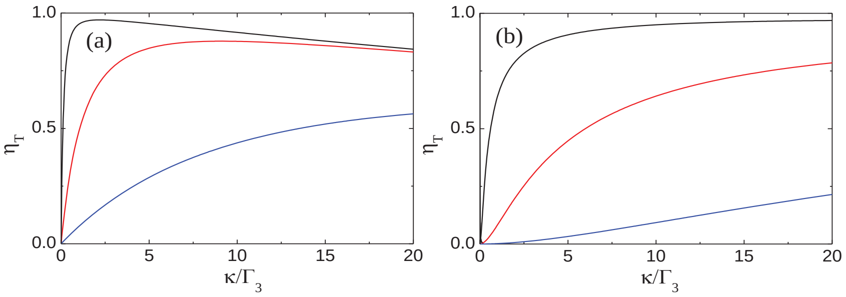

Fig. 6 (a) Switching efficiency of the reflected signal light η_R and (b) switching efficiency of the transmitted signal light η_T versus κ/Γ_3 with γ_{12}=0.0001Γ_3 (black lines), γ_{12}=0.001Γ_3 (red lines) and γ_{12}=0.01Γ_3 (blues lines), respectively. Other parameters are $g\sqrt{N}$ =15Γ_3, and Ω=0.2Γ_3.

on γ_{21}. For the high efficiency operation of the all-optical switching, it is desirable to have an atomic system with small decay rate of the ground state coherence. It has been shown [36] that in cold Rb atoms, the decoherence rate as small as $\gamma_{12}=10^{-4}\Gamma_3$ has been observed. Therefore, it is possible to achieve high switching efficiencies in experiments with cold alkaline atoms as the optical medium.

Fig. 6 plots η_R and η_T versus the cavity decay rate κ and shows that it is more efficient to operate the all-optical switching of the cavity-atom system at a relatively high κ value. Therefore, a moderate to low Q value of the cavity provides a better switching efficiency. Overall, the highly efficient all-optical switching at low control intensities requires a moderately large collective coupling coefficient $g\sqrt{N}$, a sufficiently large cavity decay rate κ, and a small decoherence rate γ_{21}. These requirements can be readily fulfilled experimentally. As a numerical example, consider cold Rb atoms (Γ_3=3 MHz) confined in a 5 cm cavity with a finesse of 150 (κ=10 MHz), with $g\sqrt{N}$ =50 MHz (N≈10^4 atoms), γ_{21}=10 KHz(~0.003Γ_3), and Ω=1.5 MHz (corresponding to a control intensity $I_c = c\varepsilon_0 E^2 = c\varepsilon_0 (h\Omega/\mu_{23})^2 \approx$0.3 mW/cm^2 that is about 5 times smaller than the Rb saturation intensity of 1.6 mW/cm^2), the switching efficiency is derived to be η_R=0.83 and η_T=0.56. Fig. 3-6 also show that the switching efficiencies for the two output channels are different. Under the normal operating conditions discussed here, $\eta_R > \eta_T$: it is more efficient to operate the all-optical switch of the cavity-atom system in the reflection mode.

2-2 Cross-phase modulation

When the control laser is tuned away from the resonance ($\Delta \neq 0$, but keeping $\Delta_c = \Delta_p = 0$), the phase shift of the intra-cavity signal field is introduced by the control laser. We first derive the analytical results with γ_{12}=0 (neglecting the decoherence. The effect of decoherence with $\gamma_{12} \neq 0$ will be considered in the numerical calculations) for the two output channels of the signal light. Specifically, the light field transmitted through the cavity is $a_T = \alpha_T e^{i\Phi_T} a_p^{in}$. The amplitude ratio of the transmitted signal field to the input signal field is given by

$$\alpha_T = \frac{\kappa((\Omega^4 + (\Delta\Gamma_3)^2)^{1/2}}{((\kappa\Omega^2)^2 + ((g^2N + \kappa\Gamma_3)\Delta)^2)^{1/2}} , \tag{8}$$

and the phase shift Φ_T is given by

$$\Phi_T = \tan^{-1} \frac{(\Gamma_3\kappa - g^2N)\Omega^2\Delta}{g^2N\Delta^2\Gamma_3 + \kappa\Omega^4} . \tag{9}$$

Under the condition of the strong collective coupling ($g^2N >> \kappa\Gamma_3$) and for a weak control field ($\Omega^2 << \Delta\Gamma_3$. For the near resonant nonlinear optics studied here, $\Delta<\Gamma_3$, the phase shift of the signal field is $\Phi_T = -\frac{\Omega^2}{\Delta\Gamma_3}$, and the amplitude ratio of the transmitted field to the input signal field is $\alpha_T = \frac{\kappa\Gamma_3}{g^2N}$. The phase shift is proportional to the control laser intensity and corresponds to the standard Kerr nonlinearity. Under the strong collective coupling condition, a large phase shift of the transmitted field can be obtained but the transmitted field amplitude is very small.

The reflected signal field from the cavity is $a_R = \alpha_R e^{i\Phi_R} a_p^{in}$. The amplitude ratio of the reflected signal field to the input signal field is given by

$$\alpha_R = \frac{g^2N\Delta}{((\kappa\Omega^2)^2 + (g^2N + \kappa\Gamma_3)^2\Delta^2)^{1/2}} , \tag{10}$$

and the phase shift Φ_R of the reflected signal field is

$$\Phi_R = \tan^{-1} \frac{\kappa\Omega^2}{(g^2N + \kappa\Gamma_3)\Delta} . \tag{11}$$

As an example, with a weak, near resonant control (Ω=0.5Γ_3 and Δ=0.01Γ_3), $g\sqrt{N} = 10\Gamma_3$, and κ=2Γ_3, the amplitude ratio of the reflected signal field to the input signal field is ~ 79% and the

XPM phase shift of the reflected field from the control laser is ~0.5 rad. In the limit of $\kappa\Omega^2 << |(g^2N + \kappa\Gamma_3)\Delta|$, one derives $\alpha_R = \frac{g^2N}{g^2N + \kappa\Gamma_3}$ and $\Phi_R \approx \frac{\kappa\Omega^2}{(g^2N + \kappa\Gamma_3)\Delta}$, which is the phase shift from the Kerr nonlinearity induced by the weak control field on the reflected cavity field. The reflected field amplitude can be nearly equal to the input signal field when the condition of strong collective coupling is satisfied ($g^2N >> \kappa\Gamma_3$), but the phase shift Φ_R approaches zero. Therefore the optimal performance of the cavity system for XPM is achieved for moderate g^2N values. As it will be further clarified in the numerical calculations that by inducing cavity EIT in the coherently coupled cavity-atom system, the XPM can be obtained near the atomic resonance with the control detuning $\Delta<<\Gamma$, which leads to a larger Φ_R value, but at the same time still maintains a sufficiently large amplitude of the transmitted/reflected field. This is in sharp contrast with the conventional XPM systems in which the laser fields have to be tuned far away from the atomic resonance to minimize the absorption loss.

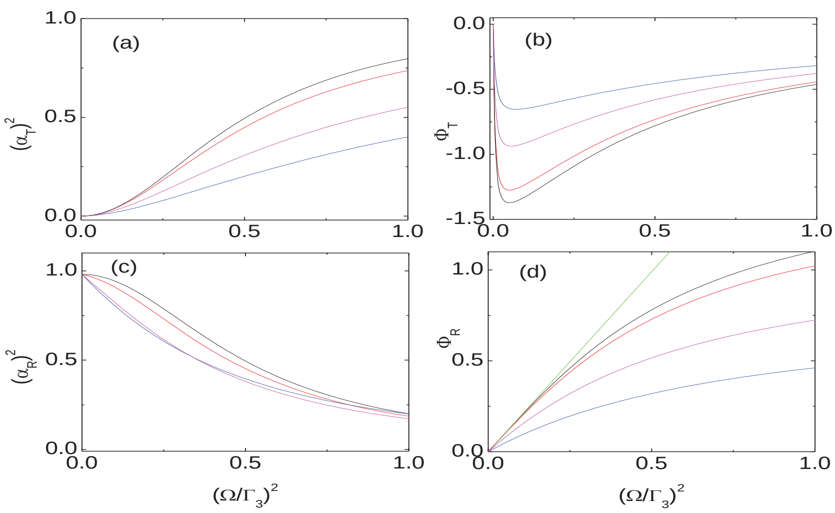

Fig. 7 (a) The intensity ratio α_T^2 and (b) phase shift Φ_T of the cavity transmitted signal field versus $(\Omega/\Gamma_3)^2$. (c) The intensity ratio α_R^2 and (d) phase shift Φ_R of the cavity reflected field versus $(\Omega/\Gamma_3)^2$. The parameters used in the calculations are $g\sqrt{N} = 10\Gamma_3$, $\kappa = 2\Gamma_3$, Δ=0.01Γ_3, and $\Delta_c = \Delta_p = 0$. The decoherence rate γ_{12}=0, 0.001Γ_3, 0.005Γ_3, and 0.01Γ_3 for the black lines, red lines, purple lines, and blues lines respectively.

It is necessary to quantify the XPM performance of the cavity-atom system by comparing the phase shit and the field amplitude. A useful system has to be able to produce large XPM phase shits and at the same time, provides sufficiently large field amplitudes. For studies of nonlinear optics at low light intensities, such requirements have to be obtained at the condition of low control field intensities. Fig. 7 plots $\frac{I_T}{I_{in}} = \alpha_T^2, \frac{I_R}{I_{in}} = \alpha_R^2$, Φ_T, and Φ_R versus the square of the control Rabi frequency Ω^2, which is proportional to the control light intensity $I = c\varepsilon_0(h\Omega/\mu_{23})^2$.

It can be seen that for the reflected field, the XPM phase shift increases with the control light intensity and exhibits saturation at high control intensities while the reflected light intensity decreases with the control intensity and approaches zero at high control intensities. For comparison, the green curve in Fig. 4(c) plots the phase shift given by $\Phi_R = \frac{\kappa\Omega^2}{(g^2N+\kappa\Gamma_3)\Delta}$, (Eq. (10) with $\kappa\Omega^2 << |(g^2N+\kappa\Gamma_3)\Delta|$). For the transmitted field, the XPM phase shift exhibits rapid change at low control intensities near zero and also saturates at high control intensities while the transmitted intensity increases with the control intensity.

The ground state decoherence degrades the system performance of the light control light. γ_{12} ultimately determines the minimum linewidth of the cavity EIT and also the lower limit of the control Rabi frequency Ω. As the decoherence rate γ_{12} increases, the amplitudes of the transmitted field and the reflected field decrease; the phase shifts of the transmitted field and the reflected field also decrease. It is preferable to select an atomic system with a small γ_{12}. We note that in cold Rb atoms, the decoherence rate $\gamma_{12}=10^{-4}\Gamma_3$ has been observed [24]. Therefore, a possible experimental system may consist of cold Rb atoms ($\Gamma_3 \approx 6$ MHz) and a cavity with a moderate finesse (with $\kappa = 2\Gamma_3$ and a cavity length of 5 cm, the required finesse is F=124). With a weak control laser, the linewidth of the cavity EIT is much smaller than Γ_3 (with $\Omega=0.5\Gamma_3$ and $\gamma_{12}=0.001\Gamma_3$, the linewidth of the cavity EIT is ~ $0.01\Gamma_3$), it is necessary to have the control laser and the signal laser with a linewidth much smaller than $0.01\Gamma_3$. For the Rb atoms, the required laser linewdth is in the KHz range, which can be obtained with the frequency stabilized diode lasers.

Fig. 7(a) and 7(c) show that at low control intensities, the reflected field amplitude is greater than the transmitted field amplitude. To achieve a large field amplitude, it is desirable to operate the cavity-atom system with the reflected light field. At low control intensities, the XPM phase shift is proportional to the control intensity, which is in the Kerr nonlinearity regime. A large φ_R value can be obtained at a sufficiently large α_R^2 value with a weak control light. As a numerical example, In an cavity-atom system with $\gamma_{12}=0.001\Gamma_3$, the XPM phase shift Φ_R ~0.5 rad. and the reflected intensity ratio α_R^2 ~ 70% can be obtained with practical parameters $\Omega=0.5\Gamma_3$ $g\sqrt{N}=10\Gamma_3$, $\kappa=2\Gamma_3$, $\Delta=0.01\Gamma_3$, and $\Delta_c=\Delta_p=0$.

The nonlinear optics at low light intensities requires a weak control field below the saturation intensity ($\Omega<\Gamma$). The presented calculations are valid under the condition ρ_{11}~1, i.e., the atomic population is concentrated in |1>, which requires $g<<\Omega$. Therefore, the cavity-atom coupling coefficient $g<<\Gamma$, consequently, this is the weak coupling regime of the cavity QED with single atoms (the bad cavity regime).

It is interesting to compare the light-control-light scheme based on the cavity EIT here with the earlier published scheme based on the cavity polariton interference [19-20]. The cavity polariton scheme works also with three-level atoms, but the free-space control laser and the signal laser are tuned to the polariton resonances (the normal mode) that are separated from the respective atomic resonances by the vacuum Rabi splitting. The control laser induces the destructive interference for the polariton excitation and renders the cavity-atom system opaque to the signal light [25]. For the cavity EIT scheme presented here, the control laser and the signal laser are tuned to the atomic resonant transitions of the three-level system and create the EIT condition. EIT renders the medium transparent to the signal laser. Therefore, the scheme in Ref. 18 and 19 is based on electromagnetically induced opaque in the normal mode excitation while the scheme here is based on electromagnetically induced transparency. The two schemes result in comparable phase shifts and amplitudes for the transmitted field under the conditions of a weak control laser and moderate optical density.

Our earlier study of the cavity-atom polariton system only analyzed the cavity-transmitted field. Here the analysis is carried out for both the cavity-transmitted field and the cavity-reflected field. The phase shift and amplitude of the cavity-reflected field as well as the cavity-transmitted field are analyzed and compared. The results show that the cavity reflected field and cavity transmitted field are complementary in their performance merits. The cavity system provides the versatility of choosing either the reflected field or transmitted field for the studies of all-optical switching and the cross-phase modulation according to their respective performance merits in a given parameter regime.

There are several studies of the EIT enhanced Kerr nonlinearities and XPM at low light intensities in multi-level atomic systems in recent years [26-33]. Most of these studies were done in four-level N-type systems coupled by three laser fields in free space. They can be divided into 4 types: a four-level N type coupled by three laser fields (Ref. 26-28), a five-level M type coupled by four laser fields (Ref. 26), a four-level tripod type coupled by three laser fields (Ref. 30-31), and a modified five-level tripod type coupled by 4 laser fields (Ref. 32). Ref. 28 proposed to inject a signal light (that induces the Kerr nonlinearity on a free-space signal light) into a Michelson-type double cavity containing the four-level atoms and therefore increase its coupling time with the free-space signal light. The double cavity is a passive device used solely for the signal light. Ref. 33 explores basically the N-type in the time domain through the EIT light storage process. Ref. 34 deals with the self Kerr nonlinearity (self phase modulation) in the three-level Λ type system confined in a travelling-wave cavity. In contrast with the EIT schemes in free space with multi-level atoms coupled by at least three laser fields, our scheme requires only two laser fields coupled with three-level atoms, but needs to operate in a cavity (it differs from Ref. [28] in that the signal light is coupled into the cavity and the cavity QED effect measured by the vacuum Rabi splitting plays an essential role). Due to the cavity feedback enhancement, our scheme produces a large XPM phase shift (~ 0.5 rad. and with a transmitted light intensity about 50% of the input light intensity under practical conditions). In the fee-space EIT schemes, in order to produce the comparable XPM phase shift, a much greater optical depth of the atomic medium is required (a few orders of magnitude greater than our scheme, see Ref. 26, 28 and 32).

The XPM phase shift in our scheme is very small when the medium optical depth becomes very large ($\sqrt{N}g \gg \kappa\Gamma_3$). There is no such limiting factor for the EIT schemes. If very large optical depths of the atomic medium are available, the EIT scheme may produce a greater XPM phase shift (maybe even reaching a value of π as suggested in Ref. 31 and 32). Therefore, our scheme performs better with an atomic medium of low to moderate optical depths while the free-space EIT schemes perform better with an atomic medium of very high optical depths.

3. Conclusion

In conclusion, we have shown that coherently coupled cavity-atom system can be used to study resonant nonlinear optics at low light intensities. The system is well suited for studies of all-optical switching and XPM at low control light intensities. In the weak coupling regime of the cavity QED, the interplay of the EIT and the collective coupling of atoms with the cavity mode enable the nonlinear control of the intra-cavity light field through a free-space control laser. The analytical results and numerical calculations show that large optical nonlinearities can be induced by the control laser near the narrow resonance of cavity EIT, which can be used to control the amplitude and phase of the cavity transmitted and reflected fields.

Acknowledgement

This work is supported by the National Science Foundation under Grant No. 1205565.

References

[1] *Cavity Quantum Electrodynamics*, 1994 edited by P. R. Berman (Academic, San Diego).
[2] J. M. Raimond, M. Brune, and S. Haroche 2001 Rev. Mod. Phys. **73** 565.
[3] H. Tanji-Suzuki, W. Chen, R. Landig, J. Simon, V. Vuletić 2011 Science **333** 1266.
[4] G. Rempe, R. J. Thompson, R. J. Brecha, W. D. Lee, and H. J. Kimble 1991 Phys. Rev. Lett. **67** 1727.
[5] P. Grangier, J. F. Roch, J. Roger. L. A. Lugiato, E. M. Pessina, G. Scandroglio, P. Galatola 1992 Phys. Rev. A **46** 2735.
[6] S. E. Harris 1997 Phys. Today **50** 36.
[7] M. D. Lukin, M. Fleischhauer, M. O. Scully, and V. L. Velichansky 1998 Opt. Lett. **23** 295.
[8] H. Wang, D. J. Goorskey, W. H. Burkett, and M. Xiao 2000 Opt. Lett. **25** 1732.
[9] G. Hernandez, J. Zhang, and Y. Zhu 2007 Phys. Rev. A **76** 053814.
[10] H. Wu, J. Gea-Banacloche, and M. Xiao 2008 Phys. Rev. Lett. **100** 173602.
[11] M. Mücke, E. Figueroa, J. Bochmann, C. Hahn, K. Murr, S. Ritter, C. J. Villas-Boas, and G. Rempe 2010 Nature **465** 755.
[12] A. Joshi and M. Xiao 2003 Phys. Rev. Lett. **91** 143904.
[13] T. Kampschulte, W. Alt, S. Brakhane, M. Eckstein, R.Reimann, A. Widera, and D. Meschede 2010 Phys. Rev. Lett. **105** 153603.
[14] J. K. Thompson, J. Simon, H. Loh, and V. Vuletic 2006 Science **313** 74.
[15] J. Sheng, H. Wu, M. Mumba, J. Gea-Banacloche, and M. Xiao 2011 Phys. Rev. A **83** 023829.
[16] M. Albert, A Dantan, and M. Drewsen 2011 Nature Photonics **5** 633.
[17] A. E. B. Nielsen, J. Kerckhoff 2011 Phys. Rev. A **84** 043821.
[18] Y. Zhu 2010 Opt. Lett. **35** 303
[19] X. Wei, J. Zhang, Y. Zhu 2010 Phys. Rev. A **82** 033808.
[20] Y. Wang, J. Zhang, and Y. Zhu 2012 Phys. Rev. A **85** 013814.
[21] W. Chen, K. M. Beck, R. Bücker' M. Gullans, M. D. Lukin, H. Tanji-Suzuki, V. Vuletić, Science 2013 **341** 768.
[22] D. F. Walls and G. J. Milburn 1994 *Quantum Optics*, (Springer-Verlag, Berlin, Heidelberg).
[23] B. Zou and Y. Zhu 2013 Phys. Rev. A **87** 053802.
[24] R. Zhao, Y. O. Oudin, S. D. Jenkins, C. J. Campbell, D. N. Matsukevich, T. A. B. Kenndedy, and A. Kuzmich 2009 Nature Phys. **5** 100.
[25] J. Zhang, G. Hernandez, and Y. Zhu 2008 Optics Express **16** 7860.
[26] H. Schmidt and A. Imamoglu 1996 Opt. Lett. **21** 1936.
[27] H. Kang and Y. Zhu 2003 Phys. Rev. Lett. **91** 93601.
[28] T. Opatrny and D. G. Welsch 2001 Phys. Rev. A **64** 023805.

Journal of Physics: Conference Series **497** (2014) 012007 doi:10.1088/1742-6596/497/1/012007

[29] A. B. Matsko, I. Novikova, G. R. Welch, and M. S. Zubairy 2003 Opt. Lett. **28** 96.
[30] D. Petrosyan and G. Kurizki 2002 Phys. Rev. A **65** 033833.
[31] D. Petrosyan and Y. P. Malakyan 2004 Phys. Rev. A **70** 023822.
[32] Z. B. Wang, K. P. Marzlin, B. C. Sanders 2006 Phys. Rev. Lett. **97** 063901.
[33] Y. F. Chen, C. Y. Wang, S. H. Wang, and I. A. Yu 2006 Phys. Rev. Lett. **96** 043603.
[34] H. Wang, D. Goorskey, M. Xiao 2001 Phys. Rev. Lett. **87** 073601.