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Third-Party Budget Breakers and Side Contracting in Team Production

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Abstract

In a team production problem with unverifiable effort, budget breaking is essential to implementing efficient levels of effort. This short paper considers the use of a third party, who does not exert effort, in a setting with general contracts that can include message games, as a way to remove resources from the team. We show that if side contracting can influence behavior in a message game in the original contract, the addition of the third party is not helpful. Additionally, we compare our view of side contracting with that of Baliga and Sjostrom (2009) to explore the nature of side contracting that is needed in order for the third party to be useful for budget breaking.

JEL Classification: C70, D74, K10.

Holmstrom's (1982) pioneering work on team production, with unverifiable effort, showed that budget breaking, as a way to punish all team members in the event of sub-optimal output (meaning someone shirked), is essential to inducing efficient levels of effort. Renegotiation prevents the team members from agreeing to destroy output in the event of sub-optimal effort. So contracting with a third party who undertakes no productive activity, but receives a transfer from the team members when someone shirks has been explored as a way to break the budget. However, if the third party can collude with one of the team members so that the team member shirks and then the two divide the transfers that the third party receives, budget breaking is not effective for implementing efficient levels of effort. Holmstrom noted that this could pose a problem, and Eswaran and Kotwal (1984) showed that the third party is not helpful because of this potential for collusion. Neither of these papers considered general contracts that could include message games, and there has been recent interest in whether the addition of message games provides a way to avoid the collusion problem.

In this short paper we study the use of a third party as a budget breaker in a deterministic team-production setting when contracts can contain message games and evaluate whether this allows the third party to be effective as a budget breaker. We focus on the opportunity for one of the players to side contract with the third party, and allow for exactly the same contracting technology in both contracts—the original contract between the n players in the

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team and the third party, and the side contract between a team member and the third party. Specifically, we assume that contracts can condition on the value of output and on public messages sent by players. We find that, in this situation, the third party is not effective as a budget breaker.

The contract between team members and the third party specifies a division of verifiable output as a function of public messages and the level of output; we refer to this contract as the original contract. After contracting, the team members choose effort, which deterministically determines the level of output and is commonly observed by the players. The players then send messages, on which the original contract conditions. If there is a side contract in place, it also conditions on these same messages. One practical interpretation of this modeling of messages is that the original contract and the side contract can each condition on the messages of the other contract.

We allow for two side contracting opportunities. In the first, which occurs after the original contract is agreed to and before team members choose effort, a team member and the third party may side contract. Such a side contract can be used to shape the colluding players' behavior in both the team production and the original-contract message game. In the second, which occurs after the realization of output and before the original contract message game, any subset of the players may side contract. This side contract can be used to shape the colluding players' incentives in the original-contract message game.

Recognizing that side contracting will disrupt the effectiveness of the third party as a budget breaker, we require that original contracts are *impervious to side contracting*. We show that for any original contract that satisfies the impervious to side contracting condition, the third party is useless for budget breaking.

We then compare our model of side contracting with that of Baliga and Sjöström (2009) who find that the third party is effective for budget breaking. The different findings are due to the differences in our assumptions about how effectively the players can side contract. Baliga and Sjöström assume that a team member and the third party can only side contract over the level of output and private side contract messages. That is, they do not allow for a side contract to condition on the same messages that the original contract does, or to condition on the allocation imposed under the original contract. We assume that a side contract can condition on the same messages as the original contract, but not on the allocation imposed under the original contract. Further, we show that when a side contract can condition on the allocation imposed under the original contract (but not on the same messages as the original contract) that the third party is ineffective for budget breaking. Our aim in this direction is to understand the qualifications for Baliga and Sjöström's novel result, and to broaden their analysis.

The paper is organized as follows. Section 1 sketches the model from Holmstrom's (1982) seminal paper. In Section 2 we incorporate message games, side contracts, and the third party. We discuss the relationship with Baliga and Sjöström (2009) in Section 3. We then briefly conclude in Section 4.

1 The Standard Model

We consider a version of Holmstrom's (1982) deterministic team production model that explicitly allows for a third-party budget breaker and message games. Players $1, 2, \ldots n$ choose costly effort, which determines the value of production. Player i's effort is denoted by $a_i \in A_i$, and we let $A = \times_{i=1,\ldots n} A_i$. The value of production, joint monetary output, is given by $q : A \to \mathbf{R}$, and q is strictly increasing, concave, and differentiable with $q(0) = 0$. Player i's cost of effort is given by $v_i : A_i \to \mathbf{R}$, and v_i is strictly convex, differentiable, and increasing with $v_i(0) = 0$. Player i's payoff is given by $t_i - v_i(a_i)$, where t_i denotes the transfer (or share of $x = q(a)$) that player i receives.

Holmstrom does not consider message games or side-contracts with a third party who is used as a budget breaker. His model has each player receive a share of the joint monetary outcome that is given by $s_i(x)$. Note that a is unverifiable and that x is verifiable. An externally enforced contract is a function s from output to shares, satisfying that the sum of the shares does not exceed output. His Theorem 1 shows that when budget balance is required, $\sum_{i=1,\ldots n} s_i(x) = x$, it is not possible to implement the optimal level of investment a^*. The optimal investment levels a^* solve

$$\max_{a \in A} [q(a) - v_1(a_1) - v_2(a_2) - \cdots - v_n(a_n)].$$

However, when the budget does not have to be balanced, Holmstrom's Theorem 2 shows that a^* can be implemented. The example in his proof specifies

$$s_i(x) = \begin{cases} b_i & \text{if } x \geq q(a^*) \\ 0 & \text{if } x < q(a^*) \end{cases} .$$

The b_i's are chosen so that $\sum_{i=1,\ldots,n} b_i = q(a^*)$ and $b_i > v_i(a_i^*) > 0$.

Thus, Holmstrom shows that if all of the output can be given to a third party, it is possible to implement the efficient investment a^*. Holmstrom noted that if collusion between one of the players and the third party was possible, the usefulness of the third party as a budget breaker may be negated. Eswaran and Kotwal (1984) show this to be the case. However, neither of these studies allows for general contracts with message games. We allow for more general contracts and formally model the side contracting. In incorporating the third party into the game as a player, we assume that the sum of the shares must exactly equal x.

2 Incorporating Messages and Side-Contracting

We allow for a message game in both the original contract and the side contract. We present the model by first considering the interaction without the side contracting and then add it. As in the Holmstrom setting, the team members' effort choices, which are unverifiable, deterministically lead to verifiable output x. Although a is unverifiable, we assume that it is commonly observed by the players.[1]

Prior to choosing effort, the team members and the third party contract. The timing of the players' interaction is illustrated in Figure 1. The contract specifies an allocation

[1]It is straight forward to extend our results to the case where effort choices are private information.

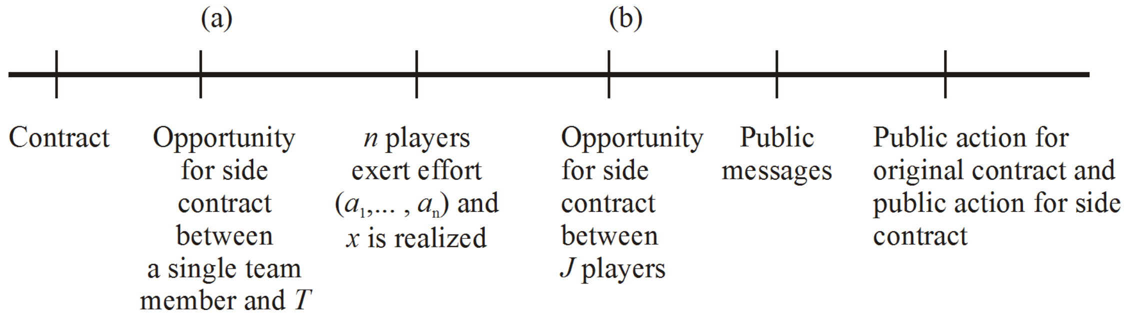

Figure 1: Timing of events.

of the output x between all of the agents, which also represents transfers because negative allocations are allowed, as a function of verifiable items. The verifiable items include both the realized output x and messages that the players send following the realization of x. The players select messages from a grand message space $M \equiv \times_{i \in N} M_i$, where M_i is the set of feasible messages for player i and $N \equiv \{1, \dots n, T\}$. A contract is $F : M \times \mathbf{R} \rightarrow \mathbf{R}^{n+1}$, where $F(m, x)$ is the allocation imposed given message profile m and output x. Player i's allocation, given a message profile and realized output is denoted by $F_i(m, x)$. We require, due to the possibility of renegotiation, that the players' allocations are balanced meaning they sum to x.

We now turn to opportunities for side contracting. Side contracting is not observed by the other players, and a side contract may contain a message game. A side contract is $f : M \times \mathbf{R} \rightarrow \mathbf{R}^{n+1}$, with $\sum_{j \in N} f_j = 0$ and $f_k = 0$ for all k not involved in the side contract. Note that the message profile that the side contract conditions on is the same one on which the original contract conditions. One interpretation is that both the original contract and the side contract condition on the same court proceedings and the information that comes out of those proceedings. Note that for a sufficiently rich grand message space, it could be as though there are messages for the original contract and messages for the side contract, and that both contracts can condition on the messages of the other. Importantly, we do not allow the original contract to condition on whether there is a side contract, and neither contract can condition on the public action imposed by the other contract.

We allow for two opportunities for side contracting; these are labeled (a) and (b) in Figure 1. Recognizing that side contracting at either of these opportunities will disrupt using the third party to break the budget, we require that the contract the players agree to be *impervious to side contracting* at either opportunity. We follow Bull and Watson's (2004) treatment of coalitions for settings with externally-enforced side contracts.

We will describe the equilibrium/solution concept, which requires that contracts are impervious to side contracting, shortly. It is helpful to first examine what side contracts can achieve, given fixed F and behavior. This allows us to consider the potential for a side contract between one of the team members and the third party, which is the opportunity described in (a) in Figure 1. Fix F and behavior of the $-i$ team members at $a_{_i}$, and assume that the messages of the $-i$ team members are functions of x and a. We denote player j's message choice by $\mu_j(a, x)$. It is possible for player i and the third party to use the side contract as a "forcing" contract to induce any effort level by player i, a_i, and any messages by player i and the third party as a function of x, $\mu_i(x)$, and $\mu_T(x)$. We focus on pure strategies. The forcing contract relies on the other team members (the $-i$ players) playing the anticipated a_{-i}, given the original contract. By conditioning on the realized x, which will correspond to the anticipated a_{-i} and side contracted a_i, player i and the third party can side contract so as to punish player i for deviating, by requiring that she pay an arbitrarily large transfer to the third party, holding fixed a_{-i}, from their agreed upon a_i. Then either player i or the third party can be punished, again with an arbitrarily large transfer paid to the other member of the coalition, for unilaterally deviating from $(\mu_i(x), \mu_T(x))$. Note that, although we normally consider the players choosing their messages as a function of the effort levels, which they observe, we consider the players side contracting over messages as a function of output because output is verifiable and effort is not.

Lemma 1: *Consider a side-contract between player i and the third party T, given an original contract and holding fixed the associated a^*_{-i} that would be induced without a side-contract. Any a_i, $\mu_i(x)$, and $\mu_T(x)$ can can be implemented with a forcing contract between i and T.*

Proof: Hold fixed, the behavior of the players other than player i and the third party (the $-i$ players). Note that it is possible to induce any messages from i and T, as a function of x, with a side-contract. Given x the side-contract can specify an arbitrarily large transfer from a player who unilaterally deviates from $(\mu_i(x), \mu_T(x))$ to the non-deviating player. Given this, it is also possible to specify any attainable x, given the anticipated play of a^*_{-i}, with a forcing contract. This simply requires imposing a large transfer from player i to T if the level of x that corresponds to the desired a_i and anticipated a^*_{-i} is not observed. That is, if $q(a_i, a^*_{-i})$ is not realized, require that i pay an arbitrarily large transfer to T. *Q.E.D*

Thus, a team member i and the third party T can effectively act as single player. Note that side contracts, which induce shirking, between several team members and the third party are possible in practice. To simplify the exposition we focus primarily on the potential for a single team member and the third party, but note that our results will generalize to side deals in which several team members agree to shirk.

We also consider side deals, between two or more players, concerning only messages and assume that these can be formed after the realization of output x but before the disclosure of messages. This opportunity is denoted by (b) in Figure 1. These side deals are also private and are not observed by the other players until after messages have been presented. Thus, the point of writing a side contract is to induce members of the coalition to change their behavior in the original-contract message game in a way that benefits the coalition. To evaluate whether this is possible, define $F_J(m) \equiv \sum_{j \in J} F_j(m)$, where $J \subset N$ is the coalition, and let $f_J(m)$ be defined analogously. Suppose that following effort profile a, and the corresponding output x, the players would coordinate on disclosure of messages $\mu(a)$ in the absence of side contracting. Further suppose that, by side contracting, a coalition J can induce its members to disclose messages $\hat{m}_J$. Then the coalition strictly gains from the side deal if and only if

$$F_J(\mu_{-J}(a), \hat{m}_J) + f_J(\mu_{-J}(a), \hat{m}_J) > F_J(\mu(a)) + f_J(\mu(a)),$$

which is equivalent to $F_J(\mu_{-J}(a), \hat{m}_J) > F_J(\mu(a))$ because $f_J = 0$. Here, "$-J$" stands for the complement of J.

In a manner similar to that used to induce shirking and then coordination on messages, the parties to such a side deal can force disclosure of specific messages. Specifically, a coalition J can specify f so that any player $j \in J$ who does not disclose a specified message must pay an amount y to each of the other players in the coalition.[2] Thus, after the realization of x and before the disclosure of messages, coalitions can effectively spot contract on which messages to disclose. In practical terms, a side contract may amount to joint disclosure of messages. Furthermore, side contracts can be used by a coalition to rearrange transfers internally.

We require that the original contract is impervious to side contracting at both opportunities, (a) and (b), for side contracting. Following Bull and Watson (2004), we use the term impervious to side contracting instead of *coalition-proof Nash equilibrium* because the latter is defined for self-enforced contracts (Nash equilibria) of standard non-cooperative games, while we require a version that examines externally enforced contracts.[3] We examine F and behavioral rule a, $\mu : A \to M$, and require that it passes the impervious to side contracting test.

Definition 1: *Given an externally-enforced contract F, a behavior rule a and μ is called* ***impervious to side contracting (ISC)*** *if*

(i) $F_{i,T}(\mu(a), q(a)) \geq F_{i,T}(\mu(a'_i, a_{-i}), q(a))$, *for each i and a'_i, and*

(ii) $F_J(\mu(a'), q(a')) \geq F_J(m', q(a'))$ *for each a', each $q(a')$, each coalition $J \subset N$, and each $m' \in M$ that is a J-deviation from $\mu(a')$ at a' and $q(a')$.*

Note that the ISC condition includes deviations by a single player, so every ISC disclosure rule is also an equilibrium disclosure rule.

The combination of the ISC condition and the constant-sum aspect of externally-enforced transfers prevents the existence of *message game phenomena.* Thus, the externally-enforced transfers imposed following a given level of output x must be the same for any effort levels a that led to x.

Lemma 2: *Consider any a' and a'' such that $x = q(a') = q(a'')$. For any ISC message rule $\mu(a)$, it must be that $F(\mu(a'), x) = F(\mu(a''), x)$.*

Proof: Suppose not. Then it must be for some $a' \neq a''$ such that $x = q(a') = q(a'')$ that $F(\mu(a'), x) \neq F(\mu(a''), x)$ and for some $J \subset N$ $F_J(\mu(a'), x) < F_J(\mu(a''), x)$.

Consider the message profile $(\mu_J(a''), \mu_{-J}(a'))$. Note that coalition J can induce this as a J-deviation from $\mu(a')$ and coalition $-J$ can induce this as a $-J$-deviation from $\mu(a'')$. In

[2] Then any sub-coalition $K \subset J$ will lose at least y when one or more of its members deviates from the prescription of f. The number y can be set large enough so that this loss is greater than any gain the sub-coalition can get by way of the original contract F.

[3] That is, ISC is Bernheim, Peleg, and Whinston's (1987) definition applied to externally enforced contracts. Following Bernheim, Peleg, and Whinston, we view a side contract as viable only if it is immune to disruption by sub-coalitions (who have to pass the same test). However, the issue of sub-coalitions is easily handled in our model, because forcing contracts, as described above, can always be designed to stifle any further side dealing by sub-coalitions.

order for μ to be ISC, given F, it must be that such a J-deviation is deterred following a', which requires

$$F_J(\mu_J(a''), \mu_{-J}(a'), x) \leq F_J(\mu(a'), x).$$

Additionally, deterring the $-J$-deviation from $\mu(a'')$ requires

$$F_{-J}(\mu_J(a''), \mu_{-J}(a'), x) \leq F_{-J}(\mu(a''), x),$$

which, because transfers must be balanced, is equivalent to

$$F_J(\mu_J(a''), \mu_{-J}(a'), x) \geq F_J(\mu(a''), x).$$

Combining yields

$$F_J(\mu(a'), x) \geq F_J(\mu_J(a''), \mu_{-J}(a'), x) \geq F_J(\mu(a''), x),$$

but we have assumed $F_J(\mu(a'), x) < F_J(\mu(a''), x)$. So this is a contradiction. *Q.E.D.*

This means it is only possible to punish one of the team members following the realization of a given suboptimal value of output. Since any suboptimal value of output, $x < q(a^*)$, can be induced by any of the team members unilaterally deviating from a^* this prevents the contract from implementing the optimal effort level a^*.

Corollary 1: *Adding a third party to this team production problem does not allow efficient levels of effort a^* to be implemented.*

In fact, we show a stronger result: adding the third party does not change the set of implementable effort profiles.

Theorem 1: *The set of implementable effort profiles a is the same with or without a third party.*

Proof: Without the third party, consider message game $F(\mu(a), x)$. In order for a to be implementable, there must be an $F(\mu(a), x)$ and an ISC message rule $\mu(a)$ such that selecting a is an equilibrium given F.

For $\mu(a)$ to be an ISC message rule, we need

$$F_J((\mu_J(a_J), \mu_{-J}(a_{-J}), x) \geq F_J((\mu_J', \mu_{-J}(a_{-J}), x), \text{ for all } \mu_J', a, x = q(a), J.$$

Given the ISC message rule, define $g(a) \equiv F(\mu(a), x)$. For a to be an equilibrium, given $g(a)$ requires

$$g(a_i, a_{-i}) - v_i(a_i) \geq g(a_i', a_{-i}) - v_i(a_i'), \text{ for all } a_i, i.$$

Since we can view a colluding player and the third party as one player, these constraints do not change with the addition of a third party. *Q.E.D*

We now elaborate on the role of the ISC condition and the observability, by the players, of effort choices. Lemma 2, which shows that message game phenomena are not possible, relies on the ISC condition. Since the effort choices are observed by all players the absence of the requirement that behavior satisfy the ISC condition would allow for different equilibria being played following different choices of a that lead to the same level of output x. This is because equilibrium is only concerned with unilateral deviations, but when side contracting is possible we need to be concerned with deviations by coalitions of players.

However, when individual effort choices are not commonly observed message game phenomena are not possible even with standard equilibrium conditions. This is because each player would need to observe a—not just a_i—in order for them to condition the equilibrium m that is played on a.

3 Relationship to Baliga-Sjöström

We feel that in most practical settings it is possible for a side contract to be able to condition on the messages that the original contract message game conditions on, and study a model which allows both the original contract and side contract to condition on the same set of public messages.[4] However, when it is not possible for a side contract to condition on the messages conditioned on by the original contract, there is scope for using a third party to break the budget.

Baliga and Sjöström (2009) consider a model in which the original contract and the side contract each have a private message game with private messages so the side contract cannot condition on private messages in the original contract (and vice versa). The timing in their model is the same as that of our model with the exception of the separate private message games for each contract. The message game for the original contract is played first, its allocation/transfer is imposed, and then the message game for the side contract is played. Importantly, they assume, as we do, that the side contract cannot condition on the outcome of the original contract. They find that the third-party can be used to break the budget.

Baliga and Sjöström's analysis demonstrates that the efficacy of third-party budget breaking relies on the side contract being unable to condition either on messages in the original contract or on the allocation imposed by enforcement of the original contract. In this section, we further explore the extent to which their interesting finding relies on the side contract being unable to condition on the original contract messages and allocation. We begin by describing the conditions that must be satisfied for third-party budget breaking. Then we briefly describe Baliga and Sjöström's "whistle-blowing" equilibria construction, which allows for budget breaking. Next we describe another class of equilibria construction that relies only on messages from the team members and allows for budget breaking. We then compare Baliga and Sjöström's model with ours. Following Baliga and Sjöström, the analysis here focuses on teams with two members, but extends to teams with more than two members.

Deterring Side Contracting

If the third party is to be effective for budget breaking, we must be able to overcome the incentive for a player to collude with the third party and then exert slightly less than her optimal level of effort, say $a_i^* - \varepsilon$, where ε is close to zero. This type of side contract would allow the third party to induce a player to side contract, and, thus, prevent us from inducing the incentives given by Holmstrom's budget breaking (in his Theorem 2) discussed above.

[4]As noted in Section 2, this reflects the notion that the both contracts can condition on the messages for the other contract.

Preventing side contracting, and the subsequent underinvestment, requires that we be able to use the message game of the original contract to deter side contracting. For any $x < q(a^*)$ there are two unilateral deviations from a^* that can cause x: a unilateral deviation by player 1 and a unilateral deviation by player 2. Denote these by $q(a_1', a_2^*) = q(a_1^*, a_2') < q(a^*)$. In order to prevent side-contracting, there must be two equilibria following any such $x < q(a^*)$: one such that player 1 cannot be given the incentive, through a side-contract with the third party, to deviate to a_1' and another such that player 2 cannot be given the incentive, through a side-contract with the third party, to deviate to a_2'. This means that player 1 must receive a lower payoff in the former equilibrium than she does in the latter. Likewise, player 2 must receive a lower payoff in the latter equilibrium than she does in the former.

It is not possible to provide these incentives for both players simultaneously. To see this, consider player 1's potential side-contracting and deviation to a_1'. Player 1 must be required to pay a transfer to player 2 if we are to deter the side-contract. If instead the outcome of the message game required that player 1 pay a transfer only to the third party, this could be anticipated and accounted for in the side-contract between player 1 and the third party. So, in fact, the transfer from player 1 to player 2 must be sufficiently large that it offsets any possible payment the third party would be willing to make to induce collusion. Thus, if the original contract specifies that the third party receive all of x when $x < q(a^*)$, the transfer from player 1 to player 2 must be at least x.[5] The same argument holds for player 2's potential side-contract. We cannot effectively have both players make a transfer to the other in a single equilibrium following $x = q(a_1', a_2^*) = q(a_1^*, a_2') < q(a^*)$.

Lemma 3: *Any contract with $n = 2$, with messages games, that uses a third party to break the budget must induce two equilibria after any $x < q(a^*)$. The first of these must require that player 1 pay a positive transfer to player 2, and impose a lower overall transfer for player 1 than in the second equilibrium. The second equilibrium must require that player 2 pay a positive transfer to player 1, and impose a lower overall transfer for player 2 than in the first equilibrium.*

Baliga and Sjöström recognized this and constructed "whistle-blowing" equilibria as a way to deter side contracting.[6] Note that in our setting in Section 2 by Lemma 2, it's not possible to have two such equilibria following sub-optimal effort by one of the team members.

Whistle-Blowing Equilibria

Baliga and Sjöström's whistle-blowing equilibria rely on the third party naming, in the original contract message game, the team member with whom he colluded (who then shirks) when the realized output is not optimal, $x < q(a^*)$. In such a case, the team member, say team member i, is required to pay a sufficiently large transfer to both the third party and

[5] Clearly, other schemes may work to implement a^*, ignoring the issue of collusion with the third party. However, if the proposed scheme does not work, schemes that involve a smaller share of x being given to the third party will not work.

[6] Baliga and Sjöström require that, when team member i is named by the third party, an allocation such that i receives $-2Z$, j receives Z, and the third party receives $x + Z$. They show (in their Theorem 3, there is $Z > 0$ will make this construction work.

the other team member j. As noted above, the payment to j must be large enough that the third party and i cannot agree beforehand to a transfer (from the third party to i) that makes collusion attractive to i.

This equilibrium construction requires that the third party is indifferent as to which team member he names. To see this, suppose instead that the third party is to receive a larger transfer from team member i when he names team member than he receives from j when he names j. Then team member j and the third party have the incentive to collude (so that j shirks) anticipating that the third party will name i—not j—as having colluded with him. Such an incentive would prevent the original contract message game from deterring side contracting, and, hence, prevent third-party budget breaking.

So the "whistle-blowing" equilibria deter side contracting when the third party "rats" on his partner to the side deal. However, the third party is actually indifferent as to which team member he names. Naturally, if there were scope for the third party to commit to name team member j instead of team member i, i and the third party would side contract. This required behavior by the third party does not fit with the, perhaps common, view of colluding players not "ratting" on each other.

Other Ways of Implementing

Baliga and Sjöström's "whistle-blowing" equilibria are not the only way to prevent side contracting. Original-contract message games that contain versions of the familiar method of "Nash implementation with harsh punishments" will also deter collusion. Consider the case where, following $x < q(a^*)$, only the team members—not the third party—disclose messages.[7] Let the team members' messages simply be naming which team member shirked.

When the team members' messages both name team member i as having shirked, i pays an appropriate transfer to both the third party and player j.[8] If their messages do not agree, they each are required to pay a sufficiently large (larger than the sum of the transfers that are paid by the agreed upon shirker when they agree) to the third party.[9] Thus, the team members are essentially forced to agree on which of them colluded or each must pay a large transfer to the third party. The team members both naming the one who shirked is an equilibrium.

Note that collusion with the third party cannot disrupt this equilibrium because there is no way for, say, team member i and the third party to commit to behavior in the original contract message game. This is because the side contract cannot condition on either messages in the original contract or on the allocation imposed under the original contract. Importantly, if the side contract could condition on either of these, a team member could commit to shirk and then to not "agree" in the message game. Further, she could be given the incentive to do this because the third party could be required, under the side contract, to pay a transfer,

[7] A similar construction that relies on messages from both team members and the third party is also possible.

[8] Recall from above that deterring side contracting requires that the shirking team member pay a positive transfer to both the third party and the other team member.

[9] Note that any original contract that conditions only on messages of the two team members (and x) and implements the two equilibria (following a given value of x) must involve a payment by both team members when i's message corresponds to one equilibrium and j's message corresponds to the other equilibrium. Otherwise, there is no way to deter unilateral deviations from both equlibria.

which would offset the large transfer that she would be required to pay to the third party, to the shirking team member.

Of course, this method of deterring side contracting may be considered problematic because, as with the typical use of harsh punishment Nash implementation, there are multiple equilibria. Baliga and Sjöström do not consider this type of construction because they are concerned with strong implementation. We don't regard multiple equilibria to be a problem in general. So we wouldn't use this as a reason to dismiss the construction. Instead, we assert that this construction reveals how limited the form of side contacting that Baliga and Sjöström assume is. Their form of side contracting, where the side deal is not able to condition on anything but x, cannot have any impact on rational behavior in the message phase of the original contract.

Side Contracting – Middle Ground

Our model essentially assumes that the side contract can condition on the original-contract messages. Baliga and Sjöström's model assumes that it cannot. Both have assumed that the side contract can condition on x, but not on the allocation imposed under the original contract.

A reasonable "middle ground" is to assume that the side contract cannot condition on the original-contract messages, but can condition on the allocation imposed under the original contract (as well as the value of x). Some practical motivation for this, in terms of a court as the external enforcer for the original contract, is that the decisions of courts are typically publicly available. However, even with this middle ground assumption on the scope for side contracting, our main result holds: the third party is useless as a budget breaker. That the side contract can condition on the allocation imposed under the original contract provides a way for the colluding players (a team member and the third party) to shape their behavior in the original-contract message game, and this allows them to effectively side contract, which disrupts budget breaking.

Theorem 2: *Suppose players can side contract over the allocation imposed under the original contract, $F(m, x)$—in addition to side contracting over x, but not on the messages in the original-contract message game. Then it is not possible to deter side contracting with an original-contract message game and the third party is not useful for budget breaking.*

Proof: To prove this, we consider the different possible constructions of equilibria. Recall that following any suboptimal x there must be two equilibria so that each team member can be deterred from colluding with the third party. We can look at differentiate classes of original-contract message games based on which players send messages, or, analogously, which players messages differ between the two equilibria following a given value of x.

This yields four cases to consider:

(a) only the third party's message "matters"

(b) only the team members' messages "matter"

(c) only the message of one team member and the third party's message "matter"

(d) the messages of both team members and the third party's message "matter."

Note that the case where F conditions on only the message of one team member and x will not work because there is no way to have two different equilibria of the nature that is needed. That team member will always prefer one of the equilibria to the other, and, thus, will always present the message that corresponds to her preferred equilibrium.

We have already dealt with cases (a) and (b) in our discussion above, and described how the ability to side contract over F would disrupt play of the desired equilibria. We briefly describe how this results in team member i and the third party side contracting before the team members select effort. For case (a), i and the third party can commit prior to the effort selection that the third party will select the message that leads to j being punished (due to the ability to side contract over F). So i and the third party would collude and there would not be effective budget breaking.

In case (b), player i and the third party would agree that i would shirk and then play an out of equilibrium message in the original-contract message game so that the team members would pay a transfer to the third party. Since they can condition their side contract on F, they could side contract so that the third party would then pay a transfer to i. That transfer could be set so that both i and the third party do better than if a^* had been selected by the team members.

We now turn to case (c). Suppose, without loss of generality, that team member i and the third party are the players who send messages—that is, team member j's message doesn't matter. Consider two equilibrium message profiles that, following $x < q(a^*)$, implement a^*. Denote the message profile for when i has shirked by (m_i^i, m_T^i), and similarly the message profile when j has shirked is (m_i^j, m_T^j). We will show that, given the properties of F at these two equilibria that are needed to implement a^* and for these to be equilibria, i and the third party will side contract prior to the team members selecting effort, and i will exert effort that is less than a_i^*.

Suppose that i and the third party collude so that i is to choose a_i arbitrarily close to a_i^* to induce $\tilde{x}$ arbitrarily close to $q(a^*)$, and then they are choose message profile (m_i^j, m_T^j). First consider whether they would side contract so as to induce $F(m_i^j, m_T^j, \tilde{x})$. That (m_i^j, m_T^j) is an equilibrium means that neither has the incentive to unilaterally deviate. Further, each can be given the strict incentive not to unilaterally deviate. Turning to the issue of whether i and the third party jointly do better by $\tilde{x}$ being produced instead of $x = q(a^*)$, recall that by assumption the cost savings from shirking is advantageous and since this side contracting limits us to only one allocation following $\tilde{x}$ we cannot prevent both i and j from having the incentive to collude with the third party.

We now evaluate case (d). Consider the case where player i has shirked and the players are to send the messages for the equilibrium that punishes player i. Denote this message profile by (m_i^i, m_j^i, m_T^i), where, as above, subscripts denote the player sending the message and superscripts denote the player that shirked. Using the equilibrium condition for player i and that this equilibrium punishes player i, it must be that

$$F_j(m_i^i, m_j^j, m_T^j) \leq F_j(m_i^j, m_j^j, m_T^j) < F_j(m_i^i, m_j^i, m_T^i).$$

Since transfers are balanced this implies that player i and the third party could jointly do better by each sending their message for the equilibrium that punishes player j. Further, since it's possible for i to choose effort arbitrarily close to a_i^* to induce x arbitrarily close to

$q(a^*)$ there is incentive for i the third party to collude before the choice of effort. They can effectively side contract to do so because the side contract can condition on F.

We have covered each case and have showed that a team member and the third party have the incentive to side contract and are able to effectively do so. In each case this disrupts the use of the third party to break the budget. *Q.E.D.*

4 Conclusion

In this short paper, we have explored the use of message games to potentially allow a third party to be useful for breaking the budget in a team production problem. We have shown that when it is possible for the side contract to condition on the same messages as the original contract, the third party is of no use for breaking the budget and does not help in attaining efficient investment. Due the difference in assumptions about the effectiveness of side deals, our main result differs from that of Baliga and Sjöström (2009). We explored these differences, and found that a more middle ground assumption that side deals can condition on the allocation imposed under the original contract also yields our main result.

References

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