Female Labor Force Participation and Welfare if Status Conscious with Multiple Reference Groups

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Female Labor Force Participation and Welfare if Status Conscious with Multiple Reference Groups

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Abstract
I develop a model with status concerns to analyze how different economic factors affect female participation, average household income and wage, as well as the welfare of both stay-at-home and working wives. Reductions in the price of domestic goods and increases in female wages have positive effects on female participation. Increases in male wages have different effects on female participation depending on whether they affect female wages or not. Events that lead to increases in female participation are usually associated with decreases in the welfare of stay-at-home wives, but are not necessarily associated with increases in welfare of working wives.

JEL classification: D62; E24; J16

Keywords: female labor force participation; relative income

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1. Introduction

Women’s labor force participation has increased dramatically over the second half of the twentieth century. The economic literature has put forward various explanations for this phenomenon, explanations that are mostly associated with positive developments that took place in women’s lives during this time. This paper shows that, paradoxically some of the events that increase female participation and in most economic models are expected to improve women’s welfare, such as decreases in gender wage gap and the price of domestic goods, might have had negative unintended consequences. The “paradox of declining female happiness” as put forward by Stevenson and Wolfers (2009) can thus be justified by status concerns with respect to multiple reference groups in a similar fashion that “the Easterlin paradox” was justified by relative income concerns (Clark, Frijters and Shields 2008).

Recent literature on female labor force participation focused on dynamic changes in culture as an engine of increased participation (Fernandez 2007, Escriche 2007, Vendrik 2003, Fortin 2008). Thus, as more women participate in the labor force the expectations regarding what is the role of women in society adjusts and it is easier for the subsequent generations of women to join the labor force. Since more and more married women have joined the labor force throughout the twentieth century, the dominating premise is that women prefer working outside the home. By changing the culture that kept married women at home, these women were “freed” from domestic drudgery and given a wider set of options that allowed them to choose the ones that maximize their welfare.

In this paper, I develop a model that incorporates several factors that are widely believed to have contributed to the increase in labor participation of married women throughout the second half of the twentieth century and analyze how these factors affect average household income and wage in the economy, as well as the welfare of both stay-at-home and working wives. I argue that once women start joining the labor force, other women will be tempted to join the labor force as well due to increased inequalities of household income. Their joining the labor force comes from what they perceive to be a necessity of having an additional income in the household due to changes in what the society deems an acceptable standard of living. In this sense, this paper is related to the literature on the "keeping up with the Joneses" where households care about their relative standard of living.
The model allows for heterogeneity in women's preference for home production, incorporates comparisons at household level, and potential feelings of envy and inadequacy due to wage gaps between women in the labor force and the men with whom they work. Thus women who work are subject to both improvements in their welfare brought about by the income that they receive, and decreases in welfare due to having to outsource their home production and having to compete with the men that already are in the workforce. Each generation of women decides on their labor participation at the beginning of their life based on wages, male and female, and price of “home goods” that are prevailing at that time.

In this context I analyze how changes in external factors, such as price of “home goods” and gender wage gaps affect the decisions of married women to join the labor force and the effect that these events have on overall welfare of both stay-at-home and working wives.

These experiments are inspired by empirical studies that showed that throughout the twentieth century price of the “home goods” relative to female wages, as well as wage gender gap have decreased noticeably. Attanasio, Low and Sanchez-Marcos (2008) show that conditional on having positive child care costs, the ratio of child care costs to women’s earning is lower in 1988 compared to 1975. They also show that wages of child care workers, which constitute the most important element of child care costs, have decreased over time. Ferrrero Martinez and Iza (2004) also argued that recent skill-biased technological change lead to an increase in skill premium and a relative decrease in the market value of child caring. Greenwood, Seshadri and Yorukoglu (2005) argue that the technological revolution that led to a dramatic decrease in the price of labor savings household durable goods is behind the increase in the female labor participation. My results are consistent with these studies that found a direct positive relationship between these events and increased female participation.

Reductions in the price of “home goods” and increases in female wages have a positive effect on female participation rate. Increases in male wages have different effects on female participation depending on whether they affect female wages or not. For instance, if female wages are independent variables, an increase in male wages decrease female participation rate. Alternatively, I assume that female wages are positive functions of male wages and past female participation rates. These conjectures are based on the fact that technological advances in the market place affect the demand for labor in general, being it male or female and wages are also influenced by the historical trajectory in the labor market of different categories of workers. In
this case an increase in male wages leads to increased female participation rate even if it is not associated with a decrease in gender wage gap.

The effects of these economic shocks on the average household and individual incomes are of particular interest. Thus an increase in female wage will always be associated with an increase in average household income, but not necessarily with an increase in average wage. The increase in female wage leads to increased female participation, and depending on the elasticity of participation with respect to female wage, it could lead to an increase or decrease in average wage in the economy. An increase in male wage leads to an increase in the average wage in the economy, but not necessarily of the average household income depending on whether female wages are independent or positive functions of male wages. If female wages are (not) independent, an increase in male wage leads to (increased) decreased female participation, and to a(n) (increase) decrease in average household income.

Assuming that female wages are positive functions of male wages and past participation rates, I analyze what types of equilibria are likely to occur in this model economy. Depending on whether female wage are increasing, decreasing, or linear function of the past participation rate, the type of hysteresis generated are associated either with the entrance or the exit from the labor market of married women. If female wage is an increasing function of past participation rates, hysteresis is possible if the economy starts from full participation rate. Exiting the labor market would be possible only if certain low (high) thresholds of male wage (price of “home goods) are reached. If female wage is a linear or diminishing function of past participation rate hysteresis is likely if the economy starts from an equilibrium where no women participate in the labor market. Entering the labor market in this case would require that certain high (low) thresholds of the male wage (price of “home” good) be reached.

Most economic models would predict that an increase in female labor supply that is a consequence of what would be generally considered positive events in their lives is associated with an increase in the welfare of the working women. However the welfare effects of some of these events on the working woman can be paradoxically negative. I analyze these welfare effects focusing on both the stay-at-home and working wife. Given that the participation rate changes over time, a woman with certain characteristics (preferences for the production of the “home” good) can either work or stay at home depending on the values of parameters of interest: male, female wage and price of “home” good. Thus the welfare analysis follows what I call a
“perennial” woman (i.e. whose behavior of either working or staying at home before and after the economic shock is constant) as well as an “average” woman (i.e. working or staying at home wives at a certain time $t$).

A reduction in the price of “home good” decreases the welfare of the “perennial” stay-at-home wives and has an ambiguous effect on the utility of the “perennial” working wives, depending on how much they care about their household relative income. As the price of “home goods” decreases, the working wives can buy more of the market good. The effect on the relative household income is ambiguous. Their household standing improves given that the value of the stay-at-home wives’ households decreases by the value of the “home good” as expressed by price. However, as more women join the labor force the average household income in the economy might increases, and thus their relative position might deteriorate. Moreover, if female wage is a function of participation rates, as more women join the labor force their wage improves. Thus, even though most factors point in the direction that a decrease in “home goods” price would improve the welfare of the “perennial” working women, the overall welfare effect is uncertain.

In a similar fashion I find that decreases in male wages or increases in female wages have negative effects on the welfare of the stay-at-home wife if female wages are independent of male wages. However if female wages are a positive function of male wages and past participation rates, a decrease in male wages is associated with a decrease in female wages and participation, so that the overall welfare effect on the stay-at-home wife becomes ambiguous.

One major factor that affects the welfare of the “average” working women is compositional. Since women have heterogeneous and time invariant preferences for producing their own “home goods”, changes in the proportion and thus composition of the women that are in the labor force have a significant impact on the average welfare. Thus, if more women drop from the labor force, the effect on the welfare of the “average” working wife is more positive or less negative than the effect on the “perennial” working wife. If more women join the labor force, the effect on the welfare of the “average” working wife is more negative or less positive than on the “perennial” working wife.

The model incorporates comparisons at the household level, as well as at the individual level in the labor market. In section 3.2 I model female wage to be a positive function of past female
participation. These conjectures are well supported by the economic literature which will be briefly reviewed below.

2. Related literature

2.1 Status concerns with different reference groups

The economic literature confirming the importance of income comparisons in the utility function is quite extensive and keeps growing. Using individual level panel data, Luttmer (2004) and Ferrer-i-Carbonell (2005) show that individuals are happier the larger their income is in comparison with the income of the reference group and that higher earnings of neighbors are associated with lower levels of self reported happiness. Dynan and Ravina (2007) find that people’s happiness is positively related to how well their group is doing relative to the average in their geographic area, even after controlling for the level of their own income. Ravina (2007) estimates an Euler equation derived from a preference specification that includes individual consumption and the average level of consumption of a geographical reference group. Her results imply that the strength of positional concerns, captured by the fraction of the consumption of the reference group that enters the utility function, is close to one third. Frank (2007) provides a plethora of anecdotal evidence regarding the effects of status concerns on individual behavior.

The implications of the “keeping up with the Joneses” models are used in most instances to explain saving, consumption behavior as well as labor supply decisions at intensive margins at the individual level, in a gender-neutral fashion. Some authors focus on relative income concerns to explain labor force participation decisions of married women. Neumark and Postlewaite (1998) develop a neoclassical model that incorporates relative income concerns into households’ utility functions in order to explain women’s employment decisions. They also test the model empirically and show that indeed the women's employment decisions are affected by the incomes of other women with whom relative income comparisons might be important, such as sisters and sisters-in-law (i.e. married women are 16 to 25% percent more likely to have an outside employment if their sister’s husbands earn more than their own husbands). Park (2005) suggests that married women’s employment

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1 Abel (1990) and Gali (1994) are concerned with the effects of preference interdependence on asset pricing, Caroll, Overland and Weil (1997, 2000), Liu and Turnovsky (2005) are concerned with their effects on capital accumulation, savings and growth, Alvarez-Cuadrado and Long (2009), Garcia-Penalosa and Turnovsky (2008) with their effects on income inequality.
decisions are influenced by their household’s relative income. He shows that married women are more likely to be in the labor force when their husbands’ relative income is low, and in particular, married women who worked previously are more likely to stay in the labor market when their husband’s relative income is low. Using data on Italian households, Del Boca and Pasqua (2003) show that the pattern of women employment during 1977-1998 had the effect of reducing inequality in family incomes.

These concerns about relative income at household level have a major impact on women’s decision to join the labor force. However these studies ignore the additional reference group that is brought about when these women join the labor force (i.e. they reflect only the additional income and thus the household income advantage that the stay-at-home wives capture from joining the labor force). Working women however face an additional reference group once they are in the labor market. As workers, they compare their salaries with their peers and these comparisons affect their job satisfaction and their welfare and should therefore be incorporated in their decision process.

The economic literature identifies two aspects of these labor market comparisons. On the one hand, relatively low wages makes people feel worse off as they feel that their work is not compensated at a “fair” value. On the other hand, relatively low wages could make them feel better off as they interpret it as a signal of what they could achieve in their profession or work organization. Using data on British workers, Clark and Oswald (1996) show that workers' reported satisfaction levels are weakly correlated with their absolute wages, but are significantly negatively correlated with their comparison wage rates. Bygren (2004) focuses on what constitutes the reference wage that affects the job satisfaction. He finds that Swedish workers primarily compare their pay with that of similar others (i.e., others with the same education and work experience) in their occupation and in the labor market as a whole. More importantly, for the purpose of this paper, is that his empirical analysis show that the higher the average wage of the reference groups, the lower the probability of being satisfied with one’s own. Sloane and Williams (2000) and Lévy-Garboua and Montmarquette (2004) using data on British, and respectively Canadian workers also found that job satisfaction is negatively related to comparison wages. Brown, Gardner, Oswald and Qian (2008) show that job satisfaction and well-being depend on relative pay and the ordinal rank of an individual’s wage within a comparison group.
Clark, Kristensen and Westergaard-Nielsen (2009) provide evidence that in some cases information effects could be stronger than comparison effects, i.e. workers do not object if their peers earn more as they feel that those higher earnings are a good indication of their future prospects. Clark, Kristensen and Westergaard-Nielsen perform the empirical analysis with data from Denmark, country that has a very high income and wage mobility. Using colleagues within the same establishment as a reference group they find that job satisfaction is positively correlated both with own earnings as well as the average earnings of other workers from within the same organization. However, if the reference group is uninformative with respect to one’s earning potential, it is unlikely that signal outweighs the status motive and thus more likely that low relative income has a negative effect on one’s well-being.

### 2.2 Gender wage gap as function of female participation

In section 3.2, I argue that female participation in the labor market is associated with externalities regarding female wages (i.e. increased female participation increases the return to working of all women). In this section I review two lines of arguments regarding why this might happen.

One argument focuses on network theories in the labor market. Recently, several studies (Calvo–Armengol and Jackson 2004, 2007, Van der Leij and Buhai 2008) address the issue of the effects of social networks on employment, wage inequality and occupational segregation. They argue that when individuals have “an inbreeding bias”, a tendency to associate more closely with individuals that share the same characteristics, there is sustained inequality in wages and employment rates across different groups. Calvo–Armengol and Jackson (2004) show that two groups (i.e. male and female) with two different networks have different employment rates due to the endogenous decision to drop out of the labor force. Arrow and Borzekowski (2004) show that, differences in the network connection between black and whites can explain about 15% of the unexplained variation in wages.

Given the evidence that indicate that differences in social networks have an important effect on wage inequality across groups, the question remains whether males and females belong to different labor networks. The empirical evidence shows that even though male and female share what is considered the social space (i.e. live in the same neighborhoods, and go to the same schools), they don’t share their networks in terms of the labor market. Berger (1995) provides...
evidence that women are at a disadvantage in the labor market due to their inferior social network. She shows that in 1982, women used mostly women contacts in the labor market: 30% of women use female contacts and only 17% of the women use male contacts. As a comparison 47% of men use male contacts and only 9% use female contacts. In general, she shows that women have lower network intensity, since 56% of men use labor contacts and only 47% of women use contacts. Fernandez and Sosa (2005) use a dataset documenting the recruitment and hiring process for an entry level job at a call center of a large US bank. In an environment that was mostly female dominated, they found that referrers of both genders tend to produce same-sex referrals (females referred females 75.1% of the time and males referred females only 56.1% of the time).

Another line of arguments that supports my conjecture that female wages increase with female labor force participation is based on statistical discrimination theories (Phelps 1972, Lundberg and Starz 1983, Coate and Loury 1993 etc.). De la Rica, Dolado and García-Peñalosa (2008) and Escriche (2007) propose models of self-fulfilling expectations in which gender wage gaps are results of statistical discrimination. If firms believe that women are more likely to quit when they have children, and training workers for highly paid careers is expensive, they will be less likely to offer the same training opportunities to women as they do to men. This in turn leads to the existence of wage gaps which discourages even further female participation and dynamically reinforces the mechanism that associates low participation and high gender wage gaps. Albanesi and Olivetti (2009) suggest that firms’ expectations that women spend more time in home production induces them to offer women a lower wage, which consequently leads women to spend less time in the market, validating firms’ beliefs. Since the intra-household allocation of labor, which affects labor attachment to the market and workers’ effort, is not observed by the firms, the incentive constraints in the labor market lead naturally to statistical discrimination. If women were to spend less time undertaking home tasks, and their labor market participation were to increase exogenously, firms’ beliefs about their likelihood to drop out would adjust eventually and gender wage differentials would diminish.

3. The Model

I consider a model with discrete non-overlapping generations. Population is a continuum and each individual lives for one period. Each household consists of two adults that remain married,
and husbands always work in the market and receive a fixed salary. Wives can work at home and produce the “home good” or in the market in which case they receive a salary and the “home good” is bought in the market. I assume that the level of “home good” is constant, not a choice variable and it can be entirely produced outside the home. Household’s consumption is made up of non-rival public “home” and market goods to which both spouses have equal access. Each generation of women decides on their labor participation at the beginning of their life based on the wages, male and female, and price of “home goods” that are prevailing at that time.

In Section 3.1 I analyze the labor supply decisions of women when female wages are exogenous. In Section 3.2 I assume that female wage depends positively on male wage and on female participation rate in the previous period.

**Preferences**

Women derive utility from consumption of “home good”, market good and their relative standing in the society. The “home good” (food, child care, etc) can be produced at home or can be bought in the market if the woman works. The market good is bought with the husband’s income if the woman stays at home, or with whatever income is left after paying for the “home good” if the woman works. Women care about their household relative standing in the society and about their individual relative standing in the workforce if they work.

The utility function of working wives is:

\[
u_{w,it} = (1 - \mu_i)u_1(c_{h,it}) + \alpha_2 u_2(c_{m,it}) + \alpha_3 u_3 \left( \frac{y_{hh}^{it}}{y_{hh}} \right) + \alpha_4 u_4 \left( \frac{y_{ind}^{it}}{y_{ind}} \right)
\]  

(1)

Where

- \(\mu_i\) reflects women’s preference for producing her own “home good”
- \(c_{h,it}\) represents the “home” good
- \(c_{m,it}\) represents the market good
- \(y_{hh}^{it}\) represents the household income
- \(\bar{y}_{hh}^{it}\) represents the average household income
- \(y_{it}^{ind}\) represents the female wage
- \(\bar{y}_{it}^{ind}\) represents the average wage

Finally, \(\alpha_i\) are the weights that reflect the importance of different components of the utility function.
The parameters \( \mu_t \) reflect the fact that women’s preferences for the “home good” bought in the market are heterogeneous and time invariant. The “home good” that is bought in the market is a generic good, it deviates from the exact specifications that the woman would have achieved if she was producing it herself (formula vs. breastfeeding, supermarket muffins vs. homemade muffins, surrogate mothers vs. pregnancy etc.). Some women care about these deviations more than others and thus derive less utility from the consumption of the “home” good if they buy it in the market.

I assume that all “home” goods can be bought in the market and that consumption of “home good” is constant and equal to \( C_{h,t} \). The price of that good is equal to \( p_t \) and can change only exogenously, i.e. as a consequence of technological change in the production of home appliances.

I let the price of the market good to be the numéraire. I also assume that all working wives receive an equal wage, \( w_{f,t} \), and that all husbands receive an equal wage \( w_{m,t} \).\(^2\) Thus, the quantity of the market good consumed in the household with the working wives is:

\[
C_{m,t} = w_{m,t} + w_{f,t} - p_t C_{h,t}
\]

Their household’s relative standing in the society depends on their household income \( Y_{t}^{hh} = w_{m,t} + w_{f,t} \) compared to the average household income, \( Y_{t}^{hh} \).

Their individual standing in the workforce depends on their individual income, \( Y_{t}^{ind} = w_{f,t} \) compared to the average wage, \( Y_{t}^{ind} \). Since all working women receive the same wage and all husbands work and receive identical wages, the subscript \( i \) can be dropped.

The utility function of stay-at-home wives is given by:

\[
u_{s,lt} = \alpha_1 u_1(C_{h,lt}) + \alpha_2 u_2(C_{m,lt}) + \alpha_3 u_3 \left( \frac{Y_{t}^{hh}}{Y_{t}^{hh}} \right)\]

Since the “home good” in this case is produced at home by the woman herself, she derives utility fully from its consumption. The quantity of market good consumed in the household with the stay-at-home wives is only a function of the husband’s wage: \( C_{m,t} = w_{m,t} \)

\(^2\) Suppose that everyone who decides to join the labor force supplies one unit of labor inelastically. Firms use Ricardian technology with constant marginal productivity. Female wage can be justified to be lower that male wage either by assuming that firms retain as surplus the difference between the marginal product of labor and female wage or by assuming that women are less productive than men (Section 2.2).
As she does not belong to the workforce, her relative standing in the society is defined only as a member of the household. She cares about her household’s implicit income \( Y_{t}^{hh} = w_{m,t} + p_{t}C_{h,t} \) compared to the average household income, \( \overline{Y}_{t}^{hh} \).

Women join the labor force if the utility from working exceeds that from staying home. Let \( \bar{\mu}_{t} \) be the cut-off for which women with \( \mu_{i} \leq \bar{\mu}_{t} \) work and women with \( \mu_{i} > \bar{\mu}_{t} \) stay-at-home.

Assuming that \( \mu_{i} \sim U[0,1] \) and taking \( \bar{\mu}_{t} \) as given, the average household income is:

\[
\overline{Y}_{t}^{hh} = w_{m,t} + \int_{0}^{\bar{\mu}} w_{f,t}d\mu + \int_{\bar{\mu}}^{1} p_{t}C_{h,t}d\mu = w_{m,t} + \bar{\mu}_{t}w_{f,t} + (1 - \bar{\mu}_{t})p_{t}C_{h,t} \tag{3}
\]

All households have the husband’s wage equal to \( w_{m,t} \). \( \bar{\mu} \) is the fraction of the households that have wives working and receiving a wage \( w_{f,t} \), and \( 1 - \bar{\mu} \) is the fraction of households that have wives staying at home and producing the “home” good worth: \( p_{t}C_{h,t} \).

The average individual income is:

\[
\overline{Y}_{t}^{ind} = \frac{\int_{0}^{\bar{\mu}} w_{m,t}d\mu + \int_{\bar{\mu}}^{1} w_{f,t}d\mu}{1 + \bar{\mu}_{t}} = \frac{w_{m,t} + \bar{\mu}_{t}w_{f,t}}{1 + \bar{\mu}_{t}} \tag{4}
\]

The labor force is made of men who all work and receive a wage \( w_{m,t} \), and a fraction \( \bar{\mu}_{t} \) of the women who receive a wage \( w_{f,t} \).

Based on historical evidence and for the purpose of having an interesting discussion regarding female participation, I assume that:

\[
w_{f,t} > p_{t}C_{h,t} \tag{5a}
\]

\[
w_{m,t} \geq w_{f,t} \tag{5b}
\]

Plugging (3) and (4) and the budget constraints into (1) and (2), and choosing functional forms for the utility function, the utility functions of both working and stay-at-home wives can be rewritten as:

\[
u_{w,lt} = (1 - \mu_{i,t})\alpha_{1}C_{h,t} + \alpha_{2}ln(w_{m,t} + w_{f,t} - p_{t}C_{h,t}) + \alpha_{3}ln\left(\frac{w_{m,t}w_{f,t}}{w_{m,t} + \bar{\mu}_{t}w_{f,t} + (1 - \mu_{i,t})p_{t}C_{h,t}}\right) + \\
\alpha_{4}ln\left(\frac{w_{f,t}(1 + \bar{\mu}_{t})}{w_{m,t} + \bar{\mu}_{t}w_{f,t}}\right) \tag{1’}
\]

Given the log utility function, the third term is always positive because \( w_{f,t} > p_{t}C_{h,t} \) and thus household status consideration always works in the favor of women who choose to work.

Alternatively, the last term is always negative because \( w_{f,t} < w_{m,t} \), thus individual status considerations always work against the women who choose to work.
\[ u_{s,t} = \alpha_1 c_{h,t} + \alpha_2 \ln(w_{m,t}) + \alpha_3 \ln\left(\frac{w_{m,t} + p_t c_{h,t}}{w_{m,t} + \tilde{\mu}_t w_{f,t} + (1-\tilde{\mu}_t)p_t c_{h,t}}\right) \tag{2'} \]

Given the log utility function the third terms is always negative because \( w_{f,t} > p_t c_{h,t} \) and thus household status consideration always works against women who choose to stay at home. I define the function \( f(\mu_{i,t}) \) as the difference between (2') and (1'), where \( \tilde{\mu}_t \) needs to be determined

\[
f(\mu_{i,t}) = \mu_{i,t} \alpha_1 c_{h,t} - \alpha_2 \ln\left(\frac{w_{m,t} + w_{f,t} - p_t c_{h,t}}{w_{m,t}}\right) - \alpha_3 \ln\left(\frac{w_{m,t} + w_{f,t}}{w_{m,t} + p_t c_{h,t}}\right) - \alpha_4 \ln\left(\frac{w_{f,t}(1 + \mu_{i,t})}{w_{m,t} + \mu_{i,t} w_{f,t}}\right) \tag{6a} \]

The cut-off \( \tilde{\mu}_t \) that gives us the fraction of women that are in the labor force is the value of \( \mu_{i,t} \), such that \( f(\tilde{\mu}_t) = 0 \). \tag{6b}

A woman whose preference for producing her own home good is given by \( \tilde{\mu}_t \) is indifferent between working and staying at home. If \( f(\mu_{i,t}) > 0 \), a woman whose preference for producing her own home good is given by \( \mu_{i,t} \) will rather stay at home than work, therefore \( \tilde{\mu}_t < \mu_{i,t} \). If \( f(\mu_{i,t}) < 0 \) a woman whose preference for producing her own home good is given by \( \mu_{i,t} \) will rather work than stay at home, thus we have \( \tilde{\mu}_t > \mu_{i,t} \). Depending on the parameter values of \( p_t, w_{m,t}, w_{f,t} \) there could be no, one or two solutions for \( \tilde{\mu}_t \). Under some mild conditions a uniquely economically interesting solution for \( \tilde{\mu}_t \) can be derived as a function of \( p_t, w_{m,t}, w_{f,t} \) (see Appendix and Fig.1).

### 3.1 Exogenous female wages

**Proposition 1 (Comparative statics):** The share of women that work \( (\tilde{\mu}_t) \) increases with a reduction in the price of “home good” \( p_t \) \( \left(\frac{d\tilde{\mu}_t}{dp_t} \leq 0\right) \), a decrease in male wage, \( w_{m,t} \) \( \left(\frac{d\tilde{\mu}_t}{dw_{m,t}} \leq 0\right) \), and an increase in the female wage, \( w_{f,t} \) \( \left(\frac{d\tilde{\mu}_t}{dw_{f,t}} \geq 0\right) \).

All proofs are contained in the appendix. A decrease in the price of “home good” will make women more likely to work as they pay less to acquire the home good” from the market, and it also improves their household income relative to that of the stay-at-home wives. The effect on equilibrium female labor force participation is thus much stronger than in the case where we do not incorporate relative standing of household income in female utility function.

A decrease in male wage increases female labor participation for several reasons. Firstly, all households suffer a loss in income, and thus a decrease in the consumption of the market
good, but since the household income of working wives is higher, their utility loss is lower. Secondly, as male wage represents a smaller proportion of their household income, their household position improves relative to that of the stay-at-home wives. And finally, their relative wage increases as male wage decreases, and their own wage stays constant.

An increase in female wage makes women more likely to join the labor force as they can consume more market good, their household income increases relative to that of the stay-at-home wives and their individual income increases as well relative to that of their male colleagues.

**Corollary 1:** Female participation is more sensitive to changes in female wages than to changes in male wages.

\[
\left| \frac{d\bar{\mu}_t}{dw_{m,t}} \right| < \left| \frac{d\bar{\mu}_t}{dw_{f,t}} \right|
\]

**Lemma 1:** The share of women that work \((\bar{\mu}_t)\) increases if relative household concerns become more important for welfare, \((\frac{d\bar{\mu}_t}{d\alpha_s} \geq 0)\) and if relative individual wage concerns become less important \((\frac{d\bar{\mu}_t}{d\alpha_4} \leq 0)\).

Different types of relative concerns have different effects on female participation. Increase in importance of relative household position makes women more likely to work, as the household status consideration always works in the favor of women who choose to work. Increase in the importance of relative individual position makes women less likely to work as individual status considerations always work against the women who choose to work.

**Proposition 2:** A decrease in \(p_t\) has a negative effect on the average wage \((\frac{d\gamma_{v;\text{ind}}}{dp_t} > 0)\), and ambiguous effects on the average household income \((\frac{d\gamma_{v;\text{hh}}}{dp_t} < 0)\). Also, an increase in \(w_{m,t}\) has a positive effect on the average wage \((\frac{d\gamma_{v;\text{ind}}}{dw_{m,t}} > 0)\), and ambiguous effects on average household income \((\frac{d\gamma_{v;\text{hh}}}{dw_{m,t}} < 0)\). Finally, an increase in \(w_{f,t}\) has ambiguous effects on the average wage \((\frac{d\gamma_{v;\text{ind}}}{dw_{f,t}} < 0)\) and a positive effect on the average household income \((\frac{d\gamma_{v;\text{hh}}}{dw_{f,t}} > 0)\).

A decrease in \(p_t\) leads to an increase in female participation (Proposition 1). The average income of the household where the wife stays at home decreases. Depending on how many women join the labor force and the extent of their additional income (difference between gained
wages and the value of the home good that they now buy), the overall effect on the average household income can be either positive or negative. Additionally, as more women are in the labor force, the average wage in the economy decreases.

An increase in $\nu_m$ leads to an increase in the income of each household. However, the same increase also leads to a decrease in female participation (Proposition 1) and thus income loses for each household whose woman switches to produce a home good whose value is less that the wage she used to receive. Additionally, as fewer women remain in the labor force and the male wage increases, the average wage in the economy increases.

An increase in $\nu_f$ leads to an increase in the income of each household with working women. It also leads to increased female participation (Proposition 1), and thus brings additional income due to the difference between female wage and the value of the home good. Additionally, as more women enter the labor force the average wage should decrease. Given that the wages of the women that were already working increased however the overall effect on the average wage in the economy is ambiguous.

As long as there is an interior solution for the fraction of women working (i.e. $\bar{\psi}_t \in (0,1)$) I define the “perennial” working woman as the woman who would work regardless of whether the shocks studied occur (i.e. $\mu_t = 0$). Similarly I define the “perennial” stay-at-home wife as the woman who would stay at home regardless of whether the shocks studied occur (i.e. $\mu_t = 1$). Thus all the results that refer to the “perennial” woman, either working or stay-at-home, refer to cases when we have interior solutions for $\bar{\psi}_t$ both before and after the economic shock\(^3\). The limiting cases, when $\bar{\psi}_t = 0$ or $\bar{\psi}_t = 1$, are discussed separately when they offer interesting insights.

**Proposition 3 (Welfare implications of a change in the price of “home good” on the “perennial” woman):** A decrease in the price of “home good” ($p_t$) has a negative effect on the welfare of the “perennial” stay-at-home wife and an ambiguous effect on the welfare of the “perennial” working wife. If all women already work (i.e. $\bar{\psi}_t = 1$), a decrease in $p_t$ has an unambiguously positive effect on the welfare of the working wife.

The decrease in welfare enjoyed by stay-at-home wives is related strictly to their household relative standing. The value of the good produced by them decreases, which in turn

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\(^3\) Since women only live for one period, the “perennial” either working or stay-at-home woman refers to the women that share the same characteristics (i.e. $\mu_t$), not to the same woman per se.
reduces both the absolute and relative value of their household and, as more women join the labor force, their relative household income deteriorates even more.

As the price of “home goods” decreases, the working wives can buy more of the market good. Since the value produced by the stay-at-home wives decreases, their relative household standing improves. Thus if participation rate stayed constant, the working women would be unambiguously better off. However, as more women join the labor force the average household income in the economy might increase, and thus their relative position might deteriorate. Additionally, as more women are in the labor force which reduces the average wage in the economy, their individual income improves. Thus, most factors point in the direction that a decrease in “home goods” price would have a positive effect on the welfare of the “perennial” working women.

**Lemma 2**: The average welfare of stay-at-home wives is equal to each stay-at-home wife’s welfare and the average welfare of working wives depends on the fraction of women that work ($\bar{\mu}_t$).

**Lemma 3**: If all women work ($\bar{\mu}_t = 1$) change in welfare of each working women is identical to the average change in welfare of working women.

**Corollary 2 (Welfare implications on the “average” woman)**: A decrease in the price of “home good” ($p_t$) has an ambiguous effect on the average utility of working wives. If all women work (i.e. $\bar{\mu}_t = 1$), a decrease in $p_t$ has an unambiguously positive effect on the average welfare of the working wives.

Compared to the welfare of the “perennial” working woman, when we analyze the welfare of the average women we have to take into consideration additional compositional effects. The women that join the labor force as a consequence of the decrease in the price of “home good” are women that suffer a stronger disutility from not producing their own “home good”. Overall these combined effects imply that the average working woman, who, it needs to be underlined, is different from the average working woman when $p_t$ was higher, may be worse off as a consequence of an decrease in $p_t$ under a wider set of parameters.

Using Proposition 3 and Lemma 3, it is apparent that if all women work both before and after the shock ($\bar{\mu}_t = 1$) a decrease in the price of “home good”, has a consistently positive effect on the average utility of working wives.
Stevenson and Wolfers (2009) argued that the decrease in women happiness, which is contemporaneous with increased female participation rates, might be explained by the fact that women in the labor force are exposed to new reference groups. Proposition 3 and Corollary 2 shows that, in fact, status concern at household level, not at the individual level, is the fundamental factor that might lead to a decrease in happiness of the average working woman. If we assume that this status concern is not present, stay-at-home wives’ utility would then stay constant when the price of “home goods” changes, since their utility would depend only on the level of goods that they produce, not their price. In this case an increase in women’s participation in the context of a decrease in the price of “home good” would be possible only if working women’s utility were higher than the utility of those wives that stayed at home, thus a higher utility than what they experienced before, when the price of “home goods” was higher.

**Proposition 4 (Welfare implications of a change in the price of male wages on the “perennial” woman):** An increase in male wage ($w_{m,t}$) has a positive effect on the welfare of the “perennial” stay-at-home wife and an ambiguous effect on the welfare of the “perennial” working wife.

Stay-at-home wives enjoy an increase in welfare as a consequence of the increase in their husband’s wages since they have access to more of the market good. Additionally their relative household position improves for two reasons: their husband’s income represents a bigger share of their household worth and as the share of women working decreases, the average household income becomes closer to their own household income.

Working wives enjoy more of the market good, but their relative status as individual workers deteriorates as gender gap increases and there are fewer women in the working force which pushes the average wage up. The effect on their household relative position is ambiguous: on the one hand, as their husband’s income represents a smaller share of their household worth, their household status deteriorates, but on the other hand as fewer women work, their household income is proportionally higher than the average in the economy.

**Corollary 3 (Welfare implications on the “average” woman):** An increase in male wage, ($w_{m,t}$) has an ambiguous effect on the average welfare of working wives.

Compared to the welfare of the “perennial” working woman when we analyze the welfare of the average woman, we have to take into consideration additional compositional effects. The women that drop out the labor force as a consequence of the increase in male wages are women
that suffer a stronger disutility from not producing their own “home good”. Thus on the average the working women are more likely to be better off than the “perennial” woman if male wages increase.

**Proposition 5 (Welfare implications of a change in the female wages on the “perennial” woman):** An increase in female wage ($w_{f,t}$) has negative effects on the welfare of the “perennial” stay-at-home wife and an ambiguous effect on the welfare of the “perennial” working wife.

The decrease in the welfare enjoyed by stay-at-home wives is related strictly to their household relative standing. The increased wages that working women earn diminish their household relative worth and, as more women join the labor force, their relative household standing decreases even more.

As their wage increases, working wives can buy more of the market good. Also their individual standing improves due to the reduction in the gender gap and the fact that more women joining the labor forces brings the average wage in the economy closer to their own wage. The effect on the relative household income is ambiguous. Their absolute household income increases as their wage increases, but as more women join the labor force the average household income in the economy increases as well, and thus their relative position might deteriorate. If all women work (i.e. $\bar{\mu}_t = 1$), an increase in $w_{f,t}$ has an unambiguously positive effect on the welfare of the working wife. Thus, most factors point in the direction that an increase in the female wage would improve the welfare of the “perennial” working women.

**Corollary 4 (Welfare implications of a change in the female wages on the “average” woman):** An increase in female wage has an ambiguous effect on the average welfare of working wives.

When we analyze the welfare of the average woman, we have to take into consideration additional compositional effects compared to the welfare of the “perennial” working woman. The women that join the labor force as a consequence of the increase in female wages are women that suffer a stronger disutility from not producing their own “home good”. Overall these combined effects imply that the average working woman may be worse off as a consequence of an increase in $w_f$ under quite realistic assumptions. However, if all women already work ($\bar{\mu}_t = 1$) an increase in $w_{f,t}$ has an unambiguously positive effect on the welfare of the working wife.
This analysis was made assuming that female wages are exogenous. In Section 3.2 I assume that female wages are a function of male wages and past female labor force participation. These conjectures are based on the fact that technological advances in the market place are going to affect the demand for labor in general, being it male or female. I assume that female and male labor would be perfect substitutes. However, given the different historical trajectory in the labor market, women are at a disadvantage in obtaining the same jobs that men do, and thus are being paid a lower salary. If all women were in the labor market, then female and male wages would be identical.

3.2 Endogenous female wages

I assume that female wages are a function of male wages and female labor force participation in the past:

\[ w_{f,t} = \theta w_{m,t} + (1 - \theta)\bar{\mu}_{t-1}^\beta w_{m,t} \] (7)

Where \( \beta \) can be greater than, less than, or equal to one, thus female wage can be increasing, decreasing or a linear function of the past participation rate. The three different cases will have different implications on the hysteresis associated with the entrance or exit from the labor market of married women.

Given that in equilibrium \( \bar{\mu}_{t-1}^\beta = \bar{\mu}_t^\beta \), equilibrium female participation can be derived using (6b) and (7). The slope as given by (7) is \( \frac{dw_f}{d\bar{\mu}_{t-1}} = \beta (1 - \theta)\bar{\mu}_{t-1}^{\beta-1} w_{m,t} > 0 \) and its curvature depends on the level of \( \beta \): \( \frac{d^2w_f}{d\bar{\mu}_{t-1}^2} = \beta(\beta - 1)(1 - \theta)\bar{\mu}_{t-1}^{\beta-2} w_{m,t} \leq (>)0 \) iff \( \beta \leq (>)1 \). Given that female wages as given by both (6b) and (7) are upward sloping in \( \bar{\mu}_t \) (see also Fig 2a,b and c), there exists at most one stable solution for \( (w_{f,t}, \bar{\mu}_t) \) under some mild stability conditions (see Appendix). If we start from a disequilibrium such that \( w_{f,t} \) as given by (7) is higher than \( w_{f,t} \) as given by (6b), a higher \( w_{f,t} \) leads to an increase in \( \bar{\mu}_t \) (Proposition 1), which leads to an increase in \( w_{f,t} \) (Eq. 7), and after several iterations equilibrium is reached. If we start from a disequilibrium such that \( w_{f,t} \) as given by (7) is lower than \( w_{f,t} \) as given by (6b), the lower \( w_{f,t} \) leads to a decrease in \( \bar{\mu}_t \) (Proposition 1), which leads to an decrease in \( w_{f,t} \) (Eq. 7), and after several iterations equilibrium is reached.
**Proposition 6 (Comparative statics):** A reduction in the price of “home good” \((p_t)\) leads to an increase in the share of women that work \(\left( \frac{d\hat{\mu}_t}{dp_t} \leq 0 \right)\), as well as female wages \(\frac{dW_{f,t}}{dp_t} \leq 0\) and a decrease in gender wage gap \(\frac{d(w_{f,t}-w_{m,t})}{dp_t} \leq 0\). An increase in male wage, \(w_{m,t}\) leads to an increase in female participation \(\left( \frac{d\hat{\mu}_t}{dw_{m,t}} \geq 0 \right)\), female wage \(\frac{dW_{f,t}}{dw_{m,t}} \geq 0\), and has ambiguous effects on gender wage gap \(\frac{d(w_{f,t}-w_{m,t})}{dw_{m,t}} \ll 0\).

A decrease in the price of “home good” leads to an increase in the share of women that work (Proposition 1) and given (7) to an increase in female wages in equilibrium. As male wages stay constant the gender gap decreases.

An increase in male wage instantaneously leads to an increase in female wage which would encourage women to join the labor force (Proposition 1). Simultaneously, the increase in male wage, by increasing the utility of stay-at-home wives relative to working wives, discourages women from joining the labor force (Proposition 1). In this set-up, with the conditions attached to the existence of a stable equilibrium, the first effect on female participation always dominates.

What is interesting about these results is that a change in male wages has the opposite effect on female participation than it did when female wages were exogenous. Male wages are directly affecting female wages and participation, and female wages are subsequently affecting their participation. This indirect effect on participation turns out to be more important than the direct one if a stable equilibrium is to be reached. Intuitively this result is due to the fact that abstracting from the feedback effect that participation has on female wages, female participation is much more sensitive to changes in female wages than to changes in male wages (Corollary 1).

**Lemma 4:** The share of women that work \((\hat{\mu}_t)\) and female wage \((w_{f,t})\) increases if relative household concerns become more important for welfare \(\left( \frac{d\hat{\mu}_t}{d\alpha_3} \geq 0 \right)\) and if relative individual wage concerns become less important \(\left( \frac{d\hat{\mu}_t}{d\alpha_4} \leq 0 \right)\).

An increase in the importance of relative household position makes women more likely to work, whereas increase in the importance of relative individual position makes women less likely to work (Lemma 1). Relative concerns affect female wage only through their effect on labor participation.
Proposition 7: A decrease in $p_t$ has an ambiguous effect on average household income ($\frac{dy_{h}^{hh}}{dp_t} < 0$) and average wage ($\frac{dy_{h}^{ind}}{dp_t} <= 0$). An increase in $w_{m,t}$ has a positive effect on average household income ($\frac{dy_{h}^{hh}}{dw_{m,t}} > 0$) and the average wage as ($\frac{dy_{h}^{ind}}{dw_{m,t}} > 0$).

What is remarkable compared to Proposition 2 is that an increase in $w_{m,t}$, given that it is associated with an increase in $w_{f,t}$ and $\tilde{\mu}_t$, has a clear positive effect on average household income, as opposed to an effect on the average individual income only. Also contrasting with Proposition 2, a decrease in $p_t$, by increasing female participation, affects $w_{f,t}$ positively, and these two effects combined lead to an uncertain effect on the average individual wage.

3.3.1 Dynamics and hysteresis

I first analyze the dynamic effects that a decrease in the price of “home”, $p_t$ has on female participation, $\tilde{\mu}_t$ and wage, $w_{f,t}$. A reduction in the price of “home good” $p$, encourages women to join the labor force (Proposition 1). The increase in the share of women that work, $\tilde{\mu}_t$ leads to an increase in female wages, which also has a positive effect on labor participation. Thus the adjustment to the higher levels of female participation and wages takes place monotonically.

Secondly, I analyze the dynamic effects that an increase in male wage, $w_{m,t}$ has on female participation, $\tilde{\mu}_t$ and wage, $w_{f,t}$. The increase in male wage leads to an increase in female wage which affects positively participation rate. The increase in $\tilde{\mu}_t$ in subsequent generations leads to even more increases in female wages which encourages more women to work. This dynamic process diffuses throughout several generations of women and will lead eventually to the new steady-state equilibrium where wages of women and labor force participation are higher than before.

Due to the existence of two possible equilibria, one stable and one unstable, the entrance and exit trajectory in the labor force as a consequence of changes in male wages and price of “home” goods might not be symmetrical. The shape of the labor demand function, whether it is increasing, decreasing or linear in female participation ($\beta$ can be greater than, less than, or equal to one) affects the type of hysteresis that could arise. The following sections analyze each case in detail.
### 3.3.2 Hysteresis when $\beta > 1$

If there are two solution for $\mu_t$ in the interval $[0,1]$ the stable one is the smaller one (Fig. 2a). In this case, hysteresis is likely if the economy experience full female participation.

Fig. 3a, b, c and d are the graphical representation of different cases of hysteresis when $\beta > 1$. Starting from interior solution equilibrium, changes in any of the exogenous variables lead to the expected variation in the endogenous variables (Proposition 6). There exist certain thresholds $w_m^0$ and $p^0$ such that if $w_m < w_m^0$ and respectively $p > p^0$ women’s participation is equal to zero, $\hat{\mu}_t = 0$ and $w_f = \theta w_m$. However, if the economy starts from an equilibrium with full female participation, $\hat{\mu}_t = 1$ we have two cases: (1) permanent hysteresis when full participation is irreversible and (2) temporary hysteresis when full participation is reversible as long as there exist $w_m^*$ and respectively $p^*$ at which our system has only one stable and no unstable equilibrium for $\mu \in [0,1]$. In case (2) as long as $w_m > w_m^*$ ($p < p^*$) female participation is 100% and once $w_m$ drops below $w_m^*$ or in case of the price of “home” good $p$ raises above $p^*$ women’s participation will decrease.

Thus, a small variation in the male wage (price of “home good”) from a value that implies full participation can have dramatically different effects on female participation and wages. Fig. 3a 3b, 3c, and 3d shows that for the same value of $w_m$ ($p$) between $w_m^*$ ($p^{**}$) and $w_m^*$ ($p^*$) we can have female full participation or not depending on the history.

### 3.3.3 Hysteresis when $\beta \leq 1$

If there are two solution for $\hat{\mu}_t$ in the interval $[0,1]$ the stable one is the bigger one (Fig. 2b and c). In this case, hysteresis is likely if the economy starts from zero female participation.

Fig. 4a, b, c and d are the graphical representation of different cases of hysteresis when $\beta \leq 1$. If we start from interior solution equilibrium, changes in any of the exogenous variables lead to the expected variation in the endogenous variables (Proposition 6). There exist certain thresholds $w_m^1$ and $p^1$ such that if $w_m > w_m^1$ and respectively $p < p^1$ there is full women’s participation, $\hat{\mu}_t = 1$ and $w_f = w_m$. However if we start from an equilibrium in which female participation is null $\hat{\mu}_t = 0$ there are two cases that can follow: (1) permanent hysteresis when no participation is immutable and (2) temporary hysteresis when no participation is not permanent, as long as there exist $w_m^*$ and respectively $p^*$ at which our system has only one stable and no unstable equilibrium for $\mu \in [0,1]$. In case (2) as long as $w_m < w_m^*$ ($p > p^*$) there is no female
participation, $\bar{\mu}_t = 0$, but if $w_m \geq w_m^* (p \leq p^*)$ women’s participation jumps to the equilibrium level that is implied by the parameters of the model and has no historical influence.

Thus, a small variation in the male wage (price of “home good”) from a value that implies no participation can have dramatically different effects on female participation and wages. Fig. 4a, b, c and d show that for the same value of $w_m (p)$ between $w_m^* (p *)$ and $w_m^* (p **)$, we can have female no participation or not depending on the starting equilibrium point.

**Proposition 8 (Welfare implications of a change in price of “home good” on the “perennial” woman):** A decrease in the price of “home good” ($p_t$) has a negative effect on the welfare of the "perennial" stay-at-home wife and an ambiguous effect on the welfare of the “perennial” working wife.

The decrease in welfare enjoyed by stay-at-home wives as a consequence of a reduction in the price of home goods is related strictly to their household relative standing. Incorporating the fact that female wages also increase, the reduction in welfare is more significant than when female wages were exogenous.

Similarly, the effect on the welfare of the working women is complicated by the endogeneity of female wages. The increase in female wages as a result of increased participation introduces additional sources of ambiguity with respect to the effect on women’ welfare, as Proposition 4 indicates. The main source of negativity in working women’s welfare comes from relative concerns at household level. As participation increases, the woman that was working even before the shock will face a decline in its household relative income which might counterbalance all the positive effects that are brought by increased ability to buy market good and improved individual standing.

If all women work ($\bar{\mu}_t = 1$) a decrease in $p_t$ has an unambiguously positive effect the welfare of the working wife as in Proposition 3, as neither female participation nor do wages change.

**Corollary 5 (Welfare implications of a change in price of “home good” on the average woman):** A decrease in the price of “home good” ($p_t$) has an ambiguous effect on the average welfare of working wives.

The additional compositional effects brought about by increased labor participation would tend to lower the welfare of the average working woman compared to the “perennial” one.
Proposition 9 (Welfare implications of a change in male wages on the “perennial” woman):

An increase in male wages \( (w_{m,t}) \) has an ambiguous effect on the welfare of both the” perennial” stay-at-home and working wife.

An increase in male wage leads to an improvement in stay-at-home wives’ ability to buy more market good and improve their relative status (Proposition 4). Since this change in male wages leads to an increase in female wages as well (Proposition 6), which has a negative effect on the welfare of staying-at-home wives (Proposition 5), the overall welfare effect is ambiguous.

Corollary 6 (Welfare implications of a change in male wages on the average woman) An increase in male wages \( (w_{m,t}) \) has an ambiguous effect on the average welfare of the working wives.

The additional compositional effects brought about by increased labor participation would tend to lower the welfare of the average compared to the “perennial” working woman.

4. Conclusions

Female labor participation increased dramatically over the past half of century in most industrialized countries. Some of the factors that lead to this development, by most objective criteria, should have also lead to increases in women’s welfare. Yet Stevenson and Wolfers (2009) show that during this time, women’s well-being declined, and this trend can be found across demographic groups and industrialized countries.

In this paper, I present a discrete generation model that incorporates relative concerns at household and individual level to analyze different factors that contributed to the increase in labor participation of married women, as well as their effects on average household income and wage in the economy, in addition to female welfare.

Reductions in the price of “home goods” and increases in female wages have a positive effect on the female participation rate. Increases in male wages have different effects on female participation depending on whether they affect female wages or not. For instance, if female wages are independent variables, an increase in male wages decreases female participation. However if female wages are positive functions of male wages, the increase in male wages leads to increased female participation even if they are not associated with a decrease in gender wage gap.
The effects of these economic shocks on the average household and individual incomes are of particular interest. An increase in female wages will always be associated with an increase in average household income, but not necessarily with an increase in average wage. An increase in male wage leads to an increase in the average wage in the economy, but not necessarily in the average household income’s depending on whether female wages are independent or positive functions of male wages.

Assuming that female wages are positive functions of both male wages and past participation rates, I show that female participation in the labor market can be subject to hysteresis (i.e. if female wage is an increasing (linear or diminishing) function of past participation rates, hysteresis is possible if the economy starts from full (zero) participation rate).

Furthermore, the welfare effects of different economic shocks are analyzed, both at individual levels and at averages across the economy. These experiments show that most factors that lead to increases in female participation are usually associated with decreases in the welfare of stay-at-home wives, but surprisingly are not necessarily associated with increases in the welfare of working wives. These results hold true both if the focus is on the welfare of women that were working before and after the shock, or on the average of the working women population. The ambiguities in the welfare results are consequences of the difficulty to quantify the changes that occur in relative household income and relative wage of the working women. Thus a reduction in the price of “home good” might improve the working women household standing given that the value of the stay–at-home wives’ households decreases by the value of the “home good” as expressed by price. However, as more women join the labor force, the average household income in the economy might increases, and thus its relative position might deteriorate. Similarly, changes in both male and female wages affect the participation rate, as well as the average household and individual income and relative standings. The sum of all these effects on the overall welfare of working wives) is therefore uncertain. Thus, ironically, the push for increased female participation would not necessarily lead to an increase in the working women welfare, even if these working women would enjoy the higher wages associated with higher participation.
Figures

Fig. 1

Fig. 2.a.

Fig. 2.b

Fig. 2.c
Fig. 3.a

Exit with hysteresis (p > 1)

Entry/Exit from µ = 1

Exit from µ = 1

Entry/Exit from µ < 1

Price of "home good"

Female participation

Exit with hysteresis (p > 1)

Exit from = 1

Entry/Exit from < 1

p* p0 p**

Male wage

Female participation

Exit with hysteresis (p > 1)

Exit from = 1

Entry/Exit from < 1

wm* wm0 wm**

Ratio wf/wm

Exit with hysteresis (p > 1)

Exit from = 1

Entry/Exit from < 1

wm* wm0 wm**
Fig. 4.a  

Entry with hysteresis ($\beta = 1$)  

Entry/Exit from $\mu = 0$  

Fig. 4.b  

Entry with hysteresis ($\beta = 1$)  

Entry/Exit from $\mu = 0$  

Fig. 4.c  

Entry with hysteresis ($\beta = 1$)  

Entry/Exit from $\mu = 0$  

Fig. 4.d  

Entry with hysteresis ($\beta = 1$)  

Entry/Exit from $\mu = 0$  

0.65  

0.7  

0.75  

0.8  

0.85  

0.9  

0.95  

1  

Price of "home good"  

Ratio $w_f/w_m$  

Entry with hysteresis ($\leq 1$)  

Entry from $\mu = 0$  

Entry/Exit from $\mu > 0$  

$p^*$  

$p_1$  

$p^**$  

0  

0.1  

0.2  

0.3  

0.4  

0.5  

0.6  

0.7  

0.8  

0.9  

1  

Price of "home good"  

Female participation  

Entry with hysteresis ($\leq 1$)  

Entry from $\mu = 0$  

Entry/Exit from $\mu > 0$  

$p^*$  

$p_1$  

$p^**$  

0  

0.1  

0.2  

0.3  

0.4  

0.5  

0.6  

0.7  

0.8  

0.9  

1  

Male wage  

Female participation  

Entry with hysteresis ($\leq 1$)  

Entry from $\mu = 0$  

Entry/Exit from $\mu > 0$  

$w_m^*$  

$w_m^1$  

$w_m^{**}$  

0  

0.1  

0.2  

0.3  

0.4  

0.5  

0.6  

0.7  

0.8  

0.9  

1  

Male wage  

Ratio $w_f/w_m$  

Entry with hysteresis ($\leq 1$)  

Entry from $\mu = 0$  

Entry/Exit from $\mu > 0$  

$w_m^*$  

$w_m^1$  

$w_m^{**}$
Appendix

Stability condition for $\tilde{\mu}_t$

From 6.a I define $f(\mu_t)$ as the difference between the welfare of stay-at-home and working wives. Thus, if there exist a stable interior solution for $f(\mu_t) = 0$, then $f(\tilde{\mu}_t - \epsilon) < 0$ and $f(\tilde{\mu}_t + \epsilon) > 0$. The stability condition in this case is $(f'(\mu_t)) > 0 | \mu_t = \tilde{\mu}_t$, thus $\alpha_1 C_{h,t} - \alpha_4 \frac{w_{m,t} - w_{f,t}}{(w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)} > 0$. (1A)

Proof of Proposition 1

Applying the implicit equation theory for (6b), using (5a) and (5b), and imposing the stability condition (1A):

$$\frac{d\tilde{\mu}_t}{dp} = -\frac{C_{h,t}}{\alpha_2 w_{m,t} + \frac{w_{f,t} - pC_{h,t}}{w_{m,t} + \tilde{\mu}_t w_{f,t}}} \leq 0 \quad (2A.a)$$

$$\frac{d\tilde{\mu}_t}{dw_{m,t}} = -\frac{\alpha_2 w_{f,t} - pC_{h,t}}{\alpha_1 C_{h,t} - \alpha_4 (w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)} \leq 0 \quad (2A.b)$$

$$\frac{d\tilde{\mu}_t}{dw_{f,t}} = \frac{\alpha_2 w_{f,t} - pC_{h,t}}{\alpha_1 C_{h,t} - \alpha_4 (w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)} \geq 0 \quad (2A.c)$$

Proof of Corollary 1

Direct consequence of (5a), (5b) applied to (2A.b) and (2A.c)

Proof of Lemma 1

Applying the implicit equation theory for (6b), using (5a) and (5b), and imposing the stability condition (1A):

$$\frac{d\tilde{\mu}_t}{d\alpha_3} = \frac{\ln(\frac{w_{m,t} + \tilde{\mu}_t w_{f,t}}{(w_{m,t} + \tilde{\mu}_t w_{f,t})})}{\alpha_1 C_{h,t} - \alpha_4 (w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)} \geq 0 \quad (2A.d)$$

$$\frac{d\tilde{\mu}_t}{d\alpha_4} = \frac{\ln(\frac{w_{f,t} + \tilde{\mu}_t w_{f,t}}{(w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)})}{\alpha_1 C_{h,t} - \alpha_4 (w_{m,t} + \tilde{\mu}_t w_{f,t})(1 + \tilde{\mu}_t)} \leq 0 \quad (2A.e)$$

Proof of Proposition 2

Using (3) and (2A.a):

$$\frac{dy_{hh}}{dp} = \frac{d\tilde{\mu}_t}{dp} (w_{f,t} - pC_{h,t}) + (1 - \tilde{\mu}_t)C_{h,t} \neq 0 \quad (3A.a)$$

Using (4) and (2Aa):
Using (3) and (2A.b.):
\[
\frac{d \gamma_{t}^{\text{ind}}}{dp} = \frac{d \bar{\mu}_{t}}{d w_{m}} (w_{f,t} - p C_{h,t}) + 1 < 0
\]
(3A.b)
Using (4) and (2Ab):
\[
\frac{d \gamma_{t}^{\text{ind}}}{d w_{m}} = \frac{d \bar{\mu}_{t}}{d w_{m}} \frac{(w_{f,t} - p C_{h,t}) + 1 + \bar{\mu}_{t}}{(1 + \bar{\mu}_{t})^2} > 0
\]
(3A.c)
Using (3) and (2Ac):
\[
\frac{d \gamma_{t}^{\text{ind}}}{d w_{f}} = \frac{d \bar{\mu}_{t}}{d w_{f}} (w_{f,t} - p C_{h,t}) + \bar{\mu}_{t} > 0
\]
(3A.d)
Using (4) and (2Ac):
\[
\frac{d \gamma_{t}^{\text{ind}}}{d w_{f}} = \frac{d \bar{\mu}_{t}}{d w_{f}} \frac{(w_{f,t} - p C_{h,t}) + \bar{\mu}_{t}}{(1 + \bar{\mu}_{t})^2} < 0
\]
(3A.e)

Proof of Proposition 3
Using (2'), (5a), and (2Aa):
\[
\frac{d \text{Welfare } \delta,t}{dp} = \frac{\alpha_{3}}{(w_{m,t} + \bar{\mu}_{t} w_{f,t} + (1 - \bar{\mu}_{t}) p C_{h,t})} \left[ C_{h,t} \frac{w_{m,t}(1 + \bar{\mu}_{t}) + w_{f,t} \bar{\mu}_{t} + p C_{h,t}}{(w_{m,t} + p C_{h,t})} - \frac{d \bar{\mu}_{t}}{dp} (w_{f,t} - p C_{h,t}) \right] > 0
\]
Using (1'), (5a), (5b) and (2Aa):
\[
\frac{d \text{Welfare } \varnothing,t}{dp} = - \frac{\alpha_{2} C_{h,t}}{w_{m,t} + w_{f,t} - p C_{h,t}} - \frac{\alpha_{3} (1 - \bar{\mu}_{t}) C_{h,t}}{w_{m,t} + \bar{\mu}_{t} w_{f,t} + (1 - \bar{\mu}_{t}) p C_{h,t}} - \frac{d \bar{\mu}_{t}}{dp} \left[ \frac{\alpha_{3} (w_{f,t} - p C_{h,t})}{(w_{m,t} + \bar{\mu}_{t} w_{f,t} + (1 - \bar{\mu}_{t}) p C_{h,t})} \right] < 0
\]
If \( \frac{d \bar{\mu}_{t}}{dp} = 0 \) then \( \frac{d \text{Welfare } \varnothing,t}{dp} < 0 \)

Proof of Lemma 2
As stay-at-home wives produce their own “home” good using (2’) it is inferred that their welfare is equal such that:

\[
\text{Average Welfare } \bar{s}, t = \text{Welfare } \bar{s}, t
\]
As working wives buy their “home” good and have heterogeneous preferences for producing it themselves, using (1’) and (6) their average welfare can be written as:
Average Welfare $\bar{w}, t =$  
\[ \alpha_1 C_{h,t} \left(1 - \frac{\bar{\mu}_t}{2}\right) + \alpha_2 \ln(w_{m,t} + w_{f,t} - p_{C,h,t}) + \alpha_3 \ln\left(\frac{w_{m,t} + w_{f,t}}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}}\right) + \alpha_4 \ln\left(\frac{w_{f,t}(1 + \bar{\mu}_t)}{w_{m,t} + \bar{\mu}_t w_{f,t}}\right) \]

**Proof of Lemma 3**

\[ \frac{d\text{Average Welfare} \bar{w}, t}{dx} = \frac{d\alpha_1 C_{h,t}}{dx} + \frac{d\alpha_2 \ln(w_{m,t} + w_{f,t} - p_{C,h,t})}{dx} + \frac{d\alpha_3 \ln\left(\frac{w_{m,t} + w_{f,t}}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}}\right)}{dx} + \frac{d\alpha_4 \ln\left(\frac{w_{f,t}(1 + \bar{\mu}_t)}{w_{m,t} + \bar{\mu}_t w_{f,t}}\right)}{dx} \]

\[ \frac{d\bar{\mu}_t}{dx} + \frac{1}{2} \alpha_1 C_{h,t} + \frac{d\text{Welfare} \bar{w}, t(\mu_t)}{dx} = \frac{d\text{Average Welfare} \bar{w}, t(\mu_t)}{dx} \]

for any $\mu_t$ given that $\bar{\mu}_t = 1$ and thus $\frac{d\bar{\mu}_t}{dx} = 0$;

**Proof of Corollary 2**

Using Lemma 2, assumptions (5a) and (5b), and (2Aa):

\[ \frac{d\text{Average Welfare} \bar{w}, t}{dp} = \alpha_2 \frac{C_{h,t}}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}} - \alpha_3 \frac{(1 - \bar{\mu}_t)C_{h,t}}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}} - \alpha_4 \frac{w_{m,t} - w_{f,t}}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}} \]

\[ \frac{d\bar{\mu}_t}{dp} \leq 0 \]

If $\frac{d\bar{\mu}_t}{dp} = 0$ then $\frac{d\text{Average Welfare} \bar{w}, t}{dp} < 0$.

**Proof of Proposition 4**

Using (2'), (5a) and (2Ab):

\[ \frac{d\text{Welfare} \bar{w}, t}{dw_{m,t}} = \frac{\alpha_2}{w_{m,t}} + \frac{\alpha_3}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}} \left(\bar{\mu}_t - \frac{d\bar{\mu}_t}{dw_{m,t}}\right)(w_{f,t} - p_{C,h,t}) > 0 \]

Using (1'), (5a), (5b) and (2Ab):

\[ \frac{d\text{Welfare} \bar{w}, t}{dw_{m,t}} = \frac{\alpha_2}{w_{m,t} + w_{f,t} - p_{C,h,t}} - \frac{\alpha_3}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}}(1 - \bar{\mu}_t) - \frac{\alpha_4(1 + \bar{\mu}_t)}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)} \]

\[ \frac{d\bar{\mu}_t}{dw_{m,t}} \left(\frac{\alpha_2}{w_{m,t} + w_{f,t} - p_{C,h,t}}(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}) - \frac{\alpha_4(1 + \bar{\mu}_t)}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)}\right) < 0 \]

**Proof of Corollary 3**

Using Lemma 2, (5a), (5b) and (2Ab):

\[ \frac{d\text{Average Welfare} \bar{w}, t}{dw_{m,t}} = \frac{\alpha_2}{w_{m,t} + w_{f,t} - p_{C,h,t}} - \frac{\alpha_3}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}}(1 - \bar{\mu}_t) - \frac{\alpha_4(1 + \bar{\mu}_t)}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)} \]

\[ \frac{d\bar{\mu}_t}{dw_{m,t}} \left[\frac{1}{2} \alpha_1 C_{h,t} + \frac{\alpha_3}{w_{m,t} + w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}}(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p_{C,h,t}) - \frac{\alpha_4(1 + \bar{\mu}_t)}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)}\right] < 0 \]

**Proof of Proposition 5**
Using (1'), (5a) and (2Ac):
\[ \frac{d\text{Welfare } \$t}{dw_{f,t}} = - \frac{\alpha_3 \left( \bar{\mu}_t + \frac{d\bar{\mu}_t}{dw_{f,t}}(w_{f,t} - p\bar{c}_{h,t}) \right)}{(w_{m,t} + p\bar{c}_{h,t})(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p\bar{c}_{h,t})} < 0 \]

Using (2'), (5a), (5b) and (2Ac):
\[ \frac{d\text{Welfare } \bar{\omega},t}{dw_{f,t}} = \frac{\alpha_2}{w_{m,t} + w_{f,t} - p\bar{c}_{h,t}} + \frac{\alpha_3 (w_{m,t} + p\bar{c}_{h,t})(1 - \bar{\mu}_t)}{(w_{m,t} + w_{f,t})(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p\bar{c}_{h,t})} + \frac{\alpha_4 (w_{m,t} - \bar{\mu}_t w_{f,t})}{(w_{m,t} + \bar{\mu}_t w_{f,t})w_{f,t}} + \]
\[ \frac{d\bar{\mu}_t}{dw_{f,t}} \left[ \frac{\alpha_3 (w_{f,t} - p\bar{c}_{h,t})}{w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p\bar{c}_{h,t}} + \frac{\alpha_4 (w_{m,t} - w_{f,t})}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)} \right] <> 0 \]

If \( \frac{d\bar{\mu}_t}{dw_{f,t}} = 0 \) then \( \frac{d\text{Welfare } \bar{\omega},t}{dw_{f,t}} > 0 \).

**Proof of Corollary 4**

Using Lemma 2 and Proposition 4:
\[ \frac{d\text{Average Welfare } \bar{\omega},t}{dw_{f,t}} = \frac{\alpha_2}{w_{m,t} + w_{f,t} - p\bar{c}_{h,t}} + \frac{\alpha_3 (w_{m,t} + p\bar{c}_{h,t})(1 - \bar{\mu}_t)}{(w_{m,t} + w_{f,t})(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p\bar{c}_{h,t})} + \frac{\alpha_4 (w_{m,t} - \bar{\mu}_t w_{f,t})}{(w_{m,t} + \bar{\mu}_t w_{f,t})w_{f,t}} + \]
\[ \frac{d\bar{\mu}_t}{dw_{f,t}} \left[ \frac{1}{2} \alpha_1 \bar{c}_{h,t} + \frac{\alpha_3 (w_{f,t} - p\bar{c}_{h,t})(w_{m,t} + w_{f,t})}{(w_{m,t} + w_{f,t})(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t)p\bar{c}_{h,t})} - \frac{\alpha_4 (w_{m,t} - w_{f,t})}{(w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)} \right] <> 0 \]

If \( \frac{d\bar{\mu}_t}{dw_{f,t}} = 0 \) then \( \frac{d\text{Average Welfare } \bar{\omega},t}{dw_{f,t}} > 0 \).

**Stability condition for \( \bar{\mu}_t \)**

I derive the slope of \( w_{f,t} \) as a function of \( \bar{\mu}_{t-1} \) as:
\[ \frac{d w_{f,t}}{d \bar{\mu}_{t-1}} = \beta (1 - \theta) \bar{\mu}_{t-1}^{-1} w_{m,t} \]  \( (4Aa) \)

I rewrite (2Ac) to derive the slope:
\[ \frac{d w_{f,t}}{d \bar{\mu}_t} = \left( \frac{\alpha_1 \bar{c}_{h,t} - \alpha_3 (w_{m,t} + \bar{\mu}_t w_{f,t})(1 + \bar{\mu}_t)}{w_{m,t} + w_{f,t} - p\bar{c}_{h,t}} \right) + \frac{\alpha_4 (w_{m,t} - \bar{\mu}_t w_{f,t})}{(w_{m,t} + \bar{\mu}_t w_{f,t})w_{f,t}} \geq 0 \]  \( (4Ab) \)

The curvature of the function as given by (7) depends on \( \beta \)
\[ \frac{d^2 w_{f,t}}{d \bar{\mu}_{t-2}^2} = \beta (\beta - 1)(1 - \theta) \bar{\mu}_{t-1}^{-2} w_{m,t} \leq (>) 0 \text{ iff } \beta \leq (>) 1 \]  \( (4Ac) \)

Whereas the function as given by (6b) is always concave.
If there exist an interior solution for \( \hat{\mu}_t \) the stability condition for it is that \( \frac{d \omega_{f,t}}{d \hat{\mu}_t} \) as given by (7) < 0.

**Proof of Proposition 6**

Plug (10) into (6b) to determine \( \omega_{f,t} \) and \( \hat{\mu}_t \), as well as the partial derivatives that reflect the effect that changes in the exogenous parameters \( w_{m,t} \) and \( p \) have on female labor force participation and female wages.

\[
D d \hat{\mu}_t =
\frac{d w_{m,t}}{d \hat{\mu}_t} \left[ \alpha_2 \frac{p c_{h,t}}{w_{m,t}(1+\theta+1-\theta) \mu_t^\beta} - p c_{h,t} \right] + \alpha_3 \frac{p c_{h,t}}{w_{m,t}(1+\theta+1-\theta) \mu_t^\beta} - dp \left[ \alpha_2 \frac{c_{h,t}}{w_{m,t}(1+\theta+1-\theta) \mu_t^\beta} - p c_{h,t} \right] + \alpha_3 \frac{c_{h,t}}{w_{m,t}(1+\theta+1-\theta) \mu_t^\beta} 
\]

where

\[
D = \alpha_1 c_{h,t} - \alpha_2 \frac{w_{m,t}(1-\theta) \mu_t^{\beta-1}}{w_{m,t}(1+\theta+1-\theta) \mu_t^\beta} - \alpha_3 \frac{\beta(1-\theta) \mu_t^{\beta-1}}{1+\theta+1-\theta} - \alpha_4 \left[ \frac{1-\theta-(1-\theta) \mu_t^\beta}{(1+\theta(1-\theta) \mu_t^\beta)} \right] \geq 0 \quad \text{From the stability condition (4Ac).}
\]

Using (5Aa) and (4Ac) it is thus shown that if a stable interior solution exists

\[
\frac{d \hat{\mu}_t}{d w_{m,t}} \geq 0 \quad (5Ab)
\]

and \( \frac{d \hat{\mu}_t}{dp} \leq 0. \quad (5Ac) \)

Using (10), (5Ab) and (5Ac):

\[
\frac{d \omega_{f,t}}{d w_{m,t}} = \theta + (1-\theta) \mu_t^\beta + \beta(1-\theta) \mu_t^{\beta-1} \frac{d \hat{\mu}_t}{d w_{m,t}} > 0 \quad (6Aa)
\]
\[ \frac{d w_{f,t}}{dp} = \beta (1 - \theta) \bar{\mu}_t^{\beta - 1} \frac{d \bar{\mu}_t}{dp} \leq 0 \]  

(6Ab)

\[ \frac{d (w_{f,t} - w_{m,t})}{dw_{m,t}} = \theta + (1 - \theta) \bar{\mu}_t^\beta + \beta (1 - \theta) \bar{\mu}_t^{\beta - 1} \frac{d \bar{\mu}_t}{dw_{m,t}} - 1 < 0 \]  

(6Ac)

**Proof of Lemma 4**

\[ \frac{d \bar{\mu}_t}{d \alpha_3} = \frac{1}{D} \ln \left( \frac{w_{m,t} (1 + \theta + (1 - \theta) \bar{\mu}_t^\beta)}{w_{m,t} + p C_{h,t}} \right) > 0 \]

\[ \frac{d \bar{\mu}_t}{d \alpha_4} = \frac{1}{D} \ln \left( \frac{(\theta + (1 - \theta) \bar{\mu}_t^\beta)(1 + \mu_t)}{1 + \mu_t (\theta + (1 - \theta) \bar{\mu}_t^\beta)} \right) < 0 \]

\[ \frac{d w_{f,t}}{d \alpha_3} = \beta (1 - \theta) \bar{\mu}_t^{\beta - 1} \frac{d \bar{\mu}_t}{d \alpha_3} > 0 \]

\[ \frac{d w_{f,t}}{d \alpha_4} = \beta (1 - \theta) \bar{\mu}_t^{\beta - 1} \frac{d \bar{\mu}_t}{d \alpha_4} < 0 \]

**Proof of Proposition 7**

Using (3), (5Ac), (6Ab) and assumptions (5a) and (5b):

\[ \frac{d Y_{th}}{dp} = \frac{d \bar{\mu}_t}{dp} (w_{f,t} - p C_{h,t}) + \bar{\mu}_t \frac{dw_{f,t}}{dp} + (1 - \bar{\mu}_t) C_{h,t} \iff 0 \]  

(7Aa)

Using (4), (5Ac), (6Ab) and assumptions (5a) and (5b):

\[ \frac{d Y_{ind}}{dp} = \frac{\frac{d \bar{\mu}_t}{dp} (w_{f,t} - w_m) + \bar{\mu}_t \frac{dw_{f,t}}{dw_m}}{(1 + \bar{\mu}_t)^2} \iff 0 \]  

(7Ab)

Using (3), (5Ab), (6Aa) and assumptions (5a) and (5b):

\[ \frac{d Y_{th}}{dw_m} = 1 + \frac{d \bar{\mu}_t}{dw_m} (w_{f,t} - p C_{h,t}) + \bar{\mu}_t \frac{dw_{f,t}}{dw_m} > 0 \]  

(7Ac)

Using (4), (5Ab), (6Aa) and assumptions (5a) and (5b):

\[ \frac{d Y_{ind}}{dw_m} = \frac{\frac{d \bar{\mu}_t}{dw_m} (w_{f,t} - w_m) + (\bar{\mu}_t \frac{dw_{f,t}}{dw_m} + 1)(1 + \bar{\mu}_t)}{(1 + \bar{\mu}_t)^2} > 0 \]  

(7Ad)

**Proof of Proposition 8**

Using (2'), (5a), (5Ac) and (6Ab):

\[ \frac{d Welfare \ $t}{dp} = \frac{\alpha_3 \left[ c_h \bar{\mu}_t^{w_{m,t} + w_{f,t} + p c_h} \frac{d \bar{\mu}_t}{dp} (w_{f,t} - p C_h) - \frac{dw_{f,t}}{dp} \bar{\mu}_t \right]}{(w_{m,t} + \bar{\mu}_t w_{f,t} + (1 - \bar{\mu}_t) p C_h)} > 0 \]

Using (1'), (5a), (5b), (5Ac) and (6Ab) it can be proven that:
Proof of Corollary 5

Using Proposition 8 and Lemma 2:

\[
\frac{d \text{Average Welfare } \bar{w}, t}{dp} = \frac{\alpha_2}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) - \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{d \tilde{u}_t}{dp} \frac{1}{\alpha_1 C_{h,t}} + \frac{\alpha_2}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) - \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{d \tilde{u}_t}{dp} \frac{w_{m,t} - \tilde{w}_{f,t}}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} + \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{\alpha_4}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) \quad < 0
\]

Proof of Proposition 9

Using (2'), (5a), (5Ab) and (6Aa):

\[
\frac{d W_{f,t}}{dw_{m,t}} = \frac{\alpha_2}{w_{m,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) - \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{d \tilde{u}_t}{dp} \frac{1}{\alpha_1 C_{h,t}} + \frac{\alpha_2}{w_{m,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) - \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{d \tilde{u}_t}{dp} \frac{w_{m,t} - \tilde{w}_{f,t}}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} + \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{\alpha_4}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) \quad < 0
\]

Proof of Corollary 6

Using Proposition 8 and Lemma 2:

\[
\frac{d \text{Average Welfare } \bar{w}, t}{dw_{m,t}} = -\frac{d \tilde{u}_t}{dp} \frac{1}{\alpha_1 C_{h,t}} + \frac{\alpha_2}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) - \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{d \tilde{u}_t}{dp} \frac{w_{m,t} - \tilde{w}_{f,t}}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} + \frac{\alpha_3}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right)
\]

\[
\frac{\alpha_4}{w_{m,t} + \tilde{w}_{f,t} + \tilde{p}C_{h,t}} \left( \frac{dw_{f,t}}{dp} \right) \quad < 0
\]
References


Frank, R.H. (2007), Falling Behind: How Income Inequality Harms the Middle Class, Berkeley: University of California Press.


