3-17-2015

Relative Price Variability and Inflation: New evidence

Deniz Baglan  
*Department of Economics, Howard University, Washington, DC 20059, USA.*, deniz.baglan@howard.edu

M. Ege Yazgan  
*Department of Economics, Kadir Has University, Istanbul 34060, Turkey.*, ege.yazgan@khas.edu.tr

Hakan Yilmazkuday  
*Department of Economics, Florida International University, Miami, FL 33199, USA.*, hyilmazk@fiu.edu

Follow this and additional works at: [http://digitalcommons.fiu.edu/economics_wps](http://digitalcommons.fiu.edu/economics_wps)

Recommended Citation

[http://digitalcommons.fiu.edu/economics_wps/93](http://digitalcommons.fiu.edu/economics_wps/93)
Relative Price Variability and Inflation:
New evidence

Deniz Baglan, M. Ege Yazgan, Hakan Yilmazkuday*

March 17, 2015

Abstract

This paper investigates the relationship between relative price variability (RPV) and inflation using monthly micro price data for 128 goods in 13 Turkish regions/cities for the period 1994–2010. The unique feature of this data set is the inclusion of annual inflation rates ranging between 0 % and 90 %. Nonparametric estimations show that there is a hump-shaped relationship between RPV and inflation, where the maximum RPV is achieved when annual inflation is approximately 20 %. It is shown that this result is consistent with a region- or city-level homogenous menu cost model featuring Calvo pricing with an endogenous contract structure and non-zero steady-state inflation.

JEL Classification: E31, E52

Key Words: Relative price variability; Calvo pricing; Menu costs

*Baglan: Department of Economics, Howard University, Washington, DC 20059, USA. deniz.baglan@howard.edu. Yazgan: Department of Economics, Kadir Has University, Istanbul 34060, Turkey. ege.yazgan@khas.edu.tr. Yilmazkuday: Department of Economics, Florida International University, Miami, FL 33199, USA. hyilmazk@fiu.edu.
1 Introduction

Given the implications for the welfare cost of inflation and monetary neutrality, the relationship between inflation and relative price variability (RPV) has long been debated in the literature. Although theoretical models have generally predicted a positive relationship\(^1\), the direction and functional form of this linkage has not always been verified by empirical studies. Despite the existence of a large body of empirical studies reporting a positive relationship\(^2\), a number of studies have supported a reverse relation between RPV and inflation. Reinsdorf (1994) found that this relation is negative during the 1980s for the US Fielding and Mizen (2000), and Silver and Ioannidis (2001) reported the same result for several European countries. Starting with the work of Parks (1978), who first noted that RPV increases more during periods of price decreases than during periods of price increases, the asymmetric or generally nonlinear effects of RPV on inflation have attracted some attention in the literature. This new direction of research has questioned the underlying functional form of the relationship and provided evidence of a quadratic relationship or threshold effects. The evidence of threshold effects differs somewhat by countries, depending on the nature of the inflation-RPV nexus. Jaramillo (1999) showed that in the U.S., the impact of inflation on RPV, while always positive, is stronger when it is below zero. Similarly, Caraballo et al. (2006) report that for Spain and Argentina, the positive effect is stronger when inflation is high and exploded during the hyperinflationary period in Argentina. Using data from Turkey, Caglayan and Filiztekin (2003) also showed that the association is significantly different during low and high inflation periods. Contrary to these aforementioned studies, during highly inflationary episodes, the association between inflation and inflation variability is significantly lower. However, Bick and Nautz (2008) found that for the US, both positive and negative effects of inflation on RPV are observed in the sense that in-

\(^1\)Whereas menu cost or Lucas-type confusion models predict linear and positive associations between inflation and RPV, recent monetary search and Calvo-type models (see Head and Kumar (2005) and Choi (2010)) predict an inflation-RPV nexus with a U-shape form

flation increases RPV only if it exceeds a threshold value. Results for the Euro area presented by Nautz and Scharff (2012), indicate that inflation significantly increases RPV only if inflation is either very low or very high in the range of their sample values. More recently, conformable with recent monetary search and Calvo-type model predictions (see Head and Kumar, 2005 and Choi, 2010), evidence has been provided of a U-shaped relationship between inflation and RPV by Choi (2010) for the US and Japan, Choi and Kim (2010) for the US, Canada and Japan, Becker (2011) for a panel of European countries, and Fielding and Mizen (2008) for the US. Moreover, in a more recent study of the effect of inflation targeting (IT) on the inflation-RPV nexus, Choi et al. (2011) analyzed a data set of twenty industrial and developing countries consisting of 12 targeters and eight non-targeters, including Turkey, during the so-called great moderation period. They show that the underlying relationship between inflation and RPV is U-shaped in most cases under study, in line with the findings by Choi and Kim (2010) and Fielding and Mizen (2008).

In this paper, we contribute to this body of literature by estimating the relationship between RPV and inflation using a semi-parametric method that allows us to estimate varying coefficients capturing changing effects of inflation, if they exist, on RPV at different levels of inflation. In this respect, we use an estimation method similar to those of Choi (2010), Choi and Kim (2010), Choi et al. (2011) and Fielding and Mizen (2008) in a panel data context by introducing further regional dimensions in addition to goods levels. This unique data set covers quite a large range of (an-
annual) inflation levels varying from 0% to 90%. In our opinion, this specific feature of the data constitutes an important opportunity to examine the inflation-RPV nexus in different inflationary environments.\(^6\)

The empirical evidence provided clearly indicates the fact that the relation between RPV and inflation is nonlinear and varies significantly with the level of inflation. However, unlike the previous studies, our empirical evidence indicates a hump-shaped relation between inflation and RPV, where the maximum dispersion is achieved when annual inflation is approximately 20%. We show that this result is consistent with a region- or city-level homogeneous menu cost model. This homogeneous menu cost model features Calvo pricing with an endogenous contract structure and non-zero steady-state inflation, where the Calvo parameter is determined through optimization. This model is capable of generating a hump-shaped relation between RPV and inflation and significantly differs from the model of Choi (2010), which produces a U-shaped relationship. Choi (2010)'s model, unlike ours, uses sectoral heterogeneity in an exogenous contract setting in which the Calvo parameter is determined in an ad-hoc manner and is assumed to differ across sectors.\(^7\) In the following sections, we present our data and estimation results. After presenting the model, we conclude.

### 2 Data and Estimation

Our empirical analysis uses the monthly price data of the 128 seasonally adjusted good-level prices published by the Turkish Statistical Institute (TurkStat) for a panel of 13 cities from January (M1) 1994 to December (M12) 2001 and from 2003:M1 to 2010:M12.\(^8\) We compute the annual inflation for each month, with respect to the

---

\(^6\)Only a few previous studies covered such high rates of inflation along with considerably lower values. In this regard, Choi et al. (2011) constitutes the main exception together with Caraballo et al. (2006) and Caglayan and Filiztekin (2003).

\(^7\)Choi (2010) notes that the shape of the inflation-RPV nexus depends on the average degree of price rigidity. For sectors in which the average degree of price rigidity is high, the relationship is U-shaped, but this link weakens when price adjustment is highly flexible (see Becker (2011)).

\(^8\)Detailed descriptions of our good-level price data are given in Appendix A.
corresponding month from the previous year, and year over year (yoy) inflation rates for each month, starting in 1995:M1 and 2004:M1. Therefore, we have a data set covering the period 1995:M1–2010:M12 with a two-year gap for 2002 M1–2003 M12. Due to this discontinuity in our data, we conduct two separate estimations for the periods 1995:M1–2001:M12 and 2004:M1–2010:M12. One important feature of these data is that the inflation levels of these two periods do not overlap. In other words, the high-inflation period’s rates never reach levels as low as those observed during the low-inflation period. The time-varying nature of our estimation procedure and this feature of the data help to justify the interpretation of the results of these two separate estimations as a single entity (see Section 2.2 below).9

Figure 1 displays the median, minimum and maximum city-specific inflation rates calculated as the good-level averages with appropriate weights for two periods of Turkish inflation. Between 1995:M1 and 2001:M12, inflation exceeds 90 percent in some cities but approaches 25% in others. During this first era, median inflation is unstable and fluctuates around 54 percent. However, during the period 2004:M1–2010:M12, inflation rates are as high as 18 percent in some of the cities and approach zero in others. The median inflation rate remains as low as 10 percent during this period.

We follow the empirical literature and measure the RPV as

$$RPV_{it} = \sqrt{\sum_{j=1}^{128} \omega_j (\pi_{ij,t} - \pi_{i,t})^2},$$  

(1)

where $i$ and $t$ refer to city and time indexes such that $i = 1, \ldots, N = 13$ at time $t = 1, \ldots, T = 84$ for both data sets. $\pi$ denotes the yoy annual inflation rate for

---

9Because our work uses data on the same country we should compare our data to those of Caglayan and Filiztekin (2003), Caglayan et al. (2008) and Choi et al. (2011). Caglayan et al. (2008)’s data consist of monthly price observations for 58 individual products sold by individual sellers in 15 neighborhoods (boroughs) in Istanbul during the period 1992:M10–2000:M6 when the average inflation rate was high but relatively stable at approximately 60 percent per annum. Caglayan and Filiztekin (2003), however, use long-term disaggregated annual price data for 22 food products collected from the 19 largest provinces in Turkey over the period 1948–1997. Choi et al. (2011) cover the period 1986:M1–2009:M9 for 5 different products.
good \( j = 1, \ldots, 128 \), calculated as \( \pi_{ij,t} = \ln P_{ij,t} - \ln P_{ij,t-12} \), where \( P_{ij,t} \) is the corresponding price level, \( \pi_{it} = \sum_{j=1}^{128} \omega_j \pi_{ij,t} \) denotes the inflation rate for city \( i \) at period \( t \), and the weight of the \( j \)-th good is denoted by \( \omega_j \) such that \( \sum_{j=1}^{128} \omega_j = 1 \).\(^{10}\)

Following the terminology introduced by Lach and Tsiddon (1992, p. 354), this measure of RPV is referred to as the intermarket RPV, where the relevant concept is the dispersion of the product inflation rates around an aggregate rate of inflation in a given city. An alternative measure would be intramarket RPV, which can be defined as the variability of relative prices of a given product across cities or stores. The empirical literature uses either intermarket or intramarket measures of RPV depending on data availability or, if possible, consider both measures at the same time.\(^{11}\) In the theoretical model presented in Section 4, RPV is defined as the intermarket RPV; hence, we use the RPV of Equation (1) in our empirical model.

### 2.1 Empirical Model

In this section, we investigate the relationship between RPV and inflation using a nonparametric model. As is widely accepted, the functional form of the relationship between variables is generally unknown, and parametric models are only implemented due to their simple estimation procedures and ease of interpretation. However, the shape of the relationship between the variables could be highly complicated, and a parametric model may present a deceptive picture of this relationship. To avoid the potential disadvantages of adopting a parametric model, we utilize a semi-parametric approach, which consists of a combination of parametric and nonparametric models. Semi-parametric estimation procedures are appealing because they preserve both the simplicity of parametric and the flexibility of nonparametric models. They are also more informative than their alternatives, such as threshold models, which impose a piecewise linear structure on the inflation function.

\(^{10}\)The weights are taken from TurkStat.\(^{11}\)Reinsdorf (1994, p. 728) states that the theoretical literature refers specifically to relative price-level variability rather than the relative price-change (inflation) variability as defined in (1). These two dispersion measures are not equivalent and can have different relationships with inflation. However, Reinsdorf’s statement refers to intramarket RPV rather than intermarket RPV for which the relevant measure should be changes rather than levels.
Specifically, we consider a partially linear regression model in which inflation has an unknown functional form and other regressors enter the model linearly. Hence, we estimate the following partially linear panel data model:

\[ RPV_{it} = \alpha_i + x'_{it} \gamma + m(\pi_{it}) + u_{it}, \]  

(2)

where \( m(\cdot) \) refers to the unknown smooth function that determines the underlying functional form of the relationship between inflation and \( RPV \). The \( r + k \) vector of regressors \( x \) include the lagged terms of \( RPV \) and \( \pi \), in particular \( x'_{it} = \{ RPV_{it-1}, \ldots, RPV_{it-r}, \pi_{i,t-1}, \ldots, \pi_{it-k} \} \). Finally, the \( \alpha_i \)'s capture the city-specific individual fixed effects. We estimate the unknown function \( m(\cdot) \) and \( \gamma \) with the profile least squares of Su and Ullah (2006). The procedure provides a coefficient estimate for each observation of inflation in our sample. We have a balanced panel of 13 cities for 84 months, and hence, 1092 observations for yearly inflation rates for each period for which we perform the estimation.

Because asymptotic normality approximation may perform badly for both the distribution of estimated parameters and the nonparametric component in finite samples, proper inference is assured by employing a fixed-design wild bootstrap procedure, which is also robust to the presence of cross-sectional and temporal clustering of the residuals. Further technical details of the econometric methodology are given in Appendix B.

12We use the Gaussian kernel function and smoothing parameter \( h \) based on the normal reference rule-of-thumb. We also implement a least squares cross-validation approach and Hurvich et al. (1998)’s expected Kullback-Leibler criteria for selecting the bandwidth. Our results are robust to the choice of bandwidth selection criteria.

13This is a sufficiently large sample size for applying the semi-parametric panel data model estimated below.

14Gonçalves and Kilian (2004) provide a detailed analysis of fixed- and recursive-design wild bootstrapping methods in autoregressive models. Su and Chen (2013) and Li et al. (2013) demonstrate fixed-design wild bootstrapping in parametric and nonparametric panel data models. Appendix B provides the details of our bootstrapping procedure.
2.2 Results

As indicated above, our semi-parametric approach permits us to estimate the unknown inflation function, \( m(\pi_t) \), at all points of inflation in our samples. Figure 2 illustrates the nonparametric estimate of \( m(\pi_t) \) along with the bootstrapped 99 percent confidence intervals for both the initial and recent sample periods of 1995:M1–2001:M12 and 2004:M1–2010:M12, respectively. Overall, the effect of inflation on RPV is hump-shaped. While the effect is decreasing with very low inflation levels, i.e., when inflation is between 2 and 6 %, it starts to increase when inflation reaches between 6 and 18%.\(^{15}\) The effect declines again when inflation is in the range of 22-30 %. This second negative impact of inflation on \( RPV \) attains its minimum and disappears when inflation is approximately 30 %. Then, as inflation increases, its effect on \( RPV \) becomes positive, although small in magnitude, as the slightly positively sloped \( \hat{m}(\pi_t) \) function indicates.\(^{16}\)

As mentioned in the introduction above, in a recent study, Choi et al. (2011) suggested that the inflation-RPV nexus seem linear for high inflation regimes but shows a nonlinear U-shape in more stable inflation environments. Overall, for a very wide range of inflation levels, our study indicates a hump-shaped relation, which is a result consistent with this evidence. During the high inflation episode, the relation approximates a linear form, while it is reminiscent of a U-shaped relation during the low inflation period.\(^{17}\)

\(^{15}\)There is discontinuity in the curve at approximately 18-22 % inflation as neither samples covers inflations in this range.

\(^{16}\)Model selection, i.e., the choice of the number of lags for the linear part of our semi-parametric model, is accomplished based on the Akaike and Bayesian information criteria (see Claeskens and Hjort (2008) for cross-validation of some well-known model selection criteria for the semiparametric models). For the initial period of 1995:M1–2001:M12, both criteria indicated that only the first lag of \( RPV \) are appropriate to include in the model. However, for the recent period 2004:M1–2010:M12, they indicate that both the first lag of \( RPV \) and first two lags of inflation should enter into the linear component of the model.

\(^{17}\)Choi et al. (2011) indicated that in high inflation countries not adopting IT, including Turkey, a decrease in inflation has not led to a shift to U-shaped relationship from a linear one. However, this shift has occurred in IT countries with high inflation. Indeed, our results indicate that this shift may have occurred. Given that their data for the period 1986:M1–2009:M9 cover only 5 products without a regional dimension, it is possible that the shift cannot be captured by their data.
3 Model

Having provided evidence of a hump-shaped relation between RPV and inflation, in this section, we show that this result is consistent with a region- or city-level homogenous menu cost model. This homogenous menu cost model features Calvo pricing with an endogenous contract structure and non-zero steady-state inflation. We obtain this disaggregated model by expanding the aggregate model of Levin and Yun (2007) into a multi-region framework. While the model of Levin and Yun (2007) focuses on relative price-level variability, our model analyzes the effect of inflation on relative price change (inflation) variability.

3.1 Implications of the Model for RPV

To save some space, the micro-foundations of the model are illustrated in Appendix C. In the following, we focus on the implications of the model for the relative price variability $\phi^r$ at the region or city level, which is given by the following steady-state expression:

$$
\phi^r = \text{var}_g (\pi^r_g | r) = \frac{\theta^r (2 - \theta^r) (\pi^r)^2}{(1 - \theta^r + (\theta^r)^2) (1 - \theta^r)},
$$

(3)

where $\pi^r_g$ is the steady-state inflation of good $g$ in region/city $r$, $\text{var}_g (\pi^r_g | r)$ is the variance of $\pi^r_g$ across goods (for any given $r$) that is consistent with RPV definition used in the empirical analysis above, $\pi^r$ is the steady-state gross inflation for region/city $r$, and $\theta^r$ measures the endogenously determined price stickiness in region/city $r$ determined according to the following discounted sum of profits, which is common across all firms in region/city $r$ according to a symmetric Nash equilibrium:

$$
\Omega^r_k = \frac{1 - \theta^r \beta}{1 - \beta} \left[ \sum_{k=0}^{\infty} (\beta \theta^r)^k \left[ \left( \frac{\tilde{P}^r}{(\Pi^r)^k} \right)^{1 - \varepsilon} - MC^r \left( \frac{\tilde{P}^r}{(\Pi^r)^k} \right)^{-\varepsilon} \right] - \omega \right],
$$

(4)

where $\beta$ is the discount factor, $\Pi^r (= \exp \pi^r)$ is the gross inflation, $\varepsilon$ is the elasticity.
of substitution across goods, \( \omega \) is the share of menu cost in output for firms with non-zero constant menu cost, and \( \tilde{P}_r^* \) is the relative price of profit maximizing price given by the following expression in the steady state:

\[
\tilde{P}_r^* = \frac{P_t^r}{P_r^r} = \left( \frac{1 - \theta^r (\Pi^r)^{\varepsilon-1}}{(1 - \theta^r)} \right)^{\frac{1}{\varepsilon}},
\]

where \( P_t^r \) is the newly set price and \( P_r^r \) is the price index in region/city \( r \); \( MC_r^r \) is the marginal cost given by the following expression in the steady state:

\[
MC_r^r = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( \frac{1 - \theta^r \beta (\Pi^r)^{\varepsilon}}{1 - \theta^r \beta (\Pi^r)^{\varepsilon-1}} \right) \tilde{P}_r^*.
\] (5)

### 3.2 Simulation of the Model

To show the implications of the model for RPV, we must parameterize the discount factor \( \beta \), elasticity of substitution \( \varepsilon \), and share of the menu cost in output \( \omega \). Accordingly, we follow Levin and Yun (2007) to set \( \beta = 0.984 \) and \( \omega = 0.029 \); we also consider alternative values of \( \varepsilon \) ranging between 5 and 25 to test for robustness. As in Levin and Yun (2007), we assume that there are no endogenous fluctuations of real output, which implies that we search for a value of \( \theta^r \) satisfying \( \theta^r = \arg \max \Omega_k^r \), where \( \Omega_k^r \) is given in Equation 4.\(^{18}\)

The implications of the model for the relation between the frequency of price change (i.e., \( 1 - \theta^r \)) and the inflation rate are presented in Figure 3. As is evident, firms change their prices more frequently as the level of inflation increases, independent of the value of \( \varepsilon \) considered. Using the obtained \( \theta^r \) values, we obtain the implications of the model for the relation between RPV and inflation according to Equation 3. The results given in Figure 4 indicate that the model successfully replicated the hump-shaped relation between RPV and inflation, independent of the value of \( \varepsilon \) considered.

\(^{18}\)We set the number of periods to 100 while searching for \( \theta^r = \arg \max \Omega_k^r \) in Equation 4.
4 Conclusions

In this paper, we presented empirical evidence of a hump-shaped relation between RPV and inflation that is shown to be consistent with a homogenous menu cost model featuring Calvo pricing, an endogenous contract structure, and non-zero steady-state inflation. This evidence indicates that the inflation-RPV nexus exhibits quite different dynamics depending on the inflationary environment, consistently with Choi et al. (2011), where the inflation-RPV relation is found to be linear in high inflation regimes, while nonlinear and U-shaped in more stable environments. Although this hump-shaped relation seems inconsistent with the U-shaped relation found in the empirical literature (e.g., Choi, 2010, Choi and Kim, 2010, and Fielding and Mizen, 2008), because this study covers periods with much higher levels of inflation (ranging between 0 % and 90 %), this result may be considered a generalization of the results in earlier studies, suggesting that the U-shaped relation can be confined to periods with relatively low levels of inflation but not long-lasting high inflation.
References


Appendix A: Data

We use seasonally adjusted good-level prices for cities and regions in Turkey that were obtained from the Turkish Statistical Institute (TurkStat). The monthly prices are reported at the retail level. The total number of retail stores throughout Turkey is 22,886, but the number of stores varies by region. The prices for each good in each region were averaged across retail stores to calculate region-specific good prices; these raw retail prices are used to calculate the consumer price index in Turkey.

A change in the collection of price data in 2003 created two sample periods. The first covers monthly periods between 1994:M1 and 2001:M12 and includes 554 good prices from 23 regions in Turkey. The second covers monthly periods between 2003:M1 and 2010:M12 and includes 449 good prices from 26 regions in Turkey. Because our main objective is to create a single data set covering both periods, we focus on the common set of cities/regions and goods, which includes the prices of 128 goods and 13 cities/regions. This is the same data set used by Yazgan and Yilmazkuday (2014) to compare the convergence properties of price levels across high- and low-inflation periods.

Appendix B: Econometric Methodology

The semiparametric panel data model of interest is given by

\[ y_{it} = \alpha_i + x'_{it} \gamma + m(z_{it}) + u_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T, \]  

(B.1)

where \( \alpha_i \)'s are fixed effects, \( x_{it} \) is a \( p \)-dimensional vector of regressors, \( m(\cdot) \) is a smooth function, \( z_{it} \) is a \( q \)-dimensional vector of exogenous regressors, and \( u_{it} \) are zero mean \( i.i.d \) innovations with variance \( \sigma_u^2 \). Therefore, we include heterogeneity through individual fixed effects and analyze the nonlinear relationship of interest.

19These stores do not change over time unless a store closes or a particular product is no longer available in that store.

20The link between the good-level price data utilized in this paper and aggregate CPI data is achieved through good- and region-specific weights assigned to the individual prices.
without imposing a specific functional form, controlling other important explanatory variables. For identification, we assume $\sum_{i=1}^{N} \alpha_i = 0$.

Taking a first order Taylor expansion of (B.1) at point $z$ yields

$$y_{it} \approx \alpha_i + x_{it}'\gamma + m(z) + (z_{it} - z)\beta(z) + u_{it}$$

where $Z_{it}(z) = (1 (z_{it} - z)' \beta(z) + (z_{it} - z)\delta(z) + u_{it}$.

In vector form, we have

$$Y \approx D\alpha + X\gamma + Z(z)\delta(z) + U,$$

where $Y = (y_{11}, \ldots, y_{1T}, \ldots, y_{n1}, \ldots, y_{nT})'$, and $Z(z) = (Z_{11}(z), \ldots, Z_{1T}(z), \ldots, Z_{n1}(z), \ldots, Z_{nT}(z))'$, $\alpha = (\alpha_2, \ldots, \alpha_n)'$, $D = (I_n \otimes t_T)d_n$, $d_n = [-t_{n-1} I_{n-1}]'$, and $t_a$ is an $a \times 1$ vector of ones.

Su and Ullah (2006) propose estimating the model in (B.2) using the profile least squares method. Their approach assumes that the individual effects parameter $\alpha$ and linear component $\gamma$ are initially known and thus estimate $\delta(z)$ by minimizing the following criterion function:

$$\frac{1}{h^q}K_h(z)K_h(z)Y - D\alpha - X\gamma - Z(z)\delta(z),$$

where $K_h(z) = h^{-q}K(z/h)$, $K$ is a kernel function and $h$ is a bandwidth parameter. This procedure profiles out the model parameters and considers the concentrated least squares for $\delta(z)$. Defining the smoothing operator as $S(z) = [Z(z)'K_h(z)Z(z)]^{-1}Z(z)'K_h(z)$ and letting $\theta = (\alpha', \gamma')'$,

$$\delta_{\theta}(z) = S(z)(Y - D\alpha - X\gamma).$$

In particular, the estimator for $m(z)$ is

$$m_{\theta}(z) = s(z)'(Y - D\alpha - X\gamma),$$
where \( s(z)^t = e'S(z) \), and \( e = (1, 0, \ldots, 0)^t \) is a \((q + 1) \times 1\) vector. However, \( \delta_\theta(z) \) depends on the unknown parameter vector \( \theta \) and hence is not operational. To operationalize \( \delta_\theta(z) \), linear parameter \( \gamma \) and the fixed effects are estimated with the profile least squares method as follows:

\[
\hat{\gamma} = \left[ X'^*M^*X^* \right]^{-1}X'^*M^*Y^*,
\]

\[
\hat{\alpha} \equiv (\hat{\alpha}_2, \ldots, \hat{\alpha}_n) = \left[ D'^*D^* \right]^{-1}D'^*(Y - X^*\hat{\gamma}),
\]

where \( D^* = (I_{nT} - S)D \), \( Y^* = (I_{nT} - S)Y \), \( X^* = (I_{nT} - S)X \), \( M^* = I_{nT} - D'[D'^*D^*]^{-1}D'^* \), \( S = (s_{11}, \ldots, s_{1T}, s_{21}, \ldots, s_{nT}) \), and \( s_{it} = s(z_{it}) \). Finally, the profile likelihood estimator for \( \delta(z) \) is given by

\[
\hat{\delta}(z) = \begin{bmatrix} \hat{m}(z) \\ \hat{\beta}(z) \end{bmatrix} = S(z)(Y - D\hat{\alpha} - X\hat{\gamma}).
\]

Estimation of the smoothing parameter (bandwidth) is crucial in a semiparametric analysis. Selecting a very small \( h \) may produce an under-smoothed (low bias, high variance) estimator while choosing a very large \( h \) may generate an over-smoothed (high bias, low variance) estimator. There exist many selection procedures to estimate the optimal bandwidth in practice. Due to its computational simplicity and attractiveness to practitioners, we utilize the normal reference rule-of-thumb \( h = 1.06s_z(NT)^{-1/(4+q)} \), where \( s_z \) is the sample standard deviation of \( \{z_j\}_{j=1}^{NT} \).

**Bootstrap**

Following Su and Chen (2013) and Li et al. (2013), we implement a fixed-design wild bootstrapping procedure. The bootstrap confidence intervals are obtained via the following steps:

1. For each \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \), obtain the bootstrap error \( u_{it}^* = \hat{u}_{it}\epsilon_{it} \), where \( \hat{u}_{it} = y_{it} - \hat{y}_{it} \) and \( \epsilon_{it} \) are i.i.d \( N(0, 1) \) across \( i \) and \( t \), and \( \hat{y}_{it} \) is the fitted value of \( y_{it} \) obtained from equation (B.1).
2. Generate the bootstrap sample \( y_{it}^* = \hat{y}_{it} + u_{it}^* \) for \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \).

3. Given a bootstrap sample for the dependent variable \( \{ (y_{it}^*, z_{it}, x_{it}), i = 1, \ldots, N, t = 1, \ldots, T \} \) obtain the estimators of \( m(.) \) and \( \gamma \) and denote the resulting estimates by \( \hat{m}^*(.) \) and \( \hat{\gamma}^* \).

4. Repeat steps (1)–(3) a large number (\( B \)) of times to obtain the bootstrap samples \( \hat{m}_{b}^*(.) \) and \( \hat{\gamma}_{b}^* \), \( b = 1, \ldots, B \). The estimators \( \text{Var}^*(\hat{m}(.) ) \) and \( \text{Var}^*(\hat{\gamma}) \) are the sample variances of \( \hat{m}_{b}^*(.) \) and \( \hat{\gamma}_{b}^* \), respectively.

5. Compute \( T_{m,b}^* = \frac{|\hat{m}_{b}^*(z) - \hat{m}(z)|}{\{\text{Var}^*(\hat{m}(z))\}^{1/2}} \) and \( T_{\gamma,b}^* = \frac{|\hat{\gamma}_{b}^* - \hat{\gamma}|}{\{\text{Var}^*(\hat{\gamma})\}^{1/2}} \) for \( b = 1, \ldots, B \).

6. Use the upper \( \alpha \) percentile of \( T_{m,b}^* \) and \( T_{\gamma,b}^* \), to estimate \( c_{m,\alpha} \) and \( c_{\gamma,\alpha} \).

7. Construct the \((1 - \alpha) \times 100\%\) bootstrapped confidence intervals as follows:

\[
\hat{m}(z) \pm \{\text{Var}(\hat{m}^*(z))\}^{1/2} c_{m,\alpha} \\
\hat{\gamma} \pm \{\text{Var}(\hat{\gamma}^*)\}^{1/2} c_{\gamma,\alpha}
\]

Appendix C: Microfoundations of the Model

The representative individual in region/city \( r \) is assumed to maximize her utility:

\[
U^r_t = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C^r_t)^{1-\sigma} - 1}{1 - \sigma} - \frac{(N^r_t)^{1+\kappa}}{1 + \kappa} \right), \quad (C.1)
\]

where \( \beta \) is the discount factor, \( N^r_t \) is the number of hours worked, and \( C^r_t \) is an index of composite goods given by:

\[
C^r_t = \left( \int_0^1 (C^r_{g,t})^{\frac{\eta}{\kappa}} dg \right)^\frac{1}{\frac{\eta}{\kappa}}
\]

where \( C^r_{g,t} \) is the consumption of good \( g \) and \( \eta > 1 \) is the elasticity of substitution across goods. The optimization results in the following demand functions:
\[ C_{g,t}^r = \left( \frac{P_{g,t}^r}{P_t^r} \right)^{-\varepsilon} C_t^r, \]

where \( P_t^r \) and \( P_{g,t}^r \) are the prices corresponding to \( C_t^r \) and \( C_{g,t}^r \), respectively, which satisfy

\[ P_t^r = \left( \int_0^1 (P_{g,t}^r)^{1-\varepsilon} \, dg \right)^{\frac{1}{1-\varepsilon}}. \]  

The individual in region/city \( r \) chooses consumption \( C_t^r \) and labor supply \( N_t^r \) according to Equation (C.1) with respect to the following budget constraint:

\[ C_t^r + E_t \left( Q_{t,t+1} \frac{B_t^r \left( \frac{P_t^r}{P_{t+1}^r} \right)^{1-\varepsilon} \, dg \right)^{\frac{1}{1-\varepsilon}} = B_t^r \frac{W_t^r N_t^r}{P_t^r} + T_t^r, \]

where \( Q_{t,t+1} \) is the stochastic discount factor for computing the real value at period \( t \) of one unit of consumption of goods in period \( t+1 \), \( B_t^r \) is the nominal bonds portfolio, and \( T_t^r \) represents transfers/dividends. The optimization results in

\[ Q_{t,t+1} = \beta \left( \frac{C_{t+1}^r}{C_t^r} \right)^{-\sigma} \left( \frac{P_t^r}{P_{t+1}^r} \right). \]

The firm producing good \( g \) in region \( r \) has the following market clearing condition:

\[ Y_{g,t}^r = C_{g,t}^r, \]

where \( Y_{g,t}^r \) is output. For the optimization problem of the firm, following Levin and Yun (2007), we consider deterministic steady states with constant real quantities over time and a symmetric Nash equilibrium with individual firms choosing the same frequency of price adjustments; this serves our purposes of analyzing the steady-state relationship between inflation and price dispersion across regions. We also assume that there are fixed costs associated with changing prices that are proportional to the output produced: \( F_{g,t}^r = \omega Y_{g,t}^r \). In formal terms, a recursive representation of the present value of current and future profits for firms re-optimizing their prices at period \( t - k \) is given by
\[ \Omega^r_{g,k} (\theta^r_g, \theta^r_r) = \left( \left( \frac{\tilde{P}^*_g}{\Pi^k} \right)^{1-\varepsilon} - MC^r \left( \frac{\tilde{P}^*_g}{\Pi^k} \right)^{-\varepsilon} \right) Y^r_g \right) - I_{\{k=0\}} \omega Y^r_g \]

One-Period Profit

\[ + \beta \{ \theta^r_g \Omega^r_{g,k+1} (\theta^r_g, \theta^r_r) + (1 - \theta^r_g) \Omega^r_{g,0} (\theta^r_g, \theta^r_r) \}, \]

Recursive Term

where \( \theta^r_g \) is the measure of price stickiness (\( \theta^r_g \) is the mean measure across firms in region/city \( r \)), \( \tilde{P}^*_g \) is the relative price of the profit maximizing price (where \( \tilde{P}^r_{g,t} \) is the newly set price), \( MC^r \) represents the marginal cost of production, and \( I_{\{k=0\}} = 1 \) only if \( k = 0 \). The firm chooses both \( \theta^r_g \) and \( \tilde{P}^*_g \). First, \( \tilde{P}^*_g \) is determined by the following first-order condition:

\[ \tilde{P}^*_g = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \left( \frac{1 - \theta \beta (\Pi^r)^{\varepsilon-1}}{1 - \theta \beta (\Pi^r)^{\varepsilon}} \right) MC^r. \] (C.3)

Second, the discounted sum of profits, given by

\[ \Omega^r_{g,k} = \frac{1 - \theta^r_g \beta}{1 - \beta} \left[ \sum_{k=0}^{\infty} (\beta \theta^r_g)^k \left( \left( \frac{\tilde{P}^*_g}{\Pi^k} \right)^{1-\varepsilon} - MC^r \left( \frac{\tilde{P}^*_g}{\Pi^k} \right)^{-\varepsilon} \right) \right] - \omega \] (C.4)

is maximized by choosing \( \theta^r_g \), which results in \( \tilde{P}^*_g = \tilde{P}^*_g^* \) and \( \theta^r_g = \theta^r_r \), given that all other firms choose \( \tilde{P}^*_g^* \) and \( \theta^r_r \), according to a symmetric Nash equilibrium.

Furthermore, Calvo pricing leads to the following price dynamics due to \( \theta^r_g = \theta^r_r \) and Equation (C.2):

\[ P^*_t = \left( (1 - \theta^r) \left( \frac{P^*_t}{P^*_{t-1}} \right)^{1-\varepsilon} + \theta^r (P^*_{t-1})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \]

This corresponds to the following steady-state expression:
\[ \tilde{P}_r^* = \frac{\bar{P}_r}{\hat{P}_r} = \left( \frac{1 - \theta^r (\Pi^r)^{\varepsilon-1}}{(1 - \theta^r)} \right)^{\frac{1}{\varepsilon}}, \]

which can be combined with Equation (C.3) to obtain an expression for steady-state marginal costs

\[ MC^r = \frac{\varepsilon - 1}{\varepsilon} \left( \frac{1 - \theta^r \beta (\Pi^r)^{\varepsilon}}{1 - \theta^r \beta (\Pi^r)^{\varepsilon-1}} \right) \tilde{P}_r^*, \quad (C.5) \]

Using Equation (C.5), we can numerically solve \( \theta^r \) through Equation (C.4), given \( \beta \) and \( \omega \).

Finally, for each region, we define the price dispersion across goods as follows:

\[ \phi_t^r = \text{var}_g (\pi_{g,t}^r | r, t) = \text{var}_g (\log P_{g,t}^r - \log P_{g,t-1}^r | r, t), \quad (C.6) \]

which measures relative price variability (RPV) \( \phi_t^r \). To show the relation between \( \phi_t^r \) and the inflation level, first define

\[ \tilde{P}_t^r = E_g \log P_{g,t}^r, \quad (C.7) \]

which implies through Calvo pricing that

\[ \tilde{P}_t^r - \tilde{P}_{t-1}^r = E_g \left( \log P_{g,t}^r - \tilde{P}_{t-1}^r \right) \]
\[ = \theta^r E_g \left( \log P_{g,t-1}^r - \tilde{P}_{t-1}^r \right) + (1 - \theta^r) \left( \log P_{g,t}^r - \tilde{P}_{t-1}^r \right) \]
\[ = (1 - \theta^r) \left( \log P_{g,t}^r - \tilde{P}_{t-1}^r \right). \quad (C.8) \]

Now, we can rewrite Equation (C.6) as follows:
\[ \phi^r_t = \theta^r \left( \frac{\bar{P}_{t-1}^r - \bar{P}_{t-2}^r}{2} \right)^2 + (1 - \theta^r) \theta^r \left( \log \bar{P}_{g,t}^r - \log P_{g,t-1}^r + \bar{P}_{t-2}^r \right)^2 + (1 - \theta^r)^2 \left( \log \bar{P}_{g,t}^r - \log \bar{P}_{g,t-1}^r + \bar{P}_{t-2}^r \right)^2 \]

Using Equations (C.7) and (C.8), it is further implied that

\[ \phi^r_t = \theta^r \left( \frac{\bar{P}_{t-1}^r - \bar{P}_{t-2}^r}{2} \right)^2 + \theta^r \left( \frac{\bar{P}_{t}^r - \bar{P}_{t-1}^r}{1 - \theta^r} \right)^2 + (1 - \theta^r) \theta^r \phi^r_{t-2}. \]

Finally, using the log-linear approximation of \( \bar{P}_t^r = \log P_t \), we obtain the following expression for relative price variability:

\[ \phi^r_t = \theta^r \left( \pi_{t-1}^r \right)^2 + \theta^r \frac{\left( \pi_{t}^r \right)^2}{(1 - \theta^r)} + (1 - \theta^r) \theta^r \phi^r_{t-2}. \]

It is implied that in the steady state, we have

\[ \phi^r = \frac{\theta^r (2 - \theta^r) \left( \pi^r \right)^2}{(1 - \theta^r + (\theta^r)^2) (1 - \theta^r)}, \]

where, as mentioned above, \( \theta^r \) is numerically solved using Equation (C.4), given \( \beta \) and \( \omega \).
Figure 1: Inflation Rates across the cities in Turkey

Figure 2: Confidence Interval for \( m(\pi) \)

Notes: Figure 2 displays the nonparametric estimate of \( m(\pi) \) for 1995:M1–2001:M12 (dots) and Jan 2004:M1–2010:M12 (circles). Dashed lines represent the 99% confidence interval for the nonparametric estimates.
Figure 3: The Implication of the Model: Frequency of Price Change versus Inflation

Notes: The vertical axis represents the percentage of firms changing their prices. $\varepsilon$ represents the elasticity of substitution across goods.
Figure 4: The Implication of the Model: Relative Price Variability versus Inflation

Notes: The size of menu cost is set to be 2.9% of labor input (i.e. $\omega = 0.029$); since we assume that the production function is linear in labor, it means that menu cost is 2.9% of real output.