Illustrating Cognitive Connections between Math and Language in Pre-Service Teachers' Perceptions of Common or Everyday Terms

Bryan Moseley and Charles Bleiker Florida International University, USA

> Kimberly Gibson University of Miami, USA

Abstract: This paper introduces a new construct that we term Math Mediated Language (MML) focusing on the notion that common or everyday terms with mathematical meanings are important building blocks for students' mathematical reasoning. A survey given to 96 pre-service early childhood educators indicated clear patterns of perceptions of these terms.

The notion of meaningful mathematical communication assumes that teachers are sensitive to the words that meaningfully link symbols and operations to the child's broader experience and growing academic knowledge (Pimm, 1987). Without the cultivation of a shared sensitivity to the notion that words embody mathematical concepts, students and teachers can be left with mathematical knowledge that is highly compartmentalized and limited in its utility to convey mathematical concepts. Further, students could be faced with the perplexing situation of understanding why the same word has mathematical meaning in one context but then loses that meaning in another. For example, in working a fraction problem a teacher may use the term equal to refer to identical quantities but at snack time that same teacher may use the term equal to imply that everyone gets the amount they want, which may vary on factors other than quantity and not provide a truly equal distribution. Worse, teachers may not be sensitive to the underlying mathematical meanings of words and may inadvertently neglect or actually restrict the range of mathematical vocabulary that is accepted in their classrooms to rudimentary operation terms. This limits the types of mathematical and linguistic communication that can provide depth to both domains. Therefore, we feel strongly that the types of mathematical understandings that teachers possess of common words are crucial yet undocumented components of the ways mathematical understandings are facilitated or undermined. This study looks at the extent to which pre-service teachers associate the common or everyday terms that students use with the domain of mathematics. We feel that these common words often provide the elemental components for much of children's mathematical communication, and that teachers should be sensitive to the implications that common terms have for mathematical discourse and reasoning. In this paper we document the extent to which pre-service elementary and early childhood educators perceive these terms as being associated with the domain of mathematics. Mathematics Across the Curriculum

Teachers' sensitivity to young children's mathematical language has been identified as a key component in the development of mathematical reasoning (Munn, 1998). However previous types of research have tended to be focused on the language that students use during explicit math lessons. We feel that this approach, while it has contributed a wealth of information about children's mathematical communication, may be too limiting in the sense that this type of inquiry does not look at the range of experiences that students are likely to receive with mathematical language. Mathematical terms and concepts do not only arise in the confines of

curricula that are designed to explicitly emphasize mathematics, but rather are experienced implicitly throughout many parts of the elementary and early childhood curricula. Therefore instead of identifying specific lessons as mathematical or not mathematical we take the approach of investigating the teacher knowledge that is foundational to the proper facilitation of many different types of activities. This research looks at pre-service teachers' sensitivities to mathematical properties in common words that might be used in a number of different school lessons or activities.

Math Mediated Language

This research lays the groundwork for construct that is termed math mediated language. The essential notion of this construct is that the co-construction of knowledge that takes place in mathematics instruction between teacher and student relies on the often subtle cognitive connections that are drawn between common words and mathematical concepts. While there certainly can be non-verbal aspects to mathematical reasoning, the vast majority of communication and demonstration of understanding of mathematics comes through its connections to language. Small changes in word choice often have sweeping effects on the underlying conceptual situation that is presented in a problem. For example, although linguistically similar, asking a child to perform the computation three divided into 12 is very different conceptually from asking for three divided by 12. Hence the ways that problems are presented, the subsequent feedback that learners receive on their problem solving, the types of problem solving schemata that are induced are theorized to be a function of the language that is used to define them. This need not be a mysterious connection between the two bodies of knowledge. Both linguistic and mathematical forms share important commonalities that are essential to cognitive reasoning (Lakoff & Nunez, 2000). Therefore the central concept of this construct is that mathematical thinking develops in consort with language such that the two are combined to produce cognitive links that are necessary for advancing meaningful understanding of both domains.

However, to be able to facilitate these cognitive connections between common language and mathematics, prospective teachers need to be sensitive to the mathematical meanings that underlie common terms so that they are able to facilitate this connection in their students' reasoning. Further this need does not vanish at the close of a mathematics lesson or activity but must be present whenever students come across mathematical concepts embedded within other bodies of knowledge. For this facilitation to take place teachers must first possess the sensitivity to mathematical meanings that are part of human communication. However little is known about how teachers perceive common words with mathematical connotations. We feel that the potential for prospective teachers to influence children's mathematical thinking is too great to leave this component of teacher knowledge unexplored. This research therefore looks to bring data to this understudied phenomenon by documenting the patterns that prospective teachers display in their sensitivity to the mathematical meanings of common words. Our hypothesis is that respondents with more training in mathematics and language will be able to perceive more of the mathematical meanings that are embedded in these common words. In keeping with this notion we further we expect that more basic mathematical terms and their synonyms will be more readily seen as mathematical than more advanced ones and their synonyms, as this requires fewer skill in both domains.

Method

Participants

A sample of 81 female and 15 male pre-service early childhood teachers enrolled at a large urban research university agreed to participate in this study. Students were pooled from five sections of a foundations course in educational psychology to produce a total of 96 respondents to the survey, 50% of which were between 21-25 years of age representing the most common age group for the profession. Of the total pool of respondents 79% identified their primary language as English and 21% as Spanish. Over 50% of the sample indicated that they planned to pursue a masters or doctorate as their terminal degree. This sample was chosen because it represented a typical range of pre-service early childhood teachers. These teachers were typically in their junior year of university study and had recently passed a standardized test of general knowledge required by the state for entry into the program. Most had recently taken both general math and English courses required to fulfill their general education requirements for the college of education. In general this group had a relatively high level of education compared with the population in general, and recent experience in the two subjects thought to be important for doing well on the tasks.

Materials

Each of the participants was given a Scantron sheet and a questionnaire developed by the authors, called the Mathematical and Verbal Educational Research Inventory Questionnaire (M.A.V.E.R.I.Q). The MAVERIQ included fifty terms, five synonyms for each of ten categories (see Appendix). The measure was developed for the specific purpose of assessing teachers' perceptions of mathematical meanings in common terms and was comprised of ten categories, seven of which included synonyms for each of the four primary operations (addition, subtraction, division, multiplication), and three relational terms (equal, less than, and more than). The remaining three categories were distracters that were included to insure that respondents made considered evaluations of each term and did not answer blindly. These distracters contained synonyms for emotion words such as happy, sad, and angry. This yields a total of 20 words expressing operations, 15 words expressing quantity relations, and 15 distracter words representing emotions (see Table 1). The survey asked, "How much do you associate the following terms with math?" and instructed participants to respond by marking a Likert scale in which 1=Very Strongly, 2=Strongly, 3=Somewhat and 4=Not at all, for each of the 50 presented terms. All of the words were randomly ordered on the page to minimize the possibility of order bias. Respondents were also asked to fill out a 12 question background survey in which they were asked about the types of courses they have taken, their age, gender and their educational plans. The MAVERIQ was developed to try to understand how pre-service teachers think verbally about basic mathematical operations and their synonyms. It is loosely modeled on the student belief questionnaire used by Schommer, Crouse, & Rhodes (1992). The use of synonyms was thought to be a reasonable way of assessing teachers' thinking about an operation across linguistic domains. Operation and relations terms were chosen as the units of analysis because they represented a best guess at what constituted useful mathematical knowledge for teaching young children.

Procedure

Each of the participants in this study was given a questionnaire and a Scantron sheet to mark their responses. A minimum of instructions were provided to the participants to reduce the chance that the researchers would influence respondents' perceptions of the listed terms on the MAVERIQ. As a result respondents were simply instructed to indicate the level to which they

associated the common terms with mathematics by marking the four point likert scale for each term on their Scantron sheet. In addition, each respondent was also asked to fill out the 12 question background survey on the same Scantron sheet. Each participant was given twenty minutes to complete the entire task.

Results/Discussion

Overall the ability to perceive mathematical meanings in all of the mathematical synonyms was significantly correlated with the number of college level language arts classes taken (.169 p<.05) and the number of college level mathematics classes taken (.179 p<.05) by the participants. This suggests that students with more training or an increased aptitude for language arts skills and mathematics were more sensitive to meanings that were embodied in these synonyms.

Further, analysis of the questionnaire data has indicated that respondents were significantly more likely to perceive some groups of words as being more mathematical than others. Synonyms for additive terms (M=8.37, SD=3.19) such as "add" and "subtract" were perceived as significantly more linked to mathematics, t(94) = 25.87 p < .05 than synonyms for multiplicative (M=9.69, SD=3.35) terms such as "multiply" and "divide." This suggests that pre-service teachers are more likely to see mathematical connections in words related to the more basic additive mathematical operations than more multiplicative complex ones. Overall, participants also perceived operation synonyms as more mathematical than synonyms for relational words $t(94) = 28.02 \text{ p} < .05 \text{ such as "equals." An analysis using multidimensional$ scaling techniques was used to document how the words were related in terms of their mathematical meanings to the respondents (see figure 1). This establishes a psychological continuum of terms that are seen to have more or less mathematical meaning to them. At the left are the four operations (addition, subtraction, multiplication, division) showing a tight relationship indicating a close psychological proximity. However with other terms such as "of" they tend to be seen as divorced from the multiplicative operation that it represents in text based problems. While many of these terms may be used in lessons given by these prospective teachers, it is clear that there are different perceptions of these terms that are at times highly dispersed from the respondents other mathematical knowledge.

While the limitations of this study do not indicate what type of instruction these respondents will provide to their students, it argues that the mathematical and linguistic knowledge that the respondents possess is often compartmentalized. If this is the case then it is difficult to expect these respondents to help their students become aware of the important cognitive connections that math and language share.

Conclusion

The knowledge that teachers bring to the classroom is an important part of the knowledge scaffolding process that takes place thought the school day. We feel that it is crucial that teachers see mathematical meanings in the words that they use across different contexts. Our data indicate that teachers have distinctly different perceptions of common terms and the levels of embedded mathematical meaning that they convey. The implications of this are wide ranging in that they provide possible reasons for the differential successes and struggles of teachers using identical curricula, and teaching similar students. Additionally they suggest a rationale for why mathematical discourse might be more difficult and less effective for more complicated concepts as those mathematical concepts are less likely to be linked to common terms in the teachers mind

and subsequently receive less grounding in their students' activities. Teachers with more training or inherent expertise in the domains of math and language may over come these challenges more frequently to provide effective scaffolding to their students that is significantly different from those that possess less training. This resonates with our results on a preliminary level such that we can theorize that if a teacher's classroom discourse were to be based around linking these terms to mathematical concepts, then that would at minimum suggest that the teacher possesses a grasp of much of the important connections between the two domains. Logically this would suggest a wider range of options for teachers facilitating classroom dialog that would support both domains and facilitate a more diverse range of student cognitions. However, more research is needed to establish the process by which common words in the classroom are used by teachers and students.

Further research in this area can look to document the types of classroom discourse that takes place in high and low performing classroom and also in culturally diverse educational settings. We feel that each of these areas hold promise for illustrating the roles that common words play in cognizing the mathematical domain and we are currently planning and conducting this type of research. Our results also provide a lens on teacher's classroom practices that can encourage them to revisit many of their assumptions about mathematics as a domain. This is important for two reasons as it provides a way that teachers can re-examine their content knowledge with a more pragmatic focus, and that it takes into account children's cognition in their reasoning about classroom content. Therefore this research suggests that important differences exist in the ways that mathematical meanings of common terms are perceived by prospective teachers and that the backgrounds and that they bring into the classroom are significant predictors of these perceptions. It is hoped that that this research will provide greater insight into the ways that teachers link these two domains and the benefits and challenges that can become available to their students as a result.

References

- Lakoff, G., & Nunez, R., E. (2000). Where mathematics comes from: How the embodied mind brings mathematics into being., New York: Basic Books.
- Munn, P. (1998). Symbolic function in pre-schoolers. In *The Development of Mathemematical Skills*. C Donlan (Ed.). UK, London: Psychology Press.
- Pimm, D. (1987) *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge.
- Schommer, M., Crouse, A. & Rhodes, N. (1992). Epistemological beliefs and mathematical text comprehension: Believing it is simple does not make it so. *Journal of Educational Psychology*, 84, 435-443.

Table 1. Mathematical Terms and Distracters

add	subtract	multiply	divide	less	more	equal	happy	sad	angry
combine	take	times	into	below	greater	same	glad	depressed	mad
and	from	by	split	inferior	superior	identical	satisfied	down	irate
plus	remove	of	partition	lower	higher	alike	content	melancholy	furious
with	reduce	replicate	share	under	over	match	elated	blue	incensed

Figure 1: Results from multidimensional scaling analysis of pre-service teachers' perceptions of operation terms and their synonyms.

