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Asset Pricing with Incomplete Information under Stable Shocks

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Asset Pricing with Incomplete Information under Stable Shocks

Abstract

We study a consumption based asset pricing model with incomplete information and α -stable shocks. Incomplete information leads to a non-Gaussian filtering problem. Bayesian updating generates fluctuating confidence in the agents' estimate of the persistent component of the dividends' growth rate. Similar results are obtained with alternate distributions exhibiting fat tails (Extreme Value distribution, Pearson Type IV distribution) while they are not with a thin-tail distribution (Binomial distribution). This has the potential to generate time variation in the volatility of model-implied returns, without relying on discrete shifts in the drift rate of dividend growth rates. A test of the model using US consumption data indicates strong support in the sense that the implied returns display significant volatility persistence of a magnitude comparable to that in the data.

Key phrases: asset pricing; incomplete information; time-varying volatility; fat tails; stable distributions;

JEL classification: G12, G13, E43

1. INTRODUCTION

We study a pure exchange Lucas (1978) asset pricing model in a setting with incomplete information on the stochastic dividends process. The stochastic setting is characterized by exogenous shocks coming from the family of α -stable distributions. This distributional assumption marks a departure from the literature on asset pricing with incomplete information in non-Gaussian settings. This literature usually employs a discrete switching process (which is necessarily non-Gaussian) to characterize the dynamics of the drift rate of dividends.

In incomplete information asset pricing models, the drift rate of the dividends process is assumed to be unobservable. Agents need to estimate this drift rate based on observed dividends in order to compute the expected future dividend payouts and hence set equilibrium asset prices. This introduces a filtering/signal extraction problem into asset pricing models.

Early work on incomplete information in asset pricing models used linear stochastic differential equations with Brownian motion increments to characterize the exogenous path of the dividends process. The unobservable drift rate of the dividends process is also characterized as a linear stochastic differential equation with Brownian motion increments. Dothan and Feldman (1986), Detemple (1986), Gennotte (1986), and more recently, Brennan and Xia (2001) study asset pricing / portfolio allocation problems in this setting. Linear Gaussian setting permits use of the Kalman filter to solve the signal extraction problem in an optimal sense.

The Kalman filter is a Bayesian updating rule that permits learning about the unobservable dividend drift rate with the arrival of new information on dividends each

period. In all the above studies, the prior distribution on the drift rate is invariably Gaussian. With such a prior in a linear Gaussian setting, the posterior distribution on the drift rate is also Gaussian. Moreover, this posterior density has a dispersion that is constant and does not react to new information after transients have died out from startup of the filter. However, it is more realistic to have time-varying dispersion on the posterior density, suggesting periods of greater or lower confidence about the state of the dividend drift (David 1997).

This limitation of the Gaussian setting was recognized in early work by Detemple (1991). He therefore uses a non-Gaussian prior distribution on the state variable in order to generate a non-Gaussian posterior distribution with time-varying dispersion. Another way to generate time-varying dispersion on the posterior distribution is by adopting a non-Gaussian stochastic setting. This is done in articles by David (1997) and Veronesi (1999, 2000). They assume that the drift rate of dividends follows a discrete state process, governed either by a Poisson arrival or Markov switching process. More recently, Veronesi (2004) generalizes by allowing the drift rate to follow a continuous-state Gaussian process subject to discrete breaks. The breaks are governed by Markov process, thus making the overall process for the drift rate non-Gaussian.

All the papers discussed above on asset pricing with incomplete information formulate the problem in continuous time. In a discrete time setting, Cecchetti et al. (2000) and Brandt et al. (2000) model dividends as a random walk driven by Gaussian innovations and a drift term that follows a discrete state Markov switching process. Thus, all extant non-Gaussian asset pricing models with incomplete information, both in continuous and discrete time, formulate the signal extraction problem facing investors as

an unobservable process for the drift rate of observable dividends that inherently involves discrete breaks / regimes.

Time-varying dispersion on the posterior density, or fluctuating confidence as in David (1997), related to the drift rate of dividends in non-Gaussian incomplete information asset pricing models, leads to time variation in the volatility of implied returns, thereby providing a mechanism for replicating this stylized fact documented in observed returns. Thus, the current literature relying on signal extraction generates volatility persistence in asset pricing models as an outcome of learning about unobservable discrete shifts or breaks in the underlying drift rate of dividends.

However, even continuous-valued dividend drift not involving discrete shifts or breaks in non-Gaussian filtering/signal extraction setting with fat tails would lead to fluctuating confidence (Kitagawa 1987), and thus presumably to time-varying volatility of implied returns. It is more realistic to model dividend drift as a continuous valued fat-tailed non-Gaussian process rather than assume that it undergoes periodic shifts that are discrete in nature.

In order to demonstrate that Bayesian learning in a continuous-valued non-Gaussian stochastic setting with fat-tails and incomplete information would lead to time-varying volatility requires an appropriate probability distribution with fat tails. One immediately runs into difficulties here because, as Geweke (2001) notes, the theory of choice under uncertainty in such settings often breaks down under the constant relative risk aversion (CRRA) utility function. Geweke (2001) specifically demonstrates the failure of the choice theory with Student- t distributions in such circumstances.

The family of α -stable distributions provides a way out of this difficulty. These distributions have a fairly long history in finance, going back to early work by Mandelbrot (1963). A comprehensive survey on the financial applications of these distributions is provided by McCulloch (1996). The $\alpha \in (0, 2]$ parameter (along with three other parameters including a skewness parameter $\beta \in [-1, 1]$) characterizes these distributions, with $\alpha = 2$ resulting in the Gaussian distribution and $\alpha < 2$ resulting in fat-tailed distributions. While the difficulty noted by Geweke (2001) also applies to the general family of α -stable distributions with arbitrary skewness β , the sub-family of these distributions with maximal negative skewness $\beta = -1$ provides an operational theory of choice under uncertainty.

In recent work, Carr and Wu (2003) use this sub-family of α -stable distributions with maximal negative skewness for capturing the observed behavior of the volatility smirk implied by S&P 500 option prices. They too are forced to work with these distributions by imposing $\beta = -1$ in order to ensure finiteness of call option values.

The α -stable distribution is also convenient in that it allows us to derive a closed-form expression for the price-dividend ratio. One may wonder, however, whether our results may be induced by the special behavior of the α -stable distribution or whether alternative fat-tailed innovations have the ability to yield similar outcomes. To investigate this issue we also apply the filter after having estimated the model with two alternative fat-tailed distributions, namely the Extreme Value distribution and the Pearson Type IV distribution. We are able to show that in both cases the fluctuation in agents' confidence is achieved, just as in the case of the α -stable distribution.

One may also wonder whether the results are simply due to the non-Gaussian nature of the error terms, in which case a thin-tail distribution would also yield similar outcomes. In order to address this concern, we apply the filter after having this time estimated a model whose α -stable shocks have been replaced by Binomially distributed innovations. The filtration reveals that such thin-tail distributed shocks are not capable of yielding fluctuation in the agent's confidence. We thus conclude that the results obtained with the α -stable distributions are not simply due to the non-Gaussian nature of the shocks but are indeed the consequence of the fat-tail nature of the distribution.

In this paper we study the asset pricing problem with incomplete information in a purely continuous state stochastic setting. We assume that the observed dividend growth rate is the sum of an unobservable persistent component and noise. The unobservable persistent component is assumed to be an autoregressive process driven by shocks that come from the family of α -stable distributions with maximal negative skewness. An incomplete information Gaussian asset pricing model is a special case. Our model with α -stable shocks allows for a simple way to numerically solve for equilibrium asset prices, and hence implied returns, a convenience generally not available under alternative non-Gaussian distributional assumptions. The solution is a simple extension of the solution to the asset pricing problem in complete information setting with α -stable shocks studied in Bidarkota and McCulloch (2003).

We characterize the solution to the asset pricing model in such a setting. We then calibrate the model to data on quarterly US per capita consumption, and study the ability of the model to replicate volatility persistence and other stylized facts in implied returns. Our model is in fact able to generate volatility persistence of a magnitude close to that in

stock returns data. It is important to note that our model does not rely on discrete shifts in the drift rate of dividend growth rates in order to generate volatility persistence, as much of the extant literature on asset pricing with incomplete information does.

Alternative approaches for endogenously generating time-varying volatility in asset pricing models include an early idea due to French and Roll (1986) that it takes time for market participants to digest newly arriving information and react to it. Using this idea, and the assumption that agents face capacity constraints in information processing, as in the rational inattention scheme of Sims (2003), Peng and Xiong (2001) endogenously generate time-varying volatility in an asset pricing model. Sims (2003) argues that outcomes resulting from information processing constraints would resemble those from signal extraction problems. In a different vein, McQueen and Vorkink (2004) develop a model of asset pricing based on prospect theory (Barberis et al. 2001) that generates volatility persistence as investors' level of risk aversion changes when their portfolio performance differs from a mental scorecard they use.

The paper is organized as follows. We describe the economic environment and the asset pricing model in section 2. We study the solution to the model in section 3. We tackle empirical issues including estimation of the model in section 4. We analyze the model implied rates of return in section 5. The last section provides some conclusions derived from the paper.

2. THE ASSET PRICING MODEL

In this section we lay out the economic environment, including specification of exogenous stochastic processes and information structure in the model.

2.1 Pure Exchange Economy

In a single good Lucas (1978) economy, with a representative utility-maximizing agent and a single asset that pays exogenous dividends of non-storable consumption goods, the first-order Euler condition is:

$$P_t U'(C_t) = \theta E_t U'(C_{t+1}) [P_{t+1} + D_{t+1}]. \quad (1)$$

Here, P_t is the real price of the single asset in terms of the consumption good, $U'(C)$ is the marginal utility of consumption C for the representative agent, θ is a constant subjective discount factor, D is the dividend from the single productive unit, and E_t is the mathematical expectation, conditioned on information available at time t .

Assume a constant relative risk aversion (CRRA) utility function with risk-aversion coefficient γ :

$$U(C) = (1 - \gamma)^{-1} C^{(1-\gamma)}, \quad \gamma \geq 0. \quad (2)$$

Since consumption simply equals dividends in this simple model, i.e. $C = D$ every period,

Equation (1) reduces to:

$$P_t D_t^{-\gamma} = E_t \theta D_{t+1}^{-\gamma} [P_{t+1} + D_{t+1}]. \quad (3)$$

On rearranging this yields:

$$P_t = E_t \theta \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} [P_{t+1} + D_{t+1}]. \quad (4)$$

Let v_t denote the price-dividend ratio, i.e. $v_t = P_t / D_t$. Then, we can rewrite Equation (4) in terms of v_t as:

$$v_t = E_t \theta \left(\frac{D_{t+1}}{D_t} \right)^{1-\gamma} [v_{t+1} + 1]. \quad (5)$$

Thus, this equation implicitly defines the solution to the asset pricing problem in this model. One specifies an exogenous stochastic process for dividends and solves for the price dividend ratio v_t .

2.2 Simplifying the Difference Equation for the P/D Ratios

Let $x_t = \ln(D_t / D_{t-1})$ denote the natural logarithm of the dividend growth rate.

Then, we can express Equation (5) as:

$$v_t = E_t \theta \exp[(1 - \gamma)x_{t+1}] (v_{t+1} + 1). \quad (6)$$

Defining $m_{t+1} \equiv \theta \exp[(1 - \gamma)x_{t+1}]$, we can rewrite Equation (6) as:

$$v_t = E_t m_{t+1} [v_{t+1} + 1]. \quad (7)$$

On forward iteration, this equation yields:

$$v_t = \sum_{i=1}^{\infty} \left(E_t \prod_{j=1}^i m_{t+j} \right) + \lim_{i \rightarrow \infty} E_t \prod_{j=1}^i m_{t+j} v_{t+i} \quad (8)$$

One solution to the above difference equation in v_t is obtained by imposing the transversality condition:

$$\lim_{i \rightarrow \infty} \left(E_t \prod_{j=1}^i m_{t+j} v_{t+i} \right) = 0. \quad (9)$$

This condition rules out solutions to the asset pricing model that imply intrinsic bubbles (Froot and Obstfeld 1991). Imposing the transversality condition on Equation (8) gives:

$$v_t = \sum_{i=1}^{\infty} \left(E_t \prod_{j=1}^i m_{t+j} \right). \quad (10)$$

Thus, the solution to the price-dividend ratio can be found by evaluating the conditional expectations on the right hand side of Equation (10), under a specified exogenous stochastic process for the dividend growth rates.

2.3 Specification of the Endowment Process

We assume that dividend growth rates stochastically evolve according to the following process:

$$x_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma^2) \quad (11a)$$

$$\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t, \quad 0 \leq \rho < 1, \quad \eta_t \sim iid S(\alpha, \beta, c, 0). \quad (11b)$$

We assume that ε_t and η_t are independent of each other contemporaneously as well as at all leads and lags. Here, $S(\alpha, \beta, c, 0)$ represents a stable distribution with characteristic exponent α , skewness parameter β , scale parameter c , and location parameter set to 0. Appendix A defines these distributions and lists some of their properties.

For technical reasons that will be evident in section 3, we need the autoregressive coefficient ρ in Equation (11b) to be non-negative. This is not a limitation of the model from an empirical viewpoint since, as we shall see in subsection 4.3, ρ is estimated to be positive and large.

We also study a benchmark case where, in Equation (11b), $\eta_t \sim iid N(0, \sigma_\eta^2)$.¹

From Appendix A, this is obtained by setting $\alpha = 2$ in the process given in Equation (11b)

¹ The model in Equations (11) has a reduced form ARMA(1,1) representation. Bansal and Yaron (2004) study such a model with conditionally heteroskedastic Gaussian errors with

above. In this case, β loses its effect on the distribution of η_t and is unidentified, and

$$\sigma_{\eta}^2 = 2c^2.$$

2.4 Incomplete Information Structure of the Economy

We assume that agents in the economy have full knowledge about the structure of the economy. They know the stochastic process governing the evolution of the dividend growth rates, including the parameters of the process. They observe the dividend stream (and hence the realized dividend growth rates x_t as well). However, we assume that agents do not ever observe the persistent component μ_t (or equivalently the noise component ε_t) of the dividend growth rates.

Agents need to form conditional expectations of μ_t in order to compute the expected future dividend payouts, and hence determine equilibrium prices. Thus, agents face a filtering/signal extraction problem. We assume that agents form conditional expectations on μ_t based on Bayesian updating rules. The signal extraction problem facing the agents is complicated here by the assumption of a non-Gaussian distribution for η_t .

In the benchmark case where η_t is Gaussian, agents face a linear Gaussian signal extraction problem. In this case, the conditional density of μ_t is Gaussian (see, for non-expected recursive utility specification of Epstein and Zin (1989). Note that in their setup, unlike ours as will become evident in the next subsection, there is complete information.

instance, Harvey 1992, Ch.3) and, therefore, completely specified by its conditional mean and variance. These are given recursively by the classic Kalman filter.

When η_t is non-Gaussian, the Kalman filter is still optimal (in a minimum mean squared error sense) but only within the class of linear estimators. The globally optimal filter turns out to be non-linear in this instance. Bayesian updating still leads to a recursive form for the conditional probability density $p(\mu_t | x_1, x_2, \dots, x_t)$.

2.5 Benchmark Case - Complete Information

In a benchmark full information economy, we assume that the innovation ε_t in Equation (11a) has zero variance (i.e. ε_t is trivially zero). In this case, $\mu_t = x_t$, and therefore agents actually observe μ_t . There is no signal extraction problem facing the agents in such an economy. This model is studied in Bidarkota and McCulloch (2003). A Gaussian version of such a full information model where $\eta_t \sim \text{iid } N(0, \sigma_\eta^2)$ is studied in Burnside (1998).

3. SOLUTION TO THE MODEL

We now proceed to evaluate Equation (10) for the price-dividend ratio under the assumed process for the dividend growth rates. The expressions for the price-dividend ratio v_t and its mean value $E(v_t)$ derived below, as well as those for returns and their mean values discussed in section 5, differ in the case when the characteristic exponent

$\alpha = 1$ from those when $\alpha \neq 1$.² In the rest of this paper we focus our attention on the more general case $\alpha \neq 1$. All the results and theorems that follow for $\alpha \neq 1$ are also applicable for $\alpha = 1$ with appropriate modifications. The required derivations and proofs of theorems for $\alpha = 1$ do not pose any additional difficulties, and can be easily adapted from those given for $\alpha \neq 1$ in this paper.

3.1 Finiteness of Conditional Expectations

As noted in the introduction, theory of choice under uncertainty with CRRA utility and fat-tailed distributions is extremely fragile (Geweke 2001). The difficulty stems from the non-existence of the conditional expectation in Equation (10) for most common leptokurtic distributions, including the Student- t . Under the assumed distribution $\eta_t \sim iid S(\alpha, \beta, c, 0)$ in the dividend growth rate process in Equation (11b), Appendix B shows that the conditional expectation in Equation (10) above is finite only when $\beta \text{sign}(1 - \gamma) = -1$. This condition is satisfied if either (i) $\beta = -1$ and $\gamma < 1$ or (ii) $\beta = +1$ and $\gamma > 1$. Thus, an operational theory of choice under uncertainty can be worked out under these conditions, despite the difficulty noted by Geweke(2001).

² This arises because of two reasons. One reason is that the expressions for Ee^x differ in the two cases (see Equation (A8) in Appendix A). A second reason is that when we aggregate iid random variables with stable distributions, the expressions for the location parameter δ for the aggregate random variable also differ in the two cases (see Equation (A7) in Appendix A).

One can in principle obtain an operational theory of choice under uncertainty without having to impose the condition $\beta \text{sign}(1-\gamma) = -1$ by truncating the α -stable distributions, as in the Dampened Power Law process of Wu (2005). However, in that case, one loses the convenience of solving the asset pricing model in a simple way as will become evident in the next subsection. Moreover, in our view, imposing $\beta \text{sign}(1-\gamma) = -1$ for the purpose of demonstrating that signal extraction in leptokurtic non-Gaussian settings would generate volatility persistence without the need for discrete breaks in dividend growth rates is not necessarily severe.

In the remainder of the paper, we assume that $\beta = -1$ and $\gamma < 1$. We shall see in the empirical section of the paper that this choice for β is consistent with negative skewness in dividends growth data. Carr and Wu (2003) refer to the $\beta = -1$ case as the finite moment log-stable process and use it for option pricing. The reason for this terminology is as follows. Briefly, Appendix B (Equation (B5)) shows that the conditional expectations in Equation (10) involve terms such as $E_t \{ \exp(\kappa \eta_{t+1}) \}$ for some constant κ . Such exponential moments do not exist for stable distributions with $\alpha < 2$ because of fat tails, except when $\beta = -1$. Equation (A8) in Appendix A provides the exact expression for such moments in this case.

3.2 Solution for the P/D Ratios

We now proceed to evaluate Equation (10) for the price-dividend ratio under the assumed process for dividend growth rates given in Equations (11). Under the

assumptions $\beta = -1$ and $\gamma < 1$, one can derive a tractable expression for the price-dividend ratio v_t . Appendix C shows that v_t in Equation (10) can be reduced to:

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot [E_t \exp\{b_i(\mu_t - \bar{\mu})\}] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\frac{1-\gamma}{1-\rho} \right]^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (12)$$

where $b_i = (1-\gamma) \left(\frac{\rho}{1-\rho} \right) (1-\rho^i)$.

It is not possible to evaluate the conditional expectation term in the above equation analytically under the assumed process for μ_t in Equation (11b).

3.3 Convergence of the P/D Ratios

The following theorem provides conditions for the infinite series in Equation (12) to converge, and hence for the price-dividend ratio to be finite.

Theorem 1. The series in Equation (12) converges if

$$r \equiv \theta \exp \left[(1-\gamma)\bar{\mu} + (1-\gamma)^2 \frac{\sigma^2}{2} + \left\{ - \left[\frac{1-\gamma}{1-\rho} \right]^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right\} \right] < 1. \quad (13)$$

Proof. See Appendix D.

Finiteness of the price-dividend ratio ensures that the expected discounted utility is finite in this model (see Burnside 1998). The next theorem derives an expression for the mean of the price-dividend ratio, i.e. the unconditional expectation of v_t in Equation (12).

It also provides conditions under which this mean is finite.

Theorem 2. The mean of the price dividend ratio is given by:

$$E(v_t) = \sum_{i=1}^{\infty} \theta^i \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \left\{ - \left[|b_i|^\alpha \cdot \frac{c^\alpha}{1-\rho^\alpha} \right] \sec\left(\frac{\pi\alpha}{2}\right) \right\} + \\ \left\{ - \left[\left| \frac{1-\gamma}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (14)$$

It is finite if $r < 1$, where r is the constant defined in Theorem 1.

Proof. See Appendix E.

3.4 Solution under Gaussian Distribution for η_t

In the benchmark case when $\eta_t \sim \text{iid } N(0, \sigma_\eta^2)$, we can obtain all the results derived in the previous subsections simply by setting $\alpha = 2$. In this case, the skewness parameter of the stable distributions β loses its significance (see Appendix A). We no longer need the restriction $\beta \text{sign}(1-\gamma) = -1$ in order to ensure finiteness of the conditional expectations term in Equation (10), discussed in subsection 3.1 for the stable case, and hence of the price-dividend ratio itself and its mean value in Equations (12) and (14), respectively.

As discussed in subsection 2.4, in the Gaussian case, the conditional density of μ_t is Gaussian, and its conditional mean and variance are given by the Kalman recursions. In this case, the conditional expectations term $E_t \exp\{b_i(\mu_t - \bar{\mu})\}$ appearing in the formula

for the price-dividend ratio given in Equation (12) can be evaluated using the formula for the moment generating function of Gaussian random variables.³

3.5 Solution under Complete Information

In the complete information benchmark case, recall from subsection 2.5 that $\mu_t = x_t$, which is observed at time t . All the analysis of section 3 goes through exactly as in the incomplete information case, with some simplifications detailed below. The condition for finiteness of conditional expectations remains unchanged in subsection 3.1. The expression for the price-dividend ratio given in Equation (12) remains the same but with $E_t \exp\{b_i(\mu_t - \bar{\mu})\} = \exp\{b_i(x_t - \bar{\mu})\}$ and $\sigma^2 = 0$. Theorem 1 goes through as before with $\sigma^2 = 0$ imposed on r defined by Inequality (13). The mean of the price-dividend ratio given in Equation (14) remains the same but with $E_t \exp\{b_i(\mu_t - \bar{\mu})\} = \exp\{b_i(x_t - \bar{\mu})\}$ and $\sigma^2 = 0$. The condition for its finiteness given by Theorem 2 remains unchanged but with $\sigma^2 = 0$ imposed on r defined by Inequality (13).

The price-dividend ratio and its related properties in the benchmark complete information model with stable distribution for η_t are derived in Bidarkota and McCulloch (2003) and with Gaussian distribution for η_t in Burnside (1998).

³ If $x \sim N(\mu, \sigma^2)$, then $E\{\exp(x)\} = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$.

4. EMPIRICAL ESTIMATION OF THE MODEL

In this section we report the data used to calibrate the theoretical model of section 2, discuss maximum likelihood estimation of the dividend growth rates process given in Equations (11), and report estimates of the parameters obtained.

4.1 Data Issues

We calibrate the asset pricing model to quarterly real per capita US consumption growth rates on non-durables and services from 1952:1 through 2004:2. Nominal seasonally adjusted per capita consumption data obtained from NIPA tables are deflated using the CPI index. Summary statistics indicate an annualized mean growth rate of 2.02 percent and a standard deviation of 1.34 percent. Skewness is estimated to be -0.40 and statistically different from 0 at the 1 percent level, indicating significant negative skewness. This provides justification for our choice of a negative value for β in subsection 3.1. Kurtosis is estimated to be 4.29 and statistically different from 3 at the 1 percent level. This provides preliminary justification for our use of fat tailed distribution in the specification of Equation (11b). The Jarque-Bera test easily rejects normality at better than the 1 percent level. The first order autocorrelation coefficient is 0.18 and statistically different from 0 at the 1 percent level. This provides preliminary empirical justification for our restriction $\rho \geq 0$ in the specification of Equation (11b).

4.2 Maximum Likelihood Estimation

The dividend growth rates process in Equations (11) constitutes a linear non-Gaussian state space model when η_t has the stable distribution. Equation (11a) is the

observation equation and Equation (11b) is the state transition equation. The non-Gaussian nature of the model renders the Kalman filter suboptimal. Recursive formulae for obtaining the conditional densities of the state variable μ_t , as well as the likelihood function, are available from an algorithm by Sorenson and Alspach (1971).

Let X_t denote the history of dividend growth rates observed at time t , i.e. $X_t \equiv \{x_1, \dots, x_t\}$. The recursive formulae for obtaining one-step ahead predictive and filtering densities, due to Sorenson and Alspach (1971), are as follows:

$$p(\mu_t | X_{t-1}) = \int_{-\infty}^{\infty} p(\mu_t | \mu_{t-1})p(\mu_{t-1} | X_{t-1})d\mu_{t-1}, \quad (15)$$

$$p(\mu_t | X_t) = p(x_t | \mu_t)p(\mu_t | X_{t-1})/p(x_t | X_{t-1}), \quad (16)$$

$$p(x_t | X_{t-1}) = \int_{-\infty}^{\infty} p(x_t | \mu_t)p(\mu_t | X_{t-1})d\mu_t. \quad (17)$$

Finally, the log-likelihood function is given by:

$$\log p(x_1, \dots, x_T) = \sum_{t=1}^T \log p(x_t | X_{t-1}). \quad (18)$$

These formulae have been applied to non-Gaussian data and extended to include a smoother formula by Kitagawa (1987).

In the Gaussian case, these integrals can be evaluated analytically and they collapse to the Kalman recursions. In most other circumstances, the integrals cannot be evaluated in closed form and one has to resort to numerical integration based either on quadrature techniques (Kitagawa 1987) or Monte Carlo methods (Durbin and Koopman 2000).

In this paper we evaluate the integrals using quadrature methods. Details on the numerical method employed and accuracy achieved are detailed in Appendix G.

The probability density for stable distributions is obtained by Fourier inversion of their characteristic function available as an exact analytical formula (Equations A.2 and A.3 in Appendix A) using the Fast Fourier Transform (FFT) methods discussed in Mittnik et al. (1999).

4.3 Parameter Estimates

Maximum likelihood parameter estimates of the consumption growth rate process (conditional on the first observation) in Equations (11) are reported in Table 1 (Panel A). Three restricted versions of this most general model, namely the incomplete information Gaussian model, the complete information stable model, and the complete information Gaussian model are reported in Panels B through D, respectively.

Parameter estimates from Panel A indicate a mean consumption growth rate of 0.49 percent per quarter, or 1.96 percent per annum. The autoregressive (AR) parameter ρ is estimated to be 0.69, somewhat lower than the value of 0.89 reported in Veronesi (2004) with an autoregressive model with asymmetric jumps. Nonetheless, it is statistically significantly different from 0 by the usual t-test at better than the 1 percent significance level. The signal-to-noise scale ratio c/c_ε (which equals $\sqrt{2}c/\sigma_\varepsilon$) is estimated to be 0.34. Parameter estimates for the incomplete information Gaussian model in Panel B are very similar, and the maximized log-likelihood value only drops slightly in this case.

Figure 1 plots the unconditional distribution of μ_t for both the stable and Gaussian models implied by Equation (11b), using the Maximum Likelihood parameter estimates reported above. With the stable index α estimated to be 1.86, so close to the value of 2 for a Gaussian distribution, the differences in the unconditional densities for the stable and Gaussian models are modest. The assumption $\beta = -1$ does not generate much skewness in the unconditional distribution for this high value of α , as evident in the figure, and thus does not seem overly restrictive.

Figure 2 plots the conditional probability densities $p(\mu_t | x_1, x_2, \dots, x_t)$. Panel A plots the densities for the stable case and Panel B for the Gaussian case. A closer examination of the density plots indicate that the conditional densities in the stable case display varied behavior, being at times even multimodal. Bidarkota and McCulloch (1998) and Bidarkota (2003) provide detailed examination of such densities in non-Gaussian state space models with stable errors fit to inflation data. On the other hand, as noted earlier, the conditional densities in the Gaussian case are Gaussian.

Figure 3 plots the mean of the filter densities $E(\mu_t | x_1, x_2, \dots, x_t)$, along with the observed consumption growth rates x_t , in Panels A and B for the stable and Gaussian incomplete information models, respectively.

Figure 4 plots the standard deviation of the filter densities for both the stable and Gaussian incomplete information models. It is clear from the figure that the variance of the filter density in the Gaussian case quickly reaches a constant value (within 10 time periods). This property of the Kalman filter was discussed in the introduction. On the other hand, the variance of the filter density in the stable case never stabilizes to a constant

value but is forever fluctuating. David (1997) refers to this as ‘fluctuating confidence’ in the investors’ estimate of the unobservable component of the dividend growth rate. This fluctuating confidence drives the time varying characteristics of returns implied by our asset pricing model, as will become evident in an analysis of the conditional moments of these returns that we undertake in the next section. The spike in the standard deviation at startup in both the stable and Gaussian models is caused by the initialization of the filter discussed in Appendix G. It neither impacts the Maximum Likelihood parameter estimates nor their standard errors much. Its possible effect on the analyses of model implications on implied returns is eliminated as will become evident in the next section.

The complete information stable model parameter estimates are reported in Panel C of Table 1. These estimates indicate a slightly higher value for α when compared to the estimates for the incomplete information stable model. However, the AR coefficient ρ is now only 0.13, as against 0.69 for the incomplete information model. This is understandable, however, because the AR process for μ_t in Equation (11b) is now combined with the iid process for ε_t in Equation (11a), and effectively an AR model is being estimated for the resulting contaminated (with iid noise) series. Nonetheless, the AR coefficient is statistically significantly different from 0 by the usual t-test at better than the 1 percent significance level.

The complete information Gaussian model parameter estimates are reported in Panel D of Table 1. These estimates change only slightly from those in Panel C. However, the maximized log-likelihood shows a large drop. The likelihood ratio (LR) test statistic

for normality (test for $\alpha = 2$) is calculated to be 4.11 and rejected.⁴ Thus, with complete information models, there is significant statistical support for stable shocks. The LR test statistic for complete information versus incomplete information model in the Gaussian case turns out to be 3.76, with a χ_1^2 p-value of 0.05. Thus, with Gaussian shocks, there is significant statistical support for the incomplete information model.

As mentioned in the introduction, one may wonder whether the fluctuations in agents' confidence may be specific to the α -stable distribution only or whether is indeed more general and thus robust to alternative fat-tailed specifications. To answer this question, we estimate and apply a filter to the model where the error terms display two additional fat-tail distributions, namely the Extreme Value distribution and the Pearson Type IV distribution.

In the Extreme Value distribution case the dividend growth rates evolve stochastically as:

$$x_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma^2)$$

$$\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t, \quad 0 \leq \rho < 1, \quad \eta_t \sim EV(\mu_{EV}, \sigma_{EV}). \quad (19)$$

where μ_{EV} and σ_{EV} are the location and scale parameters of the Extreme Value density function.

⁴ The distribution of the LR test statistic in this instance is not standard χ_1^2 because the null hypothesis lies on the admissible boundary of α . The 0.05 level critical value for such a test, available through Monte Carlo simulations from McCulloch (1997, table 4, panel b), equals 1.12.

In the Pearson Type IV distribution case the dividend growth rates evolve stochastically as:

$$x_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma^2)$$

$$\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t, \quad 0 \leq \rho < 1, \quad \eta_t \sim PearsonIV(\mu_p, m, a, \delta) \quad (20)$$

In the interest of table space, we report the results here in the text. The Extreme Value model yields parameter estimates of $\bar{\mu} = 0.006066$, $\sigma = 0.005863$, $\rho = 0.726370$, $\mu_{EV} = 0.000660$ and $\sigma_{EV} = 0.001664$ for a log-likelihood of 755.6039. The Pearson Type IV model yields parameter estimates of $\bar{\mu} = 0.003063$, $\sigma = 0.005711$, $\rho = 0.701580$, $\mu_p = 0.000846$, $m = 1.642600$, $a = 0.001809$ and $\delta = -0.20715$ for a log-likelihood of 756.3028. The standard deviations of the filtered densities are plotted in figure 5 and figure 6, indicating that even though the results do differ slightly in magnitude from the Stable case, these alternative fat-tail distributions are also capable of generating fluctuating confidence among the agents.

We also mentioned in the introduction that one may wonder whether these results are driven by the non-Gaussian nature of the shocks. This would imply that even thin-tailed distributions (thinner than Gaussian tails) could generate fluctuating confidence. To investigate this issue, we estimate and apply a filter to the model where the error terms are driven by a thin-tailed distribution, namely the Binomial distribution.

In the Binomial distribution case we loosely describe the dividend growth rates as evolving stochastically according to:

$$x_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim iid N(0, \sigma^2)$$

$$\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t, \quad 0 \leq \rho < 1, \quad \eta_t + |\min(range(\eta_t))| \sim Binom(N, P) \quad (21)$$

This abuse of notation is the result of the fact that the Binomial distribution is only valid for positive variables. We must therefore shift the empirically selected range for the innovations by the amount of its smallest negative value before applying the Binomial density. The Binomial distribution being a discrete one, we estimate the parameter P by matching the parameter N with the number of elements in the possible range of values for the shocks in a given periods. The Binomial model yields parameter estimates of $\bar{\mu} = 0.007633$, $\sigma = 0.006645$, $\rho = 0.601280$ and $P = 0.504300$ for a log-likelihood value of 750.6556. The standard deviations of the filtered densities are plotted in figure 7. We can see that, in a fashion very similar to that of the Gaussian case, the standard deviations quickly drop to a steady level, indicating that a thin-tail distribution does not seem to be capable of generating fluctuating confidence among agents. It thus appears that our previous results are not the mere effect of non-normality but indeed the work of fat tails in the distribution of the shocks.

5. ANALYSIS OF MODEL IMPLICATIONS

In this section we discuss the implications of the theoretical model of section 2 for rates of return on risky and risk free assets, set up a simulation framework for analyses of unconditional and conditional properties of model implied rates of returns, and report on the results obtained.

5.1 Model-Implied Rates of Return

Equilibrium gross equity returns R_t^e on assets held from period t through period $t+1$ are given by:

$$R_t^e = \left(\frac{P_{t+1} + D_{t+1}}{P_t} \right). \quad (22)$$

Using $v_t = P_t / D_t$ and $x_t = \ln(D_t / D_{t-1})$, this reduces to:

$$R_t^e = \left(\frac{1 + v_{t+1}}{v_t} \right) \exp[x_{t+1}]. \quad (23)$$

It is not possible to analytically evaluate the population mean of the implied equity returns, i.e. $E(R_t^e)$, in our model given the expression for v_t in Equation (12).

The price of a risk free asset P_t^f in our endowment economy guarantees one unit of the consumption good on maturity. It is given by:

$$P_t^f = \theta E_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \right). \quad (24)$$

Gross equilibrium returns on the risk free asset R_t^f are given by:

$$R_t^f = \frac{1}{P_t^f}. \quad (25)$$

Our assumption $\beta = -1$ implies that the price of the risk free asset P_t^f is infinite and hence the gross risk free returns R_t^f are zero. Appendix G has the formal proof.

Presumably, in the highly uncertain environment for the dividends process (due to fat tails on η_t), the uncertainty is so overwhelmingly unfavorable (due to negative skewness implied by $\beta = -1$) that as long as investors are risk averse, they are willing to

pay an infinite amount to guarantee themselves strictly positive consumption next period (dictated by the condition $\lim_{C \rightarrow 0} U'(C) = \infty$ for the CRRA utility function). Infinite prices for risk free assets simply mean that these assets cannot exist in the economy under the assumed stochastic process for dividends.

One can get around the difficulty of infinite risk free asset prices by truncating the α -stable distributions in Equation (11b), as in the Dampened Power Law process in Wu (2005). As noted in section 3.1, in this case, we do not need to restrict ourselves to $\beta = -1$ and $\gamma < 1$ anymore. Although truncation may be appealing, we lose the convenience of solving the asset pricing model in a simple way as described in section 3. In any case, our main objective in this paper is to demonstrate that signal extraction in fat-tailed non-Gaussian continuous-valued stochastic setting can generate volatility persistence in implied returns. Therefore, in what follows, we restrict ourselves to an analysis of implied returns on risky assets in order to ascertain the ability of the model to generate stylized facts documented for observed data.

5.2 Simulation Setup

We undertake a simulation study in order to analyze the model implications for various endogenous quantities of interest including rates of return. The simulations are performed in the following manner. We draw random numbers for ε_t and η_t in Equations (11) using parameter estimates reported in Table 1. The value of μ_0 is set to the unconditional mean of μ_t , equal to $\bar{\mu}$. We then use the simulated η_t series to generate a sequence $\{\mu_t, t = 1, 2, \dots, T\}$ using Equation (11b) with $T = 4000$. We use this sequence

and the simulated ε_t series to generate a sequence of artificial dividend growth rates $\{x_t, t = 1, 2, \dots, T\}$ according to Equation (11a).

We use the simulated sequence $\{x_t\}$ and the parameter estimates from Table 1 to obtain the posterior density $p(\mu_t | X_t)$ using the filtering Equations (15)-(17). We use this posterior density to numerically evaluate the conditional expectations terms, and hence the price-dividend ratios v_t , in Equation (12). Calculations are done for various values for the preference parameters θ (discount factor) and $\gamma < 1$ (risk aversion coefficient) that satisfy the convergence condition $r < 1$ in Equation (13). Model-implied returns on risky assets are then generated using Equation (23).

In order to eliminate any effects from startup of the Kalman filter, as in Figure 4, we drop the first ten implied returns. The following two subsections undertake analyses of the unconditional and conditional moments of the resulting implied returns series.

5.3 Analysis of Unconditional Moments

Table 2, Panel A reports unconditional moments of quarterly value-weighted real returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period 1952:1 through 2004:2. We subtract CPI inflation from nominal returns to obtain real returns, expressed in percent per annum.

Panel A indicates that quarterly real returns have a mean of 8.07 percent per annum and a standard deviation of 16.77. Panels B through E report the unconditional moments for returns implied by our theoretical model of section 2 using the simulation

setup from subsection 5.2. Moments are reported for various values of the discount factor θ and the risk aversion coefficient γ .

The maximum implied mean returns from our incomplete information stable model are only 3.90 percent and the maximum standard deviation is only 1.75 percent. Overall, it is clear from looking at all the panels that none of the models do a good job of replicating the unconditional moments of equity returns. This is simply a manifestation of the equity premium puzzle of Mehra and Prescott (1985).

It is clear from an examination of all the panels that adding incomplete information to the asset pricing model reduces the mean and raises the standard deviation of implied equity returns slightly. Generalizing from Gaussian to stable models raises the mean implied equity returns slightly in both incomplete and complete information cases. Going from Gaussian to stable models lowers the standard deviation of implied equity returns in the incomplete information case but raises it in the complete information case.

The benchmark complete information asset pricing model, with stable and Gaussian dividend growth rate processes, was studied in Bidarkota and McCulloch (2003). Panels D and E of Table 2 here replicate results reported in that study closely, although the model in that study was calibrated to a random walk process with drift, fit to annual US consumption data for the period 1890 through 1987. Our results in Table 2 thus indicate that adding incomplete information to the non-Gaussian framework of that study does not generate high enough mean equity returns to conform closely enough to the numbers in the data.

5.4 Analysis of Conditional Moments

The main assertion in our paper is that our theoretical model of section 2 with incomplete information stable shocks can replicate the stylized fact of time varying volatility documented in observed returns. We also contend that this stylized fact can neither be replicated by the incomplete information Gaussian model nor the two versions of the complete information model. We now proceed to test this assertion in the simulation setup of subsection 5.2 for $\theta=0.98$ and $\gamma=0.9$, since for these preference parameter values the unconditional mean stock returns implied by the incomplete information stable model are closest to their sample counterpart.

Let $r_t \equiv R_t^e - 1$ denote the net rates of return on risky assets, where R_t^e is the gross rate of return as in subsection 5.1. We set up the following model for analyzing net returns:

$$r_t = a_0 + \varepsilon_t, \quad \varepsilon_t \sim \sigma_t z_t, \quad z_t \sim \text{iid } N(0,1) \quad (26a)$$

The volatility of returns σ_t is modeled alternatively as a GARCH(1,1) and an asymmetric GARCH(1,1) process (AGARCH) as follows:

$$\text{GARCH}(1,1) : \sigma_t^2 = a_1 + a_2 \sigma_{t-1}^2 + a_3 |r_{t-1} - a_0|^2 \quad (26b)$$

$$\text{AGARCH}(1,1) : \sigma_t^2 = a_1 + a_2 \sigma_{t-1}^2 + a_3 |r_{t-1} - a_0|^2 + a_4 I_{t-1} |(r_{t-1} - a_0) / \sigma_{t-1}|^2. \quad (26c)$$

$$\text{where } I_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - a_0 < 0 \\ 0 & \text{otherwise} \end{cases}.$$

We restrict $a_1 > 0$, $a_2 \geq 0$, $a_3 \geq 0$, and $a_4 \geq 0$. The AGARCH(1,1) process (asymmetric GARCH) allows for leverage effects, captured by the threshold term involving the dummy variable I_{t-1} . Leverage effects, widely reported in the literature

documenting time varying volatility of stock returns, indicate that negative shocks to returns have greater effect on future volatility than do positive shocks of equal magnitude.

Table 3, Panel A reports estimates obtained by fitting the above volatility processes to quarterly value-weighted real returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period 1952:1 through 2004:2. We subtract CPI inflation from nominal returns to obtain real returns, expressed in percent per annum for estimation. Panel A indicates that, with estimates for the AGARCH process, quarterly real returns have a volatility persistence parameter a_2 of 0.44. It is well known in the literature that quarterly returns exhibit lower volatility persistence compared to higher frequency returns, such as monthly or weekly returns. The ARCH parameter a_3 is estimated to be 0.00. The leverage parameter a_4 is estimated to be 374.78.

An LR test for homoskedasticity (test for $a_2 = a_3 = a_4 = 0$) rejects easily using the χ^2_3 distribution (p-value less than 0.01). LR test for GARCH versus the AGARCH model (test for $a_4 = 0$) also rejects easily using the χ^2_1 distribution (p-value less than 0.05), indicating strong statistical significance of the leverage effect. Strictly speaking, the LR test does not have the standard χ^2 distribution for these tests because the null hypotheses lie on the boundary of admissible values for a_2 , a_3 , and a_4 (see, also, footnote 4). Andrews (2001) provides recent theoretical advances in this regard. Monte Carlo critical values for these tests generated using 1000 replications indicated even stronger rejection of the two hypotheses. We go on to examine whether our asset pricing model can replicate these stylized features of stock returns volatility.

Panels B through E of Table 3 report Maximum Likelihood estimates obtained by fitting the above volatility processes to implied returns discussed in subsection 5.3, obtained by simulation of all the four models in subsection 5.2. Estimates of the AGARCH process fit to implied returns from the incomplete information stable model indicate a volatility persistence coefficient of 0.33, an ARCH coefficient of 0.01, and a leverage parameter of 0.40. The low value for the leverage parameter as compared to that for the data is simply a reflection of the fact that our implied returns do not match the unconditional variance of returns in the data very well, as evident from the discussion of unconditional moments in subsection 5.3. LR test for homoskedasticity is strongly rejected in favor of time-varying volatility, and leverage effects are also strongly statistically significant.

Our choice of negative skewness in consumption growth rates in subsection 3.1 leads to a greater probability of large negative shocks than large positive shocks. This feature of our framework is directly responsible for generating the apparent leverage effects. Thus, our asset pricing model is able to replicate volatility persistence and leverage effects with incomplete information and stable shocks. An examination of results reported in the other panels indicates that implied returns from all other models fail to generate statistically significant volatility persistence.

Implied returns from our incomplete (and complete) information stable model are unlikely to be Gaussian. Therefore, the models for net returns in Equations (26) are likely to be misspecified, at least in the stable cases. To account for this, we also estimated versions of the GARCH(1,1) and AGARCH(1,1) models, with the scaled innovation z_t in Equation (26a) distributed as iid $S(\alpha', \beta', 1, 0)$. The volatility process in Equations (26b)

and (26c) is formulated in this case in terms of time-varying scale parameter c_t using α -powers instead of squares, as in Bidarkota and McCulloch (2004). Maximum Likelihood estimation of this model, and a subsequent LR test for $\alpha = 2$ easily rejects as suspected, indicating statistically significant non-normality of implied returns. An LR test indicates statistically significant volatility persistence with a larger volatility persistence parameter (a_2 is now estimated to be 0.48 with GARCH-stable as opposed to 0.24 with the GARCH-normal model). However, leverage effects are no longer statistically significant.

6. CONCLUSIONS

We study the consumption based asset pricing model of Lucas (1978), in incomplete information setting with stable shocks driving the exogenous stochastic dividends growth rate process. Although agents observe realized dividends (and hence their growth rates), they do not observe the persistent and noise components that make up the observed dividends. Estimation of the persistent component is important for evaluating conditional expectations of future dividends, used to set equilibrium asset prices. Its unobservability under stable shocks introduces a non-Gaussian filtering/signal extraction problem that agents solve using Bayesian updating schemes. Asset pricing with incomplete information in a Gaussian framework, with the associated filtering problem whose solution is given by the Kalman filter, is a special case. Asset pricing with complete information, in stable and Gaussian settings, is also a special case of our framework.

The non-Gaussian filtering problem leads to a recursive estimate of the persistent component of the dividend growth rate, whose conditional variance always reacts to new

data, and unlike the Kalman filter in the Gaussian setting, never settles to a constant value. This time variation in the conditional variance of the agents' estimate of the persistent component, or fluctuating confidence as in David (1997), leads to time variation in the volatility of implied returns from the model.

We test this implication of our model using quarterly per capita real US consumption data. Our results indicate strong support for our model in the sense that the implied equilibrium returns display statistically significant volatility persistence of a magnitude comparable to that in the data. Our model also replicates leverage effect noted in the time-varying volatility of observed returns, although this result is not robust to changes in the distributional assumptions about implied returns. It is important to note that our model does not rely on discrete shifts in the drift rate of dividend growth rates in order to generate volatility persistence. Neither incomplete information in a Gaussian setting, nor complete information in either Gaussian or stable settings, is able to generate these features in implied returns.

APPENDIX A

Stable Distributions and Their Properties

This section draws heavily from McCulloch (1996). Stable distributions $S(x; \alpha, \beta, c, \delta)$ are determined by four parameters. The location parameter $\delta \in (-\infty, \infty)$ shifts the distribution to the left or right, while the scale parameter $c \in (0, \infty)$ expands or contracts it about δ , so that

$$S(x; \alpha, \beta, c, \delta) = S((x - \delta)/c; \alpha, \beta, 1, 0). \quad (\text{A1})$$

The standard stable distribution function has $c = 1$ and $\delta = 0$. If a random variable X has a stable distribution, it is represented as $X \sim S(\alpha, \beta, c, \delta)$.

The characteristic exponent $\alpha \in (0, 2]$ governs the tail behavior, and therefore the degree of leptokurtosis. When $\alpha = 2$, the normal distribution results, with variance $2c^2$. For $\alpha < 2$, the variance is infinite. When $\alpha > 1$, $E(X) = \delta$; but if $\alpha \leq 1$, the mean is undefined.

The skewness parameter $\beta \in [-1, 1]$ is defined such that $\beta > 0$ indicates positive skewness. If $\beta = 0$, the distribution is symmetric stable. As $\alpha \uparrow 2$, β loses its effect and becomes unidentified.

Stable distributions are defined most concisely in terms of their log-characteristic functions:

$$\ln E \exp(iXt) = i\delta t + \psi_{\alpha, \beta}(ct) \quad (\text{A2})$$

where

$$\psi_{\alpha, \beta}(t) = \begin{cases} -|t|^\alpha (1 - i\beta \text{sign}(t) \tan(\pi\alpha/2)) & \text{for } \alpha \neq 1 \\ -|t| (1 + i\beta(2/\pi) \text{sign}(t) \ln |t|) & \text{for } \alpha = 1 \end{cases} \quad (\text{A3})$$

is the log-characteristic function for $S(\alpha, \beta, 1, 0)$.

When $\alpha < 2$, stable distributions have tails that behave asymptotically like $x^{-\alpha}$ and give the stable distributions infinite absolute population moments of order greater than or equal to α .

Let $X \sim S(\alpha, \beta, c, \delta)$ and a be any real constant. Then (A2) implies:

$$aX \sim S(\alpha, \text{sign}(a)\beta, |a|c, a\delta). \quad (\text{A4})$$

Let $X_1 \sim (\alpha, \beta_1, c_1, \delta_1)$ and $X_2 \sim (\alpha, \beta_2, c_2, \delta_2)$ be independent drawings from stable distributions with a common α . Then $Y = X_1 + X_2 \sim S(\alpha, \beta, c, \delta)$, where

$$c^\alpha = c_1^\alpha + c_2^\alpha \quad (\text{A5})$$

$$\beta = (\beta_1 c_1^\alpha + \beta_2 c_2^\alpha) / c^\alpha \quad (\text{A6})$$

$$\delta = \begin{cases} \delta_1 + \delta_2 & \text{for } \alpha \neq 1 \\ \delta_1 + \delta_2 + 2(\beta c \ln(c) - \beta_1 c_1 \ln(c_1) - \beta_2 c_2 \ln(c_2)) / \pi & \text{for } \alpha = 1. \end{cases} \quad (\text{A7})$$

When $\beta_1 = \beta_2$, β equals their common value, so that Y has the same shaped distribution as X_1 and X_2 . This is the “stability” property of stable distributions that leads directly to their role in the central limit theorem, and makes them particularly useful in financial portfolio theory. When $\beta_1 \neq \beta_2$, β lies between β_1 and β_2 .

For $\alpha < 2$ and $\beta > -1$, the long upper Paretian tail of $X \sim S(\alpha, \beta, c, \delta)$ makes Ee^X infinite. However, when $\beta = -1$,

$$\ln Ee^X = \begin{cases} \delta - c^\alpha \sec(\pi\alpha / 2), & \alpha \neq 1 \\ \delta + (2c / \pi) \ln c, & \alpha = 1 \end{cases} \quad (\text{A8})$$

This formula greatly facilitates asset pricing under log-stable uncertainty.

See also Zolotarev (1986, p.112) and McCulloch (1996).

APPENDIX B

Evaluation of Conditional Expectations in the Price-Dividend Ratio

In this appendix we derive conditions under which the conditional expectation terms that appear in Equation (10) in the text are finite. We need to derive conditions

under which $E_t \prod_{j=1}^i m_{t+j} < \infty$ where $m_{t+j} \equiv \theta \exp[(1-\gamma)x_{t+j}]$.

Let $\omega = 1 - \gamma$. Therefore, $m_{t+j} = \theta \exp[\omega x_{t+j}]$.

$$\prod_{j=1}^i m_{t+j} = \prod_{j=1}^i \theta \exp[\omega x_{t+j}] = \theta^i \exp\left[\omega \sum_{j=1}^i x_{t+j}\right]. \quad (\text{B1})$$

From dividend growth rate process in Equation (11a),

$$\sum_{j=1}^i x_{t+j} = \sum_{j=1}^i \mu_{t+j} + \sum_{j=1}^i \varepsilon_{t+j}. \quad (\text{B2})$$

From dividend growth rate process in Equation (11b), $\mu_{t+j} - \bar{\mu} = \rho(\mu_{t+j-1} - \bar{\mu}) + \eta_{t+j}$,

we have

$$\mu_{t+j} - \bar{\mu} = \rho^j (\mu_t - \bar{\mu}) + \rho^{j-1} \eta_{t+1} + \rho^{j-2} \eta_{t+2} + \dots + \rho^2 \eta_{t+j-2} + \rho \eta_{t+j-1} + \eta_{t+j}. \quad (\text{B3})$$

Therefore,

$$\begin{aligned} \sum_{j=1}^i \mu_{t+j} &= [\bar{\mu} + \rho(\mu_t - \bar{\mu}) + \eta_{t+1}] + [\bar{\mu} + \rho^2(\mu_t - \bar{\mu}) + \rho \eta_{t+1} + \eta_{t+2}] + \dots \\ &\quad \dots + [\bar{\mu} + \rho^i(\mu_t - \bar{\mu}) + \rho^{i-1} \eta_{t+1} + \rho^{i-2} \eta_{t+2} + \dots + \eta_{t+i}] \end{aligned}$$

This can be written as:

$$\sum_{j=1}^i \mu_{t+j} = i\bar{\mu} + (\mu_t - \bar{\mu}) \left[\frac{\rho(1-\rho^i)}{1-\rho} \right] + \frac{1}{1-\rho} \left[(1-\rho^i)\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \dots + (1-\rho)\eta_{t+i} \right]$$

(B4)

Therefore,

$$\prod_{j=1}^i m_{t+j} = \theta^i \exp \left(\omega \sum_{j=1}^i x_{t+j} \right)$$

$$= \theta^i \exp \left(\begin{array}{c} i\bar{\mu}\omega + \left\{ \frac{\omega\rho}{1-\rho}(1-\rho^i) \right\} (\mu_t - \bar{\mu}) + \\ \left(\frac{\omega}{1-\rho} \right) \left\{ (1-\rho^i)\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \dots + (1-\rho)\eta_{t+i} \right\} + \\ \omega \sum_{j=1}^i \varepsilon_{t+j} \end{array} \right).$$

Define $b_i = \omega \left(\frac{\rho}{1-\rho} \right) (1-\rho^i)$. From the iid nature of ε_t and η_t , we can write:

$$E_t \prod_{j=1}^i m_{t+j} = \theta^i \cdot \exp[i\bar{\mu}\omega] \cdot E_t \exp[b_i(\mu_t - \bar{\mu})] \cdot$$

$$E_t \exp \left[\frac{\omega}{1-\rho} \left\{ (1-\rho^i)\eta_{t+1} + (1-\rho^{i-1})\eta_{t+2} + \dots + (1-\rho)\eta_{t+i} \right\} \right] \cdot$$

$$E_t \exp \left[\omega \sum_{j=1}^i \varepsilon_{t+j} \right]$$

$$E_t \prod_{j=1}^i m_{t+j} = \theta^i \cdot \exp[i\bar{\mu}\omega] \cdot E_t \exp[b_i(\mu_t - \bar{\mu})] \cdot$$

$$E_t \left[\exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^i)\eta_{t+1} \right] \cdot \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^{i-1})\eta_{t+2} \right] \dots \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho)\eta_{t+i} \right] \right] \cdot$$

$$E_t \left[\exp(\omega\varepsilon_{t+1}) \cdot \exp(\omega\varepsilon_{t+2}) \dots \exp(\omega\varepsilon_{t+i}) \right]$$

(B5)

Since $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$ in Equation (11a),

$$\begin{aligned} E_t \left[\exp(\omega \varepsilon_{t+1}) \cdot \exp(\omega \varepsilon_{t+2}) \dots \exp(\omega \varepsilon_{t+i}) \right] = \\ E_t \left\{ \exp(\omega \varepsilon_{t+1}) \right\} \cdot E_t \left\{ \exp(\omega \varepsilon_{t+2}) \right\} \dots E_t \left\{ \exp(\omega \varepsilon_{t+i}) \right\} \end{aligned} \quad (\text{B6})$$

From the moment generating function of normal random variables, we have

$$E_t \left\{ \exp(\omega \varepsilon_{t+1}) \right\} = E_t \left\{ \exp(\omega \varepsilon_{t+2}) \right\} = \dots = E_t \left\{ \exp(\omega \varepsilon_{t+i}) \right\} = \exp \left\{ \frac{1}{2} \omega^2 \sigma^2 \right\}. \quad (\text{B7})$$

Since $\eta_t \sim \text{iid } S(\alpha, \beta, c, 0)$ in Equation (11b),

$$\begin{aligned} E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^i) \eta_{t+1} \right] \cdot \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^{i-1}) \eta_{t+2} \right] \dots \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho) \eta_{t+i} \right] \right\} \\ = E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^i) \eta_{t+1} \right] \right\} E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^{i-1}) \eta_{t+2} \right] \right\} \dots E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho) \eta_{t+i} \right] \right\} \end{aligned} \quad (\text{B8})$$

Using Equation (A4) from Appendix A:

$$\frac{\omega}{1-\rho} (1-\rho^i) \eta_{t+1} \sim S \left(\alpha, \text{sign} \left\{ \omega \left(\frac{1-\rho^i}{1-\rho} \right) \right\} \beta, \left| \omega \left(\frac{1-\rho^i}{1-\rho} \right) \right| c, 0 \right).$$

Since Equation (11b) also specifies that $|\rho| < 1$,

we have $\left(\frac{1-\rho^i}{1-\rho} \right) > 0$. Therefore, $\text{sign} \left\{ \omega \left(\frac{1-\rho^i}{1-\rho} \right) \right\} = \text{sign}(\omega)$. Hence,

$$\frac{\omega}{1-\rho} (1-\rho^i) \eta_{t+1} \sim S \left(\alpha, \text{sign}(\omega) \beta, \left| \omega \left(\frac{1-\rho^i}{1-\rho} \right) \right| c, 0 \right). \quad (\text{B9})$$

Similarly, we have:

$$\frac{\omega}{1-\rho} (1-\rho^{i-1}) \eta_{t+2} \sim S \left(\alpha, \text{sign}(\omega) \beta, \left| \omega \left(\frac{1-\rho^{i-1}}{1-\rho} \right) \right| c, 0 \right) \quad (\text{B10})$$

and so forth for all the other η 's in Equation (B8).

Now, $E_t \prod_{j=1}^i m_{t+j} < \infty$ if the right hand side of Equation (B5) is finite. This

requires that each of the three conditional expectation terms on the right hand side of Equation (B5) be finite. From Equation (B7), the third conditional expectation term on the right hand side of Equation (B5) is finite.

The second conditional expectation term on the right hand side of Equation (B5) is finite if each of the conditional expectation terms on the right hand side of Equation (B8) is finite. Using Equations (B9), (B10) and (A8), this happens when $\text{sign}(\omega) \cdot \beta = -1$. Or, substituting for ω , this happens when $\beta \text{sign}(1 - \gamma) = -1$.

From Equation (11b), we can solve for $\mu_t - \mu_0$ as:

$$\mu_t - \bar{\mu} = \sum_{i=0}^{\infty} \rho^i \eta_{t-i}. \quad (\text{B11})$$

Equation (11b) also specifies that $\eta_t \sim \text{iidS}(\alpha, \beta, c, 0)$. Then, using Equation (A4) from Appendix A and the fact that $0 \leq \rho$, we have $\rho^i \eta_{t-i} \sim \text{iidS}(\alpha, \beta, \rho^i c, 0)$.

Using Equations (A5), (A6), and (A7) from Appendix A, we get:

$$\sum_{i=1}^{\infty} \rho^i \eta_{t-i} \sim S \left(\alpha, \beta, \left| \sum_{i=0}^{\infty} (\rho^i)^\alpha \right|^{\frac{1}{\alpha}} c, 0 \right)$$

With $|\rho| < 1$ already assumed in Equation (11b), $\sum_{i=0}^{\infty} (\rho^i)^\alpha = \frac{1}{1 - \rho^\alpha}$.

Therefore, from Equation (B11),

$$\mu_t - \bar{\mu} = \sum_{i=0}^{\infty} \rho^i \eta_{t-i} \sim S \left(\alpha, \beta, \frac{c}{[1 - \rho^\alpha]^{1/\alpha}}, 0 \right). \quad (\text{B12})$$

From the definition of $b_i \equiv \omega \left(\frac{\rho}{1-\rho} \right) (1-\rho^i)$, $\text{sign}(b_i) = \text{sign}(\omega)$.

Therefore,

$$b_i(\mu_t - \bar{\mu}) \sim S \left(\alpha, \text{sign}(\omega)\beta, \frac{|b_i|}{(1-\rho^\alpha)^{1/\alpha}} c, 0 \right). \quad (\text{B13})$$

The unconditional expectation $E[\exp\{b_i(\mu_t - \bar{\mu})\}]$ is finite if $\text{sign}(\omega) \cdot \beta = \beta \text{sign}(1-\gamma) = -1$ from Equation (A8). It follows from the law of iterated expectations that the first conditional expectation term on the right hand side of Equation (B5) is finite if $\beta \text{sign}(1-\gamma) = -1$.

Therefore, $E_t \prod_{j=1}^i m_{t+j} < \infty$ when $\beta \text{sign}(1-\gamma) = -1$.

APPENDIX C

Derivation of the Tractable Expression for the Price-Dividend Ratio

In this appendix we derive the expression for the price dividend ratio v_t given in Equation (12).

Equation (10) gives:

$$v_t = \sum_{i=1}^{\infty} \left(E_t \prod_{j=1}^i m_{t+j} \right). \quad (C1)$$

From Appendix B, we know that $E_t \prod_{j=1}^i m_{t+j} < \infty$ when $\beta \text{sign}(1-\gamma) = -1$.

Under $\beta \text{sign}(1-\gamma) = -1$, we get from Equations (B9), (B10), and (A8):

$$E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^i) \eta_{t+1} \right] \right\} = \exp \left[- \left| \omega \left(\frac{1-\rho^i}{1-\rho} \right) \right|^{\alpha} \cdot c^{\alpha} \cdot \sec \left(\frac{\pi\alpha}{2} \right) \right] \quad (C2)$$

$$E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^{i-1}) \eta_{t+2} \right] \right\} = \exp \left[- \left| \omega \left(\frac{1-\rho^{i-1}}{1-\rho} \right) \right|^{\alpha} \cdot c^{\alpha} \cdot \sec \left(\frac{\pi\alpha}{2} \right) \right] \quad (C3)$$

and so forth for all the other η 's in Equation (B8):

$$E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho) \eta_{t+i} \right] \right\} = \exp \left[- \left| \omega \left(\frac{1-\rho}{1-\rho} \right) \right|^{\alpha} \cdot c^{\alpha} \cdot \sec \left(\frac{\pi\alpha}{2} \right) \right]. \quad (C4)$$

Substituting (C2), (C3), and (C4) into (B8) and using $|\rho| < 1$, we get:

$$\begin{aligned} E_t \left\{ \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^i) \eta_{t+1} \right] \cdot \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho^{i-1}) \eta_{t+2} \right] \dots \exp \left[\left(\frac{\omega}{1-\rho} \right) (1-\rho) \eta_{t+i} \right] \right\} \\ = \exp \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^{\alpha} \cdot c^{\alpha} \cdot \sec \left(\frac{\pi\alpha}{2} \right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^{\alpha} \right\} \end{aligned} \quad (C5)$$

Substituting (C5) and (B7) into (B5) and collecting terms results in:

$$E_t \prod_{j=1}^i m_{t+j} = \theta^i \cdot \left[E_t \exp\{b_i(\mu_t - \bar{\mu})\} \right] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\left| \frac{1-\gamma}{1-\rho} \right|^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (C6)$$

recognizing from Appendix B that $\omega = 1 - \gamma$.

Finally, substituting (C6) into (C1) gives:

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot \left[E_t \exp\{b_i(\mu_t - \bar{\mu})\} \right] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\left| \frac{1-\gamma}{1-\rho} \right|^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (C7)$$

where, from Appendix B, we have $b_i = (1-\gamma) \left(\frac{\rho}{1-\rho} \right) (1-\rho^i)$.

APPENDIX D

Proof of Theorem 1

From Equation (12),

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot [E_t \exp\{b_i(\mu_t - \bar{\mu})\}] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\left| \frac{1-\gamma}{1-\rho} \right|^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (D1)$$

or, substituting $\omega = 1 - \gamma$

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot [E_t \exp\{b_i(\mu_t - \bar{\mu})\}] \cdot \exp \left[\begin{array}{l} i\bar{\mu}\omega + i\omega^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \cdot \quad (D2)$$

$$\text{Let } v_t \equiv \sum_{i=1}^{\infty} z_i \cdot \quad (D3)$$

$$\frac{z_{i+1}}{z_i} =$$

$$\frac{\theta^{i+1} E_t \exp\{b_{i+1}(\mu_t - \bar{\mu})\} \cdot \exp \left[(i+1)\bar{\mu}\omega + (i+1) \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^{i+1} (1-\rho^j)^\alpha \right\} \right]}{\theta^i E_t \exp\{b_i(\mu_t - \bar{\mu})\} \cdot \exp \left[i\bar{\mu}\omega + i \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \right]}$$

which on simplifying becomes:

$$\frac{z_{i+1}}{z_i} = \theta \frac{E_t \exp\{b_{i+1}(\mu_t - \bar{\mu})\}}{E_t \exp\{b_i(\mu_t - \bar{\mu})\}} \exp \left[\bar{\mu}\omega + \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \cdot (1-\rho^{i+1})^\alpha \right\} \right]$$

With $|\rho| < 1$ specified in Equation (11b),

$$\lim_{i \rightarrow \infty} \left| \frac{z_{i+1}}{z_i} \right| = \theta \exp \left[\bar{\mu} \omega + \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec \left(\frac{\pi \alpha}{2} \right) \right] \right\} \right] \cdot \lim_{i \rightarrow \infty} \frac{E_t \exp \{ b_{i+1} (\mu_t - \bar{\mu}) \}}{E_t \exp \{ b_i (\mu_t - \bar{\mu}) \}}. \quad (\text{D4})$$

One can easily show that $\lim_{i \rightarrow \infty} b_{i+1} = \lim_{i \rightarrow \infty} b_i = \left(\frac{\omega}{1-\rho} \right) \rho$. Therefore, we have

$$\lim_{i \rightarrow \infty} \frac{E_t \exp \{ b_{i+1} (\mu_t - \bar{\mu}) \}}{E_t \exp \{ b_i (\mu_t - \bar{\mu}) \}} = 1. \text{ Using this in (D4), we have:}$$

$$\lim_{i \rightarrow \infty} \left| \frac{z_{i+1}}{z_i} \right| = \theta \exp \left[\bar{\mu} \omega + \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec \left(\frac{\pi \alpha}{2} \right) \right] \right\} \right] \equiv r \quad (\text{D5})$$

Substituting $\omega = 1 - \gamma$, we get:

$$r = \theta \exp \left[\bar{\mu} (1 - \gamma) + \frac{(1 - \gamma)^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{1 - \gamma}{1 - \rho} \right|^\alpha \cdot c^\alpha \cdot \sec \left(\frac{\pi \alpha}{2} \right) \right] \right\} \right]. \quad (\text{D6})$$

Proof for convergence of v_t in (D1) for $r < 1$ now follows from the ratio test

(see, for instance, Marsden 1974, Theorem 13, p.47).

APPENDIX E

Proof of Theorem 2

Derivation of Equation (14)

From Equation (12),

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot \left[E_t \exp\{b_i(\mu_t - \bar{\mu})\} \right] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\frac{1-\gamma}{1-\rho} \right]^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right]. \quad (\text{E1})$$

Therefore, from the law of iterated expectations,

$$v_t = \sum_{i=1}^{\infty} \theta^i \cdot \left[E \exp\{b_i(\mu_t - \bar{\mu})\} \right] \cdot \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \\ \left\{ - \left[\frac{1-\gamma}{1-\rho} \right]^\alpha c^\alpha \sec\left(\frac{\pi\alpha}{2}\right) \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right] \quad (\text{E2})$$

From Equation (B13), we have $b_i(\mu_t - \bar{\mu}) \sim S \left(\alpha, \text{sign}(\omega)\beta, \frac{|b_i|}{(1-\rho^\alpha)^{1/\alpha}} c, 0 \right)$.

Using Equation (A8):

$$E[\exp\{b_i(\mu_t - \bar{\mu})\}] = \exp \left[- \left\{ |b_i| \frac{c}{[1-\rho^\alpha]^{1/\alpha}} \right\}^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \quad (\text{E3})$$

Substituting into Equation (E2) gives:

$$E(v_t) = \sum_{i=1}^{\infty} \theta^i \exp \left[\begin{array}{l} i\bar{\mu}(1-\gamma) + i(1-\gamma)^2 \frac{\sigma^2}{2} + \left\{ - \left[|b_i|^\alpha \cdot \frac{c^\alpha}{1-\rho^\alpha} \right] \sec\left(\frac{\pi\alpha}{2}\right) \right\} + \\ \left\{ - \left[\frac{1-\gamma}{1-\rho} \right]^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \cdot \sum_{j=1}^i (1-\rho^j)^\alpha \right\} \end{array} \right]. \quad (\text{E4})$$

Proof of convergence of $E(v_t)$

$$\text{Let } E(v_t) \equiv \sum_{i=1}^{\infty} z_i \quad (\text{E5})$$

Using Equation (E4), one can easily show that:

$$\lim_{i \rightarrow \infty} \left| \frac{z_{i+1}}{z_i} \right| = \theta \exp \left[\bar{\mu}\omega + \frac{\omega^2 \sigma^2}{2} + \left\{ - \left[\left| \frac{\omega}{1-\rho} \right|^\alpha \cdot c^\alpha \cdot \sec\left(\frac{\pi\alpha}{2}\right) \right] \right\} \right] \cdot \lim_{i \rightarrow \infty} \exp \left[\left\{ - \left[\frac{c^\alpha}{1-\rho^\alpha} \right] \sec\left(\frac{\pi\alpha}{2}\right) \right\} \left\{ |b_{i+1}|^\alpha - |b_i|^\alpha \right\} \right]$$

Using the definition of r in Theorem 1,

$$\lim_{i \rightarrow \infty} \left| \frac{z_{i+1}}{z_i} \right| = r \cdot \lim_{i \rightarrow \infty} \exp \left[\left\{ - \left[\frac{c^\alpha}{1-\rho^\alpha} \right] \sec\left(\frac{\pi\alpha}{2}\right) \right\} \left\{ |b_{i+1}|^\alpha - |b_i|^\alpha \right\} \right] \quad (\text{E6})$$

Following from the proof of Theorem 1 in Appendix D, it suffices to show that:

$$\lim_{i \rightarrow \infty} \exp \left\{ - \left[\frac{c^\alpha}{1-\rho^\alpha} \sec\left(\frac{\pi\alpha}{2}\right) \right] \left[|b_{i+1}|^\alpha - |b_i|^\alpha \right] \right\} = 1$$

or that, $\lim_{i \rightarrow \infty} \left\{ |b_{i+1}|^\alpha - |b_i|^\alpha \right\} = 0$. With $|\rho| < 1$ specified in Equation (11b),

$$|b_i|^\alpha = \left| (1-\gamma) \left(\frac{\rho}{1-\rho} \right) \right|^\alpha [1-\rho^i]^\alpha. \text{ Therefore,}$$

$$\lim_{i \rightarrow \infty} \left\{ |b_{i+1}|^\alpha - |b_i|^\alpha \right\} = \left| (1-\gamma) \left(\frac{\rho}{1-\rho} \right) \right|^\alpha \cdot \left[\lim_{i \rightarrow \infty} \left\{ [1-\rho^{i+1}]^\alpha - [1-\rho^i]^\alpha \right\} \right] = 0.$$

APPENDIX F

Derivation of the Risk Free Asset Prices

The price of the risk free asset is given by $P_t^f = \theta E_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \right)$. With the utility function being of the constant relative risk aversion (CRRA) class and $C = D$ in the model from Section 2, this reduces to:

$$P_t^f = \theta E_t \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma}. \quad (F1)$$

Using $x_t = \ln(D_t / D_{t-1})$, we get $P_t^f = \theta E_t [\exp(-\gamma x_{t+1})]$. Substituting for x_{t+1} using Equation (11) yields:

$$P_t^f = \theta E_t [\exp\{-\gamma \bar{\mu} - \gamma \rho(\mu_t - \bar{\mu}) - \gamma \eta_{t+1} - \gamma \varepsilon_{t+1}\}].$$

Using independence of μ_t , ε_{t+1} and η_{t+1} , we can rewrite this as:

$$P_t^f = \theta \exp\{-\gamma \bar{\mu}\} E_t [\exp\{-\gamma \varepsilon_{t+1}\}] E_t [\exp\{-\gamma \rho(\mu_t - \bar{\mu})\}] E_t [\exp\{-\gamma \eta_{t+1}\}]. \quad (F2)$$

We have assumed that $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$ in Equation (11a). Therefore, using the moment generating function for the normal random variable:

$$E_t [\exp\{-\gamma \varepsilon_{t+1}\}] = \exp\left\{ \frac{\gamma^2 \sigma^2}{2} \right\}. \quad (F3)$$

We have assumed that $\eta_t \sim \text{iid } S(\alpha, \beta, c, 0)$ in Equation (11b). Therefore, using Equation (A4) we can write $-\gamma \eta_{t+1} \sim \text{iid } S(\alpha, \text{sign}(-\gamma)\beta, |-\gamma|c, 0)$. Since $\gamma \geq 0$, $-\gamma \eta_{t+1} \sim \text{iid } S(\alpha, -\beta, \gamma c, 0)$. Since it has been assumed that $\beta = -1$, it follows from Equation (A8) that:

$$E_t[\exp\{-\gamma\eta_{t+1}\}] = \infty. \quad (\text{F4})$$

From Equation (B12), $\mu_t - \bar{\mu} = \sum_{i=0}^{\infty} \rho^i \eta_{t-i} \sim S\left(\alpha, \beta, \frac{c}{[1-\rho^\alpha]^{1/\alpha}}, 0\right)$. Since $\gamma \geq 0$ and

$\rho \geq 0$, it follows from Equation (A4) that $-\gamma\rho(\mu_t - \bar{\mu}) \sim S\left(\alpha, -\beta, \frac{\gamma\rho c}{[1-\rho^\alpha]^{1/\alpha}}, 0\right)$. Since it

has been assumed that $\beta = -1$, it follows from Equation (A8) that:

$$E_t[\exp\{-\gamma\rho(\mu_t - \bar{\mu})\}] = \infty. \quad (\text{F5})$$

Equations (F2)-(F5) then imply that $P_t^f = \infty$. Gross equilibrium returns on the risk

free asset $R_t^f = \frac{1}{P_t^f}$ are consequently zero.

APPENDIX G

Numerical Implementation of Filtering Equations

The Sorenson-Alspach (1971) filter and predictive densities were evaluated at a grid of 200 points equally spaced on a truncated portion of the real line. The left truncation point was chosen to lie approximately 4.25 standard deviations of the ε shock (6 times the scale parameter) below the minimum observed consumption growth rate and the right truncation point 4.25 standard deviations above the maximum observed consumption growth rate. The likelihood and the predictive density integrals (Equations (17) and (15) respectively) were evaluated numerically by a piecewise cubic quadrature technique, as follows: integration between any two interior nodes was performed by fitting a piecewise cubic function through the four nearest nodes and approximating the required area under the integrand between those nodes by the area under the cubic. The outermost intervals employ the same cubics as the adjacent intervals. For equispaced nodes, 8 or more in number, this quadrature procedure yields the weights $8/24, 31/24, 20/24, 25/24, 1, 1, \dots, 1, 25/24, 20/24, 31/24, 8/24$ for the ordinates. The numerically computed predictive density was normalized in order to ensure that it integrated to unity. The piecewise linear interpolation and the trapezoidal rule for integration suggested by Kitagawa (1987) was not employed. Hodges and Hale (1993) propose an integration by parts procedure to speed up the Kitagawa procedure, but this was not employed either.

The accuracy of our numerical quadrature can be gauged by a comparison of the maximized log-likelihood value for the model in Equations (11) obtained from our numerical integration with α restricted to be 2, with that obtained from the Kalman filter (which is optimal in this Gaussian case), for given values of the other hyperparameters.

We verified that, with 200 nodes, our numerical approximation gives log-likelihood values accurate to four decimal places at the estimated hyperparameters of the Gaussian model. In light of this our numerical integration appears to be sufficiently accurate for drawing valid inferences from data.

In the model given in Equations (11) in the main text, x_t is the observed series,

$$p(x_t | \mu_t) = N(x_t - \mu_t; 0, \sigma^2)$$

and
$$p(\mu_t | \mu_{t-1}) = s_\alpha(\mu_t - \bar{\mu} - \rho(\mu_{t-1} - \bar{\mu}); 0, -1, c),$$

where $N(x; 0, \sigma^2)$ and $s_\alpha(x; 0, -1, c)$ are the probability densities of $N(0, \sigma^2)$ and $S(\alpha, -1, c, 0)$ distributions evaluated at x . The filter is initialized by setting the conditional mean of $p(\mu_1 | x_1)$ equal to x_1 and the conditional scale of $p(\mu_1 | x_1)$ equal to the scale of ε_1 .

Starting points for hyperparameter estimation are obtained from the Kalman filter under normality.

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Table 1
Parameter Estimates

Panel A: Incomplete Information, Stable Model					
$\bar{\mu}$	ρ	α	c	σ	Log Likelihood
0.0049	0.6889	1.8564	0.0014	0.0058	755.6100
0.0007	0.1724	0.2500	0.0007	0.0005	

Panel B: Incomplete Information, Gaussian Model					
$\bar{\mu}$	ρ	α	σ_{η}	σ	Log Likelihood
0.0050	0.7396	2.0000	0.0022	0.0058	755.4003
0.0007	0.1260	(restricted)	0.0008	0.0004	

Panel C: Complete Information, Stable Model					
$\bar{\mu}$	ρ	α	c	Log Likelihood	
0.0051	0.1343	1.9140	0.0043	755.5771	
0.0003	0.0222	0.0545	0.0002		

Panel D: Complete Information, Gaussian Model					
$\bar{\mu}$	ρ	α	σ_{η}	Log Likelihood	
0.0050	0.1758	2.0000	0.0066	753.5202	
0.0006	0.0674	(restricted)	0.0003		

This table reports the Maximum Likelihood estimates of the model for dividend growth rates, $x_t = \mu_t + \varepsilon_t$ where $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$ and where the unobserved persistent component μ_t follows

$$\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t, \quad \text{with } 0 \leq \rho < 1 \quad \text{and} \quad \eta_t \sim \text{iid } S(\alpha, \beta, c, 0)$$

The model is calibrated to quarterly real per capita US consumption growth rates on non-durables and services from the first quarter of 1952 through the second quarter of 2004. Nominal seasonally adjusted per capita consumption data obtained from NIPA tables are deflated using the CPI index. Panel A reports estimates for the most general model. Panel B reports estimates for the special case where $\eta_t \sim \text{iid } N(0, \sigma_{\eta}^2)$. Panels C and D report estimates for the complete information counterparts of panels A and B, by setting ε_t to zero.

Conditional densities of the state variable μ_t are obtained by applying the algorithm by Sorenson and Alspach (1971) in panel A and a Kalman filter in panel B. The probability density for stable distributions is obtained by Fourier inversion of their characteristic function available as an exact analytical formula using the Fast Fourier Transform (FFT) methods discussed in Mittnik et al. (1999). Standard errors are reported below each parameter estimate.

Table 2
Unconditional Moments of Returns

Panel A: Data (1952:1 to 2004:2)

	E(R)	$\sigma(R)$
Real Returns	8.07	16.77

Panel B: Incomplete Information, Stable Model

θ	γ	E(R)	$\sigma(R)$	E(Vt)
0.98	0.60	3.2901	1.7459	80.599
0.98	0.75	3.5943	1.6466	64.898
0.98	0.90	3.8986	1.555	54.321
0.99	0.60	2.2753	1.7388	406.67
0.99	0.75	2.5511	1.6363	192.95
0.99	0.90	2.8494	1.5417	123.08

Panel C: Incomplete Information, Gaussian Model

θ	γ	E(R)	$\sigma(R)$	E(Vt)
0.98	0.60	2.95	2.69	115.46
0.98	0.75	3.43	2.30	74.60
0.98	0.90	3.89	1.96	55.57
0.99	0.60	2.24	2.70	605.71
0.99	0.75	2.52	2.30	224.84
0.99	0.90	2.86	1.95	126.26

Table 2 (Continued)

Panel D: Complete Information, Stable Model

θ	γ	E(R)	$\sigma(R)$	E(Vt)
0.98	0.60	3.34	1.62	82.89
0.98	0.75	3.65	1.60	65.83
0.98	0.90	3.97	1.57	54.58
0.99	0.60	2.33	1.61	456.90
0.99	0.75	2.61	1.58	201.12
0.99	0.90	2.92	1.56	124.43

Panel E: Complete Information, Gaussian Model

θ	γ	E(R)	$\sigma(R)$	E(Vt)
0.98	0.60	3.31	1.48	82.07
0.98	0.75	3.63	1.45	65.51
0.98	0.90	3.94	1.42	54.50
0.99	0.60	2.30	1.47	438.09
0.99	0.75	2.58	1.44	198.24
0.99	0.90	2.89	1.41	123.97

Panel A reports unconditional moments of quarterly value-weighted real returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period going from the first quarter of 1952 through the second quarter of 2004. CPI inflation is subtracted from nominal returns to obtain real returns, expressed in percent per annum. Moments are reported for a range of values for the subjective discount factor θ , and the risk-aversion coefficient γ . Panels B-E report the unconditional moments of simulated returns obtained from the asset pricing model by feeding simulated consumption growth rates data using the estimated parameters from each panel of Table 1. The statistics reported in percentage per annum are the mean returns $E(R)$, standard deviation of returns $\sigma(R)$, and the mean price-dividend ratio $E(V_t)$.

Table 3
Time-Varying Volatility of Returns

Panel A: Data (1952:1 to 2004:2)					
	a_1				Log Likelihood
Homoskedastic Case	1119.1760				-1035.1148
	108.9125				
	a_1	a_2	a_3		Log Likelihood
GARCH(1,1)	300.7956	0.5814	0.1520		-1031.3912
	108.2858	0.1429	0.0769		
	a_1	a_2	a_3	a_4	Log Likelihood
AGARCH(1,1)	432.9833	0.4368	0.0000	374.7812	-1029.3003
	186.9916	0.1857	0.0871	193.4094	

Panel B: Incomplete Information, Stable Model					
	a_1				Log Likelihood
Homoskedastic Case	2.4188				-7421.9202
	0.0527				
	a_1	a_2	a_3		Log Likelihood
GARCH(1,1)	1.5272	0.2435	0.1121		-7332.3004
	0.2240	0.1036	0.0213		
	a_1	a_2	a_3	a_4	Log Likelihood
AGARCH(1,1)	1.3537	0.3291	0.0113	0.3957	-7318.2051
	0.2208	0.1060	0.0241	0.0965	

Panel C: Incomplete Information, Gaussian Model					
	a_1				Log Likelihood
Homoskedastic Case	3.8410				-8344.1956
	0.0867				
	a_1	a_2	a_3		Log Likelihood
GARCH(1,1)	2.2062	0.4115	0.0141		-8343.9301
	1.4994	0.3928	0.0158		
	a_1	a_2	a_3	a_4	Log Likelihood
AGARCH(1,1)	2.3494	0.3740	0.0143	0.0000	-8343.9351
	1.5083	0.4765	0.3770	2.0041	

Panel D: Complete Information, Stable Model					
	a_1				Log Likelihood
Homoskedastic Case	2.4774				-7469.5978
	0.0556				
	a_1	a_2	a_3		Log Likelihood
GARCH(1,1)	2.4706	0.0000	0.0025		-7468.6021
	1.1159	0.4428	0.0023		
	a_1	a_2	a_3	a_4	Log Likelihood
AGARCH(1,1)	2.4736	0.0000	0.0000	0.0069	-7468.1900
	1.7834	0.5602	0.4814	1.1496	

Table 3 (Continued)

Panel E: Complete Information, Gaussian Model

	a ₁				Log Likelihood
Homoskedastic Case	2.0171				-7059.6149
	0.0453				
	a ₁	a ₂	a ₃		Log Likelihood
GARCH(1,1)	1.7768	0.0960	0.0232		-7058.8237
	0.6118	0.3061	0.0169		
	a ₁	a ₂	a ₃	a ₄	Log Likelihood
AGARCH(1,1)	1.9709	0.0000	0.0140	0.0367	-7058.6692
	1.9305	0.9780	0.0361	0.0911	

Panel A reports the estimates of a homoskedastic model, GARCH(1,1) model and AGARCH(1,1) model fitted to quarterly value-weighted real returns on all NYSE, AMEX, and NASDAQ stocks obtained from CRSP dataset for the period going from the first quarter of 1952 through the second quarter of 2004. The models assume that returns are conditionally normally distributed. Thus

$$r_t = a_0 + \varepsilon_t, \quad \varepsilon_t \sim \sigma_t z_t, \quad z_t \sim \text{iid } N(0,1)$$

with volatility given by, respectively:

$$\text{Homoskedastic : } \sigma_t^2 = a_1$$

$$\text{GARCH(1,1) : } \sigma_t^2 = a_1 + a_2 \sigma_{t-1}^2 + a_3 |r_{t-1} - a_0|^2$$

$$\text{AGARCH(1,1) : } \sigma_t^2 = a_1 + a_2 \sigma_{t-1}^2 + a_3 |r_{t-1} - a_0|^2 + a_4 I_{t-1} |(r_{t-1} - a_0) / \sigma_{t-1}|^2$$

$$\text{where } I_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - a_0 < 0 \\ 0 & \text{otherwise} \end{cases}.$$

The restrictions $a_1 > 0$, $a_2 \geq 0$, $a_3 \geq 0$, and $a_4 \geq 0$ are enforced in all models. Panels B-E report estimates of the above models with simulated returns obtained from the asset pricing model by feeding simulated consumption growth rates data using the estimated parameters from each panel of Table 1. A subjective discount factor θ of 0.98 and a risk-aversion coefficient γ of 0.9 are used to obtain simulated returns since the unconditional mean stock returns implied by the incomplete information stable model are closest to their sample counterpart for these preference parameter values. Standard errors are reported below each volatility parameter estimate.

Figure 1. Unconditional Distribution of μ_t

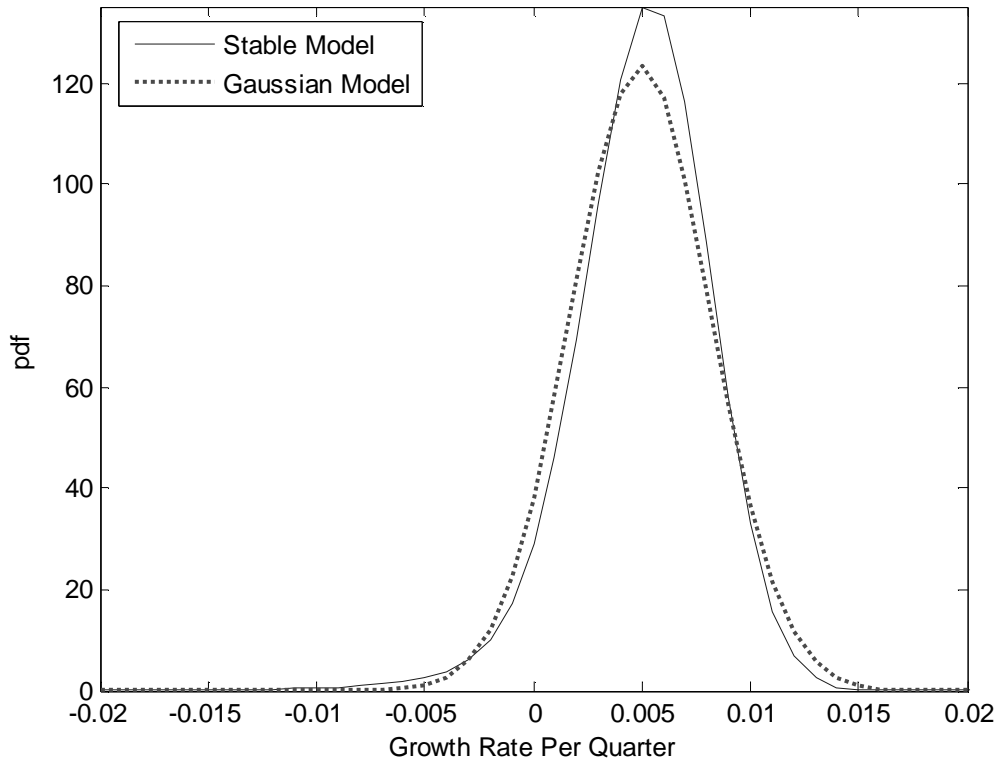


Figure 1 uses the Maximum Likelihood parameter estimates reported in Panels A and B of Table 1 to plot the unconditional distribution of μ_t , the persistent component of dividend growth rates defined by $x_t = \mu_t + \varepsilon_t$ and $\mu_t - \bar{\mu} = \rho(\mu_{t-1} - \bar{\mu}) + \eta_t$, for both the Stable model where $\eta_t \sim \text{iid} S(\alpha, \beta, c, 0)$ and the Gaussian model where $\eta_t \sim \text{iid} N(0, \sigma_\eta^2)$, with $\varepsilon_t \sim \text{iid} N(0, \sigma^2)$ and $0 \leq \rho < 1$.

Figure 2. Filtered Probability Densities $p(\mu_t | x_1, x_2, \dots, x_t)$

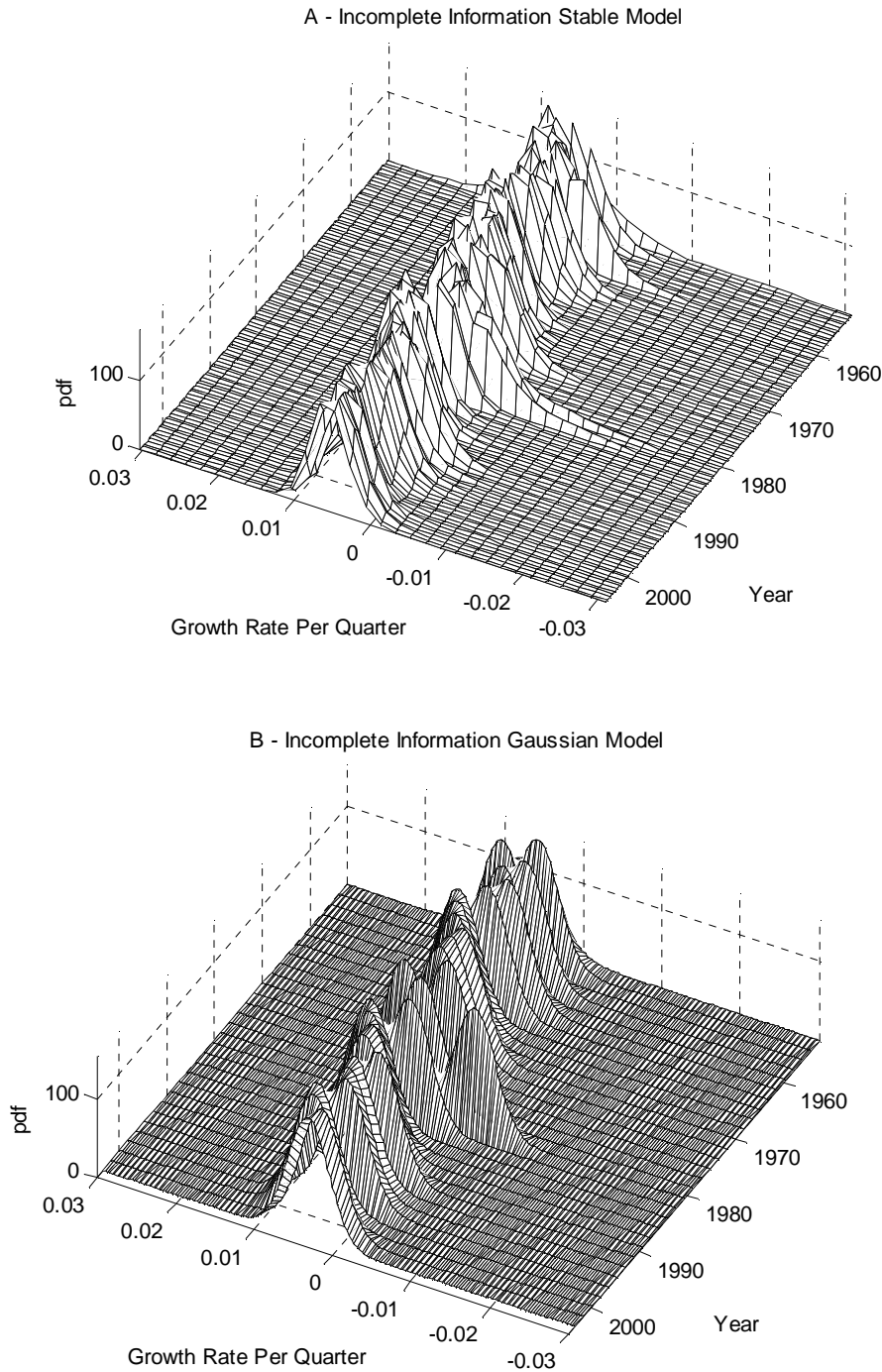


Figure 2 plots the filtered conditional probability densities $p(\mu_t | x_1, x_2, \dots, x_t)$. Panel A plots the densities for the stable case with the densities estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of Panel A in Table 1. Panel B plots the densities for the Gaussian case with the densities estimated by using a Kalman filter and the Maximum Likelihood parameter estimates of Panel B in Table 1.

Figure 3. Filtered Mean $E(\mu_t | x_1, x_2, \dots, x_t)$

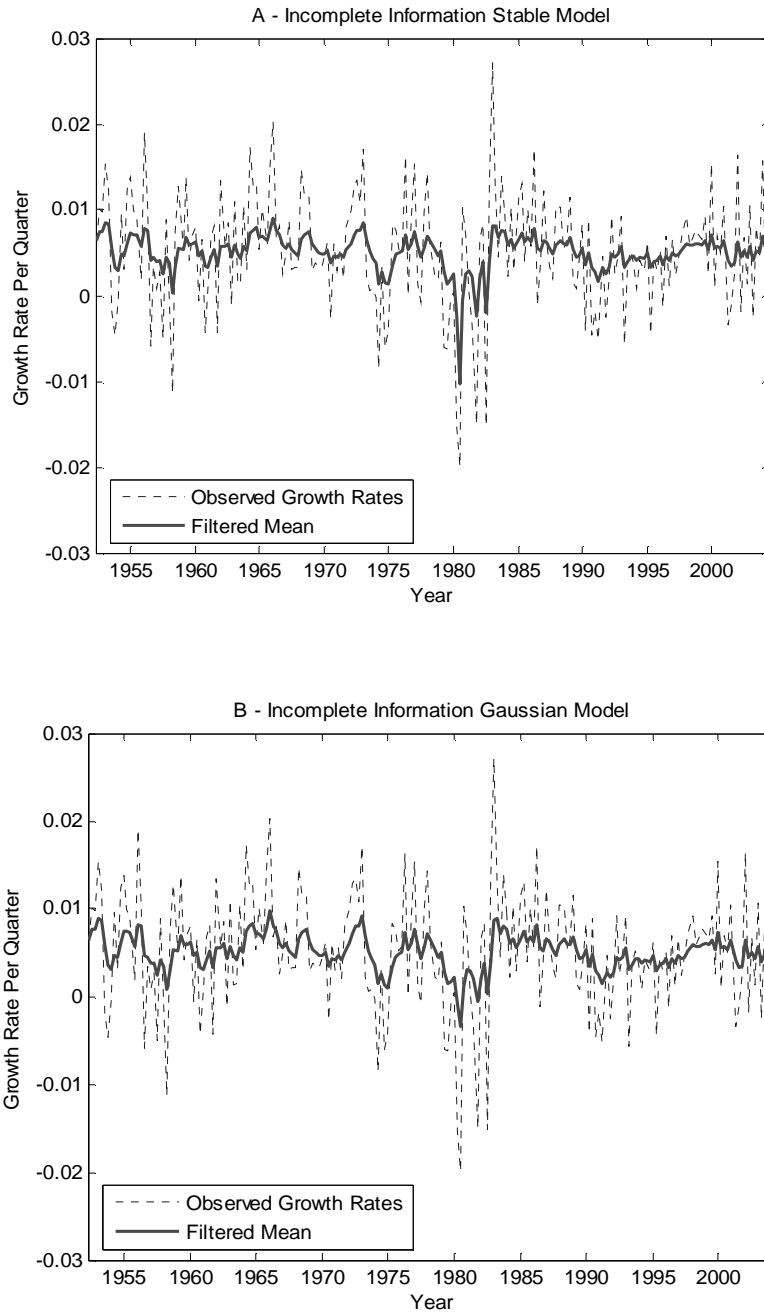


Figure 3 plots the mean of the filtered densities $E(\mu_t | x_1, x_2, \dots, x_t)$, along with the observed consumption growth rates x_t . Panel A plots the mean of the filtered densities for the stable case with the densities (plotted in Figure 2A) estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of Panel A in Table 1. Panel B plots the mean of the filtered densities for the Gaussian case with the densities (plotted in Figure 2B) estimated by using a Kalman filter and the Maximum Likelihood parameter estimates of Panel B in Table 1.

**Figure 4. Filtered Standard Deviation of $p(\mu_t | x_1, x_2, \dots, x_t)$
Fluctuating Confidence**

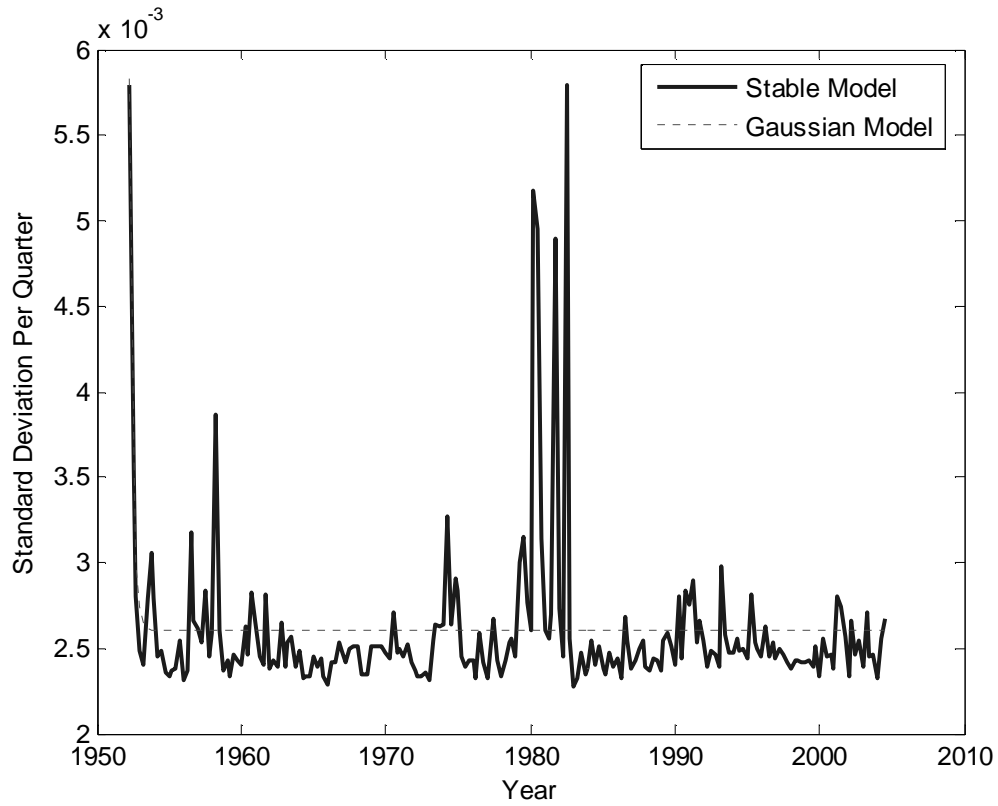


Figure 4 plots the standard deviation of the filtered densities for both the stable and Gaussian incomplete information models. The standard deviations of the filtered densities are estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of each model.

**Figure 5. Filtered Standard Deviation of $p(\mu_t | x_1, x_2, \dots, x_t)$
Fluctuating Confidence**

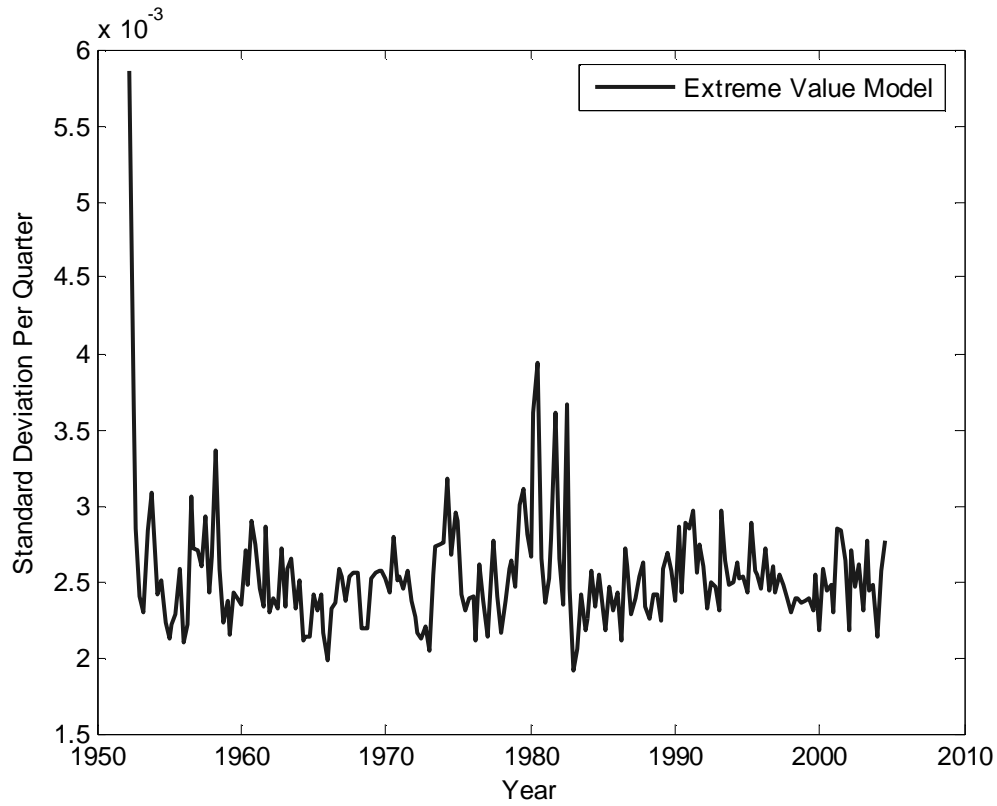


Figure 5 plots the standard deviation of the filtered densities for the Extreme Value incomplete information model. The standard deviations of the filtered densities are estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of the Extreme Value model.

**Figure 6. Filtered Standard Deviation of $p(\mu_t | x_1, x_2, \dots, x_t)$
Fluctuating Confidence**

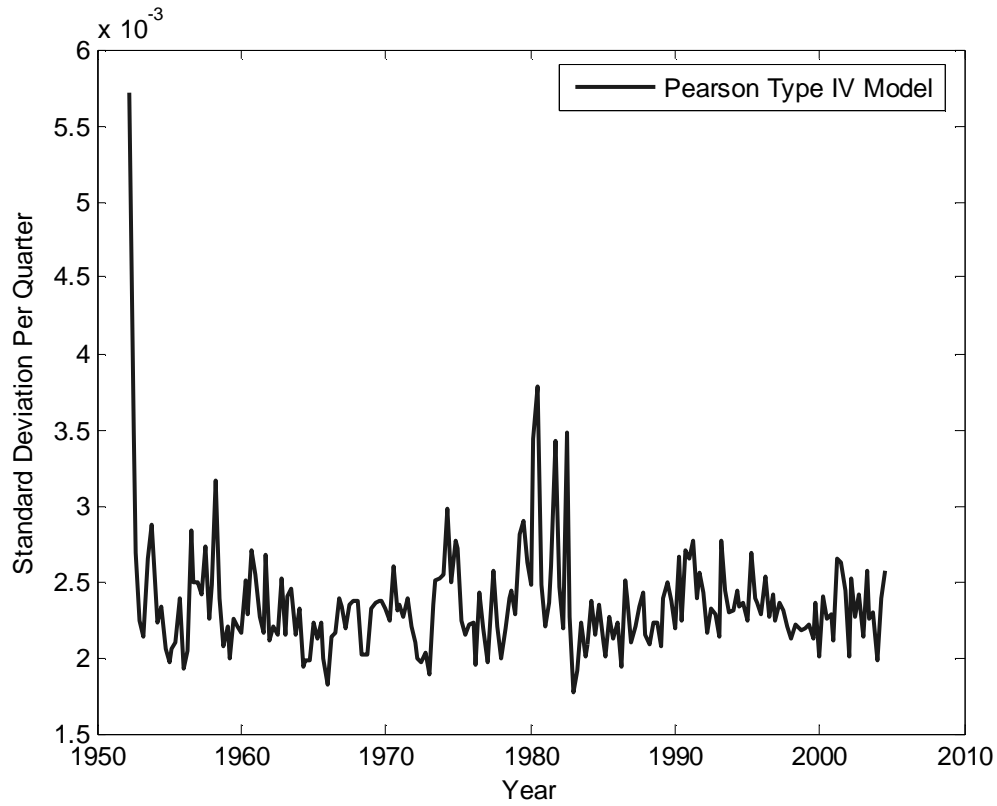


Figure 6 plots the standard deviation of the filtered densities for the Pearson Type IV incomplete information model. The standard deviations of the filtered densities are estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of the Pearson Type IV model.

**Figure 7. Filtered Standard Deviation of $p(\mu_t | x_1, x_2, \dots, x_t)$
Fluctuating Confidence**

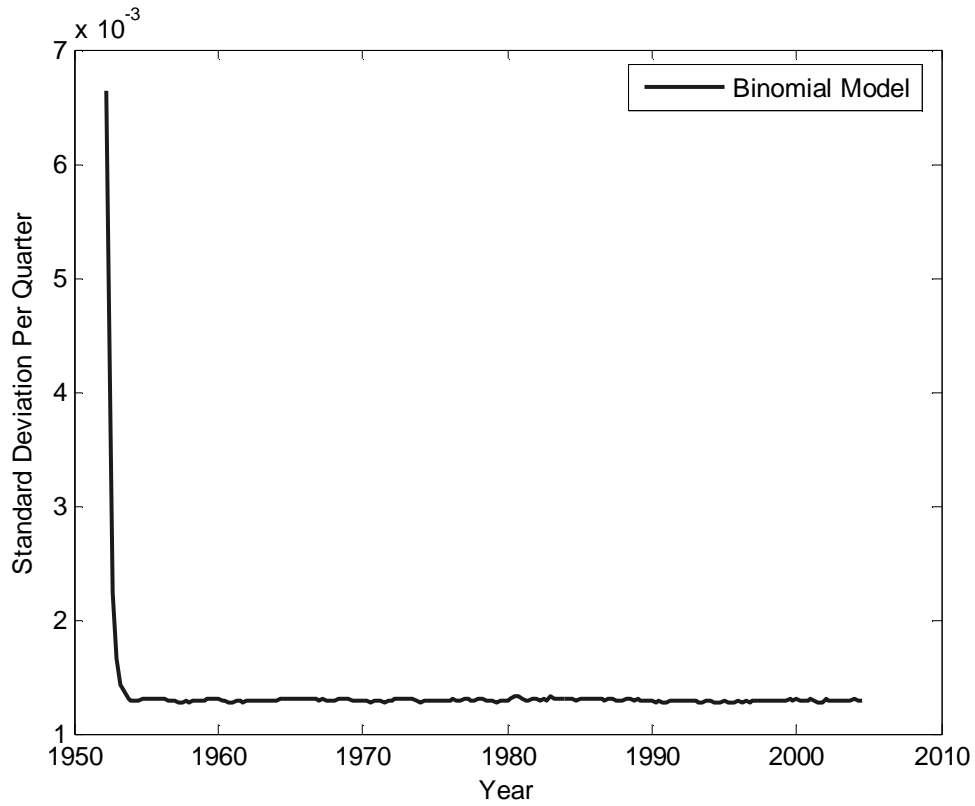


Figure 7 plots the standard deviation of the filtered densities for the Binomial incomplete information model. The standard deviations of the filtered densities are estimated by using the algorithm by Sorenson and Alspach (1971) and the Maximum Likelihood parameter estimates of the Binomial model.